



MATHS

BOOKS - CHHAYA PUBLICATION

MATHS (BENGALI ENGLISH)

**SIGNIFICANCE OF DERIVATIVE AS RATE
OF CHANGE**

Examples

1. Find the differential of the functions:

(i) $\sqrt{x^2 + 2}$ (ii) $\log(x^2 + 4)$ (iii) $\log \cos x$



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2. Find the increment and differential of the function, $f(x) = 2x^2 - 3x + 2$ when

(i) x changes to 3.02 from 3 (ii) x changes to 1.99 from 2



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3. Find the rate of change of the area of a circle with respect to its radius r when $r = 3\text{cm}$.



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4. Using differentials find the approximate value of

(i) $(82)^{\frac{1}{4}}$ (ii) $\sqrt{0.037}$



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5. Using the method of differentials, find the approximate values of

$$\sqrt{2}$$



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6. Find the rate of change of the area of a circle with respect to its radius r when $r = 4\text{cm}$.



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7. Estimate the error made in calculating the area of the triangle ABC in which the sides a and b are measured accurately as 25 cm and 16cm, while the angle C is measured as 60° but $\left(\frac{1}{2}\right)^\circ$ in error.



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8. If a triangle ABC inscribed in a fixed circle be slightly varied in such a way that its vertices

are always on the circle, show that,

$$\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0.$$



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9. Find the average rate of change of the function $y = 16 - x^2$ between $x = 3$ and $x = 4$, also find the rate of change of the function at $x = 4$.



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10. If the radius of a spherical balloon increases by 0.1 % find approximately the percentage increase in volume.



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11. Find the coordinates of the position of a particle moving along the parabola $y^2 = 4x$ at which the rate of increase of the abscissa is twice the rate of increase of the ordinate.



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12. A point is moving along the curve $x^3 = 12y$, which of the coordinates changes at a faster rate at $x = 10$?



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13. The time rate of change of the radius of a sphere is $\frac{1}{2\pi}$, when its radius is 5 cm, find the rate of change of the area of the surface of the sphere with time.



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14. A solid cube changes its volume such that its shape remains unchanged. For such a cube of unit volume show that,

rate of change of volume $= \frac{3}{2} \times$ (rate of change of area of any face of the cube.).



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15. The distance x of a particle from a fixed point at time t is given by, $x = 5 +$

$A \sin 2t + B \cos 2t$, where A and b are given to be 3 and 4 respectively. However, it is found on measurement that there is a 1% error in the maximum value of x and this is due to an error in A only. Find the percentage error in A.



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16. A spherical balloon is being inflated so that its volume increases uniformly at the rate of 40cm^3 / min. How fast is its surface area increasing, when the radius is 8 cm? Find approximately

how much the radius is 8 cm ? Find approximately how much the radius will increase during the next $\frac{1}{2}$ minute.



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17. A 5 ft long walks away from the foot of a $12\frac{1}{2}$ ft high lamp-post at the rate of 3 mi/h. find the rate at which his shadow is increasing.



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18. Water is flowing into a right circular conical vessel, 45 cm deep and 27 cm in diameter at the rate of $11 \text{ cm}^3/\text{min}$. How fast is the water-level rising when the water is 30 cm deep?



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19. A ladder 20 ft long leans against a vertical wall. If the top end slides downward at the rate of 2 ft/s, find the rate at which the lower end moves on a horizontal floor when it is 12 ft

from the wall. Find also the rate at which the slope of the ladder changes.



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20. A particle moving in a straight line covers a distance of x cm in t seconds, where $x = t^3 + 6t^2 - 15t + 18$. Find the velocity and acceleration of the particle at the end of 2 seconds. When does the particle stop ?



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21. Find the rate of change of the area of a circle with respect to its radius r when $r = 6\text{cm}$.



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22. The rate of change of the function $y = f(x)$ w.r.t. X at the point x is-

A. $\frac{1}{2} f'(x)$

B. $2f'(x)$

C. $\frac{f'(x)}{f(x)}$

D. none of these

Answer: D



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Multiple Choice Type

1. If $y = f(x)$ is a differentiable function of x .

Then-

A. $f(x + \Delta x) = f'(x)\Delta x$

B. $f(x + \Delta x) = f'(x) + f'(x)\Delta x$

C. $f(x + \Delta x) = f'(x) + \Delta x$

D. none of these

Answer: B



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2. If $y = \frac{1}{\sqrt{x+1}}$, then the rate measure of y

w.r.t. x at the point $x = 3$ is -

A. $-\frac{1}{8}$

B. $\frac{1}{16}$

C. $-\frac{1}{16}$

D. $\frac{1}{8}$

Answer: C



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3. The rate measure of the function

$y = 2x - x^2$ at $x = 4$ is -

A. -6

B. -8

C. 6

D. 8

Answer: A



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4. The differential of $\log \sin x$ is-

A. $\cot x$

B. $-\tan x dx$

C. $\cot x dx$

D. $\tan x dx$

Answer: C



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5. The approximate error in measuring the area of a square of side 10 cm due to error of 0.05 cm in measuring its side is-

A. 0.5 cm^2

B. 0.1cm^2

C. 0.2cm^2

D. 1cm^2

Answer: D



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Very Short Answer Type Questions

1. Find the differential of each of the following functions

$$y = x^3 - 3x^2 + 2x$$



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2. Find the differential of each of the following functions

$$y = (x^2 - 4x + 6)^{\frac{3}{2}}$$



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3. Find the differential of each of the following functions

$$y = e^{x^2} + a^2$$



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4. Find the differential of each of the following functions

$$y = \sin \sqrt{x}$$



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5. Find the differential of each of the following functions

$$y = e^x (\sin x + \cos x)$$



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6. Find the following differentials

$$d(x^2 - y^2)$$



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7. Find the following differentials

$$d(xy^2)$$



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8. Find the following differentials

$$d\left(\frac{x^2}{y}\right)$$



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9. Find the following differentials

$$d(x^2 \sin y)$$



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10. Find the increment and differential of the function, $y = x^2 - 2x + 3$, when

(i) x changes from 2 to 2.02, (ii) x changes from 3 to 2.97.



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11. Find the average rate of change of the function $y = f(x) = x^2$ between $x = 2$ and $x = 5$, find its instantaneous rate of change at $x = 2$.



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12. A spherical toy balloon is inflated so that its volume V (in cm^3) and surface area S (in cm^2) are functions of time t (in second), where $V = \frac{\pi}{6}t^3$ and $S = \pi t^3$. Find the rates of change of v and S at $t = 4$



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13. A ball goes through a distance of s meters in t seconds, where $s = 8t - 10t^2$, find the velocity of the ball at the end of 2 seconds.



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14. A particle moves in a straight line and its velocity v (in cm/s) at time t (in second) is $6t^2 - 2t^3$, find its acceleration at the end of 4 seconds.



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15. Find the rate of change of $s = \frac{t}{\sqrt{t+1}}$ with respect to t at $t = 3$.



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16. The radius of a circular plate increases at the rate of 0.002 cm/s.. How fast is the area changing when radius is 14 cm?



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17. The radius of a circle increases at the rate of $\frac{1}{\pi}$. Find the rate of change of its (i) circumference and (ii) area when its radius is 2 units.



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18. If the rate of change of y with respect to x is 4 and y is changing at the rate of 12 units/s find the rate of change of x per second.



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19. Let V and S be the volume and surface respectively of a sphere of radius r . Prove that,

$$2 \frac{dv}{dt} = r \frac{ds}{dt}.$$



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Short Answer Type Questions

1. Determine the approximate value of $\sin 62^\circ$ (correct to 3 places of decimals).



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2. Find the approximate value of $\sin 62^\circ$, correct to 3 places of decimal (given, $1^\circ = 0.017$).



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3. Find by the method of differentials the value of $\log_{10} 10.01$, given $\log_e 10 = 2.303$.



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4. Find the value of $\tan 44^\circ$, given $1^\circ = 0.01745$.



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5. Using the method of differentials, find the approximate values of

$$\sqrt{2}$$



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6. Using the method of differentials, find the approximate values of

$$\sqrt{0.24}$$



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7. Using the method of differentials, find the approximate values of

$$\sqrt{37}$$



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8. Using the method of differentials, find the approximate values of

$$\sqrt{0.48}$$



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9. If $f(x) = 3x^2 + 15x + 5$, then find the approximate value of $f(3.02)$, using differentials.



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10. If $y = x^4 - 12$ and if x changes from 2 to 1.99, what is the approximate change in y ?



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11. If the error in measuring a side of a cube is 5% , find the percentage error in the computation of its volume.



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12. The side of a square is measured with an error less than 0.01 cm. The area should be measured with an error less than 2 cm^2 . How large a square can be satisfactorily measured?



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13. Using the method of differential find approximately the difference between the areas of two circles of radii 7 cm and 7.02 cm



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14. Using the method of differential find approximately the difference between the volumes of two cubes of sides 4 cm and 4.05 cm,



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15. If the area of circle changes uniformly w.r.t. time , show that the rate of change of its circumference varies iversely as its radius.



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16. A particle moves along the curve $y^2 = 8x$. At what point on the curve do the abscissa and ordinate increase at the same rate?



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17. A spherical balloon is being filled with air at the rate of $25 \text{ cm}^3/\text{s}$. How fast is the radius increasing when the balloon is 20 cm in diameter?



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18. Air is expelled from a spherical balloon by decreasing the radius at the rate of $\frac{1}{8} \text{ cm/s}$. At

what rate is the air escaping when the radius is 10 cm?



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19. If γ be the increase in volume of the cube of unit volume and β be the increase in area of each surface of the cube, show that $2\gamma = 3\beta$.



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20. O is fixed point on the straight line traced out by a moving particle. If the distance of the particle from O at time t be $(a \cos nt + b \sin nt)$ [a,b and n are constants], show that the acceleration of the particle varies as its distance from O.



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21. If the side of an equilateral triangle increases at the rate of $\sqrt{3}$ cm/s and its area

at the rate of $12 \text{ cm}^2/\text{s}$, find the side of the triangle .



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22. While a train is travelling from rest to the next station, its distance x km from start in t hours is given by, $x = 90t^2 - 45t^3$.

Find its velocity and acceleration after 6 minutes.



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23. The side of a square is measured with an error less than 0.01 cm. The area should be measured with an error less than 2 square cm. How large a square can be satisfactorily measured?



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24. Q units of heat is required to raise the temperature of 1 gm. Water from $0^\circ C \rightarrow t^\circ C$, where

$$Q = t + 10^{-5} \times 2t^2 + 10^{-7} \times 3t^3$$

If the specific heat is the rate of increase of heat per unit degree rise of temperature, find the specific heat of water at 50°C .



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Long Answer Type Questions

1. What do you mean by 'differential of a function? The radius of a sphere was found by measurement as 20 cm. if the maximum error in this measurement is 0.05 cm , find the

maximum error that will occur. In the computation of the curved surface of the sphere.



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2. The measurement of angle A of the triangle ABC is 45° there is an error of 1' in the measurement of the angle. Find the percentage error in the computation of the area of the triangle. [Given : $1' = 0.00291$ radian].



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3. If in the triangle ABC, the side c and the angle C remain unchanged while the other sides and angles are changed slightly, show that,

$$\frac{da}{\cos A} + \frac{db}{\cos B} = 0$$



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4. In the triangle ABC, if the sides a, b remain constant but the base angles A and B vary,

then show that,

$$\frac{dA}{\sqrt{a^2 - b^2 \sin^2 A}} = \frac{dB}{\sqrt{b^2 - a^2 \sin^2 B}}.$$



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5. The period of oscillation T of a simple pendulum of length l is connected by the

relation $T = 2\pi \sqrt{\frac{l}{g}}$, where g is a constant.

Find approximately the percentage error in the computed value of T corresponding to an error of 1% in the value of l .



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6. The radius of sphere is found to be 20 cm with a possible error fo 0.05 cm .find the error, relative error and percentage error in the computed volume.



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7. Show that the relative error in computing the volume of cube, due to an error in

measuring its edge, is approximately equal to three times the relative error in the edge.



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8. The radius of a balloon is 7 cm. If an error of 0.01 cm is made in measuring the radius, find the error in measuring the volume of the balloon.



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9. Water is flowing into a right circular conical vessel, 24 inches deep and 12 inches in diameter at the rate of in^3/min . How fast is the water-level rising when the water is 10 inches deep?



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10. A man 6 ft tall walks away from the foot of a lamp-post 15 ft high at the rate of 3 mi/h. How fast does his shadow lengthen?





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11. A man 6 ft tall walks away from the foot of a lamp-post 15 ft high at the rate of 3 mi/h
how fast does the tip of his shadow move?



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12. The base of a water tank is a square of side 3 ft. find the rate of flow of water in the tank, if the water-level at the rate of 1 fr/min.



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13. Sand is poured onto the ground at the rate of $3 \text{ ft}^3 / \text{min}$ to form a right circular cone whose height is half the radius of the base. How fast is the height increasing when the radius of the base is 4 ft?



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14. A ladder 26 ft long leans against a vertical wall. If the top slides downwards at the rate of

10 in/s. find the rate at which the lower end moves on a horizontal floor when it is 10 ft from the wall. Find also the rate at which the slope of the ladder changes.



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15. A balloon leaves the ground 50 ft from an observer, If it rises vertically at the rate of 6 ft/s, how fast it receding from the point of observation when it is 120 ft above the ground?



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16. A light is placed on the ground at a distance of 30 ft from a house. A man 6 ft tall walks from the light towards the house at the rate of 5 ft/s find the rate at which his shadow on the wall is shortening when he is 15 ft from the wall.



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17. A kite is 160 ft high and there are 200 ft of string out. If the wind moves the kite

horizontally at the rate of 5 mi/h directly away from the man who is flying it, how fast is the string being payed out at the instant? (Assume that the string is in a straight line).



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18. Water is pumped out at a constant rate of $88m^3/\text{min}$ from a conical container held with its axis vertical, if the semi-vertical angle of the cone is 45° , find the rate of deperssion of the

water level when the depth of water level is 2 meters $\left(\pi = \frac{22}{7}\right)$.



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19. A circular ink-blot grows at the rate of $2 \text{ cm}^2/\text{s}$. find the rate at which the radius is increasing after $2\frac{6}{11}$ seconds.



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20. Two roads AB and BC intersect at B, where $\angle ABC = 60^\circ$ and $\overline{AB} = 28$ meter. A cyclist starts from A and moves towards B at the rate of 4m/sec and at the same instant another cyclist starts from B and moves along the road BC at the rate of 8 m/sec. find the rate of change of distance between them 3 seconds.



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21. Find the rate of change of the area of a circle with respect to its radius r when $r = 5\text{cm}$.



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22. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm/sec . At the instant when the radius of the circular wave is 10cm , how fast is the enclosed area increasing?





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23. An aeroplane is flying horizontally at a height $\frac{2}{3}$ km above the ground with a velocity of 15km/h. find the rate at which it is receding from a fixed point on the ground which it passed over 2 minutes ago.



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Sample Questions For Competitive Examination

1. Find the points on the curve $y = x^2 + \frac{1}{x^2}$ at which $\frac{dy}{dx} = 0$.

A. (1,2)

B. (-1,2)

C. (1,-2)

D. (-1,-2)

Answer: A::B



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2. Find the points on the curve $y = 2x^3 - 15x^2 + 24x - 10$ at which the slope of the tangent is -12.

A. (2,-19)

B. (3,-6)

C. (2,-6)

D. (3,-19)

Answer: C::D



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3. Find the increment and differential of the function, $f(x) = 2x^2 - 3x + 2$ when x changes to 1.99 from 2.



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4. The length of a side of a cube is 10 cm , if an error of 0.05 cm is made in measuring the side find the differential of volume (dv) and the approximate error.



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5. Find the increment Δy of y corresponding to the increment Δx of x for the function $y = f(x) = \sqrt{x}$ at $x = 0$ and $\Delta x = 0.0001$.



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6. The slope of the tangent to the curve represented by $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$ at the point A (2,-1) is $\frac{k}{7}$, find k .



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7. A spherical balloon is expanding. If the radius is increasing at the rate of 5 cm per minute, the rate at which the volume increases (in cubic centimeter per minute) when the radius is 10 cm is $400\pi k$, find k.



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8. A $5\frac{1}{2}$ ft long man walks away from the foot of a $12\frac{1}{2}$ ft high lamp-post at the rate of $5\frac{1}{11}$

miles/h, the rate at, which his shadow is increasing, is k miles/h, find k .



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9. The average rate of change of the function $y = 16 - x^2$ between $x = 3$, and $x = 4$ is $-k$, then what will be the value of k ?



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10. The distance x of a particle from a fixed point at time t is given by, $x = 5 + A \sin 2t + B \cos 2t$, where A and b are given to be 3 and 4 respectively. However, it is found on measurement that there is a 1% error in the maximum value of x and this is due to an error in A only. The percentage error in A is $\frac{50}{k}$, then find k .



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11. Find the rate of change of the area of a circle with respect to its radius r when $r = 7\text{cm}$.



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12. Find the rate of change of the area of a circle with respect to its radius r when $r = 12\text{cm}$.



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13. ABC is a triangle whose sides are a units, b units and c units respectively and their corresponding opposite angles are A, B and C.

If triangle ABC is inscribed in a fixed circle and is slightly varied in such a way that its vertices are always on the circle, then the value of $\cos B \cos C \frac{da}{a} + \cos A \cos C \frac{db}{b} + \cos A \cos B \frac{dc}{c}$ is -

A. 1

B. 2

C. 0

D. none of these

Answer: C



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14. ABC is a triangle whose sides are a units, b units and c units respectively and their corresponding opposite angles are A, B and C .

If in the triangle ABC, the side c and angle C remain unchanged while the other sides and angles are changed slightly then the value of

$$\frac{da}{\cos A} + \frac{db}{\cos B} \text{ is -}$$

A. 0

B. 1

C. 2

D. -1

Answer: A



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15. In triangle ABC, a, b remain constant but the base angles A and B vary, then the value of

$\frac{dA}{dB}$ is -

A. $\frac{\sqrt{a^2 - b^2 \cos^2 A}}{b^2 - a^2 \cos^2 B}$

B. $\frac{\sqrt{a^2 - b^2 \cos^2 B}}{b^2 - a^2 \cos^2 A}$

C. $\frac{\sqrt{a^2 - b^2 \sin^2 A}}{b^2 - a^2 \sin^2 B}$

D. none of these

Answer: C



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16. The radius of sphere is found to be 20 cm with a possible error of 0.05 cm.

Error made in computing the volume-

A. 4π

B. 6π

C. 80π

D. 2π

Answer: C



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17. The radius of sphere is found to be 20 cm with a possible error of 0.05 cm.

Relative error made in computing the volume-

A. 0.0075

B. 0.075

C. 0.75

D. none of these

Answer: A



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18. The radius of sphere is found to be 20 cm with a possible error of 0.05 cm.

Percentage error made in computing the volume-

A. $\frac{1}{100}\%$

B. $\frac{3}{400}\%$

C. $\frac{29}{300}\%$

D. none of these

Answer: B



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19. Each question in this section has four choices A,B,C,and D out of which only one is correct. Mark your choices as follows.

Statement-I: The error made in calculating the area of the triangle ABC where the sides a and b are measured accurately as 25cm and 16 cm respectively. while the angle C is measured as 60° with an error of $\left(\frac{1}{2}\right)^\circ$. Estimated error of the area of the triangle is $\frac{55}{63} \text{ cm}^2$.

Statement-II : Area of triangle ABC = $\frac{1}{2} ab \sin C$.

- A. Statement-I is True, Statement-II is True,
Statement -II is a correct explanation for
Statement-I
- B. Statement-I is True, Statement-II is True:
Statement-II is not a correct explanation
for Statement-I
- C. Statement-I is True, Statement-II is False
- D. Statement-I is False, Statement-II is True.

Answer: A



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20. Each question in this section has four choices A,B,C,and D out of which only one is correct. Mark your choices as follows.

Statement -I: If the radius (r) of a spherical ballon increases by 0.1%. Approximately the percentage increase in volume is 0.3.

Statement -II : Volumen of ballon = $\frac{4}{3}\pi r^3$ cubic unit.

A. Statement-I is True, Statement-II is True,

Statement -II is a correct explanation for

Statement-I

B. Statement-I is True, Statement-II is True:

Statement-II is not a correct explanation

for Statement-I

C. Statement-I is True, Statement-II is False

D. Statement-I is False, Statement-II is True.

Answer: A



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