MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

RELATION AND FUNCTIONS

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1. Fill in the blanks :

If a function $f: R \to R$ is defined by $f(x) = 2x + 3$ then $f^{-1}(19) =$ ____(R is the set real numbers).

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2. State whether the statement is true or false ?

If $A = \{a, b, c, d\}$ and $R = \{(a, c), (b, d), (b, c), (c, a), (d, b)\}$ then R is a symmertric relation on A .

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3. Let $f(x + 3) = x^2 - 3x - 1$.then find $f(x + 1)$

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4. A realation ρ is defined on set Z , a set of all integers , such that $\rho = \{(x, y) \in Z \times Z : Y - x$ is divisible by 5$\}$. Discuss whether ρ is an equialence relation .

IF $A = \{x \in Z : (2, x) \in \rho, \ -10 \leq x \leq 10\}$ then mention the elements of A .

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5. A function $f: R \to R$ is defined by $f(x) = x^2$ .Then show that f is neither nor surfection .

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6. Prove that ,

$$\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1} \frac{x + y + z - xyz}{1 - xy - yz - zx}$$

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7. IF $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}, x + y + z = \sqrt{3}$, prove that $x = y = z$

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8. Prove that $\frac{1}{2}\cos^{-1}\left(\frac{5\cos x + 3}{5 + 3\cos x}\right) = \tan^{-1}\left(\frac{1}{2}\tan\frac{x}{2}\right)$

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9. Let R be the set of real numbers . If the function $f: R \to R$ and $g: R \to R$ be defined by $f(x) = 4x + 1$ and $g(x) = x^2 + 3,$ then find (go f) and (fo g) .

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10. Prove that $\sin \cos^{-1} \tan \sec^{-1} x = \sqrt{2 - x^2}.$

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11. The relation $R = \{(1, 1), (2, 2), (4, 4), (1, 2), (1, 3), (2, 3)\}$ on the set $A = \{1, 2, 3, 4\}$ is -

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: C

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12. The value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is -

A. $\frac{7(\pi)}{6}$

B. $-\frac{\pi}{6}$

C. $\frac{\Pi}{4}$

D. $-\frac{\pi}{4}$

Answer:

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13. A binary openation * is defined on the set of all integers Z by $a*b=a+b+5, a, b \in Z$. Find whether * is associative on Z.

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14. Find the value of $\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}$. (consider prinicipal values only).

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15. Let the function $f: R \to R, g: R \to R, h: R \to R$ be defined by $f(x) = \cos x, g(x) = 2x + 1$ and $h(x) = x^3 - x - 6,$ Find the mapping h o (go h) , hence find the value of h o (go f), hence find the value of (h o (go f)) (x) when $x = \frac{\pi}{3}$ and $c = \frac{2\pi}{3}$.

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16. Prove that $\tan^{-1}\frac{1}{x+y} + \tan^{-1}\frac{y}{x^2+xy+1} = \cot^{-1}x.$

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17. Give an example of a relation defined on set of integers which is symmetric and transitive but not reflexive . Jusitfy your answer .

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18. If $\sec^{-1} x = \text{cosec}^{-1} y$, then the valuw of $\cos^{-1} \frac{1}{x} - \sin^{-1} \frac{1}{y}$ will be

A. 0

B. $\frac{2\pi}{3}$

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{2}$

Answer: A

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19. If $f(x) = \frac{x^2}{1+x^2}$, then the range of f is -

A. $[1, \infty]$

B. $[0, 1)$

C. $[-1, 1]$

D. $[0, 1]$

Answer: B

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20. $\mathbb{R}$ is a set of real numbers .If the relation R over a set A is defined such that $R = \{(a, b) : a - b < 3, a, b \in \mathbb{R}\}$. then relation R is -

A. Transitive

B. equivalence

C. reflexive

D. symmetric

Answer: C

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21. Let $\mathbb{R}$ be the set of all real numbers and for all $x \in \mathbb{R}$, the mapping $f: R \to R$ is dedined by $f(x) = ax + 2$. if (fo f) $= T_{\mathbb{R}}$, then find the value of a.

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22. Evaluate : $4\left(2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}\right)$

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23. A relation R is defined on the set of all natural numbers $\mathbb{N}$ by :

$(x, y) \in R \Rightarrow (x - y)$ is divisible by 6 for all $x, y, \in \mathbb{N}$ prove that R is an equivalence relation on $\mathbb{N}$.

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24. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1} = \pi$ then prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$

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25. The total number of injections (one -one into mappings) from $\{a_1, a_2, a_3, a_4\}$to$\{b_1, b_2, b_3, b_4, b_5, b_6, b_7\}$ is -

A. 400

B. 420

C. 800

D. 840

Answer: D

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26. Let $\mathbb{R}$ be the set of real numbers and the functions $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x^2+2x-3$ and $g(x)=x+1$, then the value of x for which $f(g(x))=g(f(x))$ is -

A. -1

B. 0

C. 1

D. 2

Answer: A

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27. Let $f(x)=ax^2+bx+c, g(x)=px^2+qx+r,$ such that $f(1)=g(1), f(2)=g(2)$ and $f(3)-g(3)=2.$ then $f(4)-g(4)$ is -

A. 4

B. 5

C. 6

D. 7

Answer: C

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28. IF $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $x^9 + y^9 + z^9 - \frac{1}{x^9 y^{-9} z^9}$ is equal to -

A. 0

B. 1

C. 2

D. 3

Answer: C

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29. Let $f(x) = 2^{100}x + 1, g(x) = 3^{100}x + 1)$

then the set of real numbers x such that $f(g(x)) = x$ is -

A. empty

B. a singleton

C. a finite set with more than element

D. infinite

Answer: B

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30. Let $F: R \to R$ be such that f is injective and $f(x)f(y) = f(x + y)$ for all x,y $\in \mathbb{R}$ If $f(x), f(y), f(z)$ are in G.P ., then x,y,z are in -

A. A.P always

B. G.P always

C. A.P depending on the values of x, y, z

D. G.P depending on the values of x, y, z

Answer: A

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31. Let $f(x-1) = x^2 + 5x - 3$.then find $f(x+1)$

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32. In the set all 3×3 real matric a relation a relation is defined as follows .A matrix A is related to a matric B if and only if there is a non - singular 3×3 matrix P such that $B = P^{-1}AP$. This relation is -

A. reflxice, symmetric but not Transitive

B. Reflexive ,Transitive but not symmetric

C. Symmetric ,Transitive but not Reflexive

D. an Equivalence relation

Answer: D

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33. For any two real numbers a and b , we define aRb if and only if $\sin^2 a + \cos^2 b = 1$, the relation R is -

A. Reflexive but not symmetric

B. symmetric but not transitive

C. transitive but not reflexive

D. an equivalence relation

Answer: D

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34. Which of the following real valued functions is / are not even functions ?

A. $f(x) = x^3 \sin x$

B. $f(x) = x^2 \cos x$

C. $f(x) = x^2 \sin x$

D. $f(x) = x - [x]$ where [x] denotes the greatest integer less than or equal to x

Answer: C::D

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35. Let $\mathbb{R}$ be the set of all real numbers and $f: R \to R$ be given by $f(x) = 3x^2 + 1$ then the set $f^{-1}(1 \quad 6)$ is -

A. $\left\{ -\sqrt{\frac{5}{3}}, 0, \sqrt{\frac{5}{3}} \right\}$

B. $\left[-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}} \right]$

C. $\left[-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right]$

D. $\left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

Answer:

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36. Let $f(x) = 2x^2 + 5x + 1$ .If we write f(x) as $f(x) = a(x+1)(x-2) + b(x-2)(x-1) + c(x-1)(x+1)$ for real numbers a , b,c then -

A. there are infinite number of chocies for a ,b,c

B. only one choice for a but infinte number of choices for b and c

C. Exactly one choice for each of a,b,c

D. More than one but finite number of choices for a,b,c

Answer: C

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37. Let $f(x) = x + \frac{1}{2}$ then the number of real values of x for which the three unequal terms $f(x), f(2x), f(4x)$ are in H.P is -

A. 1

B. 0

C. 3

D. 2

Answer: A

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38. The function $f(x) = x^2 + bx + c$, where b and c are real constants , decribes -

A. one -to -one mapping

B. onto mapping

C. not one -to but onto mapping

D. neither one - to one nor onto mapping

Answer: D

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39. If $\sin^{-1}\left(\frac{x}{13}\right) + \operatorname{cosec}^{-1}\left(\frac{13}{12}\right) = \frac{\pi}{2}$ then the values of x is -

A. 5

B. 4

C. 12

D. 11

Answer:

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40. We define a binary relation ~ on the set of all 3×3 real matrices as A ~B if and only if these exist invertible matrices P and Q such that $B = PAQ^{-1}$ .The binary relation ~ is -

A. neither reflexive nor symmetric

B. reflexive and symmetric but not transitive

C. symmetric and transitive but not reflexive

D. an Equivalence relation

Answer:

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41. For any two numbers θ and ϕ , we define $\theta R \phi$ if and only if $\sec^2 \theta - \tan^2 \phi = 1$ the relation R is -

A. Reflexive but not transitive

B. symmetric but not reflexive

C. Both reflexive and symmetric but not transitive

D. an Equivalence relation

Answer: D

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42. IF $\sin^{-1}\left(x - \frac{x^2}{2} + \frac{x^3}{4} - \frac{x^4}{8} + \ldots\ldots\right) = \frac{\pi}{6}$, when $|x| < 2$ then x =

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. $-\frac{2}{3}$

D. $-\frac{3}{2}$

Answer: A

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43. A function $f:(0,\pi/2)\to R$ is defined as :

$$f(\theta)=\begin{vmatrix}1 & \tan\theta & 1\\ -\tan\theta & 1 & \tan\theta\\ -1 & -\tan\theta & 1\end{vmatrix}$$

then the range of f is -

A. $(2,\infty)$

B. $(-\infty,-2)$

C. $(2,\infty o)$

D. $(-\infty,2)$

Answer: A

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44. Value of $2\cot^{-1}\frac{1}{2}-\cot^{-1}\frac{4}{3}$ is-

A. $-\frac{\pi}{6}$

B. $\frac{3\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

Answer: D

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45. The equation $\sin^{-1} x = 2\sin^{-1} 2a$ has real roots if

A. $|a| < \frac{1}{\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}} < |a| < \frac{1}{\sqrt{2}}$

C. $|a| > \frac{1}{2\sqrt{2}}$

D. $|a| < \frac{1}{2\sqrt{2}}$

Answer: D

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46. The function $f: N \to \mathbb{R}$ is such that $f(1) = 1$ and $g(1) + 2(f) + \ + 3f(3) + \ldots\ldots\ldots + nf(n) = n(n+1)f(n$

(N = set of natural number $\mathbb{R} =$ set of real number) then f(500) =

A. 1000

B. 500

C. 1/500

D. 1/1000

Answer: D

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47. ρ is such a relation on the set real numbers $\mathbb{R}$ where $x\rho y$ if and $xy > 0$. Then which of the following is / are true ?

A. ρ is reflexive and symmetric

B. ρ is symmetric and but not reflexive

C. ρ is symmetric and transitive

D. ρ is an equivalence relation .

Answer: A::B::D

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48. Let $\mathbb{R}$ be a relation defined on the set $\mathbb{Z}$ of all integers and xRy when $x + 2y$ is divisible by 3, then

A. R is not transitive

B. R is symmetric only

C. R is an equivalence relation

D. R is not an equivalence relation

Answer: C

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49. If the function $f: R \to R$ is defined by $f(x) = \left(x^2 + 1\right)^{35}$ for all $x \in \mathbb{R}$ then f is

A. One -one but not noto

B. onto but not one -one

C. neither one -one nor onto

D. both one-one and onto

Answer: C

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50. Let $F: X \to X$ be such that $f(f(x)) = x$ for all $x \in X$ and $X \subseteq R$, then

A. f is one to one

B. f is onto

C. f is one -to- one but not onto

D. f is onto but not one -to-one

Answer: A::B

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Jee Main Aieee Archive

1. If x, y, z are in A.P and $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in A.P then-

A. $x = y = z$

B. $x = 3y = 6z$

C. $6x = 3y = 2z$

D. $6x = 4y = 3z$

Answer: A

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2. Let $f_k = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$, where $x \in \mathbb{R}$ and $k > 1$ then $f_4(x) - f_6(x)$ equals -

A. $\frac{1}{6}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{12}$

Answer: D

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3. Let $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is -

A. $\frac{3x - x^3}{1 + 3x^3}$

B. $\frac{3x + x^3}{1 + 3x^2}$

C. $\frac{3x - x^3}{1 - 3x^2}$

D. $\frac{3x + x^3}{1 - 3x^2}$

Answer: C

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Jee Advanced Archive

1. The value of $\cot\left\{\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k-1}^{n} 2k\right)\right\}$ is

A. $\frac{23}{25}$

B. $\frac{25}{23}$

C. $\frac{23}{24}$

D. $\frac{24}{23}$

Answer: B

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2. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric function take only the priencipal values , then the correct option (s) is are -

A. $\cos\beta > 0$

B. $\sin\beta < 0$

C. $\cos(\alpha+\beta) > 0$

D. $\cos\alpha < 0$

Answer:

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3. Let $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all $x \in R$ and $g(x) = \frac{x}{2}\sin x$ for all $x \in \mathbb{R}$ .Let (fo g) (x) denote f(g(x)) and (go f) (x) denote g(f(x)) . Then which of the following is (are) true ?

A. range of f is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

B. Range of f o g is $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. $\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$

D. there is an $x \in \mathbb{R}$ such that (g o f) (x) = 1

Answer: A::B::C

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