MATHS

BOOKS - CALCUTTA BOOK HOUSE MATHS (BENGALI ENGLISH)

VARIATION

Examples

1. If $x \propto \dfrac{1}{y}$ then

    A. $x = \dfrac{1}{y}$

    B. $y = \dfrac{1}{x}$

    C. $xy = 1$

D. xy=non -zero constant

Answer:

2. if $x \propto y$, then

A. $x^2 \propto y^3$

B. $x^3 \propto y^2$

C. $x \propto y^3$

D. $x^2 \propto y^2$

Answer:

3. If $x \propto y$ and y =8 when x =2, if y=16 , then the value of x is

A. 2

B. 4

C. 6

D. 8

Answer:

4. If $x \propto y^2$ and y =4 when x =8 , if x =32 , then the positive value of y is

A. 4

B. 8

C. 16

D. 32

Answer:

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5. If y $\propto \dfrac{1}{x}$, then $\dfrac{y}{x}$ = non -zero constant ,

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6. if x $\propto z$ and y $\propto z$, then xy $\propto$ z .

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7. If $x \propto \dfrac{1}{y}$ and $y \propto \dfrac{1}{z}$, then $x \propto$

8. If $x \propto y$ then x^(n) $\propto$

9. If $x \propto y$ and $x \propto z$, then (y+z) $\propto$

10. If $(y - z) \propto \dfrac{1}{x}$, $(z - x) \propto$ and $(x - y) \propto \dfrac{1}{z}$, then find the sum of three variation constants .

11. If $x \propto y^2$ and $y = 2a \, when \, x = a,$ then find the relation between x and y .

12. If $x \propto y, y \propto z$ and $z \propto x,$ then find the product of three non-zero variation constant.

13. If $x \propto \dfrac{1}{y}$ and $y \propto \dfrac{1}{z}k,$ then find if there be any relation of direct or inverse variation between x and z .

14. If $x \propto yz$ and $y \propto zx$, then prove that z is a non- zero variation constant.

15. If b $\propto a^3$ and a increases in the ratio of 2:3 , then find in what ratio b will be increasesed .

16. If (x+y) $\propto$ (x-y), then prove that $(x^2 + y^2) \propto$ xy .

17. If (x+y) $\propto$ (x-y) , then show that $(x^3 + y^3) \propto (x^3 - y^3)$.

18. A taxi of Bipinbabu travels 14 km Path in 25 minutes . Applying theory of variation find how much path the will go in 5 hours by driving taxi with same speed .

19. A box of sweets is divided among 24 children of class one of your school , they will get 5 sweets each . Calculate by applying theory of variation how many sweets would each get, if the number of the children is reduced by 4 .

20. 50 villages had taken 18 days to dig a pond . Calculate by using theory of variation how many extra persons will be required to dig the pond in 15 days.

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21. y varies directly with square root of x and y =9 when x =9 . Find the value of x when y =6 .

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22. x varies directly with y and inversely with z. when y =5 ,z=9 then x $= \dfrac{1}{6}$. Find the relation among three variation x ,y and z if y =6 and z=$\dfrac{1}{5}$, then calculate the value of x .

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23. x varies directly with y and inversely with z if y y =4 ,z=5, then x=3 . Again ,if y=16 , z=30 , then find the value of x .

24. If (x+y) $\propto$ (x-y) , then prove that (ax +by) $\propto$ (px+qy) where a ,b ,p, q are non -zero variation constant.

25. If$a^2 + b^2 \propto ab,$ then prove that (a+b) prop (a-b).

26. If $\left(x^3 + y^3\right) \propto \left(x^3 - y^3\right)$, then prove that (x+y) $\propto$ (x-y).

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27. If 15 farmers can cultivate 18 bighas of land in 5 days, determine by using theory of variation the number of days required by 10 farmers to cultivate 12 bighas of land .

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28. Volume of a sphere varies directly with the cube of its radius. Three solid spheres having length of $1\frac{1}{2}, 2$ and $2\frac{1}{2}$ metres diameter are melted and a new sphere is formed . Find the length of diameter of the new sphere. [Consider that the volume of sphere remains same before and after melting]

29. y is sum of two variables , one of which varies directly with x and another varies inversely with x . When x=1, then y =-1 and when x =3 ,then y =5 . Find the relation between x and y .

30. If a $\propto$ b and $\propto$ c , then show that $\left(a^3b^3 + b^3c^3 + c^3a^3\right) \propto abc\left(a^3 + b^3 + c^3\right).$

31. To dig a well of x dcm deep , one part of the total expenses varies directly with x and other part varies directly with x^2. If

the expenses of digging wells of 100 dcm and 200 dcm depths are rs5000 and rs12000 respectively , calculate the expenses of digging a well of 250 dcm depth.

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32. Volume of a cylinder is in joint variation with squqre of the length of radius of base and its height . Ratio of radii of bases of two of cylinders $2:3$ and ratio of their height is $5:4$, then find the ratio of their volumes .

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33. An agricultural co- operative society of village Naraynpur has purchased a tractor . Previously 2400 bighas of land were cultivated by 25 ploughs in 36 days . Now half of the land can

be cultivated only by that tractor in 30 days . Calculate by using the theory of variation , the number of ploughs work equaly with one tractor .

34. Volume of a sphere varies directly with cube of length of its radius and surface area of sphere varies directly with the square of the length of radius. Prove that the square of volume of sphere varies directly with cube of its surface area.

35. The value of diamond varies as the square of its weight . A diamond broke into three pieces whose weights are in the ratio

$3:4:5$. For this there is a loss rs 9400. Find the value of the original diamond .

36. A heavy particle while falling freely from rest under gravitational altraction , the distance through which it falls varies directly as the square of the time taken to fall up to the distance . The particle falls through 125 metres in 5 seconds . Through what distance will it fall in 10 seconds ? What distance will it traverse in the 10th second?

37. A locomotive engine without wagons can go 24 miles an hour and its speed is diminished as the square root of the

number of wagons attached with 4 wagons its speed is 20 miles an hour. Find the maximum number of wagons with which the engine can move.

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38. The weekly expenses of a hostel are partly constant and partly vary directly as the number of inmates . If the expenses are rs 2000 when the number of inmates is 120 and rs 1700 when the number is 100 . Find the number of inmates when the expenses are rs 1880 .

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Exercise

1. If $a \propto \dfrac{1}{b}$, then

   A. $a = \dfrac{1}{b}$

   B. $b = \dfrac{1}{a}$

   C. $ab = 1$

   D. ab=non-zero constant.

Answer: D

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2. If $a \propto b$, then

   A. $a^2 \propto b^3$

   B. $a^2 \propto b^2$

C. $a^3 \propto b^2$

D. $a \propto \sqrt{b}$

Answer: B

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3. If x $\propto$ y andy =20 when x=10. if y=10 then the value of x will

be

A. 2

B. 4

C. 5

D. 8

Answer: C

4. If $x \propto \sqrt{y}$ and y=16 when x =3 . If y =64 ,then the positive value of x will be

A. 4

B. 6

C. 8

D. 10

Answer: B

5. If $x^3 \propto \sqrt{y}$ and $y = 16$ when x=4 then y=1 when x=2 .

6. If x varies inversely with y, then the value of (x+y) will be least

when x =

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7. If ax $+\dfrac{b}{y} \propto cx + \dfrac{d}{y}$ (where a ,b ,c ,d are constants),then xy=

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8. If $\left(A^2 + B^2\right) \propto \left(A^2 - B^2\right)$ then show that A $\propto$

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9. If $x^2 \propto a^2$ then $a \propto$

10. If x $\propto \dfrac{1}{\sqrt{a}}$, then $a \propto$

11. If $x^2 \propto yz, y^2 \propto zx$ and $z^2 \propto xy$, then find the product of three variation constant

12. y $\propto x^2$ and y=9 when x =9 . If y =4 then find the value of x .

13. Given that A =B +C , where B $\propto x^2$ and $C \propto x^3$. If x =1 , then A =0 and if x=-1,then A=2 . Express A as a function of x .

14. If a $\propto b^2$ and $(1 + b) \propto \sqrt{©}$ and if $a = 1, then c = 9$ and $b = 5$, then find the value of a with respect of c .

15. If A varies directly with B and C both , then show that A varies directly with (B-C) .

16. If $\dfrac{a}{b} \propto (a+b)$ and $\dfrac{b}{a} \propto (a-b)$, then show that $a^2 - b^2 = $ constant.

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17. If (x+y) $\propto z$ when y is constant and (z+x) $\propto$ y when z is constant . Then prove that $(x + y + z) \propto yz$, when when y and z are both constant.

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18. If x,y,z are variables , but (y+z-x) is a constant and if (x+y-z) $\propto (x + z - y) \propto yz$. Then prove that (x+y+z) $\propto$ yz.

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19. If x $\propto (y + z)$, $y \propto (z + x)$ and $z \propto (x + y)$ and if k, l and m are the variation constants of them respectively , then prove that $\dfrac{k}{k + 1} + \dfrac{l}{l + 1} + \dfrac{m}{m + 1} = 1.$

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20. A variable is the sum of two such quantities one of which varies directly with x and the other varies directly with x^2 . If y =3 when x =1 and y=10 when x=2 , then find the relation between x and y .

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21. A large sphere is made by melting two gold - spheres of raii r and r respectively . Find the radius of the large sphere. [(volume of sphere) $\propto (radius)^3$]

22. 2 men can cultivate 6 acres of land in 4 days . By using theory of variation , find the number of men required to cultivate 18 acres of land in 8 days.

23. 15 men can cultivate 18 bighas of land in 5 days. By using theory of variation, determine the number of bighas to be cultivated by 10 men in 10 days.

24. If 5 men can cultivate 10 acres of land in 9 days,then by usnig theory of variation, find the number of days required to cultivate 30 acres of land by 25 men .

25. In a fort there was reserved enough food of 2400 soldires for 39 days. The fort was reinforced by 800 more soldiers after 15 days without and food. Then how long will the remaining food last ?

26. If 5 men working 6 hours a day can complete a work in 20 days , in how many day s will 15 men complete the work working 8 hours a day ?

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27. The length of a pendulum varies inversely as the square of the number of beats it makes per minute. If a pendulum 16 ft long makes 27 beats per minute, find the length of the pendulum that makes 24 beats per minute by using theory of variation.

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28. If x $\propto yz^2$, $y \propto ab^2$ and z $\propto \dfrac{b}{a}$, then find how x varies with

a and b .

29. if in the variation x =ky and y=k'z, a,b,c and a' ,b',c' be two sets

of value of x,y,z show that $\dfrac{a^2 + b^2 + c^2}{aa' + \prime + \prime} = \dfrac{aa' + \prime + \prime}{a'^2 + b'^2 + c'^2}$

30. If $ax^2 + 2hxy + by^2 \propto z^2$ and (lx+my) $\propto z$ where a,b,h,l,m

are constants , then prove that x $\propto$ y .

31. If $(u^2 + v^2) \propto (x^2 + y^2)$ and $uv \propto xy$ then show that $(u + v) \propto (x + y) when \dfrac{u}{v} + \dfrac{v}{u} = \dfrac{x}{y} + \dfrac{y}{x}$.

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32. The area of circle varies directly as its radius . If the radius be 2 ft inches , the area of the circle is $17\dfrac{1}{9}$ sq-ft ,if the radius be 3 ft 6 inches , then find the area of the circle .

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33. The volume of a pyramid varies jointly as the height and the area of its base , when the area of the base is 60 sq. ft and the height 14 ft , the volume is 280 cubic -feet . What is the area of

the area of the base of a pyramid whose volume is 390 cubic feet and whose height is 26 feet ?

34. The volume of a right circualr cylinder varies as the square of the radius of its base , when its height is constant and as the height when the radius of the base is constant . The volume of a cylinder the radius of whose base and height are respectively 2 metres and 7 metres is 88 cubic metres . The volume and radius of the base of a cylinder are 396 cubic metres and 9 metres respectively . Detemine the height of the cylinder.

35. The distance through which a heavy body falls from rest varies as the square of the time of fall . If the body falls 400 feet in 5 seconds then find the distance through which it will fall in 10 seconds and in the tenth second .

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36. If a stone falls s ft in t seconds from rest s $\propto t^2$. If it is observed to fall 64 ft in 2 seconds , find how far it falls in 4 seconds ?

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37. Pressure (=P)in a liquid varies as the depth (=d) when the density (=D) is constant and it varies as density when depth is

constant .The pressure is 1 when the depth is 32 and the density 1. Find the depth at which the pressere is 2 when the density is 16 .

38. The time of going from one place to another varies directly as the distances and inversely as the speed . Two trains describe distances which are in the ratio of 5:8 and times required are in the ratio of 4:7 . Find the ratio of their speeds .

39. A locomotive engine without wagons can go 35 km per hour and its speed is diaminished by a quantity which varies as the square root of the number of wagons attached , with 9 wagons

its speed is 20 km per hour . Find the least number of wagons which the engine can move.

40. Consumption of coal by an engine varies as the square of its speed. When the speed is 50 km per hour the consumption of coal is 100 kg per hour . If the cost of 1 kg of coal be 25 P and other expenses per hour be rs 4 , find the minimum expenses when the engine runs 300 km .

41. A piece of diamond is broken into three pieces, the weights of the pieces being in the ratio 1: 2:3 . Price of a piece of diamond is proportional to the square of its weight . Find the

price of the original piece , if the loss due to breakage is rs 22000 .

42. Three rings made of gold and set with diamonds are of the same weight .The prices of the rings are rs a, rs b,and rs c and the the weight of diamond varies as the squere of its weight , show that price of one unit weight of diamond is $\frac{1}{2}(a + c - 2b)$ and that of one unit weight of gold is $\frac{1}{2}(9a + 7c + 16b)$. Given that the making charge of each of the rings are the same .

43. The value of diamond varies as the square of their weights and the squqre of the values of rubbies varies as the cube of their weight . A diamond of 'a' carats is worth m times the value of ruby of 'b'carats and both together worth 'c' rupees . Find the values of a diamond and of a ruby each weighing 'n' carats.

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44. The price of a diamond is rs 2000 . The diamond is broken into to equal weights . If the price of a diamond varies as the square of its weights , then find the amound of loss for the breakage.

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45. A valuable piece of stone whose price is rs 15600 breaks into three pieces whose weights are in the ratio 2:3:5 . If the value of the stone varies as the cube of its weights , find the percentage of loss due to breaking .

46. If x $\propto \dfrac{1}{y^2}$ then in what ratio x changes when y changes at a ratio of 9 :5 ?

47. if x $\propto z$ and y $\propto z$, then xy $\propto$ z .

48. if $\left(px^3 + qy^3\right) \propto \left(ax^3 + by^3\right)$, then show that $x \propto y$

49. If $\left(x^6 + \dfrac{1}{y^6}\right) \propto \left(x^6 - \dfrac{1}{y^6}\right)$, then prove that $y \propto \dfrac{1}{x}$.

50. If $\dfrac{1}{y^n} + \dfrac{1}{z^n} \propto \dfrac{1}{Y^n + z^n}$ then prove that $y \propto z$.

51. if x $\propto \dfrac{z}{y^2}$ and $z^2 \propto \dfrac{y}{x}$ then prove that x $\propto \dfrac{1}{y} \propto \left(\dfrac{10}{z}\right)$.

52. If $a \propto b^4$, $b^5 \propto c^6$, $c^7 \propto d^8$ and $d^9 \propto p^4$ then prove that $\dfrac{a}{p} \times \dfrac{b}{p} \times \dfrac{c}{p} \times \dfrac{d}{p}$ is constant.

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53. If $x \propto yz^2$, $y \propto pq^2$ and $z\infty\dfrac{q}{p}$ then prove that $q^4 \propto px$.

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54. If x $\propto y$ and $y \propto z$, then prove that $(x^7 + y^7 + z^7) \propto (x^5y^2 + x^3z^4 + y^5z^2)$

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55. If $x \propto y$ and $y \propto z,$ then show that

$$(x^m + y^m + z^m) \propto \left(x^3 y^{m-3} + y^3 z^{m-3} = z^3 x^{m-3}\right) \quad (\text{m} \quad =$$

constant)

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56. $(px + qy + rz) \propto \left(m\sqrt{yz} + n\sqrt{zx} + t\sqrt{xy}\right)$ (where

P,q,r,m,n,t are constants)

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57. If $a \propto b^2, b^3 \propto c^4, c^5 \propto d^6$ and $d^7 \propto e^4$, then show that $abcd \propto e^4$

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58. In the speech of a professor , the number of students present in the speech directly varies with his capacity of explaining the subjects abnd inversely with the number of speech delivered . If 54 students were present in 10 speeches of professor A , then what number of students will present in the speech delivered by B when the capacity of explanation of professor B is 3 times then that of A ?

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59. The attraction force of a satellite of any planet varies directly with the mass (M) of the planet and inversely with the square of their distances . Again the square of the period of the rotation of the satellite varies directly with the distance between them and inversely with the attraction force. if m_1, d_1, t_1 and m_2, d_2, and t_2 be two sets of corresponding

values of M (mass) , D (Distance) and T (time) , them prove that

$$m_1 t_1^2 d_2^3 = m_2 t_2^2 d_1^3.$$