MATHS

BOOKS - VIKRAM PUBLICATION (ANDHRA PUBLICATION)

QUADRATIC EXPRESSIONS

Solved Problems

1. Theorem : The roots of $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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2. Theorem : Let a, b, c $\in$ R and a $\neq$ 0. Then the roots of $ax^2 + bx + c = 0$ are non-real complex numbers if and only if $ax^2 + bx + c$ and a have the same sign for all $x \in R$.

3. Theorem : If the roots of $ax^2 + bx + c = 0$ are real and equal to $\alpha = \dfrac{-b}{2a}$, then for $\alpha \neq x \in R, ax^2 + bx + c$ and 'a' have the same sign.

4. Theorem : Let α, β be the real roots of $ax^2 + bx + c = 0$ and $\alpha < \beta$. Then

(i) $x \in R, \alpha < x < \beta \Rightarrow ax^2 + bx + c$ and $'a'$ have opposite signs.

(ii) $x \in R, x < \alpha$ or $x > \beta \Rightarrow ax^2 + bx + c$ and 'a' have the same sign.

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5. Theorem : Let $f(x) = ax^2 + bx + c$ be a quadratic function.

If $a > 0$ then f(x) has minimum value at $x = \dfrac{-b}{2a}$ and

the minimum value =

$\dfrac{4ac - b^2}{4a}$

6. Theorem : Let $f(x) = ax^2 + bx + c$ be a quadratic function.

If $a < 0$ then f(x) has maximum value at $x = \dfrac{-b}{2a}$ and the maximum value

$= \dfrac{4ac - b^2}{4a}$

7. Theorem : A necessary and sufficient condition for the quadratic equations

$a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ to

have a common roots is

$$(c_1a_2 - c_2a_1)^2 = (a_1b_2 - a_2b_1)(b_1c_2 - b_2c_1)$$

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8. Find the roots of the equation

$$3x^2 + 2x - 5 = 0$$

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9. Find the roots of the equation

$$4x^2 - 4x + 17 = 3x^2 - 10x - 17$$

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10. Find the roots of the following equations.

$$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$$

11. Find the nature of the roots of

$$4x^2 - 20x + 25 = 0$$

12. Find the nature of the roots of $3x^2 + 7x + 2 = 0$

13. For what values of m, the equation

$x^2 - 2(1 + 3m)x + 7(3 + 2m) = 0$ will have equal

roots ?

14. If α and β are the roots of $ax^2 + bx + c = 0$, find

the value of $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ in terms of a,b,c.

15. Show that the roots of the equation

$x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational,

given that p,q,r are rational.

16. Form a quadratic equation whose roots are $2\sqrt{3} - 5$ and $-2\sqrt{3} - 5$.

17. Find the quadratic equation, the sum of whose roots is 1 and the sum of squares of the roots is 13.

18. Let α and β be the roots of the quadratic equation $ax^2 + bx + c = 0, c \neq 0,$ then form the quadratic equation whose roots are $\dfrac{1-\alpha}{\alpha}$ and $\dfrac{1-\beta}{\beta}$.

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19. Solve $x^{2/3} + x^{1/3} - 2 = 0$

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20. Solve $7^{1+x} + 7^{1-x} = 50$ for real x.

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21. Solve $\sqrt{\dfrac{x}{1-x}} + \sqrt{\dfrac{1-x}{x}} = \dfrac{13}{6}$

22. Find all number which exceed their square root by 12

23. If $x^2 + 4ax + 3 = 0$ and $2x^2 + 3ax - 9 = 0$ have a common root, then find the values of a and the common root.

24. Prove that there is unique pair of consecutive positive odd integers such that the sum of their squares is 290 and find it.

25. The cost of a piece of cable wire is Rs. 35/-, If the length of the piece of wire is 4 meters more and each meter costs, Rs. 1/- less, the cost would remain un-changed. What is the length of the wire ?

26. One fourth of a herd of goats was seen in the forest. Twice the square root of the number in the herd had gone up the hill and the remaining 15 goats were on the bank of the river. Find the total number of goats.

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27. In a cricket match Anil took one wicket less than twice the number of wickets taken by Ravi. If the product of the number of wickets taken by them is 15, find the number of wickets taken by each of them.

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28. Some points on a plane are marked and they are connected pairwise by line segments. If the total number of line segments formed is 10, find the number of marked points on the plane.

29. Suppose the that quadratic equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root. Then show that $a^3 + b^3 + c^3 = 3abc$.

30. For what values of x, the expression $x^2 - 5x - 14$ is positive ?

31. For what values of x, the expression $6x^2 + 2x^{-3}$ is negative ?

32. Find the value of x at which the following expressions have maximum or minimum.

$x^2 + 5x + 6$

33. Find the value of x at which the following expressions have maximum or minimum.

$2x - x^2 + 7$

34. Find the maximum or minimum value of the quadratic expression

$2x - 7 - 5x^2$

35. Find the maximum or minimum value of the quadratic expression

$3x^2 + 2x + 17$

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36. Find the changes in the sign of $4x - 5x^2 + 2$ for $x \in R$ and find the extreme value.

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37. Find the the solution set of $x^2 + x - 12 \le 0$ by both algebaric and graphical methods.

38. Find the set of values of x for which the inequalities $x^2 - 3x - 10 < 0, 10x - x^2 - 16 > 0$ hold simultaneously.

39. Solve the inequation $\sqrt{x + 2} > \sqrt{8 - x^2}$.

40. Solve the equation

$$\sqrt{(x-3)(2-x)} < \sqrt{4x^2 + 12x + 11}$$

41. For the inequation

$$\frac{\sqrt{6+x-x^2}}{2x+5} \geq \frac{\sqrt{6+x-x^2}}{x+4}$$

42. Find the maximum value of the function

$$\frac{x^2 + 14x + 9}{x^2 + 2x + 3} \text{ over R.}$$

43. Show that none of the values of the function $\dfrac{x^2 + 34x - 71}{x^2 + 2x - 7}$ over R lies between 5 and 9.

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44. Solve the inequation

$$\sqrt{x^2 - 3x - 10} > (8 - x)$$

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Very Short Answer Questions

1. Form quadratic equation whose root $7 \pm 2\sqrt{5}$

2. Form quadratic equation whose root $-3 \pm 5i$

3. For what values of x, the following expressions are negative ?

$15 + 4x - 3x^2$

1. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the value $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ expressions in terms of a,b,c.

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2. Form quadratic equation whose root $\dfrac{p-q}{p+q}, \dfrac{-p+q}{p-q}, (p \neq \pm q)$

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3. Find the values of m for which the following equations have equal roots ?

i) $x^2 - 15 - m(2x - 8) = 0.$

4. $(m + 1)x^2 + 2(m + 3)x + (m + 8) = 0.$

5. If x is real, prove that $\dfrac{x}{x^2 - 5x + 9}$ lies between 1 and $\dfrac{-1}{11}$.

6. If $x^2 - 6x + 5 = 0$ and $x^2 - 12x + p = 0$ have a common root, then find p.

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7. If x_1, x_2 are the roots of the quadratic equation $ax^2 + bx + c = 0$ and $c \neq 0,$ find the value of $(ax_1 + b)^{-2} + (ax_2 + b)^{-2}$ in terms of a,b,c

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8. Prove that $\dfrac{1}{3x+1} + \dfrac{1}{x+1} - \dfrac{1}{(3x+1)(x+1)}$

does not lie between 1 and 4, if x is real.

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Long Answer Questions

1. $2x^4 + x^3 - 11x^2 + x + 2 = 0$

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2. For what value of x, the following expressions are

positive ?

$$x^2 - 5x + 6$$

3. For what value of x, the following expressions are positive ?

$$3x^2 + 4x + 4$$

4. For what value of x, the following expressions are positive ?

$$4x - 5x^2 + 2$$

5. For what value of x, the following expressions are positive ?

$$x^2 - 5x + 14$$

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6. Determine the range of the $\dfrac{x^2 + x + 1}{x^2 - x + 1}$ expressions.

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7. Theorem: Let $f(x) = ax^2 + bx + c$ be a quadratic function.

If $a > 0$ then f(x) has minimum value at $x = \dfrac{-b}{2a}$ and the minimum value $= \dfrac{4ac - b^2}{4a}$.

8. Theorem: Let $f(x) = ax^2 + bx + c$ be a quadratic function.

If $a < 0$ then f(x) has maximum value at $x = \dfrac{-b}{2a}$ and the maximum value $= \dfrac{4ac - b^2}{4a}$

1. Find the roots of the following equations.

$$x^2 - 7x + 12 = 0$$

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2. Find the roots of the following equations.

$$-x^2 + x + 2 = 0$$

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3. Find the roots of the following equations.

$$2x^2 + 3x + 2 = 0$$

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4. Find the roots of the following equations.

$$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$$

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5. Find the roots of the following equations.

$$6\sqrt{5}x^2 - 9x - 3\sqrt{5} = 0$$

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6. Form quadratic equation whose roots are :

2,5

7. Form quadratic equation whose roots are :

$$\frac{m}{n}, \frac{n}{m} (m \neq 0, n \neq 0)$$

8. Form quadratic equation whose roots are :

$$\frac{p-q}{p+q}, \frac{-p+q}{p-q}, (p \neq \pm q)$$

9. Form quadratic equation whose root $7 \pm 2\sqrt{5}$

10. Form quadratic equation whose roots are :

$-3 \pm 5i$

11. Find the nature of the roots of the following equation, without finding the roots.

$$2x^2 - 8x + 3 = 0$$

12. Find the nature of the roots of the following equation, without finding the roots.

$$9x^2 - 30x + 25 = 0$$

13. Find the nature of the roots of the following equation, without finding the roots.

$$x^2 - 12x + 32 = 0$$

14. Find the nature of the roots of the following equation, without finding the roots.

$$2x^2 - 7x + 10 = 0$$

15. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the values of the following expressions in terms of a,b,c.

$$\frac{1}{\alpha} + \frac{1}{\beta}$$

16. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the values of the following expressions in terms of a,b,c.

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

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17. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the values of the following expressions in terms of a,b,c.

$$\alpha^4\beta^7 + \alpha^7\beta^4$$

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18. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the values of the following expressions in terms of a,b,c.

$$\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha} \right)^2, \quad \text{if } c \neq 0$$

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19. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the values of the following expressions in terms of a,b,c.

$$\frac{\alpha^2 + \beta^2}{\alpha^{-2} + \beta^{-2}}$$

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20. Find the values of m for which the following equations have equal roots ?

i) $x^2 - 15 - m(2x - 8) = 0$.

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21. Find the values of m for which the following equations have equal roots ?

$(m + 1)x^2 + 2(m + 3)x + (m + 8) = 0$.

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22. Find the values of m for which the following equations have equal roots ?

$$x^2 + (m + 3)x + m + 6 = 0.$$

23. Find the values of m for which the following equations have equal roots ?

$$(3m + 1)x^2 + 2(m + 1)x + m = 0$$

24. Find the values of m for which the following equations have equal roots ?

$$(2m + 1)x^2 + 2(m + 3)x + (m + 5) = 0$$

25. If α, and β are the roots of $x^2 + px + q = 0$ form a quadratic equation whose roots are $(\alpha - \beta)^2$ and $(\alpha + \beta)^2$.

26. If $x^2 + bx + c = 0$, $x^2 + cx + b = 0 (b \neq c)$ have a common root, then show that $b + c + 1 = 0$

27. Prove that the roots of $(x - a)(x - b) = h^2$ are always real.

28. Find the condition that one root of the quadratic equation $ax^2 + bx + c = 0$ shall be n times the other, where n is positive integer.

29. Find two consecutive positive even integers, the sum of whose squares is 340.

30. If α, β are the roots of the quadratic equation $ax^2 + bx + c = 0,$ form a quadratic equation whose roots are $\alpha^2 + \beta^2$ and $\alpha^{-2} + \beta^{-2}.$

31. $2x^4 + x^3 - 11x^2 + x + 2 = 0$

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32. Solve $3^{1+x} + 3^{1-x} = 10$

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33. Solve $4^{x-1} - 3.2^{x-1} + 2 = 0$

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34.

$$\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}, \quad \text{when} \ \ x \neq 0 \ \text{and} \ x \neq 3$$

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35.

$$\sqrt{\frac{3x}{x+1}} + \sqrt{\frac{x+1}{3x}} = 2, \quad \text{when} \ \ x \neq 0 \ \text{and} \ x \neq -1$$

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36. Solve

$$2\left(x + \frac{1}{x}\right)^2 - 7\left(x + \frac{1}{x}\right) + 5 = 0, \quad \text{when} \ \ x \neq 0$$

37. Solve

$$\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) + 6 = 0, \quad \text{when} \quad x \neq 0$$

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38. Find a quadratic equation for which the sum of the roots is 7 and the sum of the squares of the roots is 25.

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39. If the quadratic equations $ax^2 + 2bx + c = 0$ and $ax^2 + 2cx + b = 0, (b \neq c)$ have a common root, then show that $a + 4b + 4c = 0$

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40. If $x^2 - 6x + 5 = 0$ and $x^2 - 3ax + 35 = 0$ have common root, then find a.

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41. If the equation $x^2 + ax + b = 0$ and $x^2 + cx + d = 0$ have a

common root and the first equation has equal roots, then prove that $2(b + d) = ac$.

42. Discuss the signs of the following quadratic expressions when x is real.

$$x^2 - 5x + 4$$

43. Discuss the signs of the following quadratic expressions when x is real.

$$x^2 - x + 3$$

44. For what values of x, the following expressions are negative ?

$$x^2 - 7x + 10$$

45. For what values of x, the following expressions are negative ?

$$15 + 4x - 3x^2$$

46. For what values of x, the following expressions are negative ?

$$2x^2 + 5x - 3$$

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47. For what values of x, the following expressions are negative ?

$$x^2 - 5x - 6$$

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48. Find the changes in the sign of the following expressions and find their extreme values.

$$x^2 - 5x + 6$$

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49. Find the changes in the sign of the following expressions and find their extreme values.

$$15 + 4x - 3x^2$$

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50. Find the maximum or minimum of the following expressions as x varies over R.

$$x^2 - x + 7$$

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51. Find the maximum or minimum of the following expressions as x varies over R.

$$12x - x^2 - 32$$

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52. Find the maximum or minimum of the following expressions as x varies over R.

$$2x + 5 - 3x^2$$

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53. Determine the range of the following expressions.

$$\frac{x^2 + x + 1}{x^2 - x + 1}$$

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54. Determine the range of the following expressions.

$$\frac{x + 2}{2x^2 + 3x + 6}$$

55. Determine the range of the following expressions.

$$\frac{(x-1)(x+2)}{x+3}$$

56. Determine the range of the following expressions.

$$\frac{2x^2 - 6x + 5}{x^2 - 3x + 2}$$

57. If the expressions $\dfrac{x - p}{x^2 - 3x + 2}$ takes all real value for $x \in R$, then find the bounts for p.

58. If $c^2 \neq ab$ and the roots of $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal, then show that $a^3 + b^3 + c^3 = 3abc$ or $a = 0$

59. Solve the following inequations by algebraic method.

$$15x^2 + 4x - 4 \leq 0$$

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60. Solve the following inequations by algebraic method.

$$x^2 - 2x + 1 < 0$$

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61. Solve the following inequations by algebraic method.

$$2 - 3x - 2x^2 \geq 0$$

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62. Solve the following inequations by algebraic method.

$$x^2 - 4x - 21 \geq 0$$

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63. Solve the following inequations by graphical method.

$$x^2 - 7x + 6 > 0$$

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64. Solve the following inequations by graphical method.

$$4 - x^2 > 0$$

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65. Solve the following inequations by graphical method.

$15x^2 + 4x - 4 \leq 0$

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66. Solve the following inequations by graphical method.

$x^2 - 4x - 21 \geq 0$

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67. Solve the following inequations.

$$\sqrt{3x - 8} < -2$$

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68. Solve the following inequations.

$$\sqrt{-x^2 + 6x - 5} > 8 - 2x$$

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