MATHS

BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)

PARABOLA

Solved Problems

1. Find the coordinates of the vertex and focus, and the equtions of the directrix and axes of

the following parabolas.

A. $y^2 = 16x$

B. $x^2 = -4y$

C. $3x^2 - 9x + 5y - 2 = 0$

D. $y^2 - x + 4y + 5 = 0$

Answer: B

View Text Solution

2. Find the equation of the parabola whose vertex is (3,-2) and focus is (3,1).

Watch Video Solution

3. Find the coordinates of the points on the parabola $y^2 = 2x$ whose focal distance is $\frac{5}{2}$.

Watch Video Solution

4. Find the equation of the parabola passing through the points (-1,2),(1,-1) and (2,1) and having its axis parallel to the X-axis.

Watch Video Solution

5. A double ordinate of the curve $y^2 = 4ax$ is of lengh 8a. Prove that the line from the vertex its ends are at right angles.

Watch Video Solution

6. If the coordinates of the ends of a focal chord of the parabola $y^2 = 4ax$ are $(x_1, y_1) \text{and} (x_1, y_2)$, then prove that $x_1 x_2 = a^2, y_1 y_2 = 4a^2.$

Watch Video Solution

7. For a focal chord PQ of the parabola $y^2 = 4ax$ if SP =l and SQ=l then prove that $\frac{1}{l} + \frac{1}{l} + \frac{1}{a}.$

Watch Video Solution

8. If Q is the foot of the perpendicular from a point p on the parabola $y^2 = 8(x - 3)$ to its directrix. S is an equilateral triangle then find the lengh of side of the triangle.

Watch Video Solution

9. Find the condition for the straight line lx+my+n=0 to be a tangent to the parabola $y^2 = 4ax$ and find the coordinates of the point of contact.

Watch Video Solution

10. Show that straight line 7x+6y=13 is a tangent to the parabola $y^2 - 7x - 8y + 14 = 0$ and find the point of contact.

Watch Video Solution

11. Prove that the normal chord at the point other than origin whose ordinate is equal to

its abscissa subtends a right angle at the focus.

Watch Video Solution

12. From an external point P tangents are drawn to the parabola $y(2) = 4ax$ and these tangents make angles $\theta_1, \theta_2 with its a\xi ssucht\hat{}$ cot theta_(1)+cot theta_(2)` is a constant 'a' show that P lies on a horizontal line.

Watch Video Solution

13. show that the common tangent to the parabola $y^2 = 4ax \text{and} x^2 = 4by$ is $xa^{1/3} + yb^{1/3} + a^{2/3}b^{2/3} = 0.$

Watch Video Solution

14. Prove that the area of the triangle formed by the tangents at $(x_1, y_1), (x_2) \text{and} (x_3, y_3)$ to the parabola $y^2 = 4ax (a > 0)$ is $\frac{1}{16a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$ sq.units.

Watch Video Solution

15. Prove that two parabolas $y_2 = 4ax \text{and} x^2 = 4by$ intersect (other than the origin)

at an angle of $\tan^{-1}\left[\frac{3a^{1/3}b^{1/3}}{2\left(a^{2/3}+b^{2/3}\right)}\right]$.

Watch Video Solution

16. Prove that the orthocenter of the triangle formed by any three tangents to a parabola lies on the directrix of the parabola.

View Text Solution

Exercise 3 A I

1. Find the vertex and focus of

$$4y^2 + 12x - 20y + 67 = 0$$

Watch Video Solution

Exercise 3 A

1. Find the vertex and focus of

$x^2 - 6x - 6y + 6 = 0$

Watch Video Solution

2. Find the equations of axis and directrix of the parabola $y^2 + 6y - 2x + 5 = 0$.

Watch Video Solution

3. Find the equations of axis and directrix of the parabola $4x^2 + 12x - 20y + 67 = 0$

Watch Video Solution

4. Find the equation of the parabola whose focus is S (1,-7) and vertex is A(1,-2).

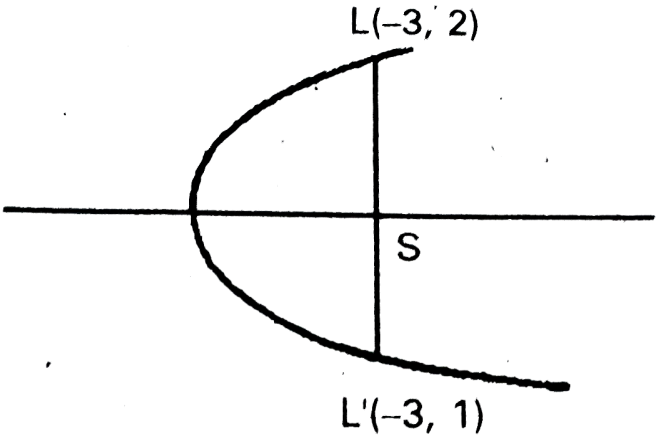
Watch Video Solution

5. Find the equation of the parabola whose focus is S (3,5) and vertex is A(1,3).

Watch Video Solution

6. Find the equation of the parabola whose latus rectum is the line segment of joining the

points (-3,2) and (-3,1).



Watch Video Solution

7. Find the position (interior or exterior or on) of the following points with respect to the parabola $y^2 = 6x$

(i) $(2, 3)$

Watch Video Solution

8. Find the co-ordinates of the point on the parabola $y^2 = 8x$ whose focal distance is 10.

Watch Video Solution

9. If $\left(\frac{1}{2}, 2\right)$ is one extermity of a focal chord of the parabola $y^2 = 8x$. Find the co-ordinates of the other extremity.

Watch Video Solution

10. Prove that the parabola $y^2 - 4ax, (a > o)$

Nearest to the focus is its vertex.

View Text Solution

Exercise 3 A Ii

1. Find the locus of the points of trisection of double ordinate of a parabola $y^2 = 4x(a > 0)$

Watch Video Solution

2. Find the equation of the parabola whose vertex and focus are on the positive X-axis at a distance of a and a' from the origin respectively.

Watch Video Solution

3. If L and L' are the ends of the latus rectum of the parabola $x^2 = 6y$ find the equations of

OL and OL' where 'O' is the origin.Also find the angle between them.

Watch Video Solution

4. Find the equation of the parabola whose axis is parallel to X-axis and which passes through these points.

(-2,1),(1,2), and (-1,3)

Watch Video Solution

5. Find the equation of the parabola whose axis is parallel to Y-axis and which passes through the points (4,5),(-2,11) and (-4,21).

Watch Video Solution

Exercise 3 A Iii

1. Find the equation of the parabola whose focus is (-2,3) and directrix is the line 2x+3y-

4=0. Also find the length of the latus rectum and the equation of the axis of the parabola.

Watch Video Solution

2. Prove that the area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_2)(y_3 - y_1)|$ sq. units where y_1, y_2, y_3 are the ordinates of its vertices.

Watch Video Solution

3. Find the co-ordinates of the vertex and focus the equation of the directrix and axis of the following parabolas.

View Text Solution

Exercise 3 B I

1. Find equation of the tangent and normal to the parablola $y^2 = 6x$ at the positive end of the latus rectum.

Watch Video Solution

2. Find the equation of the tangent and normal to the parabola $x^2 - 4x - 8y + 12 = 0$ at $\left(4, \frac{3}{2}\right)$

Watch Video Solution

3. Find the value of k if the line 2y=5x+k is a tangent to the parabola $y^2 = 6x$

Watch Video Solution

4. Find the equation of the normal to the parabola $y^2 = 4x$ which is parallel to y-2x+5=0.

View Text Solution

5. Show that the line 2x-y+2=0 is a tangent to the parabola $y^2 = 16x$. Find the point of cotact also.

View Text Solution

6. Find the equation of tangent to the parabola $y(2) = 16x$ inclined at an angle 60° with its axis and also find the point of contact.

View Text Solution

Exercise 3 B Ii

1. Find the equation of tagents to the parabola $y(2) = 16x$ which are parallel and perpendicular respectively to the line 2x-y+5=0,

also find the co-ordinates of the points of contact also.

View Text Solution

2. If lx+my+n=0 is a normal to the parabola $y(2) = 4ax$, then show that $al^3 + 2alm^2 + nm^2 = 0.$

View Text Solution

3. Show that the equations of common tangents to the circle $x^2 + y^2 = 2a^2$ and the parabola $y^2 = 8ax$ are $y = \pm(x + 2a)$.

View Text Solution

4. Find the condition for the line y=mx+c to be a tangent to the parabola $x^2 = 4ay$.

Watch Video Solution

5. Three normals are drawn (k,0) to the parabola $y(2) = 8x$ one of the normal is the axis and the remaining two normals are perependicular to each other, then find the value of k.

View Text Solution

6. Show that the locus of point of intersection of perpendicular tangents to the parabola $y(2) = 4ax$ is the directrix x+a=0.

Watch Video Solution

7. Two parabolas have the same vertex and equal lengh of latus rectum such that their axes are at right angle. Prove that the common tangents touch each at the end of latus rectum.

Watch Video Solution

8. Show that the foot of the perpendicular from focus to the tangent of the parabola

$y^2 = 4ax$ lies on the tangent at vertex.

View Text Solution

9. Show that the tangent at one extremity of a focal chord of a parabola is parallel to the normal at the other extremity.

Watch Video Solution

Exercise 3 B Iii

1. If the normal at t_1 on the parabola $y^2 = 4ax$ meet it again at t_2 on the curve then $t_1(t_1 + t_2) + 2 =$

Watch Video Solution

2. From an external point P tangents are drawn to the parabola $y(2) = 4ax$ and these tangents make angles $\theta_1, \theta_2 with it sa\xi ssucht\hat{}$ cot theta_(1)+cot theta_(2)` is a constant 'a' show that P lies on a horizontal line.

Watch Video Solution

3. Show that the common tangent to the circle $2x^2 + 2y^2 = a^2$ and the parabola $y^2 = 4ax$ intersect at the focus of the parabola $y^2 = -4ax$.

Watch Video Solution

4. The sum of the ordinates of two points on $y(2) = 4ax$ is equal to the sum of the ordinates of two other points on the same

curve. Show that the chord joining the first two points is parallel to the chord joining the other two points.

Watch Video Solution

5. If normal chord a point 't' on the parabola $y(2) = 4ax$ subtends a right angle at vertex, then prove that $t = +_{\sqrt{2}}$

Watch Video Solution