MATHS

BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)

COMPLEX NUMBERS

Solved Problems

1. Express $\frac{4+2i}{1-2i}+\frac{3+4i}{2+3i}$ in the form $a+ib, a \in Rb \in R$

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2. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$

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3. Express $(1-i)^3(1+i)$ in the form of $a+ib$

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4. Find the multiplicative inverse of $7+24i$.

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5. Determine the locus of $z, z \neq 21i$ such that Re

$$\left(\frac{z-4}{z-2i}\right) = 0$$

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6. If

$$4x + i(3x \quad y) = 3 + i \, (\quad 6)$$

, where x and y are real numbers, then find the values of x and y.

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7. If $z = 2 - 3i$, show that $z^2 - 4z + 13 = 0$

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8. Find the complex conjugate of $(3 + 4i)(2 - 3i)$

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9. show that $z_1 = \frac{2 + 1i}{25}, z_2 = \frac{-2 + i}{(1 - 2i)^2}$ are conjugate to each other.

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10. Find the square root for $(-5+12i)$

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11. polar from of $-\sqrt{7}+i\sqrt{21}$

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12. $-1-i$ in polar form with principle value of the amplitude.

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13. If the amplitude of $\left(\frac{z-2}{z-6i}\right)=\frac{\pi}{2}$ find its locus.

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14. Show that the equaton of any circle in the complex plane is of the form $z\bar{z}+b\bar{z}+b\bar{z}+c=0, (b\in C, c\in R)$

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15. Show that the complex numbers z satisfying $z^2+(\bar{z})^2=2$ constitute a hyperbola.

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16. Show that the points in the Argand diagram represented by the complex number $1+3i, 4-3i, 5-5i$ are collinear.

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17. Find the eqution of the straight line joining the points represented by $(-4+3i), (2-3i)$ in the Argand plane .

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18. $z = x + iy$ represent a point in the Argand plane, find the locus of z. such that $|z| = 2$

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19. The point P represnets a complex number z in the Argand plane. If the amplitude of z is $\frac{\pi}{4}$, determine the locus of P.

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20. If the point P denotes the complex number $z = x + iy$ in the Argand plane and if $\frac{z-i}{z-1}$ is a purelly imaginary number, find the locus of P.

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21. Describe geometrically the following subsets of C.

(i) $\{z \in C \mid |z - 1 + i| = 1\}$

(ii) $\{z \in C||z + i| \leq 3|$

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Exercise 1 A

1. If $z_1 = (2, -1), z_2 = (6, 3)$ find $z_1 - z_2$

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2. If $z_1 = (3, 5)$ and $z_2 = (2, 6)$ find, $z_1 . z_2$

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3. Write the additive inverse of the complex numbers.

$(\sqrt{3}, \sqrt{5}$

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4. Write the additive inverse of the complex numbers. ltbr $(-6, 5) + (10, -4)$

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5. Write the additive inverse of the complex numbers.

$(2, 1)(-4, 6)$

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6. If $z_1 = (6, 3), z_2 = (2, -1)$, find z_1 / z_2.

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7. If $z = (\cos\theta, \sin\theta)$, find $\left(z - \frac{1}{z}\right)$

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8. Write the multiplicative inverse of the complex numbers.

(3,4)

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9. Write the multiplicative inverse of the complex numbers.

$(\sin\theta, \cos\theta)$

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10. Write the multiplicative inverse of the complex numbers.

$(7, 24)$

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11. Write the multiplicative inverse of the complex numbers.

$(-2, 1)$

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Exercise 1 B

1. Write the following complex numbers in the form $A + iB$

$(2 - 3i)(3 + 4i)$

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2. Write the complex numbers in the form $A + iB$

$$(1 + 2i)^3$$

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3. Write the complex numbers in the form $A + iB$

$$\frac{a - ib}{a + ib}$$

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4. Write the complex numbers in the form $A + iB$

$$\frac{4 + 3i}{(2 + 3i)(4 - 3i)}$$

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5. Write the complex numbers in the form $A + iB$

$\left(-\sqrt{3} + \sqrt{2}\left(2\sqrt{3} - i\right)\right.$

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6. Write the complex numbers in the form $A + iB$

$(-5i)\left(\frac{i}{8}\right)$

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7. Write the complex numbers in the form $A + iB$

$(-1)2i$

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8. Write the complex numbers in the form $A + iB$

i^{9}

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9. Write the complex numbers in the form $A + iB$

i^{-19}

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10. Write the complex numbers in the form $A + iB$

$3(7 + 7i) + i(7 + i)$

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11. Write the complex numbers in the form $A + iB$

$$\frac{2 + 5i}{3 - 2i} + \frac{2 - 5i}{3 + 2i}$$

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12. Write the conjugate of the complex numbers

$3 + 4i$

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13. Write the conjugate of the complex numbers

$(15 + 3i) - (4 - 20i)$

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14. Write the conjugate of the complex numbers

$(2 + 5i)(- 4 + 6i)$

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15. Write the conjugate of the complex numbers

$$\frac{5i}{7+i}$$

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16. Simplifiy

$i^2 + i^4 + i^6 + \ldots. \, + (2n+1)$ terms

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17. Simplifiy

$i^{18} - 3 . i + i^2 (1 + i^4) (- i)^{26}$

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18. Find the square root of the following complex number: $-8 - 6i$

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19. Find a square root for the complex numbers.

$(3 + 4i)$

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20. Find a square root for the complex numbers.

$(-47 + i8\sqrt{3})$

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21. Find the multiplicative inverse of the complex numbers.

$\sqrt{5} + 3i$

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22. Find the multiplicative inverse of the complex numbers.

$-i$

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23. Find the multiplicative inverse of the complex numbers.

i^{-35}

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24. If $(a+ib)^2 = x+iy$, find x^2+y^2

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25. If $x+iy=\frac{3}{2+\cos\theta+i\sin\theta}$, then show that $x^2+y^2=4x-3$

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26. If $x+iy=\frac{1}{1+\cos\theta+i\sin\theta}$, show that $4x^2-1=0$

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27. If $u+iv=\frac{2+i}{z+3}$ and $z=x+iy$ find u, v

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28. If $z=3-5i$ Show that

$z^3-10z^2+58z-136=0$

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29. If $z=2-i\sqrt{7}$, then show that

$3z^3-4z^2+z+88=0$

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30. Show that $\frac{2-i}{(1-2i)^2}$ and $\left(\frac{-2-11i}{25}\right)$ are conjugate to each other

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31. $(x+iy)^{\frac{1}{3}}=(a+ib)$ then prove that $\left(\frac{x}{a}+\frac{y}{b}\right)=4(a^2-b^2)$

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32. Write $\left(\frac{a+ib}{a-ib}\right)^2-\left(\frac{a-ib}{a+ib}\right)^2$ in the fom of x+iy

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33. If x and y are real numbers of such that $\frac{(1+i)x-2i}{3+i}+\frac{(2-3i)y+i}{3-i}=i$, then determine the values of x and y.

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34. Find the least positive integer n foe which

$$\left(\frac{1+i}{1-i}\right)^{n}=1.$$

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35. If

$$\left(\frac{1+i}{1-i}\right)^{3}-\left(\frac{1-i}{1+i}\right)^{3}=x+iy,\ f\in d(x,y)$$

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36. Find the value of θ if

$(3+2i\sin\theta)/(1-2i\sin\theta)$ is purely real or purely imaginary.

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37. Find the real values of x and y if

$$\frac{x-1}{3+1}+\frac{y-1}{3-i}=i$$

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Exercise 1 C

1. Express the complex numbers modulus - amplitude form

$1 - i$

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2. Convert the complex number $\left(1 + i\sqrt{3}\right)$ into polar form.

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3. Express the complex numbers modulus - amplitude form

$-\sqrt{3} + i$

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4. Express the complex numbers modulus - amplitude form

$-1-i\sqrt{3}$

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5. Simplify $-2i(3+i)(2+4i)(1+i)$ and obtain the modulus of that complex number.

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6. If $z \neq 0$, find Arge $z + Arg\bar{z}$

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7. If $z_1 = -1$ and $z_2 = -i$, then find Arg $(z_1 z_2)$

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8. If $z_1 = -1$, $z_2 = i$ then find Arg $\left(\frac{z_1}{z_2}\right)$

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9. If

$(\cos 2\alpha + i \sin 2\alpha)(\cos 2\beta + i \sin 2\beta) = \cos\theta + i \sin\theta$

, then find the value of θ.

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10. If $\sqrt{3} + i = r(\cos\theta + i \sin\theta)$, then find the value of θ in radian measure.

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11. If $x + iy = (\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$

then value of $x^2 + y^2$

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12. If $\frac{z_2}{z_1}(z_1 \neq 0)$ is an imaginary number, then find

the value of $\left|\frac{2z_1 + z_2}{2z_1 - z_2}\right|$

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13. If $(\sqrt{3} + i)^{100} = 2^{99}(a + ib)$. Then show that

$a^2 + b^2 = 4$

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14. If $z = x + iy$ and $|z| = 1$, find the locus of z

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15. If the amplitude of $(z-1)$ is $\frac{\pi}{2}$, then find the locus of z.

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16. If the Arg $\bar{z}_1$ Arg $z_1$are$\frac{\pi}{5}$ and $\frac{\pi}{3}$ respectively find (Arg $z_1 + Argz_2$)

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17. If $z = \frac{1+2i}{1-(1-i)^2}, then\ \mathrm{a\ r\ g}(z)$ equals 0 b. $\frac{\pi}{2}$ c. π d. none of these

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18. Simplify the following complex numbers and find their modulus.

$$= \frac{(2+4i)(-1+2i)}{(-1-i)(3-i)}$$

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19. Simplify the following complex numbers and find their modulus.

$$\frac{(1+i)^3}{(2+i)(1+2i)}$$

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20. If $(1-i)(2-i)(3-i)....(1-ni) = x - iy$ then prove that $2.5 10.....(1+n)^2 = x^2 + y^2$

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21. If the real part of $\dfrac{z+1}{z+i}$ is 1, then find the locus of z.

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22. If $|z-3+i|=4$ determine the locus of z.

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23. If $|z+ai|=|z-ai|$ then find the locus of z.

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24. If $z = (x + iy)$ and if the point P in the Argand plane represent z, then desrcibe geo-metrically the locus of P satispfying the equation. $|2z - 3| = 7$

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25. If $z = (x + iy)$ and if the point P in the Argand plane represent z, then describe geo-metrically the locus of P satisfying the equation.

$|z|^2 = 4Re(z + 2)$

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26. If $z = (x + iy)$ and if the point P in the Argarand plane represent z, then desrcibe geometrically the locus of P satispfying the equation.

$|z + i|^2 - |z - i|^2 = 2$

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27. If $z = (x + iy)$ and if the point P in the Argarand plane represent z, then desrcibe geometrically the locus of P satispfying the equation.

$|z + 4i| + |z - 4i| = 10$

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28. If z_1, z_2 are two non-zero complex numbers satisfying $|z_1 + z_2| = |z_1| + |z_2|$, show that Arg z_1- Arg $z_2 = 0$

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29. $z = x + iy$ and the point P represents z, in the Argand plane and $\left|\left|\frac{z-a}{z+a}\right|\right| = 1Ra(a) \neq 0$ then find the locus of P.

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Exercise 1 D

1. Find the equation of the perpendicular bisector of the segment joining the points $7+7i, 7-7i$ in the Argand diagram.

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2. Find the equation of the straight line joining the point $-9+6i, 11-4i$ in the Argand plane.

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3. If $Z = x + iy$ and if the point P in the Argand plane represent Z, then describe geometrically the locus of z satisfying the equation.

$|z - 2 - 3i| = 5$

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4. If $Z = x + iy$ and if the point P in the Argand plane represent Z, then describe geometrically the locus of z satisfying the equation.

$2|z - 2| = |z - 1|$

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5. If $Z = x + iy$ and if the point P in the Argand plane represent Z, then describe geometrically the locus of z satisfying the equation.

Img $z^2 = 4$

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6. Find the locus of z if arg $\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$

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7. Show that the points in the Argand digraam represented by the complex numbers $2+2i, \ -2-2i, 2\sqrt{3}+2\sqrt{3}i$ are the vertices of an equilateral triangle.

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8. Find the eccentricity of the ellipse whose equation is $|z-4|+\left|z-\frac{12}{5}\right|=10$

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9. If $\dfrac{z_3 - z_1}{z_2 - z_1}$ is a real number, show that the points represented by the complex numbers z_1, z_2, z_3 are collinear.

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10. Show that the points in the Argand plnae, represented by the complex numbers $2 + i4 + 3i, 2 + 5i, 3i$ are the vertices of a square.

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11. Show that the points in the Argand plane represented by the complex numbers $-2+7i,\ -\frac{3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ are the vertices of a rhombus.

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12. The points P,Q denote the complex numbers, $\bar{z}_1, \bar{z}_2$ in the Argand diagram. O. is the origin. If $\overline{z_1 z_2}+\overline{z_2 z_1}=0$, show that $\angle POQ=90^\circ$

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13. The complex number z has argument $\theta, 0 < \theta < \frac{\pi}{2}$ and satisfy the equation $|z - 3i| = 3$. Then prove that $\left(\cot\theta - \frac{6}{z}\right) = i$

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Vsaq

1. Find the multiplicative inverse of $7 + 24i$

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2. Write the following complex numbers in the form $A + iB$. $(1 + 2i)^3$

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Saq

1. Write the conjugate of the following complex number $\frac{5i}{7 + i}$

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2. Find a squar root for the complex number $7 + 24i$

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3. Find a square root for the complex number $3 + 4i$

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4. Express of the following complex numbers in modulus amplidutes form. $1 - i$

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5. Express the complex number in modulus amplitudes form $1 + I\sqrt{3}$

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6. If the $Arg\bar{z}_1$ and $Argz_2$ are $\frac{\pi}{5}$ and $\frac{\pi}{3}$ respectively,find $(Argz_1 + Argz_2)$

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7. Write $z = -\sqrt{7} + i\sqrt{21}$ in the polar form.

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Laq

1. If the amplitude of $\left(\frac{z-2}{z-6}\right)$ is $\frac{\pi}{3}$, find its locus.

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