



MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

QUADRATIC EQUATIONS



1. IF lpha and eta be the roots of the equation $ax^2+bx+c=0$, find the values of

 $\alpha^2 + \beta^2$



2. IF α and β be the roots of the equation $ax^2 + bx + c = 0$, find the values of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$

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3. IF the roots of the equation
$$k^2 + x^2 + (kx+1)(x+k) + 1 = 0 (k \neq 0, k \neq -1)$$

are α and β , find the value of $\alpha^2 + \beta^2 + (\alpha\beta + 1)(\alpha + \beta) + 1.$

4. IF α, β be the roots of the equation $ax^2 + bx + c = 0$ find the value of $\frac{a\alpha^2}{b\alpha + c} - \frac{a\beta^2}{b\beta + c}$

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5. IF
$$\alpha, \beta$$
 be the roots of the equation
 $px^2 + qx + r = 0$, find the value of
 $\frac{1}{(p\alpha + q)^3} + \frac{1}{(p\beta + q)^3}$.

6. IF α, β and γ, δ be the roots of the equation $x^2 + px - r = 0$ and $x^2 + px + r = 0$ respectively, prove that $(\alpha - \gamma)(\alpha - \delta) = (\beta - \gamma)(\beta - \delta)$

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7. IF the ratio of the roots of equation $x^2 + px + q = 0$ be a:b prove that, $p^2ab = q(a + b)^2$ Hence, find the condition of equal roots of the given equation.

8. IF one root of the equation $x^2 - rx - s = 0$ is square of the other, prove that , $r^3 - s^2 + 3sr + s = 0.$

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9. IF $b^3 + a^2c + ac^2 = 3abc$,find the relation between

the roots of the equation $ax^2 + bx + c = 0$.

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10. Let a,b,c be real numbers with a
eq 0 and let lpha, eta be the roots of the equation $ax^2 + bx + c = 0.$

of

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11. IF lpha and eta be the roots of the equation $x^2+3x+4=0$, find the equation whose roots are $(lpha+eta)^2$ and $(lpha+eta)^2$

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12. $-\frac{lpha^2}{eta}$ and $-\frac{eta^2}{lpha}$ are the roots of the equation $3x^2 - 18x + 2 = 0$. Find the equation whose roots

are α and β (α , β real).

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13. IF α and β be the roots of $x^2 + x + 1 = 0$, form the equation whose roots are α^2 and β^2 . Account for the identify of the equation thus obtained with the original equation.

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14. IF $a^2 = 5a - 3$ and $b^2 = 5b - 3$, $(a \neq b)$, find the quadratic equation whose roots are $\frac{a}{b}$ and $\frac{b}{a}$.



coefficients whose one root is $3-\sqrt{5}$.

17. IF one root of the quadratic equation $x^2-x-1=0$ is lpha, prove that its other root is $lpha^3-3lpha.$

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18. If the ratio of the roots of equation $ax^2 + bx + c = 0$ be equal to that of the roots of $px^2 + qx + r = 0$, Prove that $rpb^2 = caq^2$.

19. In the equation $x^2 + px + q = 0$, if the coeffficient of x be taken as 17 instead of 13, the roots are obtained as (-2) and (-15). Find the roots of the original equation.

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20. IF lpha be a root of the equation $3x^2-4x+5=0$,

prove that $2lpha^2$ is a root of the equation $9x^2+28x+100=0.$

21. Dicuss the nature of the roots of the following equations:

 $3x^2 - 7x + 3 = 0$

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22. Dicuss the nature of the roots of the following equations:

 $x^2 - 18x + 81 = 0$

23. Find the nature of the roots of the equation $2x^2 - \sqrt{3}x + 2 = 0.$

24. Dicuss the nature of the roots of the following equations:

$$x^2 - 2\sqrt{7}x - 2 = 0$$

25. Dicuss the nature of the roots of the following

equations:

$$2x^2 - 3x + 5 = 0$$
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26. For what value of m the roots of the equation $x^2 - 2(5+2m)x + 3(7+10m) = 0$ are equal

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27. For what value of m the roots of the equation $x^2 - 2(5+2m)x + 3(7+10m) = 0$ are

reciprocal to one another

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28. For what value of m the roots of the equation

 $x^2-2(5+2m)x+3(7+10m)=0$ are

equal in magnitude and opposite in signs?



29. IF a,b,c are rational and a+b+c=0, show that the

roots of the equation $ax^2 + bx + c = 0$ are rational.

30. IF the roots of the equation $px^2 - 2qx + p = 0$ are real and unequal, show that the roots of the equation $qx^2 - 2px + q = 0$ are imaginary (both p and q are real).

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31. Show that the roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are equal if a,b,c are in H.P.

32. IF a,b,c are real, prove that the roots of the equation $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0$ are always real and cannot be equal unless a=b=c.

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33. Find the condition for which the roots of the quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$ are

both positive

34. Find the condition for which the roots of the quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$ are both negative

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35. Find the condition for which the roots of the quadratic equation $ax^2 + bx + c = 0 (a
eq 0)$ are

one positive and the other negative

36. Find the condition for which the roots of the quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$ are equal in magnitude and opposite in signs

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37. Find the condition for which the roots of the quadratic equation $ax^2 + bx + c = 0 (a
eq 0)$ are

reciprocal to one another

38. Find the condition for which the roots of the quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$ are one root is zero

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39. Find the condition for which the roots of the quadratic equation $ax^2 + bx + c = 0 (a
eq 0)$ are

both roots are zero.

40. IF a,b,c are real determine the nature of the roots of the equation $ax^2 + bx + c = 0$ under following conditions:

 $b^2>4ac,ab<0,ac>0$

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41. IF a,b,c are real determine the nature of the roots of the equation $ax^2 + bx + c = 0$ under following conditions:

$$b^2>4ac, ab>0, ac>0$$

42. Find the condition so that the two equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ will have a common root.

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43. Find the value of the common root.

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44. Find the values of the other roots of the two equations.



45. Also find the condition of having two common

roots.



46. Find those values of k for which the equations $x^2 - kx - 21 = 0$ and $x^2 - 3kx - 35 = 0$ have a

common root.

47. Prove that, if the equations $x^2 + px + qr = 0$ and $x^2 + qx + pr = 0 [p
eq q, r
eq 0]$ have a common root , then p+q+r=0.

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48. Prove that , if the equations $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have only non-zero common root then their other roots satisfy the equation $t^2 + at + bc = 0.$

49. If the roots of the equation $x^2 - 2px + q = 0$ are equal and y < 0 and $p \neq 1$, show that the roots of the equation $(1+y)x^2 - 2(p+y)x + q + y = 0$ are real and unequal.



50. If b > a, then prove that the equation (x-a)(x-b) - 1 = 0 has one root in $(-\infty, a)$

and the other in $(b, +\infty)$.

51. Given that a,c are the roots of the equation $px^2 - 3x + 2 = 0$ and b,d are the roots of the equation $qx^2 - 4x + 2 = 0$, find the values of p and q such that $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ and $\frac{1}{d}$ are in A.P.

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52. IF one root of the equation $px^2 + qx + r = 0$ is

the cube of the other, prove that , $rp(r+p)^2 = \left(q^2-2rp
ight)^2.$

53. Let α, β be the roots of the equation $x^2 - 4x + A = 0$ and γ, δ be the roots of the equation $x^2 - 36x + B = 0$. If α, β, γ and δ are In G.P. having positive common ratio, find the value of A and B.

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54. If x be real then prove that (x-1)(x-2)+1 is always

positive.



55. IF x is real, find the least value of $2x^2 - 3x + 5$ and the value of x for which the expression is minimum.

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56. Find for what value/values of x the value of $8x - x^2 - 15$ is

the greatest

57. Find for what value/values of x the value of $8x - x^2 - 15$ is

negative.

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58. For what real values of k, $3x^2 + kx + 39 - k^2$

cannot be negative for any real values of x?



59. IF x,a,b be real , show that

$$4(a-x)\Big(x-a+\sqrt{a^2+b^2}\Big) {
ightarrow} a^2+b^2.$$



61. IF x is real, find the greatest and the least value of

 $rac{6x^2-22x-21}{5x^2-18x+17}$, also find the corresponding values of x.



62. IF x is real and 0 < m < 1, show that the

expression
$$\displaystyle rac{x^2+2x+m}{x^2+4x+3m}$$
 is capable of assuming all

real values.

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63. IF x is real, find the real values of a which make

 $x^2 - ax + 1 - 2a^2$ always positive.

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64. Find the value of m for which the expressions $6x^2 - 7xy + 2y^2 - 9x + 7y + m$ can be resolved

into two linear factors .



65. IF $ax^2 + by^2 + cz^2 + 2ayz + 2bzx + 2cxy$ is resolvable into two linear factors, prove that either a+b+c=0 or, a=b=c.

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66. Solve by expressing the quadratic as the difference of two squares:

 $9x^2 + 25 = 0$



68. Solve by expressing the quadratic as the difference of two squares:

$$16x^2 + 24x + 13 = 0$$

69. Show that the roots of the equation $9x^2 - 24x + 25 = 0$ are complex numbers, solving the equation show that the complex roots are conjugate complex numbers.



70. Solve the following quadratic equations using Sridhar Acharya's formula:

 $2x^2+3=0$

71. Solve the following quadratic equations using Sridhar Acharya's formula:

 $4x^2 + 4x + 7 = 0$

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72. Solve the following quadratic equations using Sridhar Acharya's formula:

$$\sqrt{3}x^2-4x+2\sqrt{3}=0$$

73. Solve by factorization:

$$x^2 - ix + 12 = 0$$

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$$4x^2 + 4xi - 1 = 0$$

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75. Solve by factorization:

$$x^2+2\sqrt{2}xi+6=0$$

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76. Solve each of the following quadratic equations by comparing its roots with the roots of the general quadratic equation:

$$x^2-ig(3i-2\sqrt{3}ig)x-6\sqrt{3}i=0$$

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77. Solve each of the following quadratic quadratic equations by comparing its roots with the roots of the general quadratic equation:

 $2x^2 - 7ix + 4 = 0$
78. Solve each of the following quadratic quadratic equations by comparing its roots with the roots of the general quadratic equation:

$$3x^2 - (2-i)x + 10 - 4i = 0$$

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79. Solve each of the following quadratic quadratic equations by comparing its roots with the roots of the general quadratic equation:

$$x^{2} + (2+i)x - 2(1+7i) = 0$$



80. Solve the following equation in the complex plane C:

 $6x^2 - (18 - 5i)x + 18 + i = 0$

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81. One root of the equation $(2+3i)x^2-bx+(3+i)=0$ is 2-i. Find the other

root and the value of b.







2. The roots of the equation $ax^2 + bx + c = 0$ are

reciprocal to one another when-

A. a=c

B. a=b

C. b=c

D. a=0

Answer: A



3. IF the signs of a and c are opposite to that of b then both roots of the equation $ax^2 + bx + c = 0$ are-

A. zero

B. positive

C. negative

D. fraction

Answer: B

4. The roots of the equation $ax^2 + bx + c = 0 (a \neq 0)$ are equal in magnitude and opposite in signs when-

A. a=0

B. b=0

C. c=0

D. a=c

Answer: B

5. IF b=c=0 then both roots of the equation $ax^2+bx+c=0 (a
eq 0)$ are-

A. zero

B. positive

C. negative

D. imaginary

Answer: A



6. The maximum number of distinct roots in a quadratic equation is -

A. 1

B. 2

C. 3

D. infinite

Answer: B



7. IF a=0,b
eq 0 and c is real and rational then one root of the equation $ax^2 + bx + c = 0$ is real and rational and the other root is -

A. zero

B. real and rational

C. imaginary

D. not defined

Answer: D



8. IF a=0 and b=0 then both roots of the equation

 $ax^2 + bx + c = 0$ are-

A. zero

B. real and rational

C. imaginary

D. not defined

Answer: D



9. The roots of the equation $ax^2 + bx + c = 0$ are equal when-

A.
$$b^2-4ac < 0$$

B.
$$b^2-4ac>0$$

C.
$$b^2 - 4ac \geq 0$$

D.
$$b^2-4ac=0$$

Answer: D



10. IF a,b,c are rational numbers and $b^2 - 4ac$ is positive but not a perfect square then both roots of the equation $ax^2 + bx + c = 0$ are-

A. real

B. rational

C. irrational

D. imaginary

Answer: C

11. The minimum value of $9x^2 - 6x + 1$ is-

A. 0

B. 1

C. 2

D. 3

Answer: A



12. The maximum value of $4x - x^2 - 2$ is-

A. 0

B. 1

C. 2

D. 3

Answer: C



13. If 4 is a root of the equation $x^2+ax-12=0$,

then which of the following is its other root?

B. -2

C. 3

D. -3

Answer: D



14. State which of the following is the sum of the roots of the equation $3x^2 - 5x + 7 = 0$?

$$\mathsf{B.}-rac{5}{3}$$

C. -5

D.
$$\frac{5}{3}$$

Answer: D



15. State which of the following is the product of the roots of the equation $2x^2 - 3x + 7 = 0$?

A.
$$\frac{3}{2}$$

B. $\frac{7}{2}$
C. $-\frac{7}{2}$

D. 7

Answer: B

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16. State which of the following equation has the roots 2 and (-3)?

A.
$$x^2 - x - 6 = 0$$

B.
$$x^2+x-6=0$$

$$\mathsf{C.}\,x^2-x+6=0$$

D.
$$x^2-x-6=0$$

Answer: B



17. If the roots of the equation $5x^2 - 7x - k = 0$ are reciprocal to one another, then which of the following is the value of k?

A. -5 B. $-\frac{1}{5}$ C. 5

D.
$$\frac{-}{5}$$

Answer: A



18. If the sum of the roots of the equation $2x^2 + ax + 6 = 0$ be 5, then which of the following is the value of a ?

A. -10

B. 10

C.
$$\frac{5}{2}$$

D. $-\frac{5}{2}$

Answer: A



19. If the product of the roots of the equation $2x^2 - 7x + b = 0$ be (-3), then which of the following is the value of b?

A.
$$-\frac{3}{2}$$

B. $\frac{3}{2}$
C. -6

D. 6



20. The roots of the equation $3x^2 - 5x + p = 0$ are equal, state which of the following is the value of p?

A.
$$\frac{25}{6}$$

B. $\frac{25}{12}$
C. $-\frac{25}{6}$
D. $-\frac{25}{12}$

Answer: B





Exercise 5 A Very Short Answer Type Question

1. Find the condition for which the roots of the equation $ax^2 - (a+1)x + 1 = 0$ are always real.

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2. One root of the equation $3x^2 - 5x + c = 0$ is 2,

find its other root.

3. The product of the roots of the equationn $3x^2 + mx - (2m+3) = 0$ is 5, find m.

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4. If one root of the equation $2x^2 - 5x + k = 0$ be

twice the other, find the value of k.



5. Find the condition so that the roots of the equation $lx^2 + mx + n = 0$ are equal is magnitude and opposite in signs



6. Find the condition so that the roots of the equation $lx^2 + mx + n = 0$ are

reciprocal to one another

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7. If the ratio of the roots of the equation $x^2 - px + q = 0$ be 1:2, find the relation between p and q.

8. If the roots of the equation $qx^2 + px + q = 0$ be imaginary, where p,q > 0, then show that , 0 .

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9. Find the condition for which the quadratic equation $ax^2 + bx + c = 0$ has exactly one zero root.

10. IF the equation $4x^2 + 2bx + c = 0, b = 0$, find

the relation between the roots of the equation.

11. IF lpha be a root of the equation $ax^2 + bx + c = 0$,

show that klpha (k
eq 0) is a root of the equation $ax^2+bkx+ck^2=0.$

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12. Find the quadratic equation with real coefficients

which has 2±3i as a root $(i=\sqrt{-1})$



14. IF α and β are the roots of the equation x(x-3)=4,

find the value of $\alpha^2 + \beta^2$.

15. IF a,b,c are in G.P prove that the roots of the equation $ax^2 + 2bx + c = 0$ are equal.



16. Find m, given that the difference of the roots of

the equation $2x^2 - 12x + m + 2 = 0$ is 2.

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17. If α and β be the roots of the equation (x-a)(x-b)=c $(c \neq 0)$,prove that a and b are the roots of the equation $(x - \alpha)(x - \beta) + c = 0$.



18. IF $2 + \sqrt{3}i$ is a root of the equation $x^2 + px + q = 0$ where p and q are real, find p and q. **Vatch Video Solution**

19. Find the value of p so that the roots of the equation $3x^2 - 2(7 + 9p)x + (8 - 5p) = 0$ are reciprocal to one another.

20. IF x is real , find the signs of the each of the following expressions:

 $3x^2 - 2x + 1$

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21. IF x is real , find the signs of the each of the following expressions:

 $3x-2x^2-2$

22. IF x is real , find the signs of the each of the following expressions:

 $5x^2 - 14x + 10$

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23. IF x is real , find the signs of the each of the following expressions:

 $10x - 3x^2 - 9$

24. If x be real , find the maximum value of each of the following expressions and the corresponding values of x:

 $1 - 2x - x^2$



25. If x be real , find the maximum value of each of the following expressions and the corresponding values of x:

 $3 - 20x - 25x^2$



26. If x be real , find the maximum value of each of the following expressions and the corresponding values of x:

 $3+2x-x^2$



27. If x be real , find the least values of each of the

following expressions and the corresponding values

of x:

$$4x^2 - 4x + 1$$

28. If x be real , find the least values of each of the following expressions and the corresponding values of x:

 $3x^2 - 6x + 8$



29. If x be real , find the least values of each of the

following expressions and the corresponding values

of x:

 $3x^2 + 6x + 7$

30. For what real values of x the expressions $x^2 - 2x + 3$ is negative? Watch Video Solution

1. IF α and β are the roots of the equation x(2x-1)=1,

find the value of $lpha^2-eta^2$ and form the equation

whose roots are 2lpha-1 and 2eta-1

2. IF α and β be the roots of the equation $5x^2 + 7x + 3 = 0$, find the value of $\frac{\alpha^3 + \beta^3}{\alpha^{-1} + \beta^{-1}}$.

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3. IF p and q are the roots of the equation

$$ax^2 + bx + c = 0$$
 find the value of
 $\frac{1}{(ap^2 + c)^2} + \frac{1}{(aq^2 + c)^2}$.
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4. IF lpha,eta and γ,δ are the roots of the equations $x^2-bx+c=0$ and $x^2-px+q=0$ respectively,
$$(lpha-\gamma)(eta-\delta)-lpha.\ \gamma-eta.\ \delta=(c+q)-bp.$$

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5. If the roots of the equation $ax^2 - bx + a = 0$ be lpha and eta, show that the equation with roots $lpha^2 + 1$ and $eta^2 + 1$ will be $a^2x^2 - b^2x + b^2 = 0$

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6. IF α and β be the roots of the equation $2x^2 + x + 1 = 0$ find the equation whose roots are





7. IF α and β are the roots of the equation $x^2 + \alpha x + \beta = 0$ then find the numerical values of α and β . [Here , $\alpha \neq \beta, \alpha \neq 0, \beta \neq 0$]

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8. If the roots of the equation $ax^2 + bx + c = 0$ be α and β , find the equation whose roots are $\alpha + \frac{\alpha^2}{\beta}$ and $\beta + \frac{\beta^2}{\alpha}$

9. IF α, β be the roots of the equation $x^2 + px + q = 0$, show that $\frac{1}{\alpha + \beta}$ and $\frac{1}{\alpha} + \frac{1}{\beta}$ are the roots of $pqx^2 + (p^2 + q)x + p = 0$

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10. IF the roots of the equation $ax^2 + bx + c = 0$ be lpha and eta, find the equation whose equals are $rac{1}{lpha}+1$ and $rac{1}{eta}+1$.

11. Form the quadratic equation whose roots lpha and eta

satisfy the relations lphaeta=768and $lpha^2+eta^2=1600.$



12. Form the quadratic equation whose roots are the

squares of the roots of $x^2 + 3x + 2 = 0$.

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13. Form the quadratic equation whose roots are reciprocals of the roots of $x^2 + 3x + 4 = 0$.

14. IF $3a^2 = 4a - 5$ and $3b^2 = 4b - 5$ where a eq b,

find the value of $a^2 + b^2$.

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15. IF $3p^2 = 5p + 2$ and $3q^2 = 5q + 2$ where p $eq \,$ q , obtain the equation whose roots are (3p-2q)and (3q-2p).

16. IF α and β be the roots of the equation $x^2 - 4x + 10 = 0$, find the equation whose roots are $\frac{\alpha}{1+\beta}$ and $\frac{\beta}{1+\alpha}$. **Watch Video Solution**

17. If p and q are the roots of the equation $3x^2 + 6x + 2 = 0$, show that the equation whose roots are $\left(-\frac{p^2}{q}\right)$ and $\left(-\frac{q^2}{p}\right)$ is $3x^2 - 18x + 2 = 0$.

18. If the sum of the roots of a quadratic equation is

2 and the sum of their cubes is 27, find the equation.



19. If the sum of the roots of the equation $x^2-px+q=0$ be three times their difference show that, $2p^2=9q$.

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20. If the roots of the equation $ax^2 + bx + c = 0$ are in the ratio 3:4 show that $12b^2 = 49ac$.

21. If one root of the equation $x^2 + (5a+2)x + 5a + 2 = 0$ is five times the other

root, then find the numerical value of a.

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22. Find the quadratic equation whose roots are the

cubes of the roots of $x^2 - 4x + 3 = 0$.

23. The ratio of the roots of the equation $ax^2 + bx + c = 0$ is r:1 Prove that, $b^2r = ac(r+1)^2$ and hence find the condition so that the two roots may be equal to each other.



24. If the roots of the equation $px^2 + rx + r = 0$

are in the ratio a:b, prove that , $p(a+b)^2 = rab$.

25. In a quadratic equation of the form $x^2 + mx + n = 0$ the constant term is misprinted 56 to 54 and the roots are, therefore, obtained as 7 and 8. Find the roots of the equation correctly printed.

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26. For what value of m the roots of the equation

 $(m+1)x^2 + 2(m+3)x + m + 8 = 0$ are equal?

27. If one root of the equation $x^2 + bx + 8 = 0$ be 4 and the roots of the equation $x^2 + bx + c = 0$ are equal , find the value of c.

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28. For what value of m the roots of the equation $\frac{3}{x+3+m} + \frac{5}{x+5+m} = 1$ are equal in a magnitude and opposite in signs?

29. Find the condition so that the roots of the equation $\frac{a}{x-a} + \frac{b}{x-b} = 5$ may be equal in magnitude but oppsite in signs.

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30. Show that the roots of each of the following equations are rational(a,b,c are rational):

$$(b+c)x^2-(a+b+c)x+a=0$$

31. Show that the roots of each of the following equations are rational(a,b,c are rational):

$$(a-b+c)x^2+2cx+(b+c-a)=0$$

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32. Show that the roots of the equation $(a^2-bc)x^2+2(b^2-ca)x+c^2-ab=0$ are equal if either b=0 or , $a^3+b^3+c^3-3abc=0$

33. If one root of the equation $qx^2 + px + q = 0$ (p,q are real) be imaginary, show that the roots of the equation $x^2 - 4qx + p^2 = 0$ are real and unequal.

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34. If the roots of the equation $qx^2 + 2px + 2q = 0$ are real and unequal, prove that the roots of the equation $(p+q)x^2 + 2qx + (p-q) = 0$ are

imaginary.

35. If the roots of the equation $x^2 - 2(a+b)x + a(a+2b+c) = 0$ be equal , prove that a,b, c are in G.P,

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36. If the equation $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root , prove that , it is either $\frac{pq' - p'q}{q' - q}$ or $\frac{q' - q}{p' - p}$. Watch Video Solution **37.** Prove that if the equations $x^2 + px + r = 0$ and $x^2 + rx + p = 0$ have a common root then either p=r or , 1+p+r=0.

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38. IF x is real and the expressions $3x^2 - 17x + 20$ is always positive, show that, x cannot lie between $\frac{5}{3}$ and 4.

39. Find the limits of real value of x so that the expressions $5x^2 + 6x - 8$ is non-negative.



41. If x is real can the value of $3 + 2x - x^2$ be greater

than 4,



42. If x is real can that of $x^2 + 3x + 1$ less than $\left(-\frac{5}{4}\right)$?



43. Show that if x is real and $x^2 + 5 < 6x$, then x

must lie between 1 and 5.

44. If x is real, find the real values of p which make $4x^2 + px + 1$ always positive.



45. IF x is real, find the real values of m which make

the expressions $mx - x^2 - 1$ always negative.

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46. If
$$rac{(x-5)\left(x^2-2x+1
ight)}{(x-7)(x^2+2x+3)}$$
 is positive for all real

values of x, show that x has no value between 5 and



47. If the equation $y^2 + x^2 - 10x + 21 = 0$ is satisfied by real values of x and y, prove that x lies between 3 and 7 and y lies between (-2) and 2.

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48. If x is real , show that (x-2)(x-3)(x-4)(x-5)+2 is

always positive.

Exercise 5 A Long Answer Type Question

1. IF
$$\alpha \pm \sqrt{\beta}$$
 be the roots of the equation $x^2 + px + q = 0$, prove that $\frac{1}{\alpha} \pm \frac{1}{\sqrt{\beta}}$ will be the roots of the equation $(p^2 - 4q)(p^2x^2 + 4px) - 16q = 0.$
(Vatch Video Solution

2. IF lpha, eta and γ, δ be the roots of the equation $x^2 - bx + c = 0$ and $x^2 - cx + b = 0$ respectively,



the given one.

4. IF α , β be the roots of the quadratic equation $x^2 - px + q = 0$, find the equation whose roots are $\frac{q}{p-\alpha}$ and $\frac{q}{p-\beta}$. Accound for the identity of the equation obtained with the given equation.



5. IF
$$\frac{p^2}{q}$$
 and $\frac{q^2}{p}$ are the roots of the equation $2x^2+7x-4=0$, find the equation whose roots

are p and q (p+q real).

6. IF α, β be the roots of the equation $x^2 + 2px + 2q^2 = 0$ where p,q are rational $p^2 + q^2$ is not a perfect square, form the quadratic equation whose one root is $\alpha + \beta + \sqrt{\alpha^2 + \beta^2}$.

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7. IF α, β be the roots of the equation $cx^2 + 2bx + 2c = 0$ where b,c are real and $c^2 > b^2$, find the quadratic equation whose one root is $\alpha + \beta + \sqrt{\alpha^2 + \beta^2}$.

8. Express the roots of the equation $q^2x^2 - \left(p^2 - 2q\right)x + 1 = 0$ in terms of those of $x^2 + px + q = 0.$

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9. If the roots of the quadratics $x^2 + 2px + q = 0$ and $x^2 + 2qx + p = 0 (p \neq q)$ differ by a constant, show that p+q+1=0.

10. If one root of the equation $ax^2 + bx + c = 0$ is the square of the other, prove that , $b^3 + ac^2 + a^2c = 3abc.$

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11. If one root of the equation $ax^2+bx+c=0$ is the cube of the other, prove that , $\left(b^2-2ca
ight)^2=ca(c+a)^2$

12. If the roots of the equation $ax^2 + bx + c = 0$ are two consecutive integers then prove that, $b^2 - a^2 = 4ac.$

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13. IF α and β be the roots of $ax^2 + 2bx + c = 0$ and $\alpha + \delta, \beta + \delta$ be those of $Ax^2 + 2Bx + C = 0$, prove that , $\frac{b^2 - ac}{a^2} = \frac{B^2 - AC}{A^2}$

14. If the ratio of the roots of the equation $x^2-2px+q^2=0$ be equal to the ratio of the roots of the equation $x^2-2rx+s^2=0$, Prove that, $p^2s^2=q^2r^2.$

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15. IF α and β be the roots of the equation $x^2 + px + q = 0$ show that $\frac{\alpha}{\beta}$ is a root of the equation $qx^2 - (p^2 - 2q)x + q = 0$.

16. IF $p^3 - q(3p - 1) + q^2 = 0$, find the relation between the roots of the equation $x^2 + px + q = 0$. Watch Video Solution 17. Let a,b,c be real numbers with a
eq 0 and let lpha, etabe the roots of the equation $ax^2 + bx + c = 0$. Express the roots of $a^3x^2 + abcx + c^3 = 0$ in terms

of α, β .

18. IF α be a root of the quadratic equation $4x^2+2x-1=0$, Let prove that $4\alpha^3-3\alpha$ is the other root.

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19. Let α , β be the roots of the equation $x^2 - 3x + a = 0$ and γ , δ the roots of the equation $x^2 - 12x + b = 0$. If α , β , γ and δ are in G.P. having positive common ratio, then find the values of a and b.

20. IF a,b,c are real, show that the roots of each of

the following equations are real:

$$(x-a)(x-b) = b^2$$

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21. IF a,b,c are real, show that the roots of each of the

following equations are real:

$$(b-c)x^2+2(c-a)x+a-b=0$$

22. IF a,b, are real, show that the roots of each of the

following equations are real:

$$\frac{1}{x-a}+\frac{1}{x-b}=\frac{1}{a^2}$$

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23. IF a,b,c are real, show that the roots of each of

the following equations are real:

$$rac{1}{x-a}+rac{1}{x-b}+rac{1}{x-c}=0$$

24. If the roots of the equation
$$p(q-r)x^2 + q(r-p)x + r(p-q) = 0$$
 be equal, show that $\frac{1}{p} + \frac{1}{r} = \frac{2}{q}$.

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25. Find the condition that the roots of the quadratic equation $x^2 + px + q = 0$ should be both postitive

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26. Find the condition that the roots of the quadratic equation $x^2 + px + q = 0$ should be both negative



27. Find the condition that the roots of the quadratic equation $x^2 + px + q = 0$ should be one is positive and the other negative

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28. Find the condition that the roots of the quadratic equation $x^2 + px + q = 0$ should be equal in magnitude and opposite in signs

29. Find the condition that the roots of the quadratic equation $x^2 + px + q = 0$ should be reciprocal to one another.

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30. If the roots of the equation $x^2 + x + a = 0$ be real and unequal, then prove that the roots of the equation $2x^2 - 4(1+a)x + 2a^2 + 3 = 0$ are

imaginary (a is real).

31. Prove that, if the roots of the equation $(a^2+b^2)x^2+2(bc+ad)x+(c^2+d^2)=0$ be

real, then they are equal.

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32. Show that the roots of the equation $(a^4+b^4)x^2+4abcdx+(c^4+d^4)=0$ cannot be different, if real.
33. For what values of m the equations $3x^2 + 4mx + 2 = 0$ and $2x^2 + 3x - 2 = 0$ will have a common root?

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34. Show that the equations $px^2 + qx + r = 0$ and $qx^2 + px + r = 0$ will have a common root if p+q+r=0 or, p=q=r.

35. If the two equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ ($a \neq b$) have a common root, show that the other roots are the roots of the equation $x^2 + x + ab = 0.$



36. IF c is real, show that the roots of the quadratic equation $cx^2 + (c-1)x + 1 - 2c = 0$ are real. If the sum of the roots of the equation be equal to three times their difference, then find c.



37. IF lpha is a root of the equation $ax^2+bx+c=0$ then show that $mlpha^2(m
eq 0)$ is a root of the equation $a^2x^2+ig(2ac-b^2ig)mx+m^2c^2=0.$

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38. If the quadratic equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0 (a
eq b)$ have a common root , find the numerical value of (a+b).

39. If the roots of the equation $x^2 - px + q = 0$ be lpha, eta and the roots of the equation $x^2 - ax + b = 0$ be $lpha, \frac{1}{eta}$ then prove that, $bq(a-p)^2 = (q-b)^2$.

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40. If lpha and eta be the roots of the equation $5x^2+bx+c=0$ Show that $5x^2+bx+c=5(x-lpha)(x-eta)$

Show also that for all real values of x the expression $5x^2+bx+c$ cannot be negative if lpha and eta are real and equal

41. If α and β be the roots of the equation $5x^2 + bx + c = 0$ Show that $5x^2 + bx + c = 5(x - \alpha)(x - \beta)$ Show also that for all real values of x the expression $5x^2 + bx + c$ cannot be negative if α and β are conjugate complex.

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42. IF x is real , find the greatest and the least value

of:

 $x^2 + 14x + 9$

 $x^2 + 2x + 3$



43. IF x is real , find the greatest and the least value

of:

 $\frac{x^2 - 2x + 2}{x^2 + 3x + 9}$

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44. Show that for all real values of x the value of $\frac{x^2 - 3x + 4}{x^2 + 3x + 4}$ always lie between $\frac{1}{7}$ and 7.







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47. If x is real show that the expressions
$$\frac{(x-1)(x+3)}{(x-2)(x+4)}$$
 has no value between $\frac{4}{9}$ and 1.

48. If x is real then the expressions $\frac{x^2 + 2x - 11}{x - 3}$ assumes those values which do not lie between a and b , find a and b.



49. If x is real , show that the expressions $\frac{x^2 - ab}{2x - a - b}$ has no real value between a and b.

50. If x is real show that the expressions $\frac{3x-5}{x^2-1}$ has no value between $\frac{1}{2}$ and $\frac{9}{2}$.

51. IF x is real , find the maximum and minimum values of
$$\frac{x^2 - x + 1}{x^2 + x + 1}$$
 also find the corresponding values of x.

52. If x be real , find the greatest value of $\frac{x+2}{2x^2+3x+6}$.



53. If x is real, show that each of the following expressions is capable of assuming all real values: $2x^2 + 4x + 1$ $x^2 + 4x + 2$ Watch Video Solution

54. If x is real, show that each of the following expressions is capable of assuming all real values: $2x^2 + 5x + 2$ $x^2 + 6x + 7$

55. If x is real, show that each of the following

expressions is capable of assuming all real values:

 $\frac{p^2}{1-x}-\frac{q^2}{1+x}$

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56. IF $3x^2 + 2(p+q+r)x + pq + qr + rp$ be a

perfect square , prove that p=q=r.

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Exercise 5 B Multiple Choice Type Question

1. The least value of m which makes the roots of the equation $x^2 + 5x + m = 0$ imginary is-

A. 4

B. 5

C. 6

D. 7

Answer: C



2. The equation of the smallest degree with real coefficients having 1+i as one of the roots is-

A.
$$x^2 + x + 1 = 0$$

B. $x^2 - 2x + 2 = 0$
C. $x^2 + 2x + 2 = 0$
D. $x^2 + 2x - 2 = 0$

Answer: B



3. If the discriminant of a quadratic equation is less

than zero then the roots of this equation are-

A. both real

B. both imaginary

C. one real and another imaginary

D. none of these

Answer: B



4. If 2+3i be a root of a quadratic equation , then the equation will be-

A.
$$x^2 + 4x + 13 = 0$$

B.
$$x^2 - 4x + 13 = 0$$

$$\mathsf{C.}\,x^2 + 4x - 13 = 0$$

D.
$$x^2 - 4x - 13 = 0$$

Answer: B



5. IF -iy-x be a root of the equation $ap^2 + bp + c = 0$ then its another root will be-

A. iy+x

B.-it+x

 $\mathsf{C}.\,iy-x$

D. none of these

Answer: C



Exercise 5 B Very Short Answer Type Question

1. Express each of the following quadratic equations

as the difference of two squares and solve

 $x^2 + 1 = 0$

as the difference of two squares and solve

$$9x^2 + 16 = 0$$

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3. Express each of the following quadratic equations

as the difference of two squares and solve

 $x^2 + x + 1 = 0$

as the difference of two squares and solve

 $2x^2 + 2x + 5 = 0$

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5. Express each of the following quadratic equations as the difference of two squares and solve $3x^2 - 2x + 2 = 0$

as the difference of two squares and solve

 $8x^2 + 4x + 13 = 0$

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7. Express each of the following quadratic equations as the difference of two squares and solve $9x^2 + 12x + 10 = 0$ Watch Video Solution

as the difference of two squares and solve

$$5x^2 - 6x + 5 = 0$$

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9. Express each of the following quadratic equation

as two squares and solve

$$a^2x^2 - 2ax + 10 = 0 (a
eq 0)$$

as two squares and solve

 $4x^2 - 12xp + 25p^2 = 0$

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11. State the fundamental theorem of algebra.

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Exercise 5 B Short Answer Type Question

1. Show that the roots of each of the following quadratic equations are complex numbers. Find the solutions in each case.

 $x^2+2x+2=0$



2. Show that the roots of each of the following quadratic equations are complex numbers. Find the solutions in each case.

$$x^2 + 4x + 8 = 0$$

3. Show that the roots of each of the following quadratic equations are complex numbers. Find the solutions in each case.

 $2x^2 - 3x + 4 = 0$



4. Show that the roots of each of the following quadratic equations are complex numbers. Find the solutions in each case.

$$3x^2 - 7x + 5 = 0$$

5. Show that the roots of each of the following quadratic equations are complex numbers. Find the solutions in each case.

$$\frac{2x-1}{x-2} = \frac{x}{x-1}$$



6. Show that the roots of each of the following quadratic equations are complex numbers. Find the solutions in each case.

$$\sqrt{3}x^2+x+\sqrt{3}=0$$

7. Show that the roots of each of the following equations are conjugate complex numbers:

 $3x^2 - 4x + 3 = 0$

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8. Show that the roots of each of the following equations are conjugate complex numbers:

$$rac{1}{x-3}+rac{x}{5}=0$$

9. Show that the roots of each of the following

equations are conjugate complex numbers:

$$rac{1}{x-1} + rac{1}{x-2} = rac{2}{2x-3}$$

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10. Solve by factorization:

$$3x^2 + 8ix + 3 = 0$$



11. Solve by factorization:

$$ix^2 + x + 6i = 0$$



13. Solve by factorization:

$$2x^2 - ix + 6 = 0$$

14. Solve by factorization:

$$x^2 + 3\sqrt{2}xi + 8 = 0$$

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15. Solve by factorization:
 $12ix^2 - x + 6i = 0$
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16. Solving the equation
 $x^2 + (i - 7)x - (i - 18) = 0$, prove that the roots

of the equation are not complex conjugate numbers.



17. Solve the following quadratic equations using Sridhar Acharya's formula:

$$2x^2 + (4i - 5)x + 8 + i = 0$$

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18. Solve the following quadratic equations using Sridhar Acharya's formula:

$$y^2 - (1-2i)y + 1 + 5i = 0$$

19. Solve the following quadratic equations using

Sridhar Acharya's formula:

 $2y^2 + 3y + 8 - 6i = 0$

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20. Solve the following quadratic equations using Sridhar Acharya's formula:

$$6x^2 - (5+3i)x + 11i - 3 = 0$$

21. Solve the following quadratic equations using Sridhar Acharya's formula:

 $ix^2 - 6x - 9i = 0$

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22. Solve the following quadratic equations using Sridhar Acharya's formula:

 $3x^2 + (11i - 2)x + 4 - 8i = 0$



$$(2+i)x^2 + (i+5)x - 2(i-1) = 0$$

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24. Solve:

$$2x^2 - (3+7i)x - 3 + 9i = 0$$

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Sample Question For Competitive Exams Multiple Correct Answer Type 1. If $x^2 - kx + k + 2 = 0$ has equal roots, then the

value of k will be -

A. $2+\sqrt{20}$ B. $2-\sqrt{20}$ C. $2+\sqrt{12}$

D. $2-\sqrt{12}$

Answer: C::D



2. Let the quadratic equation $ax^2 + bx + c = 0$ has two purely complex roots. IF ap=b and aq=c, then-

A. p is purely imaginary and q is purely real

B.
$$\left|rac{1-p}{1+p}
ight|=1$$

C. $\left|q+\sqrt{q^2+1}
ight|=\left|q-\sqrt{q^2+1}
ight|$
D. $\left|q+ar{p}
ight|=\left|ar{q}+p
ight|$

Answer: B::C



3. The roots of $2(1+i)x^2 - 4(2-i)x - 5 - 3i = 0$

are-



Answer: A::D



4. The values of x satisfying the equation $|x-2|^2+|x-2|-6=0$, are-A. 4 B. 2 C. 3 D. 0 Answer: A::D **Watch Video Solution**
5. If the equaiton $ax^2+bx+c=0$ where a,b,c $\,\in\,$ R

have non-real roots then-

A.
$$c(a-b+c)>0$$

B.
$$c(a+b+c)>0$$

C.
$$c(4a-2b+c)>0$$

$$\mathsf{D}.\,b^2-4ac=0$$

Answer: A::B::C



Sample Question For Competitive Exams Integer Answer Type

1. If the equation
$$ig(K^2-5K+6ig)x^2+ig(K^2-3k+2ig)x+ig(K^2-4ig)=0$$

has more than two roots, then the value of K is-





a perfect square, then the value of K will be-

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3. If the roots of $x^2 - bx + c = 0$ be two consecutive integers, then the value of $b^2 - 4c$ will be-Watch Video Solution

4. IF x be real, then the maximum value of $5+4x-4x^2$ will be-

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5. IF $-2+i\sqrt{3}$ be a root of $x^2+px+q=0$, then

the value of p+q will be-

SampleQuestionForCompetitiveExamsComprehension Type

1. Let $f(x) = x^2 + b_1x + c_1$ and $g(x) = x^2 + b_2x + c_2$. When f(x)=0 then the real roots of f(x) are α , β and when g(x)=0 then the real roots of g(x) are $\alpha + h$, $\beta + h$. Minimum value of f(x) is $-\frac{1}{4}$ and when $x=\frac{7}{2}$ then value of g(x) will be minimum.

Minimum value of g(x) is-

A.
$$-\frac{1}{4}$$

B. -1

C.
$$-\frac{1}{3}$$

D. $-\frac{1}{2}$

Answer: A



2. Let
$$f(x) = x^2 + b_1 x + c_1$$
 and
 $g(x) = x^2 + b_2 x + c_2$. When $f(x)=0$ then the real
roots of $f(x)$ are α , β and when $g(x)=0$ then the real
roots of $g(x)$ are $\alpha + h$, $\beta + h$. Minimum value of $f(x)$
is $\frac{1}{4}$ and when $x=\frac{7}{2}$ then value of $g(x)$ will be

minimum.

Value of b_2 is-

A. -5

B. 9

C. -8

D. -7

Answer: D

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3. Let
$$f(x) = x^2 + b_1 x + c_1$$
 and $q(x) = x^2 + b_2 x + c_2$. When $f(x)=0$ then the real

roots of f(x) are α , β and when g(x)=0 then the real roots of g(x) are $\alpha + h$, $\beta + h$. Minimum value of f(x) is $-\frac{1}{4}$ and when $x=\frac{7}{2}$ then value of g(x) will be minimum.

Roots of g(x)=0 are-

A. 3,-4

B. -3, 4

C. 3,4

D. -3, -4

Answer: C



4. Consider an unknown polynomial which when divided by (x-3) and (x-4) leaves 2 and 1 as remainders, respectively, Let R(x) be the remainder when the polynomial is divided by (x-3) (x-4).

If the equation R(x)= $x^2 + ax + 1$ has two distinct real roots then values of 'a' are-

A. (-2,2)
$$B. \ (-\infty,\ -2) \cup (2,\infty)$$
C. $(-2,\infty)$

D. all real numbers

Answer: D



5. Consider an unknown polynomial which when divided by (x-3) and (x-4) leaves 2 and 1 as remainders, respectively, Let R(x) be the remainder when the polynomial is divided by (x-3) (x-4). If $R(x)=px^2 + (q-1)x + 6$ has no distinct real roots and p > 0, then least value of 3p+q is-

A. -2
B.
$$\frac{2}{3}$$

C. $-\frac{1}{3}$
D. $-\frac{4}{3}$

Answer: C



6. Consider an unknown polynomial which when divided by (x-3) and (x-4) leaves 2 and 1 as remainders, respectively, Let R(x) be the remainder when the polynomial is divided by (x-3) (x-4).

Range of f(x) =
$$=rac{\lfloor R(x)
floor}{x^2-3x+2}$$
 is-

A. [-2,2] B. $\left[-\infty - 2\sqrt{3}\right] \cup \left[-2 + \sqrt{3}, \infty\right]$ C. $\left[-\infty - 7 - 4\sqrt{3}\right] \cup \left[-7 + 4\sqrt{3}, \infty\right]$ D. none of these

Answer: C

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Sample Question For Competitive Exams Assertion Reason Type

1. Let $ax^2+bx+c=0, a
eq 0$ (a,b,c \in R) has no

real roots and a+b+2c=2

Statement-I: $ax^2+bx+c>0\,orall\,x\in R$

Statement II: a+b is positive.

A. Statement-I is true, Statement-II is true and Statement -II is a correct explanation for Statement -I.

B. Statement-I is true, Statement -II is true but

Statement-li is not a correct explanation of

Statement-I.

C. Statement-I is true, Statement-II is false

D. Statement-I is false, Statement-II Is true.

Answer: C



2. Statement I: If a > 0 and $b^2 - ac < 0$, then domain of the function f(x)= $\sqrt{ax^2 + 2bx + c}$ is R. Statement II: If $b^2 - ac < 0$ then $ax^2 + 2bx + c = 0$ has imaginary roots.

A. Statement-I is true, Statement-II is true and Statement -II is a correct explanation for Statement -I.

B. Statement-I is true, Statement -II is true but

Statement-li is not a correct explanation of

Statement-I.

C. Statement-I is true, Statement-II is false

D. Statement-I is false, Statement-II Is true.

Answer: B

