



MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

SEQUENCE AND SERIES

Illustrative Examples

1. Write first five terms of the sequence $[u_n]$ where $u_n=rac{1}{3}(3n-4)$, also

find the corresponding series.



2. Find the 5th and nth terms of the sequence [1, 4, 9, 16,....].

3. From the sequence [5, 7, 9, 11,], prepare the series of first n terms.



4. Find the first five terms of the sequence whose n th terms (u_n) is given

by

 $u_n=3n^2-2$

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5. Find the first five terms of the sequence whose n th terms (u_n) is given

by

 $u_n = nth$ prime number.

6. Determine the first four terms of the sequence defined by, $u_1=-3, u_n=rac{1}{n-1}. u_{n-1}$ for $n\geq 2.$ Also find the series of first five terms of the sequence

terms of the sequence.

7. Find the 5th and 10th terms of the sequence $[u_n]$ defined by, $u_n = \begin{cases} 2n + 7 \text{when n is odd} \\ n^2 + 1 \text{when n is even} \end{cases}$

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8. Write the series
$$\sigma_{r=1}^n \frac{2r-1}{r^2}$$
 in expanded form.

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9. If $S_n=u_1+u_2+u_3+\ldots+u_n=4^n-1$, find the first four terms of

the sequence $\{u_n\}$.



10. Write the series
$$rac{2}{3}+rac{5}{6}+rac{8}{11}+\ldots+rac{3n-1}{n^2+2}$$
 is sigma notation.

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11. For the sequence $\{u_n\}$ if $u_1=rac{1}{2}$ and $u_{n+1}=rac{u_n}{1+2u_n}(n\geq 1)$, find

the value of u_{96} .

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12. If the sum of first n terms of a series is $2n^2 + 3n$, find the series.



13. IF the nth terms of the sequence $\{u_n\}$ is $u_n = (-1)^{n-1} \cdot 2^{-n}$, find

the corresponding series up to first five terms.

14. The rth terms of an A.P is n and it's nth term is r, show that its mth term is r+n - m.

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15. Prove that in an A.P. of finite number of terms the sum of any two terms equidistant from the beginning and the end is equal to the sum of the first and last terms.

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16. Fill in the gaps (indicated by '___') in the following A.P. :

34, ___, ___, ___, 48,....

17. Find the middle terms (or terms) and the sum of the following arithmetic series :

3+7+11+15+....95



18. The fifth terms of an A.P. is 30 and its twelfth term is 65, find the sum

of its first 20 terms.

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19. The sum of n terms of an A.P. is $3n^2 + 5n$. Find the number of the term

which is equal to 152.



20. How many terms of the series {22+18+14+10+...} must be added to get

the sum 64? Explain the double answer.



21. The nth term of an A.P. is p, show that, the sum of its first (2n-1) terms

is (2n-1)p.

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22. Insert 7 arithmetic means betweem 1 and 41.



23. Find the sum of all natural numbers between 500 and 1000 which are

divisible by 13.

24. Find the sum of all odd numbers, which are perfect squares between 50 and 10,000.



25. The sum of first m terms of the A.P. u_1, u_2, u_3, \ldots is denoted by S_m . If

 $u_m=4, u_{4m}=24$ and $S_{4m}=44S_m$, find u_1 and m.

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26. If a, b, c be respectively the sums of p, q, r terms of an A.P., show that,

$$rac{a}{p}(q-r)+rac{b}{q}(r-p)+rac{c}{r}(p-q)=0$$

27. The sum of p terms of two A.P.'s are in the ratio (2p+1) : (2p-1). Find the

ratio of their 8th terms.

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28. The sum of four integers in A.P. is 20 and the sum of their squares is

120, find them.

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29. Find the sum of first n terms of the following series :

 $1.2^2 + 2.3^2 + 3.4^2 + \dots$

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30. Insert n arithmetic means between x and y.





32. There are n arithmetic means between 14 and 38 such that, second means : last mean = 4:7, find n.

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33. If
$$a^2, b^2, c^2$$
 are in A.P., prove that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are also in A.P.

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34. If $\frac{b^2 + c^2 - a^2}{2bc}$, $\frac{c^2 + a^2 - b^2}{2ca}$ and $\frac{a^2 + b^2 - c^2}{2ab}$ are in A.P. then show that, a(b + c - a), b(c + a - c) and c(a + b - c) are also in A.P. $[a + b + c \neq 0]$.



35. Find the sum to n terms of each of the following series :

$$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \dots$$

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36. Find the sum to n terms of each of the following series :

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots$$

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37. If $a_1 \geq 0$ for all t and $a_1, a_2, a_3, \ldots, a_n$ are in A.P. then show that,

$$rac{1}{\sqrt{a_1}+\sqrt{a_2}}+rac{1}{\sqrt{a_2}+\sqrt{a_3}}+rac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=rac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

38. If
$$a_1 \ge 0$$
 for all t and $a_1, a_2, a_3, \dots, a_n$ are in A.P. then show that,

$$\frac{1}{a_1a_3} + \frac{1}{a_3a_5} + \dots + \frac{1}{a_{2n-1} \cdot a_{2n+1}} = \frac{n}{a_1a_{2n+1}}$$
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39. Find the sum of the series,

$$\frac{1^{3}}{1} + \frac{1^{3} + 2^{3}}{1 + 3} + \frac{1^{3} + 2^{3} + 3^{3}}{1 + 3 + 5} + \dots \text{ upto nth term.}$$
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40. If S_r be the sum of the cubes of first r natural numbers, then show

that
$$\Sigma_{r=1}^n rac{2r+1}{S_r} = rac{4n(n+2)}{\left(n+1
ight)^2}$$
, for any natural number n.

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41. Find the sum of all possible products of first n natural numbers.

42. The base of a right angled triangle is 12 cm and the three sides are in

A.P. Find the length of the hypotenuse.



43. Find the sum of the numbers in the nth bracket of the following sequence :

(1), (2, 3), (4, 5, 6), (7, 8, 9, 10),....

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44. A man borrows Rs. 840 and agrees to repay with a total interest of Rs.

240 in 12 instalments, each instalment being less than the preceding one

by Rs. 8. What should be his first instalment?

45. The cost of constructing a 500 feet road is as follows : Rs. 22 for the first foot and an additional Rs. 2 for every subsequent foot. Find the cost of constructing the last foot of the road and that for the entire road.

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46. Two posts were offered to a man. In the first one the starting salary was Rs. 120 per month and the annual increment was Rs. 8, In the second post the salary commenced at Rs. 85 per month but the annual increment was Rs. 12. The man decided to accept that post which would give him more total earnings in the first twenty years of his service. Which post was acceptable to him ? Justify your answer.

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47. A workman agrees to accept certain wages for the first month, on the understanding that his pay is to be increased one rupee every subsequent month untill the maximum of Rs. 300 is reached. At the end

of the month for which he received Rs. 300 for first time, he resigns and finds that his wages during his period of service have averaged Rs. 288 a month. How long has he served.



48. A sum of money is distributed amongst a certain number of persons. The second receives Rs. 1 more than the first, the third Rs.2 more than the second, the fourth Rs. 3 more than the third find and so an. If the first person gets Re. 1 and the last person Rs. 67, what is the number of persons and the sum distributed?

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49. The sum of first n terms of a series is $an^2 + bn + c$. Show that c = 0 and the series is in A.P.

50. If the sum of the first m terms of an A.P. is equal to the sum of either the next n terms or the next p terms, prove that, $(m+n)\left(\frac{1}{m}-\frac{1}{p}\right) = (m+p)\left(\frac{1}{m}-\frac{1}{n}\right)$

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51. The sum of first 6 terms of a G.P. is 9 times the sum of its first 3 terms.

Find the common ratio of the G.P.

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52. The mth terms of a G.P is n and its nth term is m. Find its (2m - n)th

term.

O Watch Video Solution

53. How many terms of the series 1+3+9+27+... must be taken so that the

sum is 9841?

54. If a be the first term, I the nth and p the product of first n terms of a

G.P. show that, $p^2 = (al)^n$.

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55. Find three numbers in G.P. whose sum is 35 and product is 1000.

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56. The sum of three numbers in G.P. is $\frac{13}{3}$ and the sum of their squares is $\frac{91}{9}$, find the numbers.

57. If a, b, c are in G.P., show that

 $a^2+b^2, ab+bc$ and b^2+c^2 are also in G.P.

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58. If a, b, c are in G.P., show that

$$a^2b^2c^2igg(rac{1}{a^3}+rac{1}{b^3}+rac{1}{c^3}igg)=a^3+b^3+c^3.$$

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59. Insert :

Six geometric means between 56 and
$$\left(-\frac{7}{16}\right)$$
.



60. Insert :

n geometric means between a and b.

61. Insert :

Let $rac{x^{n+1}+y^{n+1}}{x^n+y^n}$ be the geometric mean of x and y , find n.

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62. Prove that the sum of the G.P. (a+b+....l) is
$$\frac{bl-a^2}{b-a}$$
.

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63. If
$$x = a - \frac{1}{a}$$
, $y = b - \frac{1}{b}$ and $z = c - \frac{1}{c}$ and a, b, c are in G.P., then show that $\frac{x+z}{y} = r + \frac{1}{r}$, where r is the common ratio of the G.P.

64. Sri S. Roy borrows Rs. 32760 without interest and agrees to pay back in 12 monthly instalments, each instalment being twice the preceding one. Find the amounts of second and last instalments.



65. A bouncing tennis ball rebounds each time to a height equal to one half height of the previous bounce. If it is dropped from a height of 16 metres, find the total distance it has travelled when it hits the ground for the 10th time.

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66. If a, b, c are in G.P. and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, show that x, y, z are in A.P.



70. Find the sum to n terms :

0.7 + 0.77+0.777+....



71. Find the sum to n terms of the following series :

 $1+2a+3a^2+4a^3+\ldots, [a
eq 1]$

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72. Find the nth terms and the sum to n terms of the following series :

5 + 13 + 37 + 109 + 325 +

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73. Find three unequal positive integers a, b, c, such that 2, a, b, form an

A.P. and a, b, c form a G.P.

74. A man saves Rs. 75 in the first month and in each of the succeeding months he saves twice as much as in the previous month. This process continues for 8 months. From the 9th month and onwards he could save Rs. 75 less than in the previous month. Find his total savings in 16 months.

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75. Prove that in a G.P. of finite number of terms the product of any terms equidistant from the beginning and the end is constant and is equal to the product of the first and last terms.

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76. Show that the arithmetic means of two positive numbers can never be less than their geometric mean. The arithmetic and geometric means of two positive numbers are 15 and 9 respectively. Find the numbers.

77. Prove that the difference between the first term and the fourth term of a G.P. Is at least times as great as the difference between the second and the third terms. Assume that both the first term and common ratio are positive and the common ratio $\neq 1$.

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78. If the sum of the first n terms of a G.P. = p and the sum of the first 2n terms = 3p, show that the sum of first 3n terms = 7p.

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79. The sum of 2n terms of a series in G.P. is 2R and the sum of the reciprocals of those terms is R , find the continued product of the terms.

80. If the ratio of A.M. of two numbers x and y to their G.M. is p : q, show

that,

$$x\!:\!y=\left(p+\sqrt{p^2-q^2}
ight)\!:\!\left(p-\sqrt{p^2-q^2}
ight)$$

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81. Find the sum to n terms of the series :

$$1 + (1 + 4) + (1 + 4 + 4^2) + (1 + 4 + 4^2 + 4^3) + \dots$$

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82. Find the sum of the series :

 $1^2 + 2^2 x + 3^2 x^2 + 4^2 x^3 + \dots \infty$

83. If the sides of a trianglr are three consecutive terms of a G.P. then show that $0 < r < \frac{1}{2}(\sqrt{5}+1)$, where r is the common ratio of the G.P.

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84. Express each of the following recurring decimals as a rational number

: 0.36

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85. Express each of the following recurring decimals as a rational number

: 2.025



86. If $S_1, S_2, S_3, \ldots, S_n$ are the sums to infinity of n infinite geometric series whose first terms are 1,2,3,... n and whose common ratios are

$$rac{1}{2},rac{1}{3},rac{1}{4},\ldots,rac{1}{n+1}$$
 respectively, show that, $S_1+S_2+S_3+\ldots,S_n=rac{1}{2}n(n+3)$

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87. If
$$x = 1 + a + a^2 + a^3 + \dots \infty$$
 and $y = 1 + b + b^2 + b^3 + \dots \infty$
show that $1 + ab + a^2b^2 + a^3b^3 + \dots \infty = \frac{xy}{x + y - 1}$ where 0 It alt1 and Oltblt1.

88. Show that
$$3^{1/2}.9^{1/4}.(27)^{1/8}.(81)^{1/16}...\infty = 9$$

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 89. Insert 6 harmonic means between 3 and
$$\frac{6}{23}$$

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90. The 10th and 14th terms of a H.P. are $\frac{2}{20}$ and $\frac{2}{41}$ respectively. Find the nth term of the H.P. Watch Video Solution **91.** If A, G and H be respectively quantities, then show that, (i) $AH = G^2$ (ii) A > G > H. Watch Video Solution **92.** If a, b, c are in H.P., prove that $\frac{a}{b+c}$, $\frac{b}{c+a}$ and $\frac{c}{a+b}$ are also in H.P. Watch Video Solution

93. A man travels from Kalighat to Tarakeswar with a uniform speed of u km/h and returns from Tarakeswar to Kalighat with uniform speed of v

km/h. Show that his average speed is the harmonic mean between u and
v.
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94. Two numbers are 4 and 18 ,find its arithmetic mean.
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Example
1. Find the fifteenth term of the A.P. {18, 12, 6,}.
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2. The 5th and 13th terms of an A.P. are 16 and 28 respectively. Find the A.P.
Vatch Video Solution

3. Find the sum of the following arithmetic series :

1+4+7+10+13+.... to 20 terms

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4. Find the 10th and pth terms of the G.P.

{2, 6, 18, 54,...}

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5. The 3rd and the 7th terms of a G.P. are 4 and 64 respectively. Find the

G.P.



6. Find the sum of the following geometric series :

3-9+27-81+... to 8 terms



7. Find the sum of the following geometric series :

$$1 + rac{1}{4} + rac{1}{16} + rac{1}{64} + \dots$$
 to 10 terms

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Exercise 9 A Multiple Choice Questions

1. The 6th term of the sequence {1, 4, 9, 16, ...} is -

A. 25

B. 36

C. 32

D. 64

Answer: B

2. The 5th term of the sequence $\left\{3, 1, \frac{1}{3}, \frac{1}{9}, \ldots\right\}$ is -

A.
$$\frac{1}{27}$$

B. $\frac{1}{15}$
C. $\frac{1}{81}$
D. $\frac{1}{12}$

Answer: A

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3. The 8th term of the sequence $\{-8, -6, -4, -2, \ldots\}$ is-

A. 2

B.4

C. 6

Answer: C



4. The nth term of the sequence
$$\left\{1, \frac{1}{8}, \frac{1}{27}, \frac{1}{64}, \ldots\right\}$$
 is -



Answer: C

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Exercise 9 A Very Short Answer Type Questions

1. Write the first five terms of the sequence defined by

 $u_n = (n) + (n+1)$

Also find the corresponding series in each case.



3. Write the first five terms of the sequence defined by

$$u_n = 2n^2 - 3n$$

Also find the corresponding series in each case.

4. Find the 6th and rth terms of the sequence {1, 8, 27, 64,..}.

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5. Prepare a series to first n terms of the sequence $\left\{\frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \ldots\right\}$.
Vatch Video Solution
6. Find the 11th term of the sequence $\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{5}, \frac{1}{8}, \frac{1}{12}, \ldots\right\}$.
Watch Video Solution
7. Find the 25th and 50th terms of the sequence whose nth terms is given

by

$$u_n = \left\{ egin{array}{c} rac{n}{n+1} ext{when n is odd} \ rac{n+1}{n+2} ext{when n is even} \end{array}
ight.$$

8. Write the series $\sum_{r=1}^n rac{r^2+1}{2r^2-1}$ in expanded form.



9. Determine the first five terms of the sequence defined by, $u_1=-2, u_2=2 ext{ and } u_n=rac{n}{n-2}u_{n-1}, n>2.$

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10. Write the series
$$\frac{3}{2} + 1 + \frac{7}{10} + \frac{9}{17} + \ldots + \frac{2r+1}{r^2+1}$$
 in sigma notation.
Notation.

11. Determine the first five terms of the sequence defined by, $u_1=4$ and $u_n=3u_{n-1}+2$ for $n\geq 2.$ Also find the series of first five terms of the

sequence.
12. Write the series
$$\Sigma_{r=1}^n rac{2r+1}{r^2+1}$$
 in expanded form.

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13. If
$$S_n = u_1 + u_2 + \ldots + u_n = n^2 + 2n$$
, find the first four terms of

the series.

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14. If the rth term of the sequence $\{u_n\}$ is $u_r = {(-1)}^{r-1}$. 3^{2-r} , find the

first five terms of the sequence, also find the corresponding series.



15. The sum of first r terms of a series is $ar^2 + br$, find the first and 12th

terms of the series.



16. For the sequence $\{u_n\}$ if $u_1 - 2$ and $u_{r+1} = u_r + r + 2$, for all natural number r, find the 10th term of the sequence.

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17. For the sequence
$$\{u_n\}$$
 if $u_1=rac{1}{4}$ and $u_{n+1}=rac{u_n}{2+u_n}$, find the value of $rac{1}{u_{50}}.$

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18. Find the least value of n, for which the nth term a_n of the sequence given by $a_n = n^3 - n^2 - 5n - 3$ is non-negative.

19. A sequence whose nth term is given by $a_n = 2n^2 + pn - 3p^2$ is such that $a_3 = 0$. Determine the sequence explicitly and then show that $a_n > 0$ for all $n \ge 4$.

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Exercise 9 B Multiple Choice Questions

1. Three numbers are in A.P. and their sum is 21, then the middle number

is-

A. 5

B. 6

 $\mathsf{C.}\,6.5$

D. 7

Answer: D
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2. Five numbers are in A.P. and their sum is 50, then its 3rd number will be
A. 2
B. 5
C. 10
D. 15
Answer: C
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3. If x, 2x+1 and 14 are in A.P., then the value of x is -

A. 2	
B.4	
C. 6	
D. 10	

Answer: B



4. 20 terms are A.P., if its first and last terms are 5 and 125 respectively, then the sum of its first 20 terms will be -

A. 1300

B. 1000

C. 2600

D. 3000

Answer: A



6. If sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers, then k =

A.
$$\frac{1}{n}$$

B.
$$\frac{n-1}{n}$$

C. $\frac{n+1}{2n}$
D. $\frac{n+1}{n}$

Answer: D

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7. If four numbers in A.P. are such that their sum is 50 and greatest no is 4

times the least, then the numbers are -

A. 5, 10, 15, 20

B. 4, 8, 10, 16

C. 3, 7, 11, 15

D. none of these

Answer: A

8. State which of the following is not true?

A. The resulting sequence obtained by adding 5 to each term of an A.P.

is also an A.P.

B. The resulting sequence obtained by subtracting 3 from each term

of an A.P. is also an A.P.

C. The common difference of an A.P. can never be negative.

D. If each term of a series in A.P. multiplied by 6 then the resulting

sequence so obtained number is also in A.P.

Answer: C

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9. The A.M. of two numbers is 10, if one number is 7 then the other number will be

A. 15		
B. 14		
C. 13		
D. 11		

Answer: C

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10. The pth term of an A.P. is 2p-7, which of the following will be its 4th

term ?

A. 1

 $\mathsf{B.}-1$

C. 3

 $\mathsf{D.}-3$

Answer: A

11. Which of the following is the 7th term of the A.P. 5, 12, 19,...?

A. 45 B. 47 C. 40 D. 33

Answer: B

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12. Which of the following is the 12th term of the series n+(n-1) + (n-2)+...?

A. n - 10

B. n - 11

C. n - 12

D. n - 13

Answer: B



Exercise 9 B Very Short Answer Type Questions

1. The 10th term of an A.P. is (-15) and the 31st term is (-57). Find the first

term and common difference of the A.P.

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2. If the pth term of an arithmetic progression is q and the qth term is p,

show that its (p+q)th term is 0.



5. Is 600 a term of the A.P. {2, 9, 16, 23,...}? Give reasons for your answer.

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6. Fill up the gaps (indicated by _____) in each of the following A.P.'s.

1, _ , _ , (-50)





11. Find the middle term (or terms) and the sum of each of the following

series :

$$rac{1}{2} + rac{1}{3} + rac{1}{6} + \ldots + \left(-rac{5}{6}
ight)$$

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12. Find the middle term (or terms) and the sum of each of the following

series :

2+2.4+2.8+...+10.4

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13. The twelfth term of an A.P. is (-13) and the sum of its first four terms is

24, find the sum of first 10 terms of the A.P.

14. The 5th and 11th terms of an A.P. are 41 and 20 respectively. What is its

first term ? Find the sum of its first eleven terms.

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15. The sum to n terms of an A.P. is n^2 . Find the common difference.
S Watch Video Solution
16. Show that the sum of n terms of the series {4+12+20+28+} is the
square of an even number.
Vatch Video Solution

17. Prove that when 1 is added to the sum of n terms of the series {8+16+24+...}, the result will be perfect square.

18. Find the sum of the series 1+3+4+8+7+13+10+18+... to 23 terms.



2. The sum of p terms of a series is $3p^2+5p$. Prove that the terms of the

series form an A.P.

3. Find the 12th term of a series in A.P., whose sum of n terms is $4n^2 + 3n$.



4. If $51 + 53 + 55 + \ldots + t_n = 5151$, then find the value of t_n .

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5. If the sum of the first n terms of an A.P. be n^2+3n , which term of it has

the value 162 ?

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6. How many terms of the series {27+24+21+...} must be added to get the

sum 132 ? Explain the double answer.

7. The first and last terms of an A.P. having a finite number of terms are respectively (-2) and 124 and the sum of the A.P. is 6161. Find the number of terms in the A.P. and also its common difference.





10. How many even numbers are there between 91 and 259? Find also the

sum of the even numbers between them.





14. The middle term of an arithmetic progression having (2n+1) terms is

m, show that the sum of its (2n+1) terms is (2n+1)m.



16. The sum of four integers in A.P. is 20 and the sum of their squares is

120, find them.

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17. Divide 21 into three parts in A.P. such that the product of the first and

second parts is 21.

18. The ratio of the 11th and 14th terms of an A.P. is 7:9, find the ratio of

the 10th term to 3rd term of the series.



21. If a, b, c are In A.P., then show that,

$$(a+2b-c)(2b+c-a)(c+a-b)=4abc$$

22. If a, b, c are In A.P., then show that,

 $a^2(b+c), b^2(c+a), c^2(a+b)$ are in A.P. (ab+bc+ca
eq 0)



24. If the pth , qth and rth terms of an A.P. be P, Q and R respectively then

show that,

$$p(Q-R) + q(R-P) + r(P-Q) = 0.$$

25. If
$$\frac{b+c}{a}$$
, $\frac{c+a}{b}$, $\frac{a+b}{c}$ are in A.P. show that $\frac{1}{a}$, $\frac{1}{b}$, $\frac{1}{c}$ are also in A.P. $(a+b+c\neq 0)$.

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27. If a+c = 2b and ab + cd + ad = 3bc, prove that the four numbers a, b, c,

d are in A.P. $(b \neq 0)$.

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28. Find the least value of n for which the sum of the series 20 + 28 + 36

+... to n terms is greater than 1000.



29. The nth term of a series in A.P. is an + b. Find the sum of the series up to n terms.

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30. Find the sum of the series $1^2 - 2^2 - 3^2 - 4^2 + 5^2 - 6^2 + \dots$ up to

2n terms.

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31. The perpendicular of a right angled triangle is 9 cm and the three

sides are in A.P. Find the integral value of the length of the hypotenuse.

32. If the xth term of an A.P. is $\frac{1}{y}$ and its yth term is $\frac{1}{x}$ then show that its xyth term is 1 and the sum of its first xy terms is $\frac{1}{2}(xy+1)$.

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33. An A.P. consists of n terms. If the sum of its first three terms is x and the sum of the last three terms is y, then show that, the sum of all the terms of the A.P. is $\frac{n}{6}(x + y)$.

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34. If S be the sum of first (2n+1) terms of an A.P and the sum of terms in

odd positions of these (2n+1) terms be S', then show that (n+1)S =

(2n+1)S'.

35. The sum of first p terms of an A.P. is $\frac{p^2}{n}$ and the sum of its first $q(\neq p)$ terms is $\frac{q^2}{n}$. Show that the sum of its first n terms will be n.

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36. If the A.M. of
$$\frac{1}{b-a}$$
 and $\frac{1}{b-c}$ is $\frac{1}{2(b-x)}$, then show that $(x-b)^2 = (x-c)(x-a).$

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37. A farmer undertakes to pay off a debt of Rs. 2700 by monthly instalments. He pays Rs. 200 as the first instalment and increases every subsequent instalment by Rs. 25 over the immediate previous instalments. In how many instalments his debt will be cleared up ?

38. A person pays Rs.975 in monthly instalments, each instalment is less than the former by Rs. 5. The amount of first instalment is Rs. 100. In what time will the entire amount be paid ?

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39. If 1 paisa is saved today, 2 paise next day, 3 paise succeeding day and

so on, what will be the total savings in 365 days?

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40. The cost of sinking a tubewell is 25 paise for the first metre and an additional 4 paise for every subsequent metres. Find the cost of last metre and the total cost of sinking the tubewell 500 metres deep.

41. The rate of monthly salary of an office assistant is increased annually in A.P. If he was drawing Rs. 2000 a month during the 11th year and Rs.3800 a month during 29th year, find out his initial salary and rate of annual increment. Find out also his salary at the time of retirement on completion of 32 years of service.

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42. The angles of a polygon are in A.P. lying common difference 5° . If the least angle be 120° , find the number of sides of the polygon.

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Exercise 9 B Long Answer Type Questions

1. The sum of first 21 terms of an A.P. is 28 and the sum of the 1st 28 terms

is 21. Show that one term of the progression is zero and find the sum of

the preceding terms.



first n terms of the A.P. 57, 59, 61,... then find n.

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3. Let S_n be the sum of first n terms of an A.P. If $S_{2n}=5s_n$, then find the

value of S_{3n} : S_{2n} .

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4. The sum of the first p terms of an A.P. is q and the sum of the first q

terms of the same A.P. is p. Find the sum of the first (p+q) terms of the A.P.

5. If the pth and qth terms of an A.P. be a and b respectively, show that the sum of first (p+q) terms of the A.P. is $\frac{p+q}{2}\left(a+b+\frac{a-b}{p-q}\right)$.

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6. If S_1 , S_2 , S_3 be the sums of n terms of three aA.P.'s, the first term of each A.P. being 1 and the respective common differences are 1, 2, 3 then show that, $S_1 + S_3 = 2S_2$.

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7. If the sums of n, 2n and 3n terms of an A.P. be S_1, S_2, S_3 respectively,

then show that, $S_3 = 3(S_2 - S_1)$.

8. The sums of n terms of two A.P.'s are in the ratio (4n-13) : (3n + 10). Find

the ratio of their 9th terms.



9. The sums of first n terms of two A.P.'s are in the ratio (3n+5): (5n - 9).

Show that their 4th terms are equal.

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10. Insert a number of arithmetic means between 4 and 34 such that the

sum of the resulting A.P. is 133.

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11. If a, b, c are in A.P. then show that, $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right)$ and $c\left(\frac{1}{a}+\frac{1}{b}\right)$ are also in A.P.

12. Three positive numbers a, b, c are in A.P. Prove that $\frac{1}{\sqrt{b} + \sqrt{c}}, \frac{1}{\sqrt{c} + \sqrt{a}}, \frac{1}{\sqrt{a} + \sqrt{b}}$ are also in A.P.

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13. If
$$a_1, a_2, a_3, \ldots, a_n$$
 be in A.P. Show that,

$$rac{1}{a_1a_2}+rac{1}{a_2a_3}+\ldots\,+rac{1}{a_{n-1}a_n}=rac{n-1}{a_1a_n}$$

Watch Video Solution

14. If
$$a_1, a_2, a_3, \ldots, a_{2k}$$
 are in A.P., prove that $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2k-1}^2 - a_{2k}^2 = rac{k}{2k-1} ig(a_1^2 - a_{2k}^2ig).$

15. If
$$(b-c)^2$$
, $(c-a)^2$, $(a-b)^2$ are in A.P. then prove that,
 $\frac{1}{b-c}$, $\frac{1}{c-a}$, $\frac{1}{a-b}$ are also in A.P.

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16. Find the sum to n terms of each of the following series :

 $2.4 + 6.8 + 10.12 + \ldots$

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17. Find the sum to n terms of each of the following series :

 $1^2 + 3^2 + 5^2 + \dots$

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18. Find the sum to n terms of each of the following series :

 $1.3.5 + 3.5.7 + 5.7.9 + \ldots$



19. Find the sum of the following series :

 $3.1^2 + 4.2^2 + 5.3^2 + \ldots + (n+2). n^2$

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20. Find the sum of the following series :

 $1+3+6+10+15+\ldots$ to n terms.

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21. Find the sum of the following series :

 $1+5+12+22+35+\ldots$. to n terms.

22. The sum of first n terms of a series is $n^2 + an + b$. Show that b = 0

and the series is in A.P.



23. If n is a positive integer, show that,

$$(n+1)^2 + (n+2)^2 + \ldots + 4n^2 = rac{n}{6}(2n+1)(7n+1)$$

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24. Find the sum to n terms of each of the following series :

$$(1) + (1+2) + (1+2+3) + \dots$$

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25. Find the sum to n terms of each of the following series :

$$\left(1^2
ight)+\left(1^2+2^2
ight)+\left(1^2+2^2+3^2
ight)+\dots$$

26. Find the sum to n terms of each of the following series :

$$\left(3^3-2^3
ight)+\left(5^3-4^3
ight)+\left(7^3-6^3
ight)+\dots$$

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27. Find the sum to n terms of each of the following series :

$$(1) + (2+3) + (4+5+6) + \dots$$

Watch Video Solution

28. Find the sum to n terms of each of the following series :

$$rac{1}{1.4} + rac{1}{4.7} + rac{1}{7.10} + \dots$$
29. Find the sum to n terms of each of the following series :

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots$$
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30. Find the sum to n terms of each of the following series :

 $n.1 + (n-1).2 + (n-2).3 + (n-3).4 + \dots$

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31. The sum of first P terms of an A.P. is equal to the sum of its first Q

terms. Show that the sum of its first (P+Q) terms is zero.



32. A series is given in the form : (1) + (2+3+4) + (5+6+7+8+9) +.... Find the

sum of the numbers in the rth bracket.



33. The first and last terms of an A.P. are a and I respectively. If S be the sum of the terms then show that the common difference is $\frac{l^2 - a^2}{2S - (l + a)}$

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34. If the first, second and last terms of A.P. be a, b, and c respectively,

show that the sum of all terms of the A.P. is $rac{(a+c)(b+c-2a)}{2(b-a)}.$

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35. If S_n be the sum of n consecutive terms of an A.P. show that,

$$S_{n+3} - 3S_{n+2} + 3S_{n+1} - S_n = 0$$

36. If S_n be the sum of n consecutive terms of an A.P. show that,

$$S_{n+4} - 4S_{n+3} + 6S_{n+2} - 4S_{n+1} + S_n = 0$$

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37. Let a_n denote the n-th term of an A.P. and p and q be two positive integers with p < q. If $a_{p+1} + a_{p+2} + a_{p+3} + \ldots + a_q = 0$ find the sum of first (p+q) terms of the A.P.

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38. Find the sum of the three-digited natural numbers which leave a remainder 2, when divided by 3.

39. Between the roots of the quadratic equation
$$3px^2 - 10px + 5q = 0\left(p > 0, rac{q}{p} < 1rac{2}{3}
ight)$$
, an odd number of A.M.S are

inserted and their sum exceeds their number by 10. Find the number of A.M.S inserted.





42. If S_1 , S_2 , S_3 are the sums of n natural numbers, their squares, their cubes respectively show that $9S_2^2 = S_3(1 + 8S_1)$.

43. If
$$a_1 = 2$$
 and $a_n - a_{n-1} = 2n(n \ge 2)$, find the value of

 $a_1 + a_2 + a_3 + \ldots + a_{20}.$

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44. Find the middle term of the series 1+5+9+...+101.

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45. If
$$\frac{b^2 + c^2 - a^2}{2bc}$$
, $\frac{c^2 + a^2 - b^2}{2ca}$, $\frac{a^2 + b^2 - c^2}{2ab}$ are in A.P. and a+b+c = 0

then prove that

a(b+c-a), b(c+a-b), c(a+b-c) are in A.P.

46. To verify the cash balance, the auditor of Jaya Bank Ltd. Employs his assistant to count cash in hand of Rs. 4500. At first he counts @ Rs. 150 per minute for 10 minutes only, at the end of which time he begins to count @ Rs.2 less every minute than he did in the previous minute. Ascertain how much time he will take to count Rs. 4500 ?

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47. A polygon has 25 sides, the lengths of which starting from the smallest side are in A.P. If the perimeter of the polygon is 1100 cm and the length of the largest side is 10 times that of the smallest, find the length of the smallest side and the common difference of the A.P.

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48. A sets out from a certain place and tavels 1 mile the first day, miles the second day, 3 miles the third day and so on. B sets out 5 days after A and travels the same road @ 12 miles a fay. How far will A travel before he is

overtaken by B? If they continue to travel, in what time will be overtaken by A?

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49. A man arranges to pay off a debt of Rs. 12000 in 30 annual instalments which form an A.P. When 20 of the instalments are paid, he dies leaving a half of the debt unpaid. Find the value of the first instalment.

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50. A man divides Rs. 24000 among his four sons in such a way that the amounts received by the four sons are in A.P. The ratio of the product of the amounts received by the 1st and the 3rd sons to the product of the amounts received by 2nd and 4th sons is 7 : 15. Calculate the amounts received by each son.

51. Two posts are offered to a man. In the first one, the starting salary is Rs. 1200 and increases annually by Rs. 80, in the second, the salary commences ar Rs. 850 and increases annually by Rs. 120. The man decides to accept the post that will give him better income in the first 16 years. Which post will he accept? Justify your answer.

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52. The salary of a man starts at Rs. 800 and increases annually by Rs. 20 until a maximum of Rs. 1120 is reached. Find the total amount of money earned by the man when he serves (i) 15 years (ii) 22 years.

A.
$$\frac{r}{b}$$

B. $\frac{b}{r}$
C. $\frac{b}{r^2}$
D. $\frac{r}{b^2}$

Answer: (i) Rs. 169200 , (ii) Rs. 263040

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Exercise 9 C Multiple Choice Type Questions

1. The third term of a G.P. is b and its common ratio is r , then its first term



=

Answer: C

2. In a G.P., the product of the first three terms is $\frac{27}{8}$, then its middle term = A. 3

B. 6
C.
$$\frac{3}{2}$$

D. 9

Answer: C



3. The geometric mean of two numbers $= \pm 12$, if one number is 16, then the other number =

A. 2

B. 6

C. 9

Answer: C



4. Five numbers are in G.P. and their product is 3^{10} , then the 3rd number will be -

A. 2 B. 5 C. 9

D. 11

Answer: C

5. If , 15 and 45 are in G.P. then x =

A. A.P.

B. G.P.

C. arithmetic-geometric series

D. none of these

Answer: B

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6. If three unequal real numbers are in G.P. then their reciprocals are in -

 $\mathsf{A.} A. M. < G. M.$

 $\mathsf{B.}\,A.\,M.\,\leq G.\,M.$

 $\mathsf{C.}\,A.\,M.\,\geq G.\,M.$

 $\mathsf{D}.\,A.\,M.\,>G.\,M.$

Answer: B

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7. Which is not true -

- A. The common ratio of a G.P. is always positive.
- B. If each term of a G.P. is multiplied by 6, the resulting terms are also

in G.P.

C. If each term of a G.P. is divided by 3, then the resulting terms are

also in G.P.

D. The numbers $rac{a}{r^2}, a$ and ar^2 are in G.P.

Answer: A

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8. State which of the following is not true?

A. 5		
B.4		
C. 3		
D. 2		

Answer: A

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9. If 3x + 1, 14 and 28 are in G.P. then which of the following is the value of

х?

A. 16

B. - 32

C.-64

D. 2

Answer: D

10. The rth term of a G.P. is $(\,-1)^{r-1}.2^{r+1}$, state the value of its 4th term

 $\mathsf{A.}\,2.3^7$

:

 $\mathsf{B}.\,2.3^6$

 $C. 2.3^8$

 $\mathsf{D}.\,2.3^9$

Answer: B

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11. State which of the following is the value of 8th term of the G.P. {2, 6, 18,

54,...}

B. - 45

 ${\rm C.}\pm15$

 $\mathsf{D}.\pm45$

Answer: A

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12. If 5, x, 405 are in G.P. then the value of x?

A. G.P.

B. A.P.

C. arithmetic-geometric series

D. none of these

Answer: D

13. If 4 is multiplied to each term of a G.P., then the resulting series is a-

A. G.P.

B. A.P.

C. arithmetic-geometric series

D. none of these

Answer: A

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14. If 4 is added to each term of a G.P., then the resulting series is a -

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Exercise 9 C Very Short Answer Type Questions

1. If 5, x, y, z, 405, are the first five terms of a G.P., find the values of x, y, z.



4. Find the 9th and qth terms of the G.P.

{4, -8, 16, -32,...}

5. Find the 14th and nth terms of the G.P.

$$\left\{\sqrt{5}, 1, \frac{1}{\sqrt{5}}, \frac{1}{5}, \dots\right\}.$$

$$(\sqrt{5}, 1, \frac{1}{\sqrt{5}}, \frac{1}{5}, \dots)$$

$$(\sqrt{5}, 1, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \dots)$$

$$(\sqrt{5}, 1, \frac{1}{\sqrt{5}}, \dots)$$

$$($$

8. If the nth term of a G.P. be p, then show that the product of its first (2n-

1) terms is p^{2n-1} .





17. Three different numbers a, b, c are both in A.P. G.P. - is it possible ? Give

reasons in support of your answer.

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Exercise 9 C Short Answer Type Questions

1. The common ratio of a series in G.P. is 3, the sum of the 1st and 3rd terms is equal to the sum of the squares of the 1st and 2nd terms, find the sum of n terms. If n = 6, show that the sum is 364.



2. If the fourth term of a series in geometric progression is 24 and the seventh term is 192, find the sum of its first ten terms.

3. How many terms of the series 64+32+16+8+... must be taken so that the sum may be $127\frac{1}{2}$?

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4. A G.P. has first term 3 and last term 48. If each term is twice the preceding, find the number of terms and the sum of the G.P.

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5. The sum of first 8 terms of a G.P. is five times the sum of the first 4

terms. Find the common ratio.



6. The sum of the first 4 terms of a G.P. is 40 and the sum of first 8 terms

is 3280. Obtain the G.P.

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7. The first and last terms of G.P. having finite number of terms are respectively 3 and 768 and the sum of the G.P. is 1533. Find the number of terms in the G.P. and its common ratio.

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8. Find three numbers in G.P. whose sum is 19 and product is 216.



9. Three numbers x, y, z are in G.P. Sum of the numbers is 65 and the

product of the first and third numbers is 225. Find the common ratio.



10. Find the three terms in G.P. whose product is 729 and the sum of their

product in pair is 351.

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11. The sum of three consecutive numbers in G.P. is 21 and the sum of their

squares is 189. Find the numbers.

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12. Find the sum to n terms of the following series :

$$(1) + (1+3) + (1+3+3^2) + \dots$$

13. Find the sum to n terms of the followig series :

1+3+7+15+31+...

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14. Find the sum to n terms of the followig series :

3+7+14+27+52+...

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15. If the nth term of a series be $\left(2.3^{n-1}+1
ight)$, find the sum of first r

terms of the series.



16. If a, b, c, be respectively the pth, qth and rth terms of a G.P., prove that

$$a^{q-r}.b^{r-p}.c^{p-q} = 1$$



17. If a, b, c and d are in G.P., show that,

$$(a-b)^2,$$
 $(b-c)^2,$ $(c-d)^2$ are in G.P.

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18. If a, b, c and d are in G.P., show that,

 $a^2+b^2, b^2+c^2, c^2+d^2$ are in G.P.

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19. If a, b, c and d are in G.P., show that,

 $a^2 + b^2 + c^2, ab + bc + cd, b^2 + c^2 + d^2$ are in G.P.

20. If a, b, c and d are in G.P., show that,

$$(b-c)^2 + (c-a)^2 + (d-b)^2 = (a-d)^2.$$

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21. If three numbers a, b, c satisfy the relation
$$(a^2+b^2)(b^2+c^2)=(ab+bc)^2$$
, show that, a, b, c are in G.P.

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22. If a, b, c are in geometric progression, then show that,

 $\frac{a^2+ab+b^2}{ab+bc+ca}=\frac{a+b}{b+c}$

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23. A boy saves 1 paisa the first day, 2 paise the second day, 4 paise the third day and 8 paise the fourth day and so on. Find his total savings at

the end of 10 days.

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24. Shri A. K. Roy borrows Rs. 6000 to repay the amount with interest of Rs. 138 in 10 monthly instalments, each instalment being twice the preceding one. Find the second and last instalments.

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25. A man borrows Rs. 19682 without interest and agrees to repay the money in 9 monthly instalments, each instalment being thrice the preceding one. After the seventh instalment has been paid, he wants to repay the balance in lump. How much has he to pay now ?

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26. The A.M. of two numbers is 15 and their G.M. is 12. Find the numbers.

27. If a, b, c are in G.P. and x, y be the A.M. between a, b and b, c

respectively, prove that,

$$\frac{a}{x} + \frac{c}{y} = 2$$

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28. If a, b, c are in G.P. and x, y be the A.M. between a, b and b, c

respectively, prove that,

$$\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$$

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29. If a, b, c be in A.P. and x, y, z be in G.P., prove that,

$$x^{b-c}$$
. y^{c-a} . $z^{a-b} = 1$.

30. The sum of three numbers in G.P. is 70. If each of the two extremes be multiplied by 4 and the mean by 5, the products are in A.P. Find the numbers.



31. Let t_r be the rth term of an A.P. If t_2 , t_8 and t_{32} are in G.P., then find the

common ratio of the G.P.

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32. If p, q, r be in arithmetic progression, then show that the pth, qth and

rth terms of any geometric series will be in geometric progression.



33. The nth term of a series is $3^n + 2n$, find the sum of first n terms of the

series.



34. If one arithmetic mean A and two geometric means p, q be inserted between two given numbers, then prove that,

$$rac{p^2}{q}+rac{q^2}{p}=2A.$$

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35. If A is the A.M. and G is the G.M. of two unequal positive real numbers

p and q, prove that A>G and $|G|>rac{G^2}{A}$

36. Between two positive real numbers a and b, one arithmetic mean A and three geometric means G_1, G_2, G_3 are inserted. Find the value of $\frac{(a^4 + b^4) + 4(G_1^4 + G_3^4) + 6G_2^4}{A^4}$

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37. If a, b, c are in G.P. and the equation $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then show that $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

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38. If a is A.M. of two positive numbers b and c and two G.M's between b and c are G_1 and G_2 , prove that $G_1^3 + G_2^3 = 2abc$.

39. If G is the G.M. between two positive numbers a and b, show that

40. If sum of the square of three different terms in G.P. is
$$S^2$$
 their sum is αS , then show that $\frac{1}{3} < \alpha^2 < 3$.

41. Show that sum of n G.M.s between a and b is

$$\left(rac{ab^{rac{1}{n+1}}-ba^{rac{1}{n+1}}}{a^{rac{1}{a+1}}-b^{rac{1}{n+1}}}
ight)$$

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42. Evaluate :
$$\Sigma_{K=1}^7 \left[\left(rac{1}{5}
ight)^{2K+1} . \left(7
ight)^{K-1}
ight]$$

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43. The A.M. between m and n and the G.M. between a and b are each equal to $\frac{ma + nb}{m+n}$. Find m and n in terms of a and b.



Exercise 9 C Long Answer Type Questions

1. If S be the sum, P the product and R the sum of the reciprocal of n

terms in G.P., prove that, $p^2=\left(rac{S}{R}
ight)^n$.

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2. If S_1, S_2, S_3 be respectively the sums of n, 2n and 3n terms of a G.P., prove that, $S_1(S_3 - S_2) = (S_2 - S_1)^2$.

3. If the sum of first n terms of G.P. is S and the sum of its first 2n terms is

5S, then show that the sum of its first 3n terms is 21S.



5. If
$$u_1, u_2, u_3, \ldots$$
 form a G.P. with common ratio k, find the sum of $u_1u_2 + u_2u_3 + \ldots + u_nu_{n+1}$ in terms of u_1 and k.

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6. Find the sum to n terms of the following series:

$$\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)^2 + \left(x^3+\frac{1}{x^3}\right)^2 + \left(x^4+\frac{1}{x^4}\right)^2 + \dots$$
7. If the first term, common ratio and the sum of first n terms of a G.P. be a, r and S_n respectively, find the value of $S_1+S_2+S_3+\ldots+S_n$.



8. When a certain golf-ball is dropped on a piece of pavement, it bounces to a height of three-fifth the height from which it fell. If the ball is dropped from a height of 100 cm, how far has it travelled when it hits the ground for the tenth time ? How high will it rise on the next bounce ?

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9. If the A.M. of two numbers be twice their G.M., prove that two numbers are in the ratio $(2 + \sqrt{3}): (2 - \sqrt{3}).$



10. If A and G be the A.M. and G.M. of two numbers , show that the numbers are $A\pm\sqrt{A^2-G^2}.$



11. If a, b, c form an A.P. and b, c, a form a G.P., show that $\frac{1}{c}$, $\frac{1}{a}$, $\frac{1}{b}$ form an

A.P.

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12. If the arithmetic mean of y and z be x and that of $\frac{1}{x}$ and $\frac{1}{y}$ be $\frac{1}{z}$, then

show that x, y, z are in G.P.



13. The sum of three numbers in A.P. is 15. If 1, 4 and 19 are added to the numbers the resulting numbers are in G.P. Find the numbers.



14. The sum of three numbers in A.P. is 18. If 2, 2, 6 are added respectively to 1st, 2nd and 3rd numbers, the resulting numbers are in G.P. Find the numbers.

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15. Three terms are in G.P. Their product is 216. If 4 be added to the first term and 6 to the second term, the resulting numbers are in A.P. Obtain the terms in G.P.

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16. Find the sum to n terms of each of the following series :

1 + 11 + 111 +

17. Find the sum to n terms of each of the following series :

2 + 22 + 222 +...

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 $18.0.8 + 0.88 + 0.888 + \dots$

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19. Find the sum to n terms of each of the following series :

 $0.6 + 0.66 + 0.666 + \dots$

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20. Find the sum to n terms of each of the following series :

 $1+4+13+40+\ldots$

21. Three unequal numbers a, b, c are in A.P. and a, $(b-a),\,(c-a)$ are in

G.P. Prove that,

a:b:c = 1:3:5.

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22. If a, c, b are in A.P. and b, c, d are in G.P. prove that, b, (b-c), (d-a)

are in G.P.

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23. The first term of an A.P. is the same as that of a G.P., the common difference of the one and the common ratio of the other are both 4. If the sum of the first three terms of each series are same, find the fourteenth term of each series.



24. Find the sum to n terms :

$$rac{1}{2} + rac{3}{2^2} + rac{5}{2^3} + \ldots + rac{2n-1}{2^n}$$

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25. Find the sum to n terms :

 $2 + 3.3 + 4.3^2 + 5.3^3 + \dots$

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26. Find the sum to n terms :

 $12 + 105 + 1008 + 10011 + \dots$



27. Find the sum to n terms :

 $12 + 14 + 24 + 58 + 164 + \dots$

28. The rth term of a series is $(2r+1)2^r$, find the sum of first n terms of

the series.

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29. If one geometric mean G and two arithmetic means p, q be inserted

between two given numbers, then prove that,

$$G^2 = (2p-q)(2q-p).$$

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30. A, B, C have together Rs. 5700 and the amount of money possessed by them form a G.P. If B had Rs. 150 more, the amounts would form an A.P. Find the amounts they possess.



31. If the (m + 1)th, (n + 1)th and (r + 1)th terms of an A.P. are in G.P. and $\frac{1}{m}$, $\frac{1}{n}$, $\frac{1}{r}$ are in A.P., then find the ratio of the first term of the A.P. to its common difference in terms of n.



32. The sum of first ten terms of an A.P. is 155 and the sum of first two terms of a G.P. is 9. The first term of the A.P. is equal to the common ratio of the G.P. and the first term of the G.P. is equal to the common difference of the A.P. Find the two progressions.

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33. The pth , qth and rth terms of an A.P. are in G.P. Prove that the common ratio of the G.P. is $\frac{q-r}{p-q}$ or $\frac{p-q}{q-r}$

34. If a_1, a_2, \ldots, a_n are in G.P., then show that

$$rac{1}{a_1^2-a_2^2}+rac{1}{a_2^2-a_3^2}+...+rac{1}{a_{n-1}^2-a_n^2}=rac{r^2}{\left(1-r^2
ight)^2}iggl[rac{1}{a_n^2}-rac{1}{a_1^2}iggr]$$

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35. If a, b, c, d are in A.P. and a, b, c, d are in G.P., show that $a^2-d^2=3ig(b^2-adig).$

A. a. $\frac{1-r^n}{1-r}$ B. a. $\frac{r^n-1}{1-r}$ C. a. $\frac{1-a}{1-r}$ D. $\frac{a}{1-r}$

Answer:

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Exercise 9 D Multiple Choice Type Questions

1. The sum of first n terms of the series
$$a + ar + ar^2 + \ldots + ar^{n-1} + \ldots \infty$$
 is -

2. If the sum of the first n terms of the series $a + ar + ar^2 + \ldots + ar^{n-1} + \ldots \infty$ is $\frac{a}{1-r}$, then -A. -1 < r < 1B. $-1 \le r < 1$ C. $-1 < r \le 1$ D. $-1 \le r \le 1$

Answer: A

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Exercise 9 D Very Short Answer Type Questions

1. Find the sum of each of the following infinite geometric series, if it

exists :

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots \infty$$

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2. Find the sum of each of the following infinite geometric series, if it

exists :

$$\frac{1}{3} - \frac{2}{9} + \frac{4}{27} - \frac{8}{81} + \dots \infty$$

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3. Find the sum of each of the following infinite geometric series, if it

exists :

$$rac{1}{2} + rac{3}{4} + rac{9}{8} + rac{27}{16} + \ldots \infty$$

4. Find the sum of each of the following infinite geometric series, if it exists :

 $1+1+1+1+\ldots\infty$

5. Find the sum of each of the following infinite geometric series, if it exists : $\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \frac{2}{5^5} + \frac{3}{5^6} + \dots \infty$

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6. Find the sum of each of the following infinite geometric series, if it

exists :

 $0.9+0.81+0.729+\ldots\infty$

7. Find the sum of each of the following infinite geometric series, if it

exists :

$$rac{1}{16} + rac{1}{12} + rac{1}{9} + rac{4}{27} + \ldots \infty$$

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8. Find the sum of each of the following infinite geometric series, if it

exists :

$$\frac{4}{7} - \frac{5}{7^2} + \frac{4}{7^3} - \frac{5}{7^4} + \frac{4}{7^5} - \frac{5}{7^6} + \dots \infty$$

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9. Express each of the following recurring decimals in the form of an infinite geometric series and obtain their values in the form of rational numbers :

0.3

10. Express each of the following recurring decimals in the form of an infinite geometric series and obtain their values in the form of rational numbers :

0.315

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11. Express each of the following recurring decimals in the form of an infinite geometric series and obtain their values in the form of rational numbers :

3.1735

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12. Express each of the following recurring decimals in the form of an infinite geometric series and obtain their values in the form of rational numbers :

 $1.\,\dot{41} + 2.2\dot{12}$

13. Express 0. 4 as an infinite geometric series and hence prove that $(0.444...)^{1/2} = 0.666...$

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14. The sum of an infinite geometric series is $\frac{1}{3}$ and its first terms is $\frac{1}{4}$,

find the series.

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15. The sum of an infinite geometric series is 6 and the sum of its first two

terms is $\frac{9}{2}$, find its common ratio.

16. In a G.P. the sum of infinite terms is 15, the sum of the squares of these

terms is 45. Find the G.P.



17. Two infinite geometric series starts from the same number. If the common ratios are 0.85 and 0.55 respectively, show that the sum of the first series is three times that of the second.

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18. The distance passed over by a certain pendulum bob in succeeding swings form the G.P. (16, 12, 9,...) cm respectively. Calculate the total distance traversed by the bob before it comes to rest.

19. In an infinite G.P., each term is equal to three times the sum of all the terms that follow it and the sum of the first two terms is 15, find the sum of the series to infinity.

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20. If
$$x = a + \frac{a}{r} + \frac{a}{r^2} + \ldots \infty \left(-1 < \frac{1}{r} < 1 \right)$$
,
 $y = b - \frac{b}{r} + \frac{b}{r^2} - \ldots \infty$ and $z = c + \frac{c}{r^2} + \frac{c}{r^4} + \ldots \infty$, show that ,
 $\frac{xy}{z} = \frac{ab}{c}$.

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21. If |a| < 1, |b| < 1, then show that

$$a(a+b)+a^2ig(a^2+b^2ig)+a^3ig(a^3+b^3ig)+\ldots=rac{a^2}{1-a^2}+rac{ab}{1-ab}$$

22. If, for real numbers a, b, c, r with
$$|r| > 1, x = a + \frac{a}{r} + \frac{a}{r^2} + \dots, \quad y = b - \frac{b}{r} + \frac{b}{r^2} - \frac{b}{r^3} + \dots$$
 and $z = c + \frac{c}{r^2} + \frac{c}{r^4} + \dots$, then show that xyc = abz.

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23. If for two reals a, b, with
$$|a| > 1$$
, $|b| > 1$, $x = 1 + \frac{1}{a} + \frac{1}{a^2} + \dots$ and $y = 1 + \frac{1}{b} + \frac{1}{b^2} + \dots$ then show that the um of the series $1 + 1ab + a^2b^2 + \dots = \frac{1 - x - y + xy}{1 - x - y}$

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24. For a real number 'a' with
$$|a|<1$$
, if $x=a-a^3+a^5-\ldots,y=1+a^2+a^4+\ldots$ and $z=rac{1}{a}+a^3+a^7+\ldots$ then show that $a^2z=xy.$



 $d=1+c+c^2+\ldots$, find the value of (d-a)

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26. If, for $0 < a < \frac{1}{2}$, the sum of series $a + a^2 + a^3 + \ldots$ is b, then show that the series $b - b^2 + b^3 - b^4 + \ldots$ is also convergent and hence find its sum.

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$$x=\sum\limits_{n=0}^{\infty}\cos^{2n} heta, \hspace{1em} y=\sum\limits_{n=0}^{\infty} an^{2n}arphi \hspace{1em} ext{an}\hspace{1em} z=\sum\limits_{n=0}^{\infty}\sin^{2n} heta an^{2n}arphi \Big(0< heta,arphi<$$

If

then show that xyz = xy + yz - z.

1. Examine whether the following sequences are in H.P. :

$$\left\{\frac{3}{2}, \frac{3}{11}, \frac{3}{17}, \frac{3}{23}, \ldots\right\}$$

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2. Examine whether the following sequences are in H.P. :



3. Examine whether the following sequences are in H.P. :

$$\left\{\frac{1}{7}, \frac{1}{9}, \frac{1}{12}, \frac{1}{16}, \ldots\right\}$$

4. Examine whether the following sequences are in H.P. :

$$\left\{\frac{1}{a-3b},\frac{1}{a-2b},\frac{1}{a-b},\frac{1}{a},\ldots\right\}$$

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5. Examine whether the following sequences are in H.P. :

$$\left\{ \frac{1}{c+3d'}, \frac{1}{c+2d'}, \frac{1}{c'}, \frac{1}{c-3d'}, \ldots \right\}$$

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6. Find the H.P whose first and second terms are $\frac{1}{3}$ are $\frac{1}{5}$ respectively.

7. If the 10th and 19th terms of a H.P. are
$$\left(-\frac{5}{37}\right)$$
 and $\left(-\frac{5}{64}\right)$

respectively, find the nth term of the H.P.



its (p + q)th term is $\frac{pq}{p+q}$.

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9. What is the value of the H.M. between (a + b) and (a - b) ?

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10. Insert two HM.s between
$$\frac{1}{3}$$
 and $\frac{1}{24}$.

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11. Insert two H.M.s between (-6) and $\frac{3}{13}$.

12. If the G.M. and H.M. of two numbers are 15 and 9 respectively, find the

numbers.



$$rac{a}{b+c-a}, rac{b}{c+a-b}, rac{c}{a+b-c}$$
 are in H.P.

16. If a, b, c are in H.P., prove that,

 $rac{1}{bc}, rac{1}{ca}, rac{1}{ab}$ are in H.P.

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17. If a, b, c are in H.P., prove that,

$$a(b+c), b(c+a), c(a+b)$$
 are in A.P.

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18. If a, b, c are in H.P., prove that,

$$rac{a}{a-b}=rac{a+c}{a-c}$$

19. If a, b, c are in A.P., show that, $\frac{bc}{a(b+c)}, \, \frac{ca}{b(c+a)}$ and $\frac{ab}{c(a+b)}$ are in

H.P.



20. If a^2, b^2, c^2 are in A.P., show that, (b+c), (c+a), (a+b) are in H.P.`



21. If four positive numbers a, b, c and d are in A.P. then prove that abc,

abd, acd and bcd are in H.P.



22. Prove that, a, b,c are in A.P., G.P. or, H.P. accordingly as $\frac{a-b}{b-c} = 1$ or $\frac{a}{b}$ or, $\frac{a}{c}$.

23. If $a^x = b^y = c^z$ and a, b, c, are in G.P., prove that x, y, z are in H.P.



24. If p, q, r are in A.P., q, r, s are in G.P. and r, s, t are in H.P., prove that p, r, t

are in G.P.

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25. If y is the H.M. between x and z, prove that,

$$\frac{1}{y-x} + \frac{1}{y-z} = \frac{1}{x} + \frac{1}{z}.$$

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26. The harmonic mean of two numbers is 4, their arithmetic mean 'A' and the geometric mean G satisfy the relation $2A + G^2 = 27$. Find the

numbers.



27. If the p-th, q-th and the r-th terms of a H.P. be x, y, z respectively, prove

that,

$$yz(q-r)+zx(r-p)+xy(p-q)=0$$

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28. A man travels a certain distance with a uniform speed of 5 km/h ad returns the same distance with a uniform speed of 4km/h. What is the average speed of the man?

A. p^{th}, q^{th} and r^{th} terms of A.P. are in G.P.

B. p^{th}, q^{th} and r^{th} terms of G.P. are in A.P.

C. p^{th}, q^{th} and r^{th} terms of H.P. are in H.P.

D. none of these

Answer:
$$4rac{4}{9}km/h$$

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Sample Questions For Competitive Exams Mcqs

1. If p, q and r are in A.P. then which of the following are true?-

A. x, y, z are in H.P
B.
$$\frac{1}{x}$$
, $\frac{1}{y}$, $\frac{1}{z}$ are in A.P.
C. x, y, z are in G.P.

D.
$$rac{1}{x}, rac{1}{y}, rac{1}{z}$$
 are in G.P.

Answer: A, B, C

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2. If
$$x^2 + 9y^2 + 25z^2 = xyz\left(\frac{15}{x} + \frac{5}{y} + \frac{3}{z}\right)$$
, then -
A. $\frac{729}{16}$
B. 6
C. 0

Answer: A,B

D. 54

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3. The next term of G.P. x, x^2+2 and x^3+10 is -

A. $S_{40} = -820$

B. $S_{2n}>S_{2n+2}$

 $C. S_{51} = 1275$

D. $S_{2n+1}>S_{2n-1}$

Answer: A,D



4. If
$$S_n = 1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots$$
 , then-

A.
$$\frac{(a+c)(3a-c)}{4a^2c^2}$$

B. $\frac{2}{bc} - \frac{1}{b^2}$
C. $\frac{2}{bc} - \frac{1}{a^2}$
D. $\frac{(a-c)(3a+c)}{4a^2c^2}$

Answer: A,B



5. If a, b and c are in H.P., then the value of $rac{(ac+ab-bc)(ab+bc-ac)}{(abc)^2}$

is -

Sample Questions For Competitive Exams Integer Answer Type

1. If 5^{th} term of a G.P. is 2 and product of first 9 terms is 298 K, then the value of K is -

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2. If x, y, z are in G.P. and (x + 3), (y + 3), (z + 3) are in H.P., then the value of

y is -

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3. Let the sum of the three terms of a G.P. is 14. If 1 is added to the 1st and the 2nd term and 1 is subtracted from the 3rd term, then they will be in A.P. The value of the smallest term is -



Sample Questions For Competitive Exams Comprehension Type

1. Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then The product of all numbers is -

Β.	0
υ.	U

C. 4

D. 2

Answer: C

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Sample Questions For Competitive Exams

1. Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then The sum of all the four numbers is -

A. 1

B. 3

C. 2

 $\mathsf{D.}-2$

Answer: D



2. Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then The common difference of the four numbers is -

A. 9 B. 8 C. 12 D. 6

Answer: A

3. In a G.P., the sum of the first and the last term is 66, the product of the second and the last but one is 128 and the sum of terms is 126.

If an increasing G.P. is considered, then the number of terms in G.P. is -

A. 4	
B. 5	
C. 6	
D 7	

Answer: C

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4. In a G.P., the sum of the first and the last term is 66, the product of the second and the last but one is 128 and the sum of terms is 126.

If the decreasing G.P. is considered, then the sum of the infinite terms are

A. 78

B. 126

C. 128

D. none of these

Answer: B

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5. In a G.P., the sum of the first and the last term is 66, the product of the second and the last but one is 128 and the sum of terms is 126.In any case, the difference of the least and the greatest term is -

A. Statement-I is true, Statement II is true and Statement II is a correct

explanation for Statement-I.

B. Statement-I is true, Statement II is true but Statement II is not a

correct explanation of Statement-I.

C. Statement - I is true, Statement - II is false.
D. Statement - I is false, Statement - II is true.

Answer: D



Sample Questions For Competitive Exams Assertion Reason Type

1. Statement I : If the arithmetic mean of two numbers is $\frac{5}{2}$, geometric mean of the numbers is 2, then the harmonic mean will be $\frac{8}{5}$. Statement II : For a group of positive numbers $(G. M.)^2 = (A. M.) \times (H. M.)$

A. Statement-I is true, Statement II is true and Statement II is a correct

explanation for Statement-I.

- B. Statement-I is true, Statement II is true but Statement II is not a correct explanation of Statement-I.
- C. Statement I is true, Statement II is false.

D. Statement - I is false, Statement - II is true.

Answer: C



