MATHS

BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)

DE MOIVRE'S THEOREM

SOLVED PROBLEMS

1. Simplify $\frac{(\cos\alpha + i\sin\alpha)^4}{(\sin\beta + i\cos\beta)^8}$

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2. If m, n integers and $x = \cos\alpha + i\sin\alpha, y = \cos\beta + i\sin\beta$, then prove that $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$ and $x^m y^n - \frac{1}{x^m y^n} = 2i\sin(m\alpha + n\beta$

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3. If n is a positive integer, show that

$$(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right).$$

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4. If n is an integer then show that

$$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n(\theta/2)\cos\left(\frac{n\theta}{2}\right)$$

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5. If $\cos\alpha + \cos\beta + \cos\gamma = 0 \sin\alpha + \sin\beta + \sin\gamma$, Prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{3}{2} = \sin^2\alpha + \sin^2\beta + \sin^2\gamma.$$

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6. Find all the values of $\left(\sqrt{3}+i\right)^{\frac{1}{4}}$.

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7. Find all the roots of the equation

$x^{11}-x^{7}+x^{4}-1=0$

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8. If $1, \omega, \omega^2$ are the cube roots of unity prove that

$\left(1-\omega+\omega^{2}\right)^{6}+\left(1-\omega^{2}+\omega\right)^{6}=128=\left(1-\omega+\omega^{2}\right)^{7}+\left(1+\omega-\omega^{2}\right)^{7}$

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9. If $1, \omega, \omega^2$ are the cube roots of unity prove that

$(a+b)\left(a\omega+b\omega^{2}\right)\left(a\omega^{2}+b\omega\right)=a^{3}+b^{3}$

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10. If $1, \omega, \omega^2$ are the cube roots of unity prove that

$x^2 + 4x + 7 = 0$ where $x = \omega - \omega^2 - 2.$

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11. If α, β are the roots of the equation $x^2 + x + 1 = 0$ then prove that

$\alpha^4 + \beta^4 + \alpha^{-1}\beta^{-1} = 0.$

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Textual Exercise (EXERCISE - 2 (a))

1. If n is integer then show that

$(1+i)^{2n} + (1-i)^{2n} = 2^{n+1} \cos. \dfrac{n\pi}{2}.$

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2. Find the values of the following :

$$\left(1+I\sqrt{3}\right)^{3}$$

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3. Find the values of the following :

$$(1-i)^{8}$$

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4. Find the values of the following :

$$(1+i)^{16}$$

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5. Find the values of the following :

$$\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{5}-\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^{5}$$

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6. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$ then for any $n \in N$ show that $\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$.

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7. If $\cos \alpha + \cos \beta + \cos \gamma = 0$

$= \sin \alpha + \sin \beta + \sin \gamma$ then show that

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$

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8. If $\cos \alpha + \cos \beta + \cos \gamma = 0$

$= \sin \alpha + \sin \beta + \sin \gamma$ then show that

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin(\alpha + \beta + \gamma)$

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9. If $\cos\alpha + \cos\beta + \cos\gamma = 0$

$= \sin\alpha + \sin\beta + \sin\gamma$ then show that

$\cos(\alpha+\beta) + \cos(\beta+\gamma) + \cos(\gamma+\alpha) = 0$

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10. If n is integer and $z = cis\theta, (\theta \neq (2n+1)\pi/2)$, then show that

$\frac{z^{2n}-1}{z^{2n}+1} = i\tan n\theta.$

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11. If $(1+x)^n = a_0 + a_1x + a_2x^2 + \dots\dots\dots + a_nx^n$, then show that

(i) $a_0 - a_2 + a_4 - a_6 + \dots\dots\dots = 2^{n/2}\cos.\frac{n\pi}{4}$

(ii) $a_1 - a_3 + a_5 - a_7 + \dots\dots\dots = 2^{n/2}\sin.\frac{n\pi}{4}$

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EXERCISE - 2 (b)

1. Find all the values of following .

$\left(1 - I\sqrt{3}\right)^{1/3}$

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2. Find all the values of following .

$(-i)^{1/6}$

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3. Find all the values of following .

$(1+i)^{2/3}$

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4. Find all the values of following .

$(-16)^{1/4}$

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5. Find all the values of following .

$(-32)^{1/5}$

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6. If A, B, C are angles of a triangle such that $x = cisA, y = cisB, z = cisC$, then find the value of xyz.

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7. If $x = cis\theta$, then find the value of $\left[x^6 + \frac{1}{x^6}\right]$.

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8. Find the cube roots of 8 .

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9. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that

$$\frac{1}{2+\omega}+\frac{1}{1+2\omega}=\frac{1}{1+\omega}.$$

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10. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that

$$(2-\omega)\left(2-\omega^2\right)\left(2-\omega^{10}\right)\left(2-\omega^{11}\right)=49.$$

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11. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that

$$(x+y+z)\left(x+y\omega+z\omega^2\right)\left(x+y\omega^2+z\omega\right)=x^3+y^3+z^3-3xyz$$

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12. Prove that $-\omega$, and $-\omega^2$ are the roots of $z^2 - z + 1 = 0$, where ω and ω^2 are the complex cube roots of unity .

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13. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$(a+b)^3 + (a\omega + b\omega^2)^3 + (a\omega^2 + b\omega)^3$

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14. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$(a+2b)^2 + (a\omega^2 + 2b\omega)^2 + (a\omega + 2b\omega^2)^2$

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15. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$\left(1-\omega+\omega^2\right)^3$

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16. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$(1-\omega)\left(1-\omega^2\right)\left(1-\omega^4\right)\left(1-\omega^8\right)$

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17. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$$\left[\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right]+\left[\frac{a+b\omega+c\omega}{b+c\omega+a\omega^2}\right]$$

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18. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$$(1+\omega)^3 + \left(1+\omega^2\right)^3$$

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19. If $1, \omega, \omega^2$ are the cube roots of unity , then find the values of the following .

$$\left(1-\omega+\omega^2\right)^5 + \left(1+\omega-\omega^2\right)^5$$

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20. Solve the following equations.

$$x^4 - 1 = 0$$

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21. Solve the following equations.

$x^5 + 1 = 0$

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22. Solve the following equations.

$x^9 - x^5 + x^4 - 1 = 0$

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23. Solve the following equations.

$x^4 + 1 = 0$

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24. Find the number of 15^{th}roots of unity, which are also 25^{th} roots of unity .

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25. If the cube roots of unity are $1, \omega, \omega^2$, then find the roots of the equation

$(x-1)^3 + 8 = 0.$

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26. Find the product of all the values of $(1+i)^{4/5}$.

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27. If $z^2 + z + 1 = 0$, where z is a complex number , prove that

$$\left(z+\frac{1}{z}\right)^2 + \left(z^2+\frac{1}{z^2}\right) + \left(z^3+\frac{1}{z^3}\right)^2 + \left(z^4+\frac{1}{z^4}\right)^2 + \left(z^5+\frac{1}{z^5}\right)^2 -$$

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28. If $1, \alpha, \alpha^2, \alpha^3, \ldots, \alpha^{n-1}$ be the n^{th} roots of unity, then prove that $1^p + \alpha^p + (\alpha^2)^p + (\alpha^3)^p + \ldots + (\alpha^{n-1})^p = \begin{cases} 0 & \text{if} \quad p \neq kn \\ n & \text{if} \quad p = kn \end{cases}$ where

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29. Prove the sum of $99t^{th}$ powers of the roots of the equation $x^7 - 1 = 0$ is zero and hence deduce the roots of $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$

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30. If n is a positive integer, show that $(P + iQ)^{1/n} + (P - iQ)^{1/n} = 2(P^2 + Q^2)^{1/2n} \cos\left(\frac{1}{n}, \tan. \frac{Q}{P}\right).$

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31. Show that one value of

$$\left(\frac{1+\sin.\frac{\pi}{8}+i\cos.\frac{\pi}{8}}{1+\sin.\frac{\pi}{8}-i\cos.\frac{\pi}{8}}\right)^{8/3} \text{ is } -1.$$

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32. Solve $(x-1)^n = x^n$, where n is a positive integer.

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Very Short Answer Questions

1. Find the values of $(1-i)^8$

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2. If $x = cis\theta$, then find the value of $\left[x^6 + \frac{1}{x^6}\right]$.

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Short Answer Questions

1. If A,B,C are angles of a triangle such that $x = cisA, y = cisB, z = cisC$, then find the value of xyz .

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2. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that $\frac{1}{2+\omega} + \frac{1}{1+2\omega} = \frac{1}{1+\omega}$.

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3. $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11}) = 49.$

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Long Answer Questions

1. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$ then for any $n \in N$ show that $\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$.

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2. Show that one value of

$$\left(\frac{1 + \sin.\frac{\pi}{8} + i\cos.\frac{\pi}{8}}{1 + \sin.\frac{\pi}{8} - i\cos.\frac{\pi}{8}}\right)^{8/3} \text{ is } -1.$$

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3. If n is a positive integer, show that

$$(1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right).$$

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4. If n is an integer then show that

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5. If $\cos\alpha+\cos\beta+\cos\gamma=0\sin\alpha+\sin\beta+\sin\gamma$, Prove that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}=\sin^2\alpha+\sin^2\beta+\sin^2\gamma.$$

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6. If $1, \omega+\omega^2$ are the cube roots of unity prove that

(i) $(1-\omega+\omega^2)^6+(1-\omega^2+\omega)^6=128$

$=(1-\omega+\omega^2)^7+(1+\omega-\omega^2)^7$

(ii) $(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega)=a^3+b^3$

(iii) $x^2+4x+7=0 \quad \text{where} \quad x=\omega-\omega^2-2.$

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