



### MATHS

# BOOKS - VIKRAM PUBLICATION ( ANDHRA PUBLICATION)

## **ELLIPSE**

**Solved Problems** 

1. Find the eccentricity, co ordinates of foci,

length of latus rectum and equation of



**2.** Find the eccentricity, co ordinates of focilength of latus rectum and equation of directrices of the folloeing ellipses.

$$3x^2 + y^2 - 6x - 2y - 5 = 0$$

**3.** Find the equation of the elipse referred to its major and minor axes as the coordinate axes x, y respectively with latus rectum of length 4 and the distance between foci  $4\sqrt{2}$ .



4. If the latus rectum of an ellipse is equal to

the half of minor axis, then find its eccentricity.



**5.** If  $0_1 0_2$  are the eccentric angles of the extremeties of a focal chord (other that the verticles) of the ellipse  $rac{x^2}{a^2}+rac{y^2}{b^2}=1~(a>b)$ and e its its eccentricity. Then show that  $e\cos{(0_1+0_2)\over 2}=\cos{(0_1-0_2)\over 2}$ Watch Video Solution

6. If  $0_10_2$  are the eccentric angles of the extremeties of a focal chord (other that the verticles) of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (a > b

and e its its eccentricity. Then show that

$$rac{e+1}{e-1} = \mathrm{cot}igg(rac{0_1}{2}igg). \ \mathrm{cot}igg(rac{0_2}{2}igg).$$

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**7.** C is the centre, AA' and BB' are major and minor axis of the ellipse.

 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$  If PN is the ordinate f a point P on the ellipse then show that  $\frac{(PN)^2}{(A'N)(AN)} = \frac{(BC)^2}{(CA)^2}$ 

**8.** S and T are the foci of an ellipse and B is one end of the minor axis. IF STB is an equilateral traingle , then find the eccentricity of the ellipse.

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**9.** Show that among the points on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (a > b),(-a > 0) is the fatthest point and (a,0) is the nearest point from the focus(ae,0).



**10.** The orbit of the Earth is an ellipse with eccentricity  $\frac{1}{60}$  with the sun at one of its foci, the major axis being approximately  $186 \times 10^6$  miles in length. Find the shortest and longest distance of the Earth from the sun.

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11. Find the equation of the tangent and normal to the ellipse  $9x^2 + 16y^2 = 144$  at the

#### end of the latus rectum in the first quadrant.

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12. If a tangent to the ellipse 
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
  
( $a > b$ ) meets its major axis and minor axis at  
M and N respectively. Then prove that  
 $\frac{a^2}{(CM)^2} + \frac{b^2}{(CN)^2} = 1$ . Where C is the centre

of the ellipse.

#### 13. Find the condition for the line

lx+my+n=0 to be a tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ Watch Video Solution

#### 14. Find the condition for the line

lx+my+n=0 to be a normal to the ellipse  $\frac{x^2}{x^2} + \frac{y^2}{y^2} = 1$ 

$$\frac{1}{a^2} + \frac{b}{b^2} =$$

15. If the normal at one end of a latus rectum of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  passes through one end of the minor axis, then show that  $e^4 + e^2 = 1$ [e is the eccentricity of the ellipse]

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**16.** If PN is the ordinate of a point P on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the tangent at P meets the X-axis at T then show (CN) (CT) = $a^2$  where C is the centre of the ellipse.



#### 17. Show that the points of intersection of the

perpendicular tangets to an ellipse lie on a

circle.

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Exercise 4 A

**1.** Find the equation of the ellipse with focus at (1,-1)  $e=\frac{2}{3}$  and directrix as x + y + 2 = 0. Watch Video Solution

2. Find the equation of the ellise in the standard form whose distance between foci is 2 and the length of latus rectum is  $\frac{15}{2}$ .



**3.** Find the equation of the ellipse in the standard form such that distance between foci is 8 and distance between directrices is 32.



**4.** Find the eccentricity of the ellipse, (in standard form), if its length of the latus rectum is equal to half of its major axis.



5. The distance of a point on the ellipse  $x^2 + 3y^2 = 6$  from its centre is equal to 2. Find the eccentric angles.



**6.** Find the equation of ellipse in the standard form. If it passes through the points(-2,2) and (3,-1).

**7.** If the ends of major axis of an ellipse are (5,0) and (-5,0). Find the equation of the ellipse in the standard form if its focus lie on the line 3x-5y-9=0.



8. If the length of the major axis of an ellipse is

three times the length of its minor axis then

find the eccentricity of the ellipse.



**9.** Find the length of major axis, minor axis, latus rectum, eccentricity co-ordinates of centre, foci and the equations of directrices of the following ellipse.

 $9x^2 + 16y^2 = 144$ 

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**10.** Find the length of major axis, minor axis, latus rectum, eccentricity co-ordinates of centre, foci and the equations of directrices of

the following ellipse.

$$4x^2 + y^2 - 8x + 2y + 1 = 0$$

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**11.** Find the length of major axis, minor axis, latus rectum, eccentricity co-ordinates of centre, foci and the equations of directrices of the following ellipse.

$$x^2 + 2y^2 - 4x + 12y + 14 = 0$$

12. Find the equation of the ellipse in the form

$$rac{\left(x-h
ight)^2}{a^2}+rac{\left(y-k
ight)^2}{b^2}=1.$$
 Given the

following data.

Centre(2,-1) one end of major axis (2,-5),  $e=\frac{1}{3}$ .

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13. Find the equation of the ellipse in the form

$$rac{\left(x-h
ight)^2}{a^2}+rac{\left(y-k
ight)^2}{b^2}=1.$$
 Given the

following data.

Centre(4,-1) one end of major axis is (-1,-1), and

passing through (8,0).



14. Find the equation of the ellipse in the form

$$rac{\left(x-h
ight)^2}{a^2}+rac{\left(y-k
ight)^2}{b^2}=1.$$
 Given the

following data.

Centre(0,-3),  $e=\frac{2}{3}$ , semi -minor axis =5.

15. Find the equation of the ellipse in the form

$$rac{\left(x-h
ight)^2}{a^2}+rac{\left(y-k
ight)^2}{b^2}=1.$$
 Given the

following data.

Centre(2,-1),  $e=\frac{1}{2}$ , length of latus rectum 4.

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16. Find the radius of the circle passing through the foci of an ellipse  $9x^2 + 16y^2 = 144$  and having least radius.

**17.** A man running on a race course notices that the sum of the distances of the two flag posts from him is always 10m and the distance between the flag posts is 8m. Find the equation of the race course traced by the man.



**18.** A line of fixed length (a+b) moves so that its ends are always on two perpendicular straight lines fixed. Prove that a marked point on the line , which divides this line in to portions of lengths a and b describes an ellipse when a=8, b=12.

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19. Prove that the equation of the chord joining the points  $\alpha$  and  $\beta$  on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

**1.** Find the equation of tangent and normal to the ellipse  $x^2 + 8y^2 = 33$  at (-1,2).

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2. Find the equation of tangent and normal to

the ellipse

$$x^2+2y^2-4x+12y+14=0$$
 at (2,-1)

**3.** Find the equation of the tangents to  $9x^2 + 16y^2 = 144$  , which makes equal intercepts on the co-ordianate axis.



**4.** Find the co-ordinates for the points on the ellipse  $x^2 + 3y^2 + 37$  at which the normal is

parallel to the line 6x-5y=2.



5. Find the value of k if 4x+y+k=0 is a tangent

to the ellipse  $x^2 + 3y^2 = 3$ .

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**6.** Find the condition for the line x cos  $\alpha$ +y sin

lpha=p to be a tangent to the ellipse $rac{x^2}{a^2}+rac{y^2}{b^2}=1\,.$ 

7. Find the equations of tangent and normal to the ellipse  $2x^2 + 3y^2 = 11$  at the point whose ordinate is 1.



8. Find the equation to the tangents to the ellipse,  $x^2 + 2y^2 + 3$  drawn from the point (1,2) and also find the angle between these tangents.

**9.** Find the equations of tangents to the ellipse  $2x^2 + y^2 = 8$  which are Parallel to x-2y-4=0

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10. Find the equations of tangents to the

ellipse  $2x^2+y^2=8$  which are

perpendicular to x+y+2=0

**11.** Find the equations of tangents to the ellipse  $2x^2 + y^2 = 8$  which are which makes an angle  $\frac{\pi}{4}$  with x-axis.

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12. A circle of radius 4, is concentric with the ellipse  $3x^2 + 13y^2 = 78$ . Prove that a common tangent is inclined to the major axis at an angle  $\frac{\pi}{4}$ .

**13.** Show that the foot of the perpendicular drawn from the centre on any tangent to the ellipse lies on the curve

$$\left(x^2+y^2
ight)^2 = a^2x^2+b^2y^2.$$

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**14.** Show that the locus of the feet of the perpendiculars drawn from foci to any tangent of the ellipse is the auxilliary circle.

15. The tangent and normal to the ellipse  $x^2 + 4y^2 = 4$  at a point  $(\theta)$  on its meets the major axis in Q and R respectively. If  $0 < \theta < \frac{x}{2}$  and QR=2, then show that  $\theta = \cos^{-1}\left(\frac{2}{3}\right)$ .