MATHS

BOOKS - CHHAYA PUBLICATION MATHS (BENGALI ENGLISH)

TANGENT AND NORMAL

Example

1. Find the equation of tangents at the specified points on each of the following corves :

$y^2 = 4ax \text{at} (0, 0)$

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2. Find the equation of tangents at the specified points on each of the following corves :

$4x^2 - 9y^2 = 36 \text{at} (3, 2)$

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3. Find the equation of tangent at the specified point on the following curve :

$y^2 = x$ at the point whose abscissa is double the ordinate

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4. Find the equation of tangent at the specified point on the following curve :

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{at}(a\sec\theta, b\tan\theta)$

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5. Find the equation of tangents at the specified points on each of the following corves :

$x^3 - 3ayx + y^3 = 0\text{at}(x, y)$

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6. Find the equation of tangents at the specified points on each of the following corves :

$x = a\cos^3\theta, y = b\sin^3\theta$ at the point θ

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7. Find the equation of tangents at the specified points on each of the following corves :

$x = ct + \frac{c}{t} = , y = ct - \frac{c}{t} \text{at } = 2$

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8. Find the equation of tangents at the specified points on each of the following curves :

$x = 1 - \cos\theta, y = \theta - \sin\theta \text{ at } \theta = \frac{\pi}{4}$

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9. Find the equation of normal at the specified point on each of the following curves :

$x^2 + y^2 - 4x - 6y - 12 = 0 \text{at}(-3, 3)$

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10. Find the equation of normal at the specified point on each of the following curves :

$y^2 = 4(x-1) \text{at} (5, 4)$

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11. Find the equation of normal at the specified point on each of the following curves :

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{at} \quad (a\cos\theta, b\sin\theta)$

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12. Find the equation of normal at the specified point on each of the following curves :

$4x^2 + 9y^2 = 72 \text{at}(3, 2)$

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13. Find the equation of normal at the specified point on each of the following curves :

$yx^2 + 4y = 8 \text{at}(x, y)$

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14. Find the equation of normal at the specified point on each of the following curves :

$xy = c^2 \text{at}\left(ct, \frac{c}{t}\right)$

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15. Find the equation of normal at the specified point on each of the following curves :

$x = 3\cos\theta - \cos^2\theta, y = 3\sin\theta - \sin^3\theta \quad \text{at} \quad \theta = \frac{\pi}{4}$

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16. Find the eauation of the normal to the hyperbola $xy = 4$ at the pint (2,2). Also ,determine the point at which the normal again intersects the hyperbola.

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17. Find the equation of the tangents and normal to the curve $x = \sin 3t, y = \cos 2t \quad \text{at} \quad t = \frac{\pi}{4}$

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18. The striaght line y=kx+3 is a tangent to the curve $7x^2 - 4y^2 = 28$ then the value of k are $\pm a$, find a.

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19. Find the condition that the straight line $x\cos\theta + y\sin\theta = p$ may touch the parabola $y^2 = 4ax$.

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20. Find the equation of the tangent to the curve $y = \sqrt{5x - 3} - 2$, which is parallel to the line $4x - 2y + 3 = 0$

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21. Find the equation of the tangent to the parabola $y^2 = 8x$ which is inclined at an angl 45° with the x-axis.

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22. Find the condition that the straight line $lx + my = n$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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23. Show that the line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y=be^{-\frac{x}{a}}$ at the point where it crosses the y-axis.

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24. At what point will the tangent to the curve $y=2x^3-15x^2+36x-21$ be paralle to the x-axis . Also find the equation of tangents to the curve at these points.

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25. Find the equation of the common tangent to the parbolas $y^2=4ax$ and $x^2=4by$

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26. Find the equation of the normal to the hyperbola $3x^2 - 4y^2 = 12$ at the point (x_1, y_1) on it. Hence, show that the straight line $x + y + 7 = 0$ is a normal to the hyperbola. Find the coordinates of the foot of the normal.

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27. A normal to the parabola $y^2 = 5x$ makes an angle 45° with the x-axis. Find the equation of the normal and the cooridnates of its foot.

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28. Find the coordinates of the points on the ellipse $3x^2 + y^2 = 37$ at the normals are parallel to

$5x - 6y + 3 = 0$.

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29. Find the euation of the normal to the parabola $y^2 = 3x$ which is perpendicular to the line $y = 2x + 4$. also find the coordinates of the foot of the normal.

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30. If the straight line $lx + my = n$ is a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ show that $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{\left(a^2 + b^2\right)^2}{n^2}$.

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31. If the normal at $\left(at_1^2, 2at_1\right)$ to $y^2 = 4ax$ intersect the parabola at $\left(at_2^2, 2at_2\right)$, prove that,

$$t_1 + t_2 + \frac{2}{t_1} = 0 (t_1 \neq 0)$$

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32. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(a\sec, \theta b\tan\theta)$. Hence show that if the tangent intercepts unit length on each of the coordinate axis than the point (a,b) satisfies the equation $x^2 - y^2 = 1$

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33. Find the length of the tangent from the point (7,2) to the circle $2x^2 + 2y^2 + 5x + y = 15$

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34. Prove that the normal at the extermities of a focal chord of a parabola intersect at right angles.

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35. Show that for all values of n, the euqation of the tangent to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ at the point (a, b), is $\frac{x}{a} + \frac{y}{b} = 2$

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36. If the straight line $\frac{x}{h}+\frac{y}{k}=1$ touches the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=1$, then show that $\left(\frac{a}{h}\right)^{\frac{n}{n-1}}+\left(\frac{b}{k}\right)^{\frac{n}{n-1}}=1$

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37. Show that the lenght of the portion of the tangent to the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ at any point of it, intercept between the coordinate axes is contant.

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38. Find the angle between the curves $x^2-y^2=2a^2$ and $x^2+y^2=4a^2$

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39. If the curves $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect at right angles then prove that $a-b=c-d$

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40. Find the eqution of the curve in which the portion of the tangent between the coordinate axes is bisected at the point of contact.

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41. If p_1, p_2 be the lenghts of perpendiculars from origin on the tangent and the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ drawn at any point

on it, show that,

$4p_1^2 + p_2^2 = a^2$

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42. Show, that the angle between the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = ab$ at a point of intersection is $\tan^{-1}\frac{a-b}{\sqrt{ab}}$

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43. Show that the curves $x = y^2$ and $xy = k$ cut at right angles, if $8k^2 = 1$

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44. If the normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ at any point makes an angle ϕ with positive direction of the x-axix, prove that, the equastion of the normal is

$y \cos \phi - x \sin \phi = a \cos 2\phi$

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45. Two tangents to the parabola $y^2 = 4ax$ meet at an angle α. Prove that the locus of their of intersections, is $y^2 - 4ax = (x + a)^2 \tan^2 \alpha$

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46. If $\tan \theta + \cot \theta = 2$, then $\tan^2 \theta + \cot^2 \theta =$

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47. Find the equation of normal to the hyperbola $4x^2 - 9y^2 = 36$, at the point (1,2)

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Multiple Choice Type Questions

1. If m is the slop of the normal to the continous curve $y = f(x)$ at the point (x_1, y_1), then m is equal to-

A. $\left(\frac{dy}{dx}\right)_{(x_1, y_1)}$

B. $\left(-\frac{dy}{dx}\right)_{(x_1, y_1)}$

C. $\left(\frac{dx}{dy}\right)_{(x_1, y_1)}$

D. $\left(-\frac{dx}{dy}\right)_{(x_1, y_1)}$

Answer: D

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2. If the tangent to the contentious curve $y = f(x) \text{at} p(a, b)$ is parallel to the x-axis, then the equation of the tangent at p is

A. $y = b$

B. $y = a$

C. $y + b = 0$

D. $y + a = 0$

Answer: A

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3. If the tangent t the curv $y = f(x) \text{at} P(x_1, y_1)$ is paralle to the y-axis, then the of the normal to the curve at P is -

A. $y = x_1$

B. $y = y_1$

C. $x = x_1$

D. $x = y_1$

Answer: B

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4. If the slopes of the tangents and normal to the curve $y = f(x)$ at the point (x,y) be $\frac{dy}{dx}$ and m respecrtively, then the value of m is -

A. $-\frac{dy}{dx}$

B. $\frac{dx}{dy}$

C. $-\frac{dx}{dy}$

D. none of these

Answer: C

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5. The eqaution of the normal to the continuous curve $y = f(x)$ at the point (x_1, y_1) is-

A. $y - y_1 = -\frac{dx}{dy}(x - x_1)$

B. $x - x_1 = -\frac{dx}{dy}(y - y_1)$

C. $y - y_1 = -\frac{dy}{dx}(x - x_1)$

D. $x - x_1 = -\frac{dy}{dx}(y - y_1)$

Answer: A

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6. If the noraml to the continous curve $y = f(x) \text{at} P(x_1, y_1)$ makes angle ψ with the positive direction of x-aixs. Then-

A. $\left(\frac{dx}{dy}\right)_{(x_1 \quad y_1)} = \tan\psi$

B. $-\left(\frac{dx}{dy}\right)_{(x_1 \quad y_1)} = \cot\psi$

C. $-\left(\frac{dx}{dy}\right)_{(x_1 \quad y_1)} = \tan\psi$

D. $\left(\frac{dx}{dy}\right)_{(x_1 \quad y_1)} = -\tan\psi$

Answer: C

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7. The slope of the normal to the parabola $x^2 = 4ay$ at $(2at, at^2)$ is -

A. $\frac{1}{t}$

B. t

C. $-t$

D. $-\frac{1}{t}$

Answer: D

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8. The slope of the normal to the rectangular hyperbola $xy = 4at\left(2t, \frac{2}{t}\right)$ is -

A. $-t^2$

B. t^2

C. $2t$

D. $-2t$

Answer: B

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9. The slope of the tangent to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ is -

A. t

B. $\frac{1}{t}$

C. $-t$

D. $-\frac{1}{t}$

Answer: B

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10. The slope of the normal to the cirlce $x^2 + y^2 = a^2$ at the point (x_1, y_1) is -

A. $\frac{x_1}{y_1}$

B. $-\frac{x_1}{y_1}$

C. $-\frac{y_1}{x_1}$

D. $\frac{y_1}{x_1}$

Answer: D

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11. The slope of the tangent to the rectangular hyperbola $xy = c^2$ at the point $\left(ct, \frac{c}{t}\right)$ is -

A. $-\frac{1}{t}$

B. $-\frac{1}{t^2}$

C. $\frac{1}{t}$

D. $\frac{1}{t^2}$

Answer: B

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12. The slope of the normal to the circle $x^2 + y^2 = a^2$ at the point $(a\cos\theta, a\sin\theta)$ is-

A. $-\cot\theta$

B. $-\tan\theta$

C. $\tan\theta$

D. $\cot\theta$

Answer: C

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13. The slop of the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(a\cos\theta, b\sin\theta)$- is

A. $\frac{b}{a}\tan\theta$

B. $\frac{b}{a}\cot\theta$

C. $-\frac{b}{a}\tan\theta$

D. $-\frac{b}{a}\cot\theta$

Answer: D

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14. The slop of the normal to the reactangular hyperbola $xy = c^2$ point (x_1, y_1) is -

A. $-\frac{x_1}{y_1}$

B. $\frac{x_1}{y_1}$

C. $-\frac{y_1}{x_1}$

D. $\frac{y_1}{x_1}$

Answer: B

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15. The slope of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is-

A. $\frac{b^2 x_1}{a^2 y_1}$

B. $\frac{b^2 y_1}{a^2 x_1}$

C. $-\frac{b^2 x_1}{a^2 y_1}$

D. $\frac{b^2 y_1}{a^2 x_1}$

Answer: A

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16. The slop of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(a\sec\theta, b\tan\theta)$ is -

A. $\frac{b}{a}\sin\theta$

B. $-\frac{b}{a}\sin\theta$

C. $\frac{a}{b}\sin\theta$

D. $-\frac{a}{b}\sin\theta$

Answer: D

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Very Short Answer Type Questions

1. If $px + qy = r$ be a tangent to the circle $x^2 + y^2 = a^2$ at any given point then find the equatin of the normal to the circle at the same point.

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2. If $lx + my + n = 0$ be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at a given point, on it, find the equation of the normal to the circle at the same point.

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3. Find the points on the ellipse $4x^2 + 9y^2 = 36$ at which the tangents are paralle to x-axis.

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4. Is there any tangent paralle to x-axis to the parabola $y^2 + 20x$? Give reasons for your answer.

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5. Find where the normal to the circle $x^2 + y^2 - 4x + 6y + 9 = 0$ is parallel to x-axis.

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6. Find the where the tangents to the parabola $y = x^2$ is paralle to the line $y = 4x - 5$

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7. Find the coordinates of points on the hyperbola $xy = c^2$ at which the normal is perpendicular to the line $x + t^2 y = 2c$

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8. Prove that the tangent to the curce $y = x^2 - 5x + 6$ at the points (2,0) and (3,0) are at right angles.

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9. Find the equation of the tangent at the specified points to each of the following curves.

the parabola $y^2 = 4xat(1, 2)$

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10. Find the equation of the tangent at the specified points to each of the following curves.

the ellipse $9x^2 + 16y^2 = 288at(4, 3)$

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11. Find the equation of the tangent at the specified points to each of the following curves.

the circle $x^2 + y^2 - 4x - 6y - 3 = 0 at (2, -1)$

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12. Find the equation of the tangent at the specified points to each of the following curves.

the reactangular hyperbola $xy = 16 \text{at}(-4, -4)$

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13. Find the equation of the tangent at the specified points to each of the following curves.

the hyperbola $\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1 \text{at}(x, y)$

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14. Find the equation of the tangent at the specified points to each of the following curves.

the parabola $y^2 = 36x$ at point whose ordinate is three times the abscissa

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15. Find the equation of the tangent at the specified points to each of the following curves.

the ellipse$x^2 + 4y^2 = 25$ at point whsoe ordinate is 2

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16. Find the equation of the tangent at the specified points to each of the following curves.

the circle $x^2 + y^2 - 6x - 2y + 6 = 0$ at point equidistant from coordinate axes

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17. Find the equation of the tangent at the specified points to each of the following curves.

the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}} \text{at}(x_1, y_1)$

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18. Find the equation of the tangent at the specified points to each of the following curves.

the ellipse $x = a\cos\theta, y = b\sin\theta$ at $\theta = \frac{\pi}{3}$

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19. Find the equation of the tangent at the specified points to each of the following curves.

the curve $x^3 + xy^3 - 3x^2 + 4x + 5y + 2 = 0 \text{at}(1, -1)$

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20. Find the equation of the normal at the specified point to each of the following curves

$y^2 = 4ax \text{at}(0, 0)$

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21. Find the equation of the normal at the specified point to each of the following curves

$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \text{at}(x, y)$

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22. Find the equation of the normal at the specified point to each of the following curves

$y^2 = 4ax$ at $\left(\frac{a}{p^2}, \frac{2a}{p}\right)$

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23. Find the equation of the normal at the specified point to each of the following curves

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{at} (a \sec\theta, b \tan\theta)$$

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24. Find the equation of the normal at the specified point to each of the following curves

$$x^2 + y^2 - 4x - 6y + 3 = 0 \text{at} (1, 6)$$

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25. Find the equation of the normal at the specified point to each of the following curves

$x^2 + y^2 = 8$ at point, where $\frac{dy}{dx} = -1$

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26. Find the equation of the normal at the specified point to each of the following curves

$x = y^2 - 4y$ at points where the curve crosses the y-axis

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27. Find the equation of the normal at the specified point to each of the following curves

$x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

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28. Find the equation of the normal at the specified point to each of the following curves

$x = at^2, y = 2at$ at the point t

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29. Find the equation of the normal at the specified point to each of the following curves

$x = a(2\cos t + \cos 2t), y = a(2\sin t - \sin 2t)$ at $t = \frac{\pi}{2}$

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30. Find the equation of the normal at the specified point to each of the following curves

$x = a\sin^3 t, y = b\cos^3 t$ at the point 't'.

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31. Find the lengths of the tangents drawn from the point.

(x_1, y_1) to the circle $x^2 + y^2 + 2x = 0$

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32. Find the lengths of the tangents drawn from the point.

$(-4, 5)$ to the circle $x^2 + y^2 = 16$

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33. Find the lengths of the tangents drawn from the point.

$(-1, 1)$ to the circle $x^2 + y^2 - 2x + 4y + 1 = 0$

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34. Find the lengths of the tangent drawn from the point.

$(2, -2)$ to the circle $3(x^2 + y^2) - 4x - 7y = 3$

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35. Find the length of the tangent from any point on the circule

$x^2 + y^2 - 4x + 6y - 2 = 0$ to the circle

$x^2 + y^2 - 4x + 6y + 7 = 0$

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Short Answer Type Questions

1. Find the points on the hyperbola $2x^2 - 3y^2 = 6$ at which the slop of the tangent line is (-1)

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2. The slope of the tangent line to the curve $x^3 - x^2 - 2x + y - 4 = 0$ at some point on it is. 1. Find the coordinates of such point or points.

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3. The slope of the normal to the parabola $3y^2 + 4y + 2 = x$ at a point it is 8. find the coordinates of the points.

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4. Find the point on the parabla $y = x^2 - 6x + 9$, where the tangent is paralle to the line joining the points $(4, 1)$ and (3,0).

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5. Find the equation of the tangent and normal of the following curves at the specified point:

$y = x^2 + 4x + 1$ at $x = 3$

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6. Find the equation of the tangent and normal of the following curves at the specified points :

$y^2 = 4ax$ at the ends of latus rectum

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7. Find the equation of the tangent of the following curve at the specified point :

$y = x^3 - 3x$ at the point (2,2)

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8. Find the equation of the tangent and normal of the following curve at the specified point :

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $(a\cos\theta, b\sin\theta)$

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9. Find the equation of the tangent and normal to each of the following curves at the specified points :

$x = a\sec\theta, y = b\tan\theta$ at the point θ

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10. Find the equation of the tangent and normal to each of the following curves at the specified points :

$xy^2 = 18$ at the point $(2, 3)$

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11. Find the equation of the tangent and normal to each of the following curves at the specified points :

$y(x-2)(x-3)+7=x$ and the point, the where curve intersect the x-axis.

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12. Find the equation of the tangent and normal to each of the following curves at the specified points :

$x=a(\theta-\sin\theta), y=a(1-\cos\theta)$ at $\theta=\pi$

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13. Find the equation of the tangent and normal to each of the following curves at the specified points :

$x^{\frac{2}{3}}+y^{\frac{2}{3}}=2at(1,1)$

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14. Show that the equation of the normal to the hyperbola $x = a\sec\theta, y = b\tan\theta$ at the point $(a\sec\theta, b\tan\theta)$ is, $ax\cos\theta + by\cot\theta = a^2 + b^2$.

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15. Find the equation of tangent to the circle $x^2 + y^2 = a^2$ at the point $(a\cos\theta, a\sin\theta)$. Hence , show that the line $y = x + a\sqrt{2}$ touches the given circle,Find the coordinats of the point of contact.

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16. If the tangent to the curve $y = x^3 + ax + b$ at (1,-6) is paralle to the line $x - y + 5 = 0$, find the a and b.

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17. The slope to the tangent to the parabola $3y^2 = 8x$ at the point $\left(\frac{2}{3}t^2, \frac{4}{3}t\right)$ is (-2), find the equation of the tangent.

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18. Find the equations of the tangents to the circle $x^2 + y^2 = 16$ having slop $\left(-\frac{4}{3}\right)$.

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19. The slope of tangent to the ellipse $x^2 + 4y^2 = 4$ at the point $(2\cos\theta, \sin\theta)$ is $\sqrt{2}$, find the equation of the tangent.

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20. Find the equation of the tangnet to the curve $y = \sqrt{3x - 2}$ which is parallel to the line $4x - 2y + 5 = 0$

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21. Find the point on the curve $y = x^3$ where the slop of the tangent is equal to x-coordinate of the point.

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22. Using calculus find the cordinates of the point on the parabola $y^2 = 12x$ at which the tangent are paralle to the line $2x + 3y = 5$

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23. Find the eqauations of the tangent to the ellipse $4x^2 + 9y^2 = 36$ at the point (x_1, y_1), hence find the coordinates of the points on this ellipse at which the tangents are parallel to the line $2x - 3y = 6$

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24. Find the equation of the tangent to the parabola $y^2 = 8x$ which is inclined at an angl 45° with the x-axis.

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25. Find the equation of the tangent to the ellise $x^2 + 16y^2 = 16$ at the point $(4\cos\alpha, \sin\alpha)$. Hence , find the equation of the tangnets to the ellipse which are inclined at an angle 60° to x-axis.

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26. Find the equation of the tangnet to the hyperbola $3x^2 - 4y^2 = 12$, which are inclined at an angle 60° to x-axis.

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27. Prove the equation of the tangent from any point on the line $3x - 8y + 2 = 0$ to the circles $x^2 + y^2 + 2x - 10y + 12 = 0$ and $x^2 + y^2 - 4x + 6y + 8 = 0$ are equal.

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28. If the length of the length drawn from (f,g) to the circle $x^2 + y^2 = 6$ be twice the length of the tangent drawn from the same point to the circle $x^2 + y^2 + 3(x + y) = 0$ then show that $g^2 + f^2 + 4g + 4f + 2 = 0$.

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29. Show that the equation of the normal to the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ at $\left(\frac{5}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$ is the line $5x-3y=8\sqrt{2}$.

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30. Show that the equation of the normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $\left(a\sqrt{2},b\right)$ is $ax+b\sqrt{2}=\left(a^2+b^2\right)\sqrt{2}$.

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31. Find the equation of normal to the hyperbola $x^2-y^2=16$ at $(4\sec\theta,4\tan\theta)$. Hence ,show that the

line $x + y\sqrt{2} = 8\sqrt{2}$ is a normal to this hyprbola . Find the coordinates of the foot of the normal.

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32. Find the equation of normal to the parabola $y^2 = 4x$, paralle to the straight line $y = 2x$.

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33. Find the equation of normal to the ellipse $x^2 + 4y^2 = 4$ at $(2, 1)$.

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34. Find the equation of that normal to the parabola $x^2 = 4ay$ which maeks an angle 60° with x-axis.

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35. Find the eqaution of that normal to the hyperbola $3x^2 - 2y^2 = 10$ at points where the line $x + y + 3 = 0$ cuts the curve.

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36. Find the equation of normal to the parabola $y^2 = 12x$ at $\left(3t^2, 6t\right)$. Hence, find the equation of the normal to this parabola which makes an angle 135° with the x-axis.

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37. Prove that the normals at the points (1,2) and (4,4) of the parbola $y^2 = 4x$ intersect on the parabola.

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38. Find the equation of the normal to the hyperbola $x^2 = 4y$ drawn at (2,3).

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39. Find the equation of the normal to the hyperbola $x^2 - y^2 = 9$ at the point p(5,4).

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40. Find the equation of the normal to the curve $x^2 = 4y$, which passes through the point (1,2).

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41. Find the equation of the normal at the points on the curve $y = \frac{x}{1 - x^2}$, where the tangent makes an angle 45° with the axis of x.

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42. A tangent is drawn to the curve $x^2(x - y) + a^2(x + y) = 0$ at the origin. Find the angle it makes with the x-axis.

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Long Answer Type Questions

1. Show that the straight line $\frac{x}{a}+\frac{y}{b}=2$ touches the curve $\left(\frac{x}{a}\right)^3+\left(\frac{y}{b}\right)^3=2$, find the coordinates of the point of contact.

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2. Prove that the line $x+y=3a$ touches the curve $x^3+y^3=3axy$, find the coordinnates of the points of contact.

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3. Find the equation of the tangent to the parabola $y^2 = 4x + 5$ which is parallel to the straight line $y = 2x + 7$

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4. Find the equation of tangnets to the hyperbola $\frac{x^2}{25} - \frac{y^2}{36} = 1$, parallel to the line $3x - 3y = 0$

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5. Find the equation of the tangents to the parabola $3y^2 = 8x$, parallel to the line $x - 3y = 0$.

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6. Find the equation of the tangent to the ellipse $2x^2 + 3y^2 = 30$, which are parallel to the straight line $x + y + 18 = 0$

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7. Find the equation of the tangents to the cirlce $x^2 + y^2 = 81$ which are perpendicular to the line $4x + 3y = 0$

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8. Find the equation of the tangent to the parabola $y^2 = 8x$ at the point $(2t^2, 4t)$. Hence find the equation of the tangnet to this parabola, perpendicular to $x + 2y + 7 = 0$

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9. Find the equations of the tangents ot the ellipse $x^2 + 3y^2 = 4$ which are perpendicular to $x + 2y + 7 = 0$.

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10. Find the equation of the tangent, perpendicular to the line $x + 2y = 0$, to the hyperbola $7x^2 - 4y^2 = 28$ and find the coordinates of the point of contact.

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11. If the straight line $lx + my + n = 0$ touches the :

circle $x^2 + y^2 = a^2$, show that $a^2\left(l^2 + m^2\right) = n^2$

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12. If the straight line $lx + my + n = 0$ touches the :

circle $x^2 + y^2 + 2agx + 2fy + c = 0$ show that,

$\left(l^2 + m^2\right)\left(g^2 + f^2 - c\right) = (gl + fm - n)^2$

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13. If the straight line $lx + my + n = 0$ touches the :

parabola $y^2 = 4ax$, prove that $am^2 = nl$

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14. Find the condition that the straight line $lx + my = n$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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15. If the straight line $lx + my + n = 0$ touches the : hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, show that $a^2l^2 - b^2m^2 = n^2$.

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16. If the straight line $y = x \sin \alpha + a \sec \alpha$ be a tangent ot the circle $x^2 + y^2 = a^2$, then show that $\cos^2 \alpha = 1$

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17. Find the condition that the straight line $x\cos\theta + y\sin\theta = p$ may touch the parabola $y^2 = 4ax$.

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18. Find the condition that the straight line $x\cos\alpha + y\sin\alpha = p$ is a tangent to the :

ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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19. hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$

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20. If the straight line $lx + my = 1$ touches th curve $(ax)^n + (by)^n = 1$. Then show that,

$$\left(\frac{l}{a}\right)^{\frac{n}{n-1}} + \left(\frac{m}{b}\right)^{\frac{n}{n-1}} = 1.$$

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21. If the straigjht line $x\cos\alpha + y\sin\alpha = p$ touches the curve $x^m y^n = a^{m+n}$, prove that.

$$p^{m+n} m^m n^n = (m+n)^{m+n} a^{m+n} \sin^n\alpha \cos^m\alpha$$

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22. Show that the tangents at the ends of latus rectum of an ellipse intersect on the major axis.

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23. Find the equation of the common tangent to the circle $x^2 + y^2 = 8$ and the parabola $y^2 = 16x$.

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24. Find the equation o of the common tangent to the parabolas $y^2 = 32x$ and $x^2 = 4y$

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25. Find the common tangents to the hyperbola $x^2 - 2y^2 = 4$ and the circle $x^2 + y^2 = 1$

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26. The equation of the tangent to the curve $y^2 = ax^3 + b$ at the point (2,3) on it is $y = 4x - 5$, find a and b.

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27. In each of the following cases find the angle between the given curves :

$x^2 + 6y = 7$ and $y = x^3$

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28. In each of the following cases find the angle between the given curves :

$x^2 - y^2 = a^2$ and $x^2 + y^2 = \sqrt{2}a^2$.

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29. Show that curves $x^3 - 3xy^2 + 2 = 0$ and $y^3 - 3x^2y + 2 = 0$ intersect at right angles.

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30. If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect at right angles then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} = \frac{1}{d}$

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31. The equation of the tangent to the curve $y = a + bx + cx$ where it meet the y-axis is $2x + y = 3$, if

the normal to the curve at the same point meets the curve again at a point whose abscissa is $\frac{5}{2}$, then find a,b and c.

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32. If x_1 and y_1 be the intercepts on the x and y-axis respectively of tangent to the curve $x = a\cos^3\theta, y = b\sin^3\theta$ at any point θ , on the then priove that,

$$\frac{x_1^2}{a^2} + \frac{y_1^2}{b^1} = 1$$

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33. Show that the length of the portion of the tangent to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ at any point on it, intercepted between

the coordinate axis in constant.

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34. Show that the sum of the intercept on the coordinates axes of tangent to the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at any point on it is constant.

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35. If h and k be the intercept on the coordinates axes of tangent to the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1$ at any point, on it, then prove that $\frac{h^2}{a^2}+\frac{k^2}{b^2}=1$

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36. Find the equation of the normal to the parabola $y^2 = 4ax$ at a point (x_1, y_1) on it. Show that three normal can be drawn to a parabola from an external point.

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37. Find the coodition that the straight line $lx + my + n = 0$ is a normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$

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38. Find the coodition that the straight line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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39. If the line $lx + my = 1$ be a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, show that, $\frac{a^2}{l^2} - \frac{b^2}{m^2} = \left(a^2 + b^2\right)^2$

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40. If the striaht line $lx + my = 1$ is a normal to the parbaola $y^2 = 4ax$ then show that,

$al^3 + 2alm^2 = m^2$

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41. If the line $lx + my = 1$ is normal to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, show that $\frac{9}{l^2} - \frac{4}{m^2} = 169$

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42. Show that the line $\frac{ax}{3} + \frac{by}{4} = c$ be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, When$15c = a^2e^2$ where e is the eccentricity of the ellipse.

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43. If the line $x\cos\alpha + y\sin\alpha = p$ be a normal to the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$, show that,

$p^2\left(a^2\sec^2\alpha - b^2\text{cosec}^2\alpha\right) = \left(a^2 + b^2\right)^2$

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44. Show that the normal to the curve $x = 3\cos\theta - \cos^2\theta, y = 3\sin\theta - \sin^3\theta$ at $\theta = \frac{\pi}{4}$ passes through the origin.

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45. Show that the normal at any point θ to the curve $x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$ is at a constant distance from the origin.

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46. Show that the normal to the rectangular hyperbola $xy = c^2$ at point t meet the curve again at t' such that $t^3t' = 1$.

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47. The angle between the two tangents drawn from a point p to the circle $x^2 + y^2 = a^2$ is 120°. Show that the locus of P is the circle $x^2 + y^2 = \frac{4a^2}{3}$

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48. Find the equation of tangent to the curve $xy^2 = 4(4 - x)$ where it meet the line $y = x$.

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49. Find the slope of normal to the ellipse , $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ at the point (a,b).

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50. Find the equation of the tangent to the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ at the point (α, β)

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A Multiple Correct Answer Type

1. Points on the curve $f(x) = \frac{x}{1-x^2}$, where the tangent is inclined at an angle of $\frac{\pi}{4}$ to x-axis, are

A. (0,0)

B. $\left(\sqrt{3}, -\frac{\sqrt{3}}{2}\right)$

C. $\left(-2, \frac{2}{3}\right)$

D. $\left(-\sqrt{3}, \frac{\sqrt{3}}{2}\right)$

Answer: A,B,D

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2. Let the parabolas y=x(c-x)and $y = x^2 + ax + b$ touch each other at the point (1,0), then-

A. a+b+c=0

B. a+b=2

C. b-c=1

D. a+c=-2

Answer: A,C,D

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3. The angle formed by the positive y-axis and the tangent to $y = x^2 + 4x - 17$ at $\left(\frac{5}{2}, \frac{-3}{4}\right)$ is-

A. $\tan^{-1}(9)$

B. $\frac{\pi}{2} - \tan^{-1}(9)$

C. $\frac{\pi}{2} + \tan^{-1}(9)$

D. $\cot^{-1}(9)$

Answer: B,C

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4. The angle between the tangents to the curves $y = x^2$ and $x = y^2$ at (1,1) is -

A. $\cot^{-1}\frac{4}{5}$

B. $\sin^{-1}\frac{3}{5}$

C. $\tan^{-1}\frac{3}{4}$

D. $\tan^{-1}\frac{1}{3}$

Answer: A,B,C

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5. If the tangent at any point $P\left(4m^2, 8m^3\right)$ of $x^3 - y^2 = 0$ normal also to the curve $x^3 - y^2 = 0$, then the value of m is-

A. $m = \frac{\sqrt{2}}{3}$

B. $m = \frac{-\sqrt{2}}{3}$

C. $m = \frac{3}{\sqrt{2}}$

D. $m = \frac{-3}{\sqrt{2}}$

Answer: A,B

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Integer Answer Type

1. The striaght line y=kx+3 is a tangent to the curve $7x^2 - 4y^2 = 28$ then the value of k are $\pm a$, find a.

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2. A normal to the paraboal $y^2 = 5x$ make and angle 45° with the x-axis. If the coordinates of its foot is $\left(\frac{5}{k}, \frac{-5}{2}\right)$, then find k.

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3. If the normal at $\left(at_1^2, 2at_1\right)\text{to}y^2 = 4ax$ intesect the parabola at $\left(at_2^2, 2at_2\right)\text{then}t_1 + t_2 + \frac{k}{t_1} = 0(t_1 \neq 0)$, find k.

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4. The length of the tangnet from the point $(7, 2)$ to the circle $2x^2 + 2y^2 + 5x - 15 = 0$ k, what will be the value of k ?

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5. For all values of n, the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$at the point (a,b) is $\frac{x}{a} + \frac{y}{b} = k$, find k.

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Comperhension Type

1. Consider the curve $x = 1 - 3t^2, y = t - 3t^2$. If tangent at point $\left(1 - 3t^2, t - 3t^2\right)$ inclined at an angle θ to the positive x-axis and another tangent at P(2,-3) cuts the cuve again at Q. The value of $\tan\theta + \sec\theta$ is equal to-

A. $3t$

B. t

C. $t - t^2$

D. $t^2 - 2t$

Answer: a

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2. Consider the curve $x = 1 - 3t^2, y = t - 3t^2$. If tangent at point $\left(1 - 3t^2, t - 3t^2\right)$ inclined at an angle θ to the positive x-axis and another tangent at P(2,-3) cuts the cuve again at Q. The point Q will be-

A. $(1, -2)$

B. $\left(\frac{-1}{3}, -\frac{2}{3}\right)$

C. $(-2, 1)$

D. $(0, 0)$

Answer: ii

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3. Consider the curve $x = 1 - 3t^2, y = t - 3t^2$. If tangent at point $\left(1 - 3t^2, t - 3t^2\right)$ inclined at an angle θ to the positive x-axis and another tangent at P(2,-3) cuts the cuve again at Q. The angle between the tangent at P and Q will be-

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

Answer: iii

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4. Find the gradient of the tagent to the curve $ay^2 - bx^2 = 1$ at the point (a,b)

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5. Find the gradient of the tagent to the curve $2y^2 + 3x^2 = 1$ at the point (-3,1)

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6. Find the gradient of the tagent to the curve $y^3 + 5x^3 + x^2 = 1$ at the point (-1,1)

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Assertion Reason Type

1. Find the equation of the tagent to the curve $y^2 - 2x^3 + 8 = 0$ at the point (2,1)

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2. Statement -II : The tagent at $x = 1$ to the curve $y = x^3 - x^2 - x + 2$ again meet the curve at $x = -1$

Statement II: When the equation of a tangent solved with the curve. Repeated roots are obtained at point of tangency.

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