



## MATHS

# BOOKS - VIKRAM PUBLICATION ( ANDHRA PUBLICATION)

## DEFINITE INTEGRALS

### Solved Problems

1. Evaluate  $\int_1^2 x^5 dx$



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2. Evaluate  $\int_0^{\pi} \sin x \, dx$



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3. Evaluate  $\int_0^a \frac{dx}{x^2 + a^2}$



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4. Evaluate  $\int_1^4 x \sqrt{x^2 - 1} \, dx$



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5. Evaluate  $\int_0^2 \sqrt{4 - x^2} dx$



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6. Evaluate  $\int_0^{16} \frac{x^{1/4}}{1 + x^{1/2}} dx$



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7. Evaluate  $\int_{-\pi/2}^{\pi/2} \sin|x| dx$



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8. Show that  $\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx$ .

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9. Evaluate  $\int_0^{\pi/2} \frac{\cos^{5/2} x}{\sin^{5/2} x = \cos^{5/2} x} dx$

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10. Show that

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1).$$

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11. Evaluate  $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

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12. Find  $\int_{-a}^a (x^2 + \sqrt{a^2 - x^2}) dx$

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13. Find  $\int_0^{\pi} \frac{x \cdot \sin x}{1 + \sin x} dx$

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14. Evaluate  $\int_0^{\pi/2} x \sin x dx$



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15. Evaluate  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \left[ \frac{n-i}{n+i} \right]$  by using the method of finding definite integral as the limit of a sum.



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16. Evaluate  $\lim_{n \rightarrow \infty} \frac{2^k + 4^k + 6^k + \dots + (2n)^k}{n^{k+1}}$

by using the method of finding definite integral as

the limit of a sum.



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17.

Evaluate

$$\lim_{n \rightarrow \infty} \left[ \left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{n}\right) \cdots \left(1 + \frac{n}{n}\right) \right]^{\frac{1}{n}}$$



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18. Let  $f: R \rightarrow R$  be a continuous periodic function and  $T$  be the period of it. Then prove that for any positive integer  $n$ ,

$$\int_0^{nT} f(x) dx = n \int_0^T f(x) dx$$



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19. Find  $\int_0^{\pi/2} \sin^4 x dx$



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20. Find  $\int_0^{\pi/2} \sin^7 x dx$



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21. Find  $\int_0^{\pi/2} \cos^8 x dx$



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22. Evaluate  $\int_0^a \sqrt{a^2 - x^2} dx$



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23. Evaluate  $\int_0^{\pi/2} \sin^4 x \cos^5 x dx$



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24. Evaluate  $\int_0^{\pi/2} \sin^5 x \cos^4 x dx$



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25. Evaluate :  $\int_0^{\frac{\pi}{2}} \sin^6 x \times \cos^4 x dx$ .



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26. Find  $\int_0^{2\pi} \sin^4 x \cos^6 x dx$



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27. Find  $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^4 x dx$



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28. Find  $\int_0^{\pi} x \sin^7 x \cos^6 x dx$

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29. Find  $\int_{-a}^a x^2 (a^2 - x^2)^{3/2} dx$

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30. Find  $\int_0^1 x^{3/2} \sqrt{1-x} dx$

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**31.** Find the area under the curve  $f(x) = \sin x$  in  $[0, 2\pi]$



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**32.** Find the area under the curve  $f(x) = \cos x$  in  $[0, 2\pi]$



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**33.** Find the area bounded by the parabola  $y = x^2$ ,  
the  $x$ -axis and the lines  $x = -1$ ,  $x = 2$ .



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**34.** Find the area cut off between the line  $y=0$  and the parabola  $y = x^2 - 4x + 3$ .



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**35.** Find the area bounded by  $y = \sin x$  and  $y = \cos x$  between any two consecutive points of intersection.



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**36.** Find the area of one of the curvilinear triangles bounded by  $y = \sin x$ ,  $y = \cos x$  and x-axis

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**37.** Find the area of the right angled triangle with base  $b$  and altitude  $h$ , using the fundamental theorem of integral calculus.

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**38.** Find the area bounded between the curves  $y^2 - 1 = 2x$  and  $x = 0$

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39. Find the area enclosed by the curve  $y=3x$  and  $y=6x-x^2$



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40. Find the area enclosed between  $y = x^2 - 5x$  and  $y = 4 - 2x$



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41. Find the area bounded between the curves  $y = x^2, y = \sqrt{x}$



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42. Find the area bounded between the curves

$$y^2 = 4ax, x^2 = 4by (a > 0, b > 0).$$



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## Exercise 7 A

1. Evaluate the integrals as limit of a sum.

$$\int_0^5 (x + 1) dx$$



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2. Evaluate the integrals as limit of a sum.

$$\int_0^4 x^2 dx$$



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3. Evaluate the integrals as limit of a sum.

$$\int_0^4 (x + e^{2x}) dx$$



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4. Evaluate the integrals as limit of a sum.

$$\int_0^1 (x - x^2) dx$$

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## Exercise 7 B

1. Evaluate the definite integrals .

$$\int_0^a (a^2x - x^3) dx$$

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2. Evaluate the definite integrals .

$$\int_2^3 \frac{2x}{1+x^2} dx$$

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3. Evaluate the definite integrals .

$$\int_0^{\pi} \sqrt{2 + 2 \cos \theta} d\theta$$



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4. Evaluate the definite integrals .

$$\int_0^{\pi} \sin^3 x \cdot \cos^3 x dx$$



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5. Evaluate the definite integrals .

$$\int_0^3 |1 - x| dx$$



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6. Evaluate the definite integrals .

$$\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1 + e^x} dx$$

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7. Evaluate the definite integrals .

$$\int_0^1 \frac{dx}{\sqrt{3 - 2x}}$$

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8. Evaluate the definite integrals .

$$\int_0^a (a + x - 2\sqrt{a}\sqrt{x}) dx$$



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9. Evaluate the definite integrals .

$$\int_0^{\pi/4} \sec^4 \theta d\theta$$



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10. Evaluate the definite integrals .

$$I = \int_0^3 \frac{x}{\sqrt{x^2 + 16}} dx$$

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11. Evaluate the definite integrals .

$$I = \int_0^1 x \cdot e^{-x^2} dx$$

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12. Evaluate the definite integrals .

$$I = \int_1^5 \frac{dx}{\sqrt{2x-1}}$$

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13. Evaluate the integrals .

$$I = \int_0^4 \frac{x^2}{1+x} dx$$



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14. Evaluate the integrals .

$$\int_{-1}^2 \frac{x^2}{x^2+2} dx$$



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15. Evaluate the integrals .

$$I = \int_0^1 \frac{x^2}{x^2+1} dx$$



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16. Evaluate the integrals .

$$\int_0^{\pi/2} x^2 \sin x dx$$



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17. Evaluate the integrals .

$$\int_0^4 |2 - x| dx$$



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18. Evaluate the integrals .

$$\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$$



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19. Evaluate the integrals .

$$\int_0^{\pi/2} \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} dx$$



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20. Evaluate the limit.

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n+1} + \sqrt{n+2} + \dots + \sqrt{n+n}}{n\sqrt{n}}$$



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21. Evaluate the limit .

$$\lim_{n \rightarrow \infty} \left[ \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{6n} \right]$$



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22. Evaluate the limit .

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left[ \frac{\tan(\pi)}{4n} + \frac{\tan(2\pi)}{4n} + \dots + \frac{\tan(n\pi)}{4n} \right]$$



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23. Evaluate the limit .

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i^3}{i^4 + n^4}$$



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24. Evaluate the limit .

$$Lt_{n \rightarrow \infty} \sum_{i=1}^n \frac{i}{n^2 + i^2}$$



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25. Evaluate the limit .

$$Lt_{n \rightarrow \infty} \frac{1 + 2^4 + 3^4 + \dots + n^4}{n^5}$$



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26. Evaluate the limit .

$$\lim_{n \rightarrow \infty} l \left[ \left(1 + \frac{1}{n^2}\right) \left(1 + \frac{2^2}{n^2}\right) \dots \dots \left(1 + \frac{n^2}{n^2}\right) \right]^{\frac{1}{n}}$$



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27. Evaluate the limit .

$$\lim_{n \rightarrow \infty} \frac{(n!)^{\frac{1}{n}}}{n}$$



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**28.** Evaluate the integral

$$\int_0^{\pi/2} \frac{dx}{4 + 5 \cos x}$$



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**29.** Evaluate the integral

$$\int_a^b \sqrt{(x - a)(b - x)} dx$$



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**30.** Evaluate the integral

$$\int_0^{1/2} \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx$$

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**31.** Evaluate the integral

$$I = \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

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**32.** Evaluate the integral

$$\int_0^{\pi/2} \frac{a \sin x + b \cos x}{\sin x + \cos x} dx$$

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**33.** Evaluate the integral

$$\int_0^a x(a-x)^n dx$$



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**34.** Evaluate the integral

$$\int_0^2 x\sqrt{2-x} dx$$



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**35.** Evaluate the integral

$$\int_0^{\pi} x \sin^3 x dx$$





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$$36. \int_0^{\pi} \frac{x}{1 + \sin x} dx.$$



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37. Evaluate the integral

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$



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**38.** Evaluate the integral

$$\int_0^1 \frac{\log(1+x)}{1+x^2} dx$$



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**39.** Evaluate the integral

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$



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**40.** Evaluate the integral

$$\int_0^{\pi/4} \log(1 + \tan x) dx$$





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41. Evaluate the integral

$$\int_0^{\frac{3}{2}} |x \sin \pi x| dx$$



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42. Evaluate the integral

$$\int_0^1 \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx$$



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**43.** Evaluate the integral

$$\int_0^1 x \tan^{-1} x dx$$



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**44.** Evaluate the integral

$$\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$



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**45.** Suppose that  $f: R \rightarrow R$  is a continuous periodic function and  $T$  is the period of it . Let  $a \in R$  . Then

prove that for any positive integer  $n$

$$\int_0^{a+nT} f(x)dx = n \int_a^{a+T} f(x)dx$$



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## Exercise 7 C

1. Find the value of the integral

$$\int_0^{\pi/2} \sin^{10} x dx$$



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2. Find the value of the integral

$$\int_0^{\pi/2} \sin^{10} x dx$$



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3. Find the value of the integral

$$\int_0^{\pi/2} \cos^7 x \cdot \sin^2 x dx$$



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4. Find the value of the integral

$$\int_0^{\pi/2} \sin^4 x \cdot \cos^4 x dx$$



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5. Find the value of the integral

$$\int_0^{\pi} \sin^3 x \cos^6 x dx$$



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6. Find the value of the integral

$$\int_0^{2\pi} \sin^2 x \cos^4 x dx$$



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7. Find the value of the integral

$$\int_{-\pi/2}^{\pi/2} \sin^2 \theta \cdot \cos^7 \theta d\theta$$



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8. Find the value of the integral

$$\int_{-\pi/2}^{\pi/2} \sin^3 \theta \cdot \cos^3 \theta d\theta$$



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9. Find the value of the integral

$$\int_0^a x (a^2 - x^2)^{\frac{7}{2}} dx$$

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10. Find the value of the integral

$$\int_0^2 x^{3/2} \sqrt{2-x} dx$$

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11. Evaluate the integral

$$\int_0^1 x^5 (1-x)^{5/2} dx$$

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12. Evaluate the integral

$$\int_0^4 (16 - x^2)^{5/2} dx$$



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13. Evaluate the integral

$$\int_{-3}^3 (9 - x^2)^{3/2} x dx$$



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14. Evaluate the integral

$$\int_0^5 x^3 (25 - x^2)^{7/2} dx$$



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15. Evaluate the integral

$$\int_{-\pi}^{\pi} \sin^8 x \cos^7 x dx$$

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16. Evaluate the integral

$$\int_3^7 \sqrt{\frac{7-x}{x-3}} dx$$

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17. Evaluate the integral

$$\int_2^6 \sqrt{(6-x)(x-2)} dx$$



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18. Evaluate the integral

$$\int_0^{\pi/2} \tan^5 x \cos^8 x dx$$



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19. Evaluate the integral

$$\int_0^1 x^{7/2} (1-x)^{5/2} dx$$

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20. Evaluate the integral

$$\int_0^{\pi} (1 + \cos x)^3 dx$$

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21. Evaluate the integral

$$\int_4^9 \frac{dx}{\sqrt{(9-x)(x-4)}}$$

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22. Evaluate the integral

$$\int_0^5 x^2 (\sqrt{5-x})^7 dx$$



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23. Evaluate the integral

$$\int_0^{2\pi} (1 + \cos x)^5 (1 - \cos x)^3 dx$$



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**Exercise 7 D**

1. Find the area of the region enclosed by the given curves .

$$y = \cos x, y = 1 - \frac{2x}{\pi}$$



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2. Find the area of the region enclosed by the given curves .

$$y = \cos x, y = \sin 2x, x = 0, x = \frac{\pi}{2}$$



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3. Find the area of the region enclosed by the given curves .

$$y = x^3 + 3, y = 0, x = -1, x = 2$$



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4. Find the area of the region enclosed by the given curves .

$$y = e^x, y = x, x = 0, x = 1$$



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5. Find the area of the region enclosed by the given curves .

$$y = \sin x, y = \cos x, x = 0, x = \frac{\pi}{2}$$



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6. Find the area of the region enclosed by the given curves .

$$x = 4 - y^2, x = 0$$



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7. Find the area enclosed with in the curve

$$|x| + |y| = 1$$



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8. Find the area enclosed with in the curve

$$x = 2 - 5y - 3y^2, x = 0$$



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9. Find the area enclosed with in the curve

$$x^2 = 4y, x = 2, y = 0$$



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10. Find the area enclosed with in the curve

$$y^2 = 3x, x = 3$$



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11. Find the area enclosed with in the curve

$$y = x^2, y = 2x$$



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12. Find the area enclosed with in the curve

$$y = \sin 2x, y = \sqrt{3} \sin x, x = 0, x = \frac{\pi}{6}$$



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13. Find the area enclosed with in the curve

$$y = x^2, y = x^3$$



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14. Find the area enclosed with in the curve

$$y = 4x - x^2, y = 5 - 2x$$



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**15.** Find the area in Sq. units bounded by the x-axis ,  
part of the curve  $y = 1 + \frac{8}{x^2}$  and the ordinates  $x=2$   
and  $x=4$

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**16.** Find the area of the region bounded by the  
parabolas  $y^2 = 4x$  and  $x^2 = 4y$

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17. Find the area bounded by the curve  $y = \ln x$  the X-axis and the straight line  $x=e$



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18.  $y = x^2 + 1, y = 2x - 2, x = -1, x = 2$



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19.  $y^2 = 4x, y^2 = 4(4 - x)$



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20.  $y = 2 - x^2, y = x^2$



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21. Show that the area enclosed between the curve

$y^2 = 12(x + 3)$  and  $y^2 = 20(5 - x)$  is  $64\sqrt{\frac{5}{3}}$



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22. Find the area of the region

$\{(x, y) / x^2 - x - 1 \leq y \leq -1\}$



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**23.** The circle  $x^2 + y^2 = 8$  is divided into two parts by the parabola  $2y = x^2$ . Find the area of both the parts.



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**24.** Show that the area of the region bounded by  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (ellipse) is  $\pi ab$ . Also deduce the area of the circle  $x^2 + y^2 = a^2$



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25. Find the area of region enclosed by curves

$$y = \sin \pi x, y = x^2 - x, x = 2$$



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26. Let AOB be the positive quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ with } OA=a, OB=b.$$

Then show that the area bounded between the

chord AB and the arc AB of the ellipse is  $\frac{(\pi - 2)ab}{4}$



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27. Prove that the curves  $y^2 = 4x$  and  $x^2 = 4y$  divide the area of the square bounded by  $x = 0$ ,  $x = 4$ ,  $y = 4$  and  $y = 0$  into three equal parts.



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### Very Short Answer Question

1.  $\int_2^3 \frac{2x}{1+x^2} dx$



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$$2. \int_0^{\pi} \sqrt{2 - 2 \cos \theta} d\theta$$



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$$3. \int_0^2 |1 - x| dx$$



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## Short Answer Question

$$1. \int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$$



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2. Evaluate the integral

$$I = \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$



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Long Answer Question

1. 
$$\int_0^{\pi/2} \frac{dx}{4 + 5 \cos x}$$



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2. Evaluate the integral

$$\int_0^{\pi} \frac{x}{1 + \sin x} dx$$



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3. Evaluate  $\int_{\pi/6}^{\pi/3} \frac{\sqrt{x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$



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