



# MATHS

## BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

### RANDOM VARIABLE AND ITS DISTRIBUTION

Example

1. Examine which one of the following represent the probability distribution of a random variable  $x$



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2. Three cards are drawn at random from a pack of 52 playing cards find the probability distribution of the number of aces



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3. Four bad oranges are mixed accidentally with 16 good oranges find the probability distribution of the number of bad oranges in a draw of two oranges



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4. A pair of dice is thrown 4 times If getting a double is considered a success find the probability distribution of number of successes



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5. A balanced coin is tossed repeatedly until a tail appears if  $x$  denotes the number of heads preceding the first tail find the probability distribution of  $x$



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6. A lot contains 36 articles of which 6 are defective A sample of 5 articles was drawn at random (i) without replacement (ii) with replacement find the probability distribution of  $x$  the number of defectives



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7. A discrete random variable  $x$  has the following probability distribution



(i) find a

(ii) Evaluate :  $P(X \leq 1)$  ,  $P(X \geq 4)$ ,  $P(0 \leq X \leq 3)$  ,  $P(0.1 \leq X \leq 2)$

(iii)  $F \in \mathbb{R}$  the minimum value of  $a$  such that

$P(X \leq 4) \geq 0.6$



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8. A random variable  $x$  has the following probability distribution



Find the probability distribution of a random variable  $y$  where  $y = 2x^2 + 3$



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9. If  $x$  is a discrete random variable and 'a' is a constant show that

(i)  $E(a) = a$

(ii)  $E(ax) = aE(x)$

(iii)  $E(x - \bar{X}) = 0$



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**10.** If  $a$  is a constant prove that

(i)  $\text{var}(a) = 0$

(ii)  $\text{var}(aX) = a^2 \text{var}(X)$

(iii)  $\text{Var}(ax + b) = a^2 \text{var}(X)$



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11. If  $x$  and  $Y$  are two independent random variables prove that ,

$\text{Var}(aX+bY)=a^2\text{Var}(X)+b^2\text{Var}(y)$  where  $a$  and  $b$  are constant



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12. Two cards are drawn simultaneously (or successively without replacement ) from a well shuffled pack of 52 cards find the mean and variance of the number of aces



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**13.** Two cards are drawn successively with replacement from a well-shuffled pack of 52 cards. Find the mean and variance the number of kings.



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**14.** A coin is tossed 4 times let  $x$  denote the number of heads find the probability distribution of  $X$  also find the mean and variance of  $X$



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**15.** A box contains 4 black and 5 white balls let  $X$  denote the number of white balls in a random draw of 3 balls find  $E(X)$  and standard deviation of  $X$



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**16.** A gambler throws two unbiased dice and stands to loss Rs 5 if he fails to throw a 5 to win Rs 10 if he throws on e five and to win Rs 25 if he throws fives is the game fair ?



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17. (i) find the variance of the number obtained on a throw of an unbiased die

(ii) what is the mathematical expectation of the sum of points on  $n$  dice ?



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18. A random variable  $X$  has the following probability distribution



Find the expectation and standard deviation of the random variable  $X$



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**19.** The probability that there is at least one error in an accounts statement prepared by A is 0.2 and for B and C they are 0.25 and 0.4 respectively. A, B and C prepared 10, 16 and 20 statements respectively. Find the expected number of correct statements in all.



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**20.** Assuming that half of the population is vegetarian so that the chance of an individual being a vegetarian is  $\frac{1}{2}$  and assuming that 100 investigators can each take a sample of 10 individuals to see whether they are vegetarians how many investigators would you expect to report that 3 people or less were vegetarians ?



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**21.** A number is chosen at random from the set 1,2,3 ...100 and another number is chosen at

random from the set  $1, 2, 3, \dots, 50$  what is the expected value of the product ?



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**22.** Obtain the expectation of the number of tails preceding the first head in the indefinite series of tosses of the same coin



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**23.** Three persons A,B and C are taking part in a shooting competition where the person hitting the target first will win Rs 94 probability of hitting by them are  $\frac{3}{5}$ ,  $\frac{2}{5}$  and  $\frac{3}{4}$  respwectively if the cometition is fair and A,B and C shoots in order then calculate the prize money each will win it is given that the total prize money of the competition is Rs 94



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**Multiple Choice Type Questions**

1. The probability function of a random variable is always

A.  $\geq 1$

B.  $0 > 1$

C. positive

D. non negative

**Answer: D**



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2. The value of  $E(10x)$  is

A.  $10+E(x)$

B.  $10 E(x)$

C.  $E(x)$

D.  $E(x)-10$

**Answer: B**



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3. The value of  $\text{Var}(4x)$  is

A.  $16 \text{ Var}(x)$

B.  $4 \text{ Var}(x)$

C.  $2 \text{ Var}(x)$

D.  $\text{Var}(x)+4$

**Answer: A**



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**4. The value of  $\text{Var}(x)$  is**

A.  $E(x^2) - E(x)$

B.  $E(x^2) - \{E(x)^2\}$

C.  $E(x^2) + E(x)$

D.  $\{E(x)\}^2 - E(x^2)$

**Answer: B**



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5. If  $F(x)$  is the probability function of a random variable  $X$  and  $X$  can assume only two values  $x_1, x_2$  then the value of  $F(x_1) + f(x_2)$  is

A.  $\geq 1$

B.  $\leq 1$

C.  $\geq 0$

D. 1

**Answer: D**



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**6. Var (ax+b) is equal to**

A.  $a \text{ Var}(x)+b$

B.  $a^2 \text{Var}(x) + b$

C.  $a^2 Var(x)$

D. none of these

**Answer: C**



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## Very Short Answer Type Questions

1. Probability Distribution of Random variable



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2. Mean or mathematical expectation of random variable



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3. Variance of random variable



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4.  $E(K) = K$  where  $K$  is a constant .TRUE or FALSE



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5. prove that  $E(ax+b)=aE(x)+b$  where  $a$  and  $b$  are constants



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6. Prove that  $E(x - \bar{x})=0$  where  $\bar{x}$  = mean of a random variable  $x$



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7.  $\text{Var}(K) = 0$  where  $k$  is a constant



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8. Prove that  $\text{Var}(a+bx) = b^2 \text{Var}(x)$



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9. Define a random variable and explain what is meant by its mean (or mathematical expectation )



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**10.** Define a discrete random variable  $X$  and its mean  $E(x)$  and variance  $\text{Var}(x)$



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**11.** What is meant by discrete probability distribution ?



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**12.** Define the probability function of a discrete random variable  $x$ . state the conditions satisfied

by it



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**13.** The variance of a random variable  $x$  is defined by the expected value of  $(x - \bar{x})^2$  where  $\bar{x}$  is the mean of  $x$  prove that  $\text{variance}(x) = E(x^2) - \{E(x)\}^2$



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14. If X and Y are independent random variable

show that  $Var(ax + by) = a^2\omega_1^2 + b^2\omega_2^2$

where  $\omega_1$  and  $\omega_2$  are standard deviation of x and y respectively and a,b are constant



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## Short Answer Type Questions

1. Define a random variable on the sample space of the random experiment of tossing two unbiased coins



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2. Define two random variable  $X$  and  $Y$  on the sample space of the random experiment of rolling a balanced die



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3. Examine which one of the following represent the probability distribution of a random variable  $X$





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4. A random experiment consists of three independent tosses of a fair coin write down the sample space let  $x$  be the number of heads obtained obtain the probability distribution of  $x$  and calculate its expectation and variance



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5. Four unbiased coins are tossed simultaneously let  $x$  be the random variable

denoting the number of heads obtained obtain the probability distribution of  $x$  find  $E(x)$  and  $\text{var}(x)$  [symbols have their usual meaning]



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6. Let  $x$  denote the sum of digits in the random experiment of throwing two balanced dice find the probability distribution of the random variable  $x$



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7. Let  $x$  denote the number of tails preceding the first head when an unbiased coin is tossed repeatedly find the probability distribution of  $x$



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8. In a box there are 5 watches of which 2 are known to be defective two watches are taken out at random let  $x$  denote the number of defective watches selected obtain the probability distribution of  $x$  also calculated mean of  $x$



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**9.** Five defective bulbs are accidentally mixed with 20 good ones it is not possible to just look at a bulbs and tell whether or not it is defective find the probability distribution of the number of defective bulbs in a draw of 4 bulbs



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**10.** From a lot of 25 articles containing 5 defective a sample of 4 articles was drawn at random (a) without replacement (b) with



replacement find the probability distribution of  $x$   
the number of defectives



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**11.** Two cards are drawn simultaneously (or successively without replacement ) from a well shuffled pack of 52 cards find the mean, variance and standard deviation of the number of kings



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12. A discrete random variable  $x$  has the following probability distribution



(a) Determine the value of  $a$

(b) Find  $P(x < 3)$ ,  $P(x \geq 4)$ ,  $P(0 < x < 5)$

(c ) Find the minimum value of  $m$  for which

$$P(x \leq m) \geq 0.6$$



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13. A random variable  $x$  has the following probability function



(a) Calculate k

(b) find  $(P(x < 2), P(x \geq 2), P(-2 < x \leq 2))$

(c) calculate the minimum value of k such that

$$P(x \leq 1) \geq 0.36$$



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**14.** For what value of k will the function  $f(x) = kx$   $x=1,2,3, \dots, n$  be a probability distribution of a random variable ?



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15. The probability distribution of a random variable  $x$  is as follows



Find the probability distribution of the variable  $y$  where  $y = x^2 + 5$



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16. Let  $x$  denote the number of heads when 10 unbiased coins are tossed find the probability distribution of  $x$



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17. In a given business venture a man can make a profit of Rs 300 with probability 0.6 incur a loss of Rs 100 with probability 0.4 calculate his expectation



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## Long Answer Type Questions

1. Find the mean of the sum of points on 2 unbiased dice



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2. A box contains 4 white and 6 red balls if 2 balls are drawn at random find the expectation of the number of red balls



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3. A bag contains 5 white and 7 black balls find the expectation of a man who is allowed to draw two balls from the bag and who is to

receive one rupee for each black ball and two rupee for each white ball drawn



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4. An urn contains 7 white and 3 red balls two balls are drawn together at random from this urn compute the probability that neither of them is white find also the probability of getting one white and one red ball hence compute the expected number of white balls



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5. What is the expectation from a throw with a single die when a score of  $n$  is rewarded with a prize of  $n^2$  rupees



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6. If a person gains or loses an amount equal to the number appearing when a balanced die is rolled once according to whether the number is even or odd how much money can he expect per game in the long run ?



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7. A gambler rolls two unbiased dice and stands to loss Rs 2 if he fails to throw a six to win Rs 4 if he throws one six and to win Rs 10 if he throws two sixes is the game fair ?



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8. A and B play a game in which they take turns to throw 4 coins simultaneously when A throws he receives Rs  $h$  where  $h$  is the number of heads shown . Find the expectation of A . when B

throws 4 heads or 4 tails he receives Rs  $k$ , otherwise he receives nothing find the value of  $k$  for the game to be fair



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9. A and B play for a prize of Rs 99 the prize is to be won by a player who first throes a '3' with one die A first throes and if he fails B throws and if he fails A again throws and so on find their respective expectations



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10. The probability distribution  $f(x)$  of a discrete random variable  $x$  is given by



Find mean and variance of  $x$



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## Multiple Correct Answer Type

1. The probability distribution of a discrete random variable  $x$  is as follows



find the expectation and S.D of X



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2. Find the mean variance and standard deviation of the following probability distribution



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3. The probability distribution of a random variable  $x$  is as follows



if  $y = 2x + 3$  find the expectation variance and standard deviation of the random variable  $y$

A.  $\frac{1}{n + 1}$

B.  $\frac{n!}{n + 1!}$

C.  $\frac{n}{n + 1}$

D.  $\frac{2n}{n + 1}$

**Answer: A::C::D**



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4. Let  $x$  and  $y$  be two independent random variables having variance  $k$  and  $2$  respectively if the variance of  $z=3y-x$  be  $25$  find  $k$

A.  $\frac{1}{52c_4}$

B.  $\frac{4c_1}{52c_4}$

C.  $\left(\frac{1}{13c_4}\right)^4$

D.  $4\frac{)}{270725}$

**Answer:**



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5. For a random variable  $x$  it is given  $E(x) = 10$  and  $Var(x) = 25$  find the positive values of  $a$  and  $b$  such that  $y = aX - b$  have expectation 0 and variance 1

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**Integer Answer Type**

1. A number is chosen at random from the set 1,2,3 ...150 and another number is chosen at random from the set 1,2,3....75 find the expected value of the product of the chosen number



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2. If  $E_1, E_2, E_3 \dots E_n$  be  $n$  independent events such that  $P(E_i) = \frac{1}{1+i}$  for  $1 \leq i \leq n$  then the chance that none of  $E_1, E_2, E_3 \dots E_n$  occur is



A.  $\frac{1}{n+1}$

B.  $\frac{n!}{(n+1)!}$

C.  $\frac{n}{n+1}$

D.  $\frac{2n}{n+1}$

**Answer: A::B**



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3. If shuffling a pack of 52 cards four cards are accidentally dropped the probability that the dropped cards are from the same suit is



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4. An unbiased die thrown twice find the probability distribution of the number of sixes



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5. The probability distribution of the number of successes in two tosses of a die where a success is defined as getting a number greater than 4 then the probability distribution of number of successes in two tosses of die are



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## Matrix Match Type

1. A random variable  $x$  takes values  $0, 1, 2, 3, \dots$ . With probability proportional to  $(x + 1) \left(\frac{1}{5}\right)^x$  then



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2. If standard deviation of a random variable  $x$  is 2 then  $\text{var}(5x)$  is

A. 25

B. 100

C. 50

D. 26

**Answer:**



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**Comprehension Type**

1. The probability distribution function of a random variable  $x$  is given by



value of  $c$  is  $\frac{1}{k}$  find  $k$



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2. In a single throw of a die if  $x$  denotes the number on its upper face the mean of  $x$  is  $\frac{k}{2}$  what will be the value of  $k$ ?



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3. If a pair of dice is thrown and  $x$  denotes the sum of the numbers on them the expectation of  $x$  is  $k$  find the value of  $k$



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4. The standard deviation of the data 6, 8, 10, 12, 14 is

A. 0

B. 1

C. 2.14

D. 2.83

**Answer: D**



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5. Two numbers  $p$  and  $q$  are chosen randomly from the set  $\{1,2,3,4,5,6,7,8,9,10\}$  with replacement match column I contain the nature of the roots of  $x^2 + px + q = 0$  with column II contains their probability



A. 0.65

B. 0.55

C. 0.3

D. 0.75

**Answer: A::B::C::D**



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6. A student has to match three historical events: dandi march, quit india movement, and mahatma gandhiji's assassination with the



years 1948 ,1930 and 1942 the student has no knowledge of the corect answeres and decides to match the events and years randomly here  $x$  denotes the number of correct answer obtained by the student



A. 0.2

B. 0.1

C. 0.3

D. 0.4

**Answer: A::B::C::D**



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## Assertion Reason Type

1. The standard deviation of the data 7, 9, 11, 13, 15 is



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2. The mean deviation of the following data 2, 5, 6, 7, 9, 10, 11, 14 is



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3. The median of  
 $0, 2, -3, 2, 2, 5, -1, 5, 5, -3, 6, 5, 6, 6$  is .



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4. A random variable  $X$  has the following probability distribution



The value of  $k$  is

- A. Statement I is true Statement II is true  
Statement II is a correct explanation for  
Statement I
- B. Statement I is true Statement II is true is  
true , Statement II is not a correct  
explanation for Statement I
- C. Statement I is true , Statement II is false
- D. Statement I is false Statement II is true

**Answer: B**



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5. A random variable  $X$  has the following probability distribution



The mean of  $X$  is



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6. A random variable  $X$  has the following probability distribution



The variance of  $X$  is



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7. The following is the data of wages per day :  
5, 4, 7, 8, 8, 7, 5, 7, 9, 5, 7, 9, 10, 8. Find the  
mode



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8. The following marks are obtained by students  
in a test :

82, 72, 91, 90, 86, 85, 92, 70, 71, 83, 89, 94, 85, 79, 62

.Find the range of the marks





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