MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

COMPLEX NUMBERS

Examples

1. Is the following computation correct? If not give the correct computation: $\left[\sqrt{(-2)}\sqrt{(-3)}\right] = \sqrt{(-2)\cdot(-3)} = \sqrt{6}$

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2. A student writes the formula $\sqrt{ab} = \sqrt{a}\sqrt{b}$. Then he substitutes $a = -1$ and $b = -1$ and finds $1 = -1$. Explain where is he wrong?

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3. Explain the fallacy

$-1 = i \times i = \sqrt{-1} \times \sqrt{-1} = \sqrt{(-1) \times (-1)} = \sqrt{1} = 1.$

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4. Evaluate.

(i) i^{1998}

(ii)i^{-9999}

(iii) $(-\sqrt{-1})^{4n=3}$,n $\neq$ N

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5. Find the value of $1 + i^2 + i^4 + i^6 + \dots + i^{2n}$,

where$i = \sqrt{-1}$ and nEN.

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6. If $a = \frac{1+i}{\sqrt{2}}$, where $i = \sqrt{-1}$, then find the value of a^{1929}.

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7. The value of the sums $\sum_{n=1}^{13} \left(i^n + i^{n+1}\right)$, where $i = \sqrt{-1}$ is : (a) i (b) $i - 1$ (c) $-i$ (d) 0

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8. Find the value of $\sum_{n=0}^{100} i^{n!}$, where $i = \sqrt{-1}$

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9. Find he value of $\sum_{r=1}^{4n+7} i^r$ where, $i = \sqrt{-1}$.

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10. Show that the polynomial $x^{4p} + x^{4q+1} + x^{4r+2} + x^{4s+3}$ is divisible by $x^3 + x^2 + x + 1, wherep, q, r, s \in n\cdot$

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11. What is the digit in the unit's place of

$(5172)^{11327}$?

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12. What is the digit in the unit's place of

$(143)^{86}$?

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13. What is the digit in unit's place of

$(1354)^{22222}$?

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14. What is the digit in the unit's place of

$(13057)^{941120579}$?

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15. What is the digit in the unit's place of $(1008)^{786}$?

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16. What is the digit in the unit's place of $(2419)^{111213}$?

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17. If $\frac{x-3}{3+i}+\frac{y-3}{3-i}=i$ where $x, y \in R$ then

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18. If $(a+ib)^5 = \text{p} + \text{iq}, where \quad i = \sqrt{-1}$,

prove that $(b+ia)^5 = q+ip$.

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19. Find the least positive integral value of

n, for which $\left(\frac{1-i}{1+i}\right)^n$, where $i = \sqrt{-1}$, is purly

imaginary with positive imaginary part.

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20. If the multicative inverse of a comlex number is $(\sqrt{3}+4i)\backslash 19$, where

$i = \sqrt{-1}$, find the complex number.

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21. Find the value of θ if $(3 + 2i\sin\theta)/(1 - 2i\sin\theta)$ is purely real or purely imaginary.

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22. Find real values of $xandy$ for which the complex numbers $-3 + ix^2yandx^2 + y + 4i$ are conjugate of each other.

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23. If $x = -5 + 2\sqrt{-4}$, find the value of $x^4 + 9x^3 + 35x^2 - x + 4.$

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24. Let z be a complex number satisfying the equation $z^2 - (3 + i)z + \lambda + 2i = 0$, whre $\lambda \in$ R and non -real root.

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25. Find the argument s of $z_1 = 5 + 5i, z_2 = -4 + 4i, z_3 = -3 - 3i$ and $z_4 2 - 2i,$ where$i = \sqrt{-1}.$

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26. Findthe principal value of the arguments of $z_1 = 2 + 2i, z_2 = -3 + 3i, z^3 = -4 - 4i$ and $z_4 = 5 - 5i, wherei = \sqrt{}$

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27. Find the argument and the principal value of the argument of the complex number $z = \dfrac{2 + i}{4i + (1 + i)^2}$ where $i = \sqrt{-1}$

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28. 1. If $|z-2+il= \quad < 2$ then find the greatest and least value of $|z|$

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29. If z is a any complex number such that $|z+4| \leq 3,$ find the greatest value of |z+1|.

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30. If $|z_1|=1, |z_2|=2, |z_3|=3, and|9z_1z_2+4z_1z_3+z_2z_3+3|=12,$ then find the value of $|z_1+z_2+z+3|\cdot$

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31. For any two complex numbers, z_1, z_2

$\left|\frac{1}{2}(z_1+z_2)+\sqrt{z_1z_2}\right|+\left|\frac{1}{2}(z_1+z_2)-\sqrt{z_1z_2}\right|$ is equal to

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32. If z_1 and z_2 are two complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\overline{z_2}}$ is unimodular whereas z_1 is not unimodular then $|z_1|$=

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33. If arg $(z_1) = \frac{17\pi}{18}$ and arg $(z_2) = \frac{7\pi}{18}$, find the principal argument of $z_1 z_2$ and(z_1 / z_2).

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34. If z_1 and z_2 are conjugate to each other , find the principal argument of $(- z_1 z_2)$.

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35. Write the value of $arg(z) + arg(z)$.

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36. Write the polar form of $-\frac{1}{2}-\frac{i\sqrt{3}}{2}$

(Where, $i=\sqrt{-1}$).

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37. Given that $|z-1|=1,$ where z is a point

on the argand planne , show that $\frac{z-2}{z}=i\tan(argz),$

where $i=\sqrt{-1}.$

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38. Let z be a non-real complex number

lying on $|z|=1,$ prove that $z=\frac{1+i\tan\left(\frac{arg(z)}{2}\right)}{1-i\tan\left(\frac{arg(z)}{2}\right)}$ (where $i=\sqrt{-1}.$)

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39. Prove that $tan\left(i(\log)_e\left(\frac{a-ib}{a+ib}\right)\right)=\frac{2ab}{a^2-b^2}(wherea, b \in R^+)$

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40. If m and x are two real numbers where $m \in I$, then $e^{2mi\cot^{-1}x}\left(\frac{x\cdot i+1}{x\cdot i-1}\right)^m$

(A) $\cos x+i\sin x$ (B) $\frac{m}{2}$ (C) 1 (D) $\frac{m+1}{2}$

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41. Express $(1+i)^{-1}$,where, i=$\sqrt{-1}$ in the form A+iB.

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42. If $\sin\left(\log_e i^i\right)=a+ib,$ where $i=\sqrt{-1},$

find a and b, hence and find $\cos\left(\log_e i^i\right)$.

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43. Find the general value of $\log_2(5i)$,

where $i = \sqrt{-1}$.

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44. If $|z_1| = |z_2|$ and arg $(z_1 / z_2) = \pi$, then

find the of $z_1 z_2$.

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45. Let z and w are two non zero complex number such that $|z| = |w|$, and $Arg(z) + Arg(w) = \pi$ then (a) $z = w$ (b) $z = \overline{w}$ (c) $\bar{z} = \overline{w}$ (d) $\bar{z} = -\overline{w}$

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46. Find the square root of

$$X + \sqrt{(-X^4 - X^2 - 1)}.$$

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47. Solve the equation $z^2 + |z| = 0$, where z is a complex number.

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48. Number of solutions of the equation $z^2 + |z|^2 = 0$, where $z \in C$, is

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49. Find the all complex numbers satisying the equation

$$2|z|^2 + z^2 - 5 + i\sqrt{3} = 0, wherei = \sqrt{-1}.$$

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50. If $z_r = \cos\left(\frac{\pi}{3_r}\right) + i\sin\left(\frac{\pi}{3_r}\right), r = 1, 2, 3, ,$ prove that $z_1 z_2 z_3 z_\infty = i.$

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51. Express $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5} \in a + ib$ Form , where $i = \sqrt{-1}.$

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52. Find all roots of $X^5 - 1 = 0.$

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53. Find all roots of the equation

$X^6 - X^5 + X^4 - X^3 + X^2 - X + 1 = 0.$

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54. if $\alpha, \beta, \gamma$are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ then $\frac{\alpha - 1}{\beta - 1} + \frac{\beta - 1}{\gamma - 1} + \frac{\gamma - 1}{\alpha - 1}$

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55. If $z = \frac{\sqrt{3 + i}}{2}$ (where $i = \sqrt{-1}$) then $\left(z^{101} + i^{103}\right)^{105}$ is equal to

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56. If $\left(\frac{3}{2} + \frac{i\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$, where x and y are reals, then the ordered pair (x,y) is given by

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57. If the polynomial $7x^3 + ax + b$ is divisible by $x^2 - x + 1$, find the value of $2a + b$.

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58. If $1, \omega, \omega^2, ...\omega^{n-1}$ are n, nth roots of unity, find the value of $(9 - \omega)\left(9 - \omega^2\right)...\left(9 - \omega^{n-1}\right)$.

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59. F $a = \cos(2\pi/7) + is \in (2\pi/7)$, then find the quadratic equation whose roots are $\alpha = a = a^2 + a^4 and \beta = a^3 = a^5 + a^7$.

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60. Find the value of $\sum_{k=1}^{10} \left[\sin\left(\frac{2\pi k}{11}\right) - i\cos\left(\frac{2\pi k}{11}\right)\right], wherei = \sqrt{-1}.$

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61. If $n \geq 3 \text{ and } 1, \alpha_1, \alpha_2, \alpha_3, ..., \alpha_{n-1}$ are the n,nth roots of unity, then find value of $\left(\sum\sum\right)_{1 \leq i < j \leq n-1} \alpha_i \alpha_j$

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62. Complex numbers $z_1, z_2 \text{ and } z_3$ are the vertices A,B,C respectivelt of an isosceles right angled triangle with right angle at C. show that $(z_1 - z_2)^2 = 2(z_1 - z_3)(z_3 - z_2)$.

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63. Complex numbers z_1, z_2, z_3 are the vertices of A,B,C respectively of an equilteral triangle. Show that $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$.

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64. If z_1, z_2 and z_3 are the vertices of an equilasteral triangle with z_0 as its circumcentre , then changing origin to z^0 ,show that $z_1^2+z_2^2+z_3^2=0$, wherez_1, z_2, z_3, are new complex numbers of the vertices.

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65. Show that inverse of a point a with respect to the circle $|z-c|=R(a$ and c are complex numbers, centre and redius R) is the point $c+\frac{R^2}{\bar{a}-\bar{c}}$,

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66. Find the perpendicular bisector of $3+4i$ and $-5+6i, wherei=\sqrt{-1}$.

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67. If z_1, z_2 and z_3 are the affixes of the vertices of a triangle having its circumcentre at the

origin. If zis the affix of its orthocentre, prove that

$Z_1 + Z_2 + Z_3 - Z = 0.$

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68. Let $z_1 z_2$ and z_3 be three complex

numbers and $a, b, c \in R,$ such that

$a + b + c = 0$ and $az_1 + bz_2 + cz_3 = 0$ then show that $z_1 z_2$ and z_3 are collinear.

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69. Show that the area of the triagle on the argand plane formed by the complex numbers Z, iz and $z + iz$is $\frac{1}{2}|z|^2$, where $i = \sqrt{-1}.$

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70. Show that the point a' is the reflection of the point a in the line $z\bar{b} + \bar{z}b + c = 0$, If $a'b + ab + c = 0$.

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71. Find the center and radius of the circle $2z\bar{z} + (3 - i)z + (3 + i)z - 7 = 0$, where $i = \sqrt{-1}$.

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72. Find all circles which are orthogonal to $|z| = 1$ and $|z - 1| = 4$.

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73. Let $z_1 = 10 + 6i$ and $z_2 = 4 + 6i$. If z is any complex number such that the argument of $\frac{(z - z_1)}{(z - z_2)}$ is $\frac{\pi}{4}$, then prove that $|z - 7 - 9i| = 3\sqrt{2}$.

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74. 1. If $|z-2+il= \ < 2$ then find the greatest and least value of $|z|$

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75. In the argand plane, the vector $z=4-3i, where i=\sqrt{-1},$ is turned in the clickwise sense through number represented by the new vector.

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76. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy $BD=2AC$.If he point D and M represent the complex numbers $1+i$ and $2-i$ respectively, then A represents the complex number......or......

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77. Find the maximum and minimum values of $|z|$ satisfying $\left|z+\frac{1}{z}\right|=2$

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78. If $\left|z+\frac{4}{z}\right|=2$, fin the maximum and minimin values of $|z|$.

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79. If $|z|\geq 3$, then determine the least value of $\left|z+\frac{1}{z}\right|$.

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80. Q. Let z_1 and z_2 be nth roots of unity which subtend a right angle at the origin, then n must be the form $4k$.

A. $4k+1$

B. $4k+2$

C. $4k+3$

D. $4k$

Answer: d

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81. If $|z|=1$ and $\omega=\dfrac{z-1}{z+1}$ (where $z \in -1$), then $\text{Re}(\omega)$ is

A. 0

B. $\dfrac{-1}{|z+1|^2}$

C. $\left|\dfrac{z}{z=1}\right|\cdot\dfrac{1}{|z+1|^2}$

D. $\dfrac{\sqrt{2}}{|z+1|^2}$

Answer: a

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82. if a, b, c, a_1, b_1 and c_1 are non-zero complexnumbers satisfying $\frac{a}{a_1}+\frac{b}{b_1}+\frac{c}{c_1}=1+i$ and $\frac{a_1}{a}+\frac{b_1}{b}+\frac{c_1}{c}=0$, where $i=\sqrt{-1}$, the value of $\frac{a^2}{a_1^2}+\frac{b^2}{b_1^2}+\frac{c^2}{c_1^2}$ is

A. 2i

B. 2+2i

C. 2

D. None of these

Answer: a

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83. Let z and ω be complex numbers. If $Re(z)=|z-2|, Re(\omega)=|\omega-2|$ and $arg(z-\omega)=\frac{\pi}{3}$, then the value of $Im(z+w)$, is

A. $\frac{1}{\sqrt{3}}$

B. $\frac{2}{\sqrt{3}}$

C. $\sqrt{3}$

D. $\frac{4}{\sqrt{3}}$

Answer: d

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84. The mirror image of the curve $arg\left(\frac{z-3}{z-i}\right)=\frac{\pi}{6}, i=\sqrt{-1}$ in the real axis

A. $arg\left(\frac{z+3}{z+i}\right)=\frac{\pi}{6}$

B. $arg\left(\frac{z-3}{z+i}\right)=\frac{\pi}{6}$

C. $arg\left(\frac{z+i}{z+3}\right)=\frac{\pi}{6}$

D. $arg\left(\frac{z+i}{z-3}\right)=\frac{\pi}{6}$

Answer: d

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85. The mirror image of curve arg $\left(\frac{z+i}{z-i}\right)=\frac{\pi}{4}$ in line x-y=0 is :

A. $arg\left(\frac{z+i}{z+1}\right)=\frac{\pi}{4}$

B. $arg\left(\frac{z+1}{z+i}\right)=\frac{\pi}{4}$

C. $arg\left(\frac{z-i}{z+1}\right)=\frac{\pi}{4}$

D. $arg\left(\frac{z+i}{z-1}\right)=\frac{\pi}{4}$

Answer: c

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86. If $z+\frac{1}{z}=1$ and $a=z^{2017}+\frac{1}{z^{2017}}$ and b is the lastdigit of the number $2^{2^n}-1$,, when the integer $n>1$, the value of a^2+b^2 is

A. 23

B. 24

C. 26

D. 27

Answer: c

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87. if$\omega and \omega^2$ are the nonreal cube roots of unity and $[1/(a+\omega)]+[1/(b+\omega)]+[1/(c+\omega)]=2\omega^2$ and $\left[1/(a+\omega)^2\right]+\left[1/(b+\omega)^2\right]+\left[1/(c+\omega)^2\right]=2\omega$, then find the value of $[1/(a+1)]+[1/(b+1)]+[1/(c+1)]$.

A. -2

B. -1

C. 1

D. 2

Answer: d

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88. If a,b,c are distinct integers and $\omega(\neq 1)$ is a cube root of unity, then the minimum value of $\left|a+b\omega+c\omega^2\right|+\left|a+b\omega^2+c\omega\right|$ is

A. $\sqrt{3}$

B. 3

C. $4\sqrt{2}$

D. 2

Answer: a

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89. If $|z-2i| \leq \sqrt{2}$, where $i=\sqrt{-1}$, then the maximum value of $|3-i(z-1)|$, is

A. $\sqrt{2}$

B. $2\sqrt{2}$

C. $2+\sqrt{2}$

D. $3+2\sqrt{2}$

Answer: C

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90. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $Re(z_1 \bar{z}_2) = 0$, then the pair ofcomplex nunmbers $\omega = a + ic$ and $\omega_2 = b + id$ satisfies

A. $|\omega_1| = 1$

B. $|\omega_2| = 1$

C. $Re(\omega_1 \overline{\omega}_2 = \ -0$

D. None of these

Answer: a,b,c

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91. The complex numbers z_1, z_2, z_3 stisfying $(z_2 = z_3) = (1 + i)(z_1 - z_3). where i = \sqrt{-1},$ are vertices of a triangle which is

A. equilateral

B. isosceles

C. right angled

D. scalene

Answer: b,c

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92. If z satisfies the inequality $| - 1| < |z + 1|,$ then one has

A. $|z - 2 - i| < |z + 2 - i|, i = \sqrt{-1}$

B. $|arg(z + i)| < \frac{\pi}{2}, i = \sqrt{-1}$

C. $Re(z) < 0$

D. $Im(i\bar{z}) > 0, i\sqrt{-1}$

Answer: a,b,d

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93. The equation $z^2 - i|z-1|^2 = 0$, where $i = \sqrt{-1}$, has.

A. no real root

B. no purely imaginary root

C. all roots inside $|z| = 1$

D. atleast two roots

Answer: a,b,c

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94. Let z_1, z_2 be two complex numbers represented by points on the circle $|z_1| =$ and and $|z_2| = 2$ are then

A. $\max . |2z_1 + z_2| = 4$

B. $\min . |z_1 + z_2| = 1$

C. $\left|z_2 + \frac{1}{z_1}\right| \leq 3$

D. $\left|z_1 + \frac{2}{z_2}\right| \leq 2$

Answer: a,b,c,d

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95. Consider a quadratic equation $az^2bz + c = 0$, where a,b and c are complex numbers.

The condition that the equation has one purely imaginary root , is

A. `(abarb-barac)(b barc+barbc)+(cbara-barca)^(2)=0

B. $(a\bar{b} + \bar{a}b)(b\bar{c} + \bar{b}c) + (c\bar{a} - \bar{c}a)^2 = 0$

C. $\left(a\bar{b}-\bar{a}b\right)\left(b\bar{c}-\bar{b}c\right)+\left(c\bar{a}+\bar{c}a\right)^2=0$

D. $\left(a\bar{b}+\bar{a}b\right)\left(b\bar{c}-\bar{b}c\right)+\left(c\bar{a}-\bar{c}a\right)^2=0$

Answer: b

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96. Consider a quadratic equaiton $az^2+bz+c=0$, where a,b,c are complex number.

The condition that the equaiton has one purely real roots is

A. $\left(a\bar{b}+\bar{a}b\right)\left(b\bar{c}-\bar{b}c\right)=\left(c\bar{a}+\bar{c}a\right)^2$

B. $\left(a\bar{b}-\bar{a}b\right)\left(b\bar{c}+\bar{b}c\right)=\left(c\bar{a}+\bar{c}a\right)^2$

C. $\left(a\bar{b}-\bar{a}b\right)\left(\bar{b}c-\bar{b}c\right)=\left(c\bar{a}-\bar{c}a\right)^2$

D. $\left(a\bar{b}-\bar{a}b\right)\left(b\bar{c}-\bar{b}c\right)=\left(c\bar{a}+\bar{c}a\right)^2$

Answer: c

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97. Consider the quadratic equation $az^2 + bz + c = 0$ where a, b, c are non-zero complex numbers. Now answer the following.

The condition that the equation has both roots purely imaginary is

A. $\frac{a}{a} = \frac{b}{b} = \frac{c}{c}$

B. $\frac{a}{a} = \frac{b}{b} = \frac{c}{c}$

C. $\frac{a}{a} = \frac{b}{b} = -\frac{c}{c}$

D. $\frac{a}{a} = -\frac{b}{b} = \frac{c}{c}$

Answer: d

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98. Let Papoint denoting a comples number z on the complex plane.

$i.e. \quad z = Re(z) + iIm(z), \quad \text{where} \quad i = \sqrt{-1}$

$\text{if} \quad Re(z) = x \text{ and } Im(z) = y, thenz = x + iy$

If Pmovew such that

$|Re(z)| + \ | Im(z) = a(a \in R^{+})$

The locus of P is

A. a parallelogram which is not arhombus

B. a rhombus which is not a square

C. a rectangle which is not a square

D. a square

Answer: d

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99. Let P point denoting a complex number z on the complex plane. $i.\ e.\ z = Re(z) + iIm(z), \quad \text{where} \quad i = \sqrt{-1}$

if $\quad Re(z) = x$ and $Im(z) = y, thenz = x + iy$.The area of the circle inscribed in the region denoted by $|Re(z)| + |Im(z)| = 10$ equal to

A. 50π sq units

B. 100π sq units

C. 55 sq units

D. 110 sq units

Answer: a

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100. Let P point denoting a complex number z on the complex plane. $i.e. z = Re(z) + iIm(z), \quad \text{where} \quad i = \sqrt{-1}$

if $Re(z) = x$ and $Im(z) = y, thenz = x + iy$ Number of integral solutions satisfying the eniquality$|Re(z)| + |Im(z)| < 21, . is$

A. 841

B. 839

C. 840

D. 842

Answer: c

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101. If $z_1, z_2 \in C, z_1^2 + z_2^2 \in R, z_1\left(z_1^2 - 3z_2^2\right) = 2$ and $z_2\left(3z_1^2 - z_2^2\right) = 11,$ the value of $z_1^2 + z_2^2$ is

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102. Consider four complex numbers $z_1 = 2 + 2i, ,$

$z_2 = 2 - 2i, z_3 = -2 - 2i \text{ and } z_4 = -2 + 2i), wherei = \sqrt{-1},$

Statement -1 $z_1, z_2, z_3 \text{ and } z_4$

constitute the vertices of a

square on the complex plane because

Statement -2 The non-zero complex numbers $z, \bar{z}, -z, -\bar{z}$

always constitute the vertices of a square.

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103. Consider z_1 and z_2 are two complex numbers

such that $|z_1 + z_2| = |z_1| + |z_2|$

Statement -1 $amp(z_1) - amp(z_2) = 0$

Statement -2 The complex numbers z_1 and z_2 are collinear.

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104. If $|z - iRe(z)| = |z - Im(z)|$, then prove that z

lies on the bisectors of the quadrants, $\text{where } i = \sqrt{-1}.$

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105. Find the gratest and the least values of $|z_1 + z_2|$,

if $z_1 = 24 + 7i$ and $|z_2| = 6$, $\text{where } i = \sqrt{-1}$

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106. If $|z-1|=1,$ where is a point on the argand plane, show that $\frac{z-2}{z}=i\tan(argz), wherei=\sqrt{-1}.$

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107. If $arg\left(z^{1/2}\right)=\frac{1}{2}arg\left(z^{2}+\bar{z}z^{1/3}\right),$ find the value of $|z|$.

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108. C is the complex numvers $f:C\to R$ is defined by $f(z)=\left|z^{3}-z+2\right|.$ Find the maximum value of $f(z), If|z|=1.$

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109. Prove that the complex numbers z_1 and z_2 and the origin form an isosceles triangle with vertical angle $\frac{2\pi}{3}$, if $z_1^2+z_2^2+z_1z_2=0.$

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110. If $\alpha = e^{i2\pi/7} and f(x) = a_0 + \sum_{k-0}^{20} a_k x^k$, then prove that the value of $f(x) + f(\alpha, x) + \quad + f(\alpha^6 x)$ is independent of α.

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111. Show that all the roots of the equation $a_1 z^3 + a_2 z^2 + a_3 z + a_4 = 3$, $(where|a_i| \leq 1, i = 1, 2, 3, 4,)$ lie

outside the circle with centre at orihin and radius $2/3$.

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112. The points A, B, C represent the complex numbers z_1, z_2, z_3 respectively on a complex plane & the angle $B\&C$ of the triangle ABC are each equal to $\frac{1}{2}(\pi - \alpha)$. If $(z_2 - z_3)^2 = \lambda(z_3 - z_1)(z_1 - z_2)(\sin^2)\frac{\alpha}{2}$ then determine λ.

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113. If z_1 and z_2 are two complex number such that $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1$, Prove that $i\frac{z_1}{z_2} = k$ where k is a real number Find the angle between the lines from the origin to the points $z_1 + z_2$ and $z_1 - z_2$ in terms of k

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114. If $z = x + iy$ is a complex number with $x, y \in Q$ and $|z| = 1$, then show that $\left|z^{2n} - 1\right|$ is a rational numberfor every $n \in N$.

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115. If a is a complex number such that $|a| = 1$, then find thevalue of a, so that equation $az^2 + z + 1 = 0$ has one purely imaginary root.

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116. If $n \in N > 1$, find the sum of real parts of the roots of the equation $z^n = (z+1)^n$.

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117. Among the complex numbers z which satisfies $|z - 25i| \leq 15$, find the complex numbers z having

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118. Two different non parallel lines cut the circle $|z| = r$ at points $a; b; c; d$ respectively . prove that these lines meet at a point $\left(\left(a^{-1} + \dfrac{b^{-1} - c^{-1} - d^{-1}}{a^{-1}b^{-1} - c^{-1}d^{-1}}\right.\right.$

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119. If n is an odd integer but not a multiple of 3, then prove that $xy(x+y)\left(x^2+y^2+xy\right)$ is a factor of $(x+y)^n - x^n - y^n$.

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120. Show that the triangle whose vertices are $z_1 z_2 z_3$ and $z_1' z_2' z_3'$ are directly similar , if $\begin{vmatrix} z_1 & z'_1 & 1 \\ z_2 & z'_2 & 1 \\ z_3 & z'_3 & 1 \end{vmatrix} = 0$

Watch Video Solution

121. if ω is the nth root of unity and Z_1, Z_2 are any two complex numbers , then probe that .

$\Sigma_{n-1}^{k=0}\left|z_1 + \omega^k z_2\right|^2 = n\left\{\left|z_1\right|^2 + \left|z_2\right|^2\right\}$ where $n \in N$

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122. If $z_1 + z_2 + z_3 + z_4 = 0$ where $b_1 \in R$ such that the sum of no two values being zero and $b_1 z_1 + b_2 z_2 + b_3 z_3 + b_4 z_4 = 0$ where z_1, z_2, z_3, z_4 are arbitrary complex numbers such that no three of them are collinear, prove that the four complex numbers would be concyclic if $|b_1 b_2||z_1 - z_2|^2 = |b_3 b_4||z_3 - z_4|^2$.

Watch Video Solution

Exercise For Session 1

1. If $(1+i)^{2n} + (1-i)^{2n} = -2^{n+1}(where, i = \sqrt{-1}$ for all those n, which are

A. even

B. odd

C. multiple of 3

D. None of these

Answer:

Watch Video Solution

2. If $i = \sqrt{-1},$ the number of values of i^{-n} for a different $n \in I$ is

A. 1

B. 2

C. 3

D. 4

Answer:

Watch Video Solution

3. If $a > 0 \text{ and } b < 0, then \sqrt{a}\sqrt{b}$ is equal to (where, $i = \sqrt{-1}$)

A. $-\sqrt{a \cdot |b|}$

B. $\sqrt{a \cdot |b| i}$

C. $\sqrt{a \cdot |b|}$

D. none of these

Answer:

Watch Video Solution

4. The value of $\sum_{r=-3}^{1003} i^r (where i = \sqrt{-1})$ is

A. 1

B. -1

C. i

D. $-i$

Answer:

Watch Video Solution

5. The digit in the unit's place of $(153)^{98}$ is

A. 1

B. 3

C. 7

D. 9

Answer:

Watch Video Solution

6. The digit in the unit's place of $(141414)^{12121}$ is

A. 4

B. 6

C. 3

D. 1

Answer:

Watch Video Solution

Exercise For Session 2

1. If $\frac{1-ix}{1+ix} = a - ib$ and $a^2+b^2=1$, $where\ a, b \in R$ and $i = \sqrt{-1}$, then xis equal to

A. $\frac{2a}{(1+a)^2+b^2}$

B. $\frac{2b}{(1+a)^2+b^2}$

C. $\frac{2a}{(1+b)^2+a^2}$

D. $\frac{2b}{(1+b)^2+a^2}$

Answer:

Watch Video Solution

2. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\left(\sec^{-1}\frac{1}{x} + \sin^{-1}x\right)$ (where, $X \neq 0, \ -1 \leq X \leq 1$ and $i = \sqrt{-1}$, is

A. 2

B. 4

C. 6

D. 8

Answer:

Watch Video Solution

3. If $z = (3+4i)^6 + (3-4i)^6$, where$i = \sqrt{-1}$, then Im (z) equals to

A. -6

B. 0

C. 6

D. none of these

Answer:

Watch Video Solution

4. If $(x+iy)^{1/3}=a+ib, \text{where} i=\sqrt{-1}, then\left(\frac{x}{a}+\frac{y}{b}\right)$ is equal to

A. $4a^2b^2$

B. $4(a^2-b^2)$

C. $4a^2-b^2$

D. a^2+b^2

Answer:

Watch Video Solution

5. If $\dfrac{3}{2+\cos\theta+i\sin\theta}=a+ib \text{where} i=\sqrt{-1}$ and $a^2+b^2=\lambda a-3,$ the v

A. 3

B. 4

C. 5

D. 6

Answer:

Watch Video Solution

6. If $z(\neq -1)$ is a complex number such that $\dfrac{z-1}{z+1}$ is purely imaginary, then $|z|$ is equal to

A. $\dfrac{1}{2}$

B. 1

C. $\sqrt{2}$

D. 2

Answer:

Watch Video Solution

7. The complex number $\sin(x) + i\cos(2x)$ and $\cos(x) - i\sin(2x)$ are conjugate to each other for

A. $x = n\pi, n \in I$

B. $x = 0$

C. $x = \left(n + \frac{1}{2}\right), n \in I$

D. 2

Answer:

Watch Video Solution

8. If $\alpha$and $\beta$are different complex numbers with $|\beta| = 1$, then find $\left|\frac{\beta - \alpha}{1 - \overline{\alpha}\beta}\right|$.

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

Answer:

Watch Video Solution

9. If $x = 3 + 4i$ find the value of $x^4 - 12x^3 - 70x^2 - 204x + 225$

A. -45

B. 0

C. 35

D. 15

Answer:

Watch Video Solution

10. If $|z_1 - 1| \leq , |z_2 - 2| \leq 2, |z_{33}| \leq 3,$ then find the greatest value of $|z_1 + z_2 + z_3|.$

A. 6

B. 12

C. 17

D. 23

Answer:

Watch Video Solution

11. The principal value of $arg(z)$, where $z = 1 + \cos\left(\frac{8\pi}{5}\right) + i\sin\left(\frac{8\pi}{5}\right)$ (where, $i = \sqrt{-1}$) is given by

A. $-\frac{\pi}{5}$

B. $-\frac{4\pi}{5}$

C. $\frac{\pi}{5}$

D. $\frac{4\pi}{5}$

Answer:

Watch Video Solution

12. If $|z_1| = 2, |z_2| = 3, |z_3| = 4 \text{ and } |z_1 + z_2 + z_3| = 5. \text{ then} |4z_2z_3 + 9z_3z_1 + 1$ is

A. 24

B. 60

C. 120

D. 240

Answer:

Watch Video Solution

13. If $|z-i| \leq 5$ and $z_1 = 5+3i$ (where, $i = \sqrt{-1}$), then greatest and least values of $|iz + z_1|$

A. 7 and 3

B. 9 and 1

C. 10 and 0

D. none of these

Answer:

Watch Video Solution

14. If z_1, z_2 and z_3, z_4 are two pairs of conjugate complex numbers then $arg\left(\frac{z_1}{z_4}\right) + arg\left(\frac{z_2}{z_3}\right) =$

A. 0

B. $\frac{\pi}{2}$

C. π

D. $\frac{3\pi}{2}$

Answer:

Watch Video Solution

Exercise For Session 3

1. The rea part of $(1-i)^{-i}$, where $i = \sqrt{-1}$ is

A. $e^{-\pi/4}\cos\left(\frac{1}{2}\log_e 2\right)$

B. $-e^{-\pi/4}\sin\left(\frac{1}{2}\log_e 2\right)$

C. $e^{-\pi/4}\cos\left(\frac{1}{2}\log_e 2\right)$

D. $e^{-\pi/4}\sin\left(\frac{1}{2}\log_e 2\right)$

Answer:

Watch Video Solution

2. The amplitude of $e^{e^{-(i\theta)}}$, where $\theta \in R$ and $i=\sqrt{-1}$, is

A. $\sin\theta$

B. $-\sin\theta$

C. $e^{\cos\theta}$

D. $e^{\sin\theta}$

Answer:

Watch Video Solution

3. If $z = i \log_e (2 - \sqrt{3})$, where$i = \sqrt{-1}$ then the cos z is equal to

A. i

B. 2i

C. 1

D. 2

Answer:

Watch Video Solution

4. If $z = (i)^{i}$ ^ $(((i)))wherei = \sqrt{-1}, then|z|$ is equal to 1 b. $e^{-\pi/2}$ c. $e^{-\pi}$ d. none of these

A. 1

B. $e^{-\pi/2}$

C. $e^{-\pi}$

D. e^{π}

Answer:

Watch Video Solution

5. $\sqrt{(-8-6i)}$ is equal to (where, $i=\sqrt{-1}$

A. $1 \pm 3i$

B. $\pm(1-3i)$

C. $\pm(1+3i)$

D. $\pm(3-i)$

Answer:

Watch Video Solution

6. $\frac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}=$

A. $-\frac{3}{2}i$

B. $\frac{3}{4}i$

C. $-\frac{3}{4}i$

D. $-\frac{3}{2}$

Answer:

Watch Video Solution

7. If $0 < amp(z) < \pi, \text{then} amp(z) - amp(-z)$ is equal to

A. 0

B. $2amp(z)$

C. π

D. $-\pi$

Answer:

Watch Video Solution

8. If $|z_1| = |z_2|$ and $amp(z_1) + amp(z_2) = 0,$ then

A. $z_1 = z_2$

B. $\bar{z}_1 = z_2$

C. $z_1 + z_2 = -0$

D. $\bar{z}_1 = \bar{z}_2$

Answer: B

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9. The solution of the equation $|z| - z = 1 + 2i$ is

A. $2 - \frac{3}{2}i$

B. $\frac{3}{2} + 2i$

C. $\frac{3}{2} - 2i$

D. $-2 + \frac{3}{2}i$

Answer: C

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10. The number of jsolutions of the equation $z^2 + \bar{z} = 0,$ is

A. 1

B. 2

C. 3

D. 4

Answer: D

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11. If

$z_r = \cos\left(\frac{r\alpha}{n^2}\right) + i\sin\left(\frac{r\alpha}{n^2}\right)$, where$r = 1, 2, 3, \ldots, n$ and $i = \sqrt{-1}$, then

is equal to

A. $e^{i\alpha}$

B. $e^{-i\alpha/2}$

C. $e^{i\alpha/2}$

D. $\sqrt[3]{e^{i\alpha}}$

Answer:

Watch Video Solution

12. If $\theta \in R$ and $i = \sqrt{-1}$, then $\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n$ is equal to

A. $\cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$

B. $\cos\left(\frac{n\pi}{2} + n\theta\right) + i\sin\left(\frac{n\pi}{2} + n\theta\right)$

C. $\sin\left(\frac{n\pi}{2} - n\theta\right) + i\cos\left(\frac{n\pi}{2} - n\theta\right)$

D. $\cos\left(n\left(\frac{\pi}{2} + 2\theta\right)\right) + i\sin\left(n\left(\frac{\pi}{2} + 2\theta\right)\right)$

Answer:

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13. If $z_4 + 1 = 0$, where$i = \sqrt{-1}$ then z can take the value

A. $\frac{1+i}{\sqrt{2}}$

B. $\cos\left(\frac{\pi}{8}\right) + i\sin\left(\frac{\pi}{8}\right)$

C. $\frac{1}{4i}$

D. i

Answer:

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14. If $\omega(\neq 1)$ is a cube root of unity, then $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)$...upto $2n$ is factors, is

A. 2^n

B. 2^{2n}

C. 0

D. 1

Answer:

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15. If α, β, γ are the cube roots of p, then for any x,y,z $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ =

A. $\frac{1}{2}(-1 - i\sqrt{3}), i = \sqrt{-1}$

B. $\frac{1}{2}(1 + i\sqrt{3}), i = \sqrt{-1}$

C. $\frac{1}{2}(1 - i\sqrt{3}), i = \sqrt{-1}$

D. none of these

Answer:

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Exercise For Session 4

1. If z_1, z_2, z_3 and z_4 are the roots of the equation $z^4 = 1$, the value of $\sum_{i=1}^{4} z i^3$ is

A. 0

B. 1

C. $i, i = \sqrt{-1}$

D. $1 + i, i = \sqrt{-1}$

Answer: A

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2. If $z_1, z_2, z_3, \ldots\ldots\ldots\ldots\ldots, z_n$ are n nth roots of unity, then for $k = 1, 2, , \ldots\ldots\ldots, n$

A. $|z_k| = k \mid z_{k+1}|$

B. $|z_{k+1}| = k \mid z_{k1}|$

C. $|z_{k+1}| = |zk| + |z_{k-1}|$

D. $|z_k| = |z_{k+1}|$

Answer: D

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3. If $1, \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{n-1}$ are n, nth roots of unity, then $(1 - \alpha_1)(1 - \alpha_2)(1 - \alpha_3)\ldots(1 - \alpha_{n-1})$ equals to

A. 0

B. 1

C. n

D. n^2

Answer: C

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4. The value of $\sum_{i=1}^{6}\left(\sin.\frac{2\pi k}{7}-i\cos.\frac{2\pi k}{7}\right)$

A. -1

B. 0

C. $-i$

D. i

Answer:

Watch Video Solution

5. If $\alpha(\neq 1)$ is a nth root of unity then $S=1+3\alpha+5\alpha^2+\ldots\ldots\ldots$ upto n terms is equal to

A. $\frac{2n}{1-\alpha}$

B. $-\frac{2n}{1-\alpha}$

C. $\frac{n}{1-\alpha}$

D. $-\frac{n}{1-\alpha}$

Answer:

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6. a and b are real numbers between 0 and 1 such that the points $Z_1 = a + i, Z_2 = 1 + bi, Z_3 = 0$ form an equilateral triangle, then a and b are equal to

A. $a = b = 2 + \sqrt{3}$

B. $a = b = 2 - \sqrt{3}$

C. $a = b = -2 - \sqrt{3}$

D. none of these

Answer: B

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7. If $|z| = 2$, the points representing the complex numbers $-1 + 5z$will lie on

A. a circle

B. a straight line

C. a parabola

D. an ellipse

Answer:

Watch Video Solution

8. If $|(z-2)/(z-3)| = 2$ represents a circle, then find its radius.

A. 1

B. $\frac{1}{3}$

C. $\frac{3}{4}$

D. $\frac{2}{3}$

Answer:

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9. If centre of a regular hexagon is at origin and one of the vertices on Argand diagram is 1+2i, where $i=\sqrt{-1}$, then its perimeter is

A. $2\sqrt{5}$

B. $6\sqrt{2}$

C. $4\sqrt{5}$

D. $6\sqrt{5}$

Answer:

Watch Video Solution

10. If z is a comlex number in the argand plane, the equation $|z-2|+|z+2|=8$represents

A. a parabola

B. an ellipse

C. a hyperbola

D. a circle

Answer: D

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11. If $|z-2-3i|+|z+2-6i|=4$ where $i=\sqrt{-1}$ then find the locus of $P(z)$

A. an ellipse

B. ϕ

C. line segment of points $2+3i$ and $-26i$

D. none of these

Answer:

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12. locus of the point z satisfying the equation $|z-1|+|z-i|=2$ is

A. a straight line

B. a circle

C. an ellipse

D. a pair of straight lines

Answer:

Watch Video Solution

13. If $z, iz \text{ and } z+iz$ are the vertices of a triangle whose area is 2units, the value of $|z|$is

A. 1

B. 2

C. 4

D. 8

Answer:

Watch Video Solution

14. If $\left|z-\frac{4}{z}\right|=2$ then the greatest value of $|z|$ is (A) $\sqrt{5}-1$ (B) $\sqrt{5}+1$ (C) $\sqrt{5}$ (D) 2

A. $\sqrt{5}-1$

B. $\sqrt{3}+1$

C. $\sqrt{5}+1$

D. $\sqrt{3}-1$

Answer:

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Exercise Single Option Correct Type Questions

1. if cos (1-i) = a+ib, where a , b $\in$ R and $i = \sqrt{-1}$, then

A. $a = \frac{1}{2}\left(e - \frac{1}{e}\right)\cos 1, b = \frac{1}{2}\left(e + \frac{1}{e}\right)\sin 1$

B. $a = \frac{1}{2}\left(e + \frac{1}{e}\right)\cos 1, b = \frac{1}{2}\left(e - \frac{1}{e}\right)\sin 1$

C. $a = \frac{1}{2}\left(e + \frac{1}{e}\right)\cos 1, b = \frac{1}{2}\left(e + \frac{1}{e}\right)\sin 1$

D. $a = \frac{1}{2}\left(e - \frac{1}{e}\right)\cos 1, b = \frac{1}{2}\left(e - \frac{1}{e}\right)\sin 1$

Answer: B

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2. Number of roots of the equation $z^{10} - z^5 - 992 = 0$ with negative real part is

A. 3

B. 4

C. 5

D. 6

Answer: C

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3. If z_1 and $\bar{z}_1$ represent adjacent vertices of a regular polygon of n sides where centre is origin and if $\frac{Im(z)}{Re(z)} = \sqrt{2} - 1$, then n is equal to: (A) 8 (B) 16 (C) 24 (D) 32

A. 2

B. 4

C. 6

D. 8

Answer: D

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4. If $\prod_{p=1}^{r} e^{ip\theta} = 1$, where $\prod$ denotes the continued product and $i = \sqrt{-1}$, the most general value of θ is (where, n is an integer)

A. $\frac{2n\pi}{r(r-1)}, n \in I$

B. $\frac{2n\pi}{r(r+1)}, n \in I$

C. $\frac{4n\pi}{r(r-1)}, n \in I$

D. $\frac{4n\pi}{r(r+1)}, n \in I$

Answer: D

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5. If $(3+i)(z+\bar{z}) - (2+i)(z-\bar{z}) + 14i = 0$, where $i = \sqrt{-1}$, then z $\bar{z}$ is equal to

A. 10

B. 8

C. -9

D. -10

Answer: A

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6. The centre of a square ABCD is at z=0, A is z_1. Then, the centroid of $\triangle ABC$ is (where, $i = \sqrt{-1}$)

A. $z_1(\cos\pi \pm i\sin\pi)$

B. $\frac{z_1}{3}(\cos\pi \pm i\sin\pi)$

C. $z_1\left(\cos\left(\frac{\pi}{2}\right) \pm i\sin\left(\frac{\pi}{2}\right)\right)$

D. $\frac{z_1}{3}\left(\cos\left(\frac{\pi}{2}\right) \pm i\sin\left(\frac{\pi}{2}\right)\right)$

Answer: D

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7. If $z=\frac{\sqrt{3}-i}{2}$, where $i=\sqrt{-1}$, then $\left(i^{101}+z^{101}\right)^{103}$ equals to

A. iz

B. z

C. $\bar{z}$

D. None of these

Answer: B

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8. Let α and β be two fixed non-zero complex numbers and 'z' a variable complex number. If the lines $\alpha\bar{z}+\bar{a}z+1=0$ and $\beta\bar{z}+\bar{\beta}z-1=0$ are mutually perpendicular, then

A. $ab+\bar{a}\bar{b}=0$

B. $ab-\bar{a}\bar{b}=0$

C. $\bar{a}b-a\bar{b}=0$

D. $a\bar{b} + \bar{a}b = 0$

Answer: D

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9. If $\alpha = \cos\left(\frac{8\pi}{11}\right) + i\sin\left(\frac{8\pi}{11}\right)$ then $Re\left(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5\right)$ is

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 0

D. None of these

Answer: B

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10. The set of points in an Argand diagram which satisfy both $|z| \leq 4$ and $0 \leq arg(z) \leq \frac{\pi}{3}$, is

A. a circle and a line

B. a radius of a circle

C. a sector of a circle

D. an infinite part line

Answer: C

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11. If $f(x) = g(x^3) + xh(x^3)$ is divisiblel by $x^2 + x + 1$, then

A. g(x) is divisible by (x-1) but not h(x) but not h(x)

B. h(x) is divisible by (x-1) but not g(x)

C. both g(x) and h(x) are divisible by (x-1)

D. None of above

Answer: C

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12. If the points represented by complex numbers $z_1 = a + ib, z_2 = c + id$ and $z_1 - z_2$ are collinear, where $i = \sqrt{-1}$, then

A. ad+bc=0

B. ad-bc=0

C. ab+cd=0

D. ab-cd=0

Answer: B

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13. Let C and R denote the set of all complex numbers and all real numbers respectively. Then show that $f : C \to R$ given by $f(z) = |z|$ for all $z \in C$ is neither one-one nor onto.

A. f is injective but not surjective

B. f is surjective but not injective

C. f is nither injective nor surjective

D. f is both injective and surjective

Answer: C

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14. Let α and β be two distinct complex numbers, such that $|\alpha| = |\beta|$. If real part of α is positive and imaginary part of β is negative, then the complex number $(\alpha + \beta) / (\alpha - \beta)$ may be

A. zero

B. real and negative

C. real and positive

D. purely imaginary

Answer: D

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15. The complex number z satisfies thc condition $\left|z - \frac{25}{z}\right| = 24$. The maximum distance from the origin of co-ordinates to the points z is

A. 25

B. 30

C. 32

D. None of these

Answer: A

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16. The points A,B and C represent the complex numbers $z_1, z_2, (1-i)z_1 + iz_2$ respectively, on the complex plane (where, $i = \sqrt{-1}$). The $\triangle ABC$, is

A. isosceles but not right angled

B. right angled but not isosceles

C. isosceles and right angled

D. None of the above

Answer: C

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17. The system of equations $|z + 1 - i| = \sqrt{2}$ and $|z| = 3$ has

A. no solution

B. one solution

C. two solution

D. None of these

Answer: A

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18. Dividing f(z) by z-i, we obtain the remainder 1-i and dividing it by z+i, we get the remainder 1+i. Then, the remainder upon the division of f(z) by z^2+1, is

A. i+z

B. 1+z

C. 1-z

D. None of these

Answer: C

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19. The centre of circle represented by $|z+1| = 2|z-1|$ in the complex plane is

A. 0

B. $\frac{5}{3}$

C. $\frac{1}{3}$

D. None of these

Answer: B

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20. If $x = 9^{\frac{1}{3}} 9^{\frac{1}{9}} 9^{\frac{1}{27}} \ldots\ldots ad$ inf $y = 4^{\frac{1}{3}} 4^{-\frac{1}{9}} 4^{\frac{1}{27}} \ldots\ldots ad$ inf and $z = \sum_{r=1}^{\infty} (1+i)^{-r}$ then , the argument of the complex number $w = x + yz$ is

A. 0

B. $-\tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

C. $-\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$

D. $\pi - \tan^{-1}\left(\frac{\sqrt{2}}{3}\right)$

Answer: B

Watch Video Solution

21. If centre of a regular hexagon is at origin and one of the vertices on Argand diagram is 1+2i, where $i = \sqrt{-1}$, then its perimeter is

A. $2\sqrt{5}$

B. $4\sqrt{5}$

C. $6\sqrt{5}$

D. $8\sqrt{5}$

Answer: C

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22. Let $|Z_r - r| \leq r, Aar = 1, 2, 3. \dots, n$. Then $\left|\sum_{r=1}^{n} z_r\right|$ is less than

A. n

B. 2n

C. n(n+1)

D. $\frac{n(n+1)}{2}$

Answer: C

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23. If $\arg\left(\frac{z_1 - \frac{z}{|z|}}{\frac{z}{|z|}}\right) = \frac{\pi}{2}$ and $\left|\frac{z}{|z|} - z_1\right| = 3$, then $|z_1|$ equals to

A. $\sqrt{3}$

B. $2\sqrt{2}$

C. $\sqrt{10}$

D. $\sqrt{26}$

Answer: C

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24. If $|z-2-i|=|z|\left|\sin\left(\frac{\pi}{4}-argz\right)\right|$, where $i=\sqrt{-1}$, then locus of z, is

A. a pair of straight lines

B. circle

C. parabola

D. ellipse

Answer: C

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25. If , $Z_1, Z_2, Z_3, \ldots\ldots. Z_{n-1}$ are n^{th} roots of unity then the value of $\frac{1}{3-Z_1}+\frac{1}{3-Z_2}+\ldots\ldots\ldots+\frac{1}{3-Z_{n-1}}$ is equal to

A. $\frac{n\cdot 3^{n-1}}{3^n-1}+\frac{1}{2}$

B. $\frac{n\cdot 3^{n-1}}{3^n-1}-1$

C. $\frac{n\cdot 3^{n-1}}{3^n-1}+1$

D. None of these

Answer: D

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26. If $z=(3+7i)(\lambda+i\mu)$, when $\lambda, \mu \in I-\{0\}$ and $i=\sqrt{-1}$, is purely imaginary then minimum value of $|z|^2$ is

A. 0

B. 58

C. $\frac{3364}{3}$

D. 3364

Answer: D

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27. Given $z = f(x) + ig(x) where f, g : (0, 1) \overrightarrow{0, 1}$ are real valued functions. Then which of the following does not hold good? $z = \frac{1}{1 - ix} + i\frac{1}{1 + ix}$ b. $z = \frac{1}{1 + ix} + i\frac{1}{1 - ix}$ c. $z = \frac{1}{1 + ix} + i\frac{1}{1 + ix}$ d. $z = \frac{1}{1 - ix} + i\frac{1}{1 - ix}$

A. $z = \frac{1}{1 - ix} + i\left(\frac{1}{1 + ix}\right)$

B. $z = \frac{1}{1 + ix} + i\left(\frac{1}{1 - ix}\right)$

C. $z = \frac{1}{1 + ix} + i\left(\frac{1}{1 + ix}\right)$

D. $z = \frac{1}{1 - ix} + i\left(\frac{1}{1 - ix}\right)$

Answer: B

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28. If $z^3 + (3+2i)z + (-1+ia) = 0, \quad \text{where} \quad i = \sqrt{-1}$, has one real root, the value of a lies in the interval (a $\in$ R)

A. (-2,-1)

B. (-1,0)

C. (0,1)

D. (1,2)

Answer: B

Watch Video Solution

29. If m and n are the smallest positive integers satisfying the relation $\left(2CiS\frac{\pi}{6}\right)^m = \left(4CiS\frac{\pi}{4}\right)^n$, where $i = \sqrt{-1}$, $(m+n)$ equals to

A. 60

B. 72

C. 96

D. 120

Answer: B

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30. Number of imaginary complex numbers satisfying the equation, $z^2 = \bar{z}2^{1 - |z|}$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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Exercise More Than One Correct Option Type Questions

1. If $\dfrac{z+1}{z+i}$ is a purely imaginary number (where $\left(i=\sqrt{-1}\right)$, then z lies on a

A. straight line

B. circle

C. circle with radius = $\dfrac{1}{\sqrt{2}}$

D. circle passing through the origin

Answer: B::C::D

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2. If z satisfies $|z-1| < |z+3|$ then $\omega = 2z+3-i$, (where $i=\sqrt{-1}$) sqtisfies

A. $|\omega - 5 - i| < |\omega + 3 + i|$

B. $|\omega - 5| < |\omega + 3|$

C. $Im(i\omega) > 1$

D. $|arg(\omega - 1)| < \frac{\pi}{2}$

Answer: B::C::D

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3. If the complex numbers is $(1 + ri)^3 = \lambda(1 + i)$, when $i = \sqrt{-1}$, for some real λ, the value of r can be

A. $\cos\frac{\pi}{5}$

B. $\cos ec\frac{3\pi}{2}$

C. $\cot\frac{\pi}{12}$

D. $\tan\frac{\pi}{12}$

Answer: B::C::D

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4. If z $\in$ C, which of the following relation(s) represents a circle on an Argand diagram? (where,$i = \sqrt{-1}$)

A. $|z-1| + |z+1| = 3$

B. $|z-3| = 2$

C. $|z-2+i| = \frac{7}{3}$

D. $(z-3+i)(\bar{z}-3-i) = 5$

Answer: B::C::D

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5. If $1, z_1, z_2, z_3, \ldots\ldots, z_{n-1}$ be the n, nth roots of unity and ω be a non-real complex cube root of unity, then $\prod_{r=1}^{n-1} (\omega - z_r)$ can be equal to

A. 1+ω

B. -1

C. 0

D. 1

Answer: A::C::D

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6. If z is a complex number which simultaneously satisfies the equations $3|z-12|=5|z-8i|$ and $|z-4|=|z-8|$, where $i=\sqrt{-1}$, then Im(z) can be

A. 8

B. 17

C. 7

D. 15

Answer: A::B

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7. If $P(z_1), Q(z_2), R(z_3)$ and $S(z_4)$ are four complex numbers representing the vertices of a rhombus taken in order on the complex plane, which one of the following is held good?

A. $\frac{z_1 - z_4}{z_2 - z_3}$ is purely real

B. $\frac{z_1 - z_3}{z_2 - z_4}$ is purely imaginary

C. $|z_1 - z_3| \neq |z_2 - z_4|$

D. $amp\left(\frac{z_1 - z_4}{z_2 - z_4}\right) \neq amp\left(\frac{z_2 - z_4}{z_3 - z_4}\right)$

Answer: A::B::C

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8. If $|z-3| = \min\{|z-1|, |z-5|\}$, then Re(z) is equal to

A. 2

B. 2.5

C. 3.5

D. 4

Answer: A::D

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9. If $arg(z+a)=\frac{\pi}{6}$ and $arg(z-a)=\frac{2\pi}{3}(a \in R^{+})$, then

A. $|z|=a$

B. $|z|=2a$

C. $arg(z)=\frac{\pi}{3}$

D. $arg(z)=\frac{\pi}{2}$

Answer: A::C

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10. If z=x+iy, where $i=\sqrt{-1}$, then the equation $\left|\left(\frac{2z-i}{z+i}\right)\right|=m$ represents a circle, then m can be

A. $\frac{1}{2}$

B. 1

C. 2

D. $\in(3, 2\sqrt{3})$

Answer: A::B::D

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11. Equation of tangent drawn to circle $|z|=r$ at the point $A(z_0)$, is

A. $Re\left(\frac{z}{z_0}\right)=1$

B. $Im\left(\frac{z}{z_0}\right)=1$

C. $Im\left(\frac{z_0}{z}\right)=1$

D. $z\overline{z_0} + z_0\bar{z} = 2r^2$

Answer: A::D

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12. z_1 and z_2 are the roots of the equaiton $z^2 - az + b = 0$ where $|z_1| = |z_2| = 1$ and a,b are nonzero complex numbers, then

A. $|a| \leq 1$

B. $|a| \leq 2$

C. $arg(a) = arg(b^2)$

D. $arg(a^2) = arg(b)$

Answer: B::D

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13. If α is a complex constant such that $\alpha^2 + z + \overline{\alpha} = 0$ has a real root then

A. $\alpha + \overline{\alpha} = 1$

B. $\alpha + \overline{\alpha} = 0$

C. $\alpha + \overline{\alpha} = -1$

D. the absolute value of real root is 1

Answer: A::C::D

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14. If the equation $z^3 + (3+i)z^2 - 3z - (m+i) = 0$, where $i = \sqrt{-1}$ and $m \in R$, has atleast one real root, value of m is

A. 1

B. 2

C. 3

D. 5

Answer: A::D

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15. If $z^3 + 3 + 2i(z + (- 1 + ia) = 0$ has on ereal roots, then the value of a lies in the interval $(a \in R)$ $(- 2, 1)$ b. $(- 1, 0)$ c. $(0, 1)$ d. $(- 2, 3)$

A. (-2,1)

B. (-1,0)

C. (0,1)

D. (-2,3)

Answer: A::B::D

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Exercise Passage Based Questions

1.

$$arg(\bar{z}) + arg(-z) = \begin{cases} \pi, & \text{if arg (z)} \quad < 0 \\ -\pi, & \text{if arg (z)} \quad > 0 \end{cases}, \text{where} - \pi < arg(z) \leq \pi$$

.

If $arg(z) > 0$, then arg (-z)-arg(z) is equal to

A. $-\pi$

B. $-\frac{\pi}{2}$

C. $\frac{\pi}{2}$

D. π

Answer: A

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2.

$$arg(\bar{z}) + arg(-z) = \begin{cases} \pi, & \text{if arg (z)} \quad < 0 \\ -\pi, & \text{if arg (z)} \quad > 0 \end{cases}, \text{where} - \pi < arg(z) \leq \pi$$

.

Let z_1 and z_2 be two non-zero complex numbers, such that $|z_1| = |z_2|$ and $arg(z_1, z_2) - \pi,$ then z_1 is equal to

A. z_2

B. $\overline{z_2}$

C. $-z_2$

D. $-\overline{z_2}$

Answer: D

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3.

$$arg(\bar{z}) + arg(-z) = \begin{cases} \pi, & \text{if arg (z)} \quad < 0 \\ -\pi, & \text{if arg (z)} \quad > 0 \end{cases}, \text{where} -\pi < arg(z) \leq \pi$$

.

If $arg(4z_1) - arg(5z_2) = \pi,$ then $\left|\frac{z_1}{z_2}\right|$ is equal to

A. 1

B. 1.25

C. 1.5

D. 2.5

Answer: B

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4. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \ \forall n \in I.$

If $\sum_{n=1}^{25} i^{n!} = a + ib, \quad \text{where} \quad i = \sqrt{-1}$, then a-b, is

A. prime number

B. even number

C. composite number

D. perfect number

Answer: A

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5. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \forall n \in I.$

If $\sum_{r=-2}^{95} i^r + \sum_{r=0}^{50} i^{r!} = a + ib,$ where $i = \sqrt{-1},$ the unit digit of $a^{2011} + b^{2012}$, is

A. 2

B. 3

C. 5

D. 6

Answer: C

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6. Sum of four consecutive powers of i(iota) is zero.

i.e.,$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \forall n \in I.$

If $\sum_{r=4}^{100} i^{r!} + \prod_{r=1}^{101} i^{r} = a + ib, \quad \text{where} \quad i = \sqrt{-1}$, then a+75b, is

A. 11

B. 22

C. 33

D. 44

Answer: B

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7. For any two complex numbers $z_1 \text{and} z_2$,

$$|z_1 - z_2| \geq \begin{cases} |z_1| - |z_2| \\ |z_2| - |z_1| \end{cases}$$

and equality holds iff origin $z_1 \quad \text{and} \quad z_2$ are collinear and z_1, z_2 lie on the same side of the origin .

If $\left|z - \frac{1}{z}\right| = 2$ and sum of greatest and least values of $|z|$ is λ, then λ^2, is

A. 2

B. 4

C. 6

D. 8

Answer: D

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8. For any two complex numbers $z_1 \text{and} z_2$, $|z_1 - z_2| \geq \begin{cases} |z_1| - |z_2| \\ |z_2| - |z_1| \end{cases}$ and equality holds iff origin z_1 and z_2 are collinear and z_1, z_2 lie on the same side of the origin . If $\left|z - \frac{2}{z}\right| = 4$ and sum of greatest and least values of $|z|$ is λ, then λ^2, is

A. 12

B. 18

C. 24

D. 30

Answer: C

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9. For any two complex numbers $z_1 \text{and} z_2$,

$$|z_1 - z_2| \geq \begin{cases} |z_1| - |z_2| \\ |z_2| - |z_1| \end{cases}$$

and equality holds iff origin $z_1 \text{ and } z_2$ are collinear and z_1, z_2 lie on the same side of the origin .

If $\left|z - \frac{3}{z}\right| = 6$ and sum of greatest and least values of $|z|$ is 2λ, then λ^2, is

A. 12

B. 18

C. 24

D. 30

Answer: A

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10. Consider the two complex numbers z and w, such that

$$w = \frac{z-1}{z+2} = a + ib, \quad \text{where} \quad a, b \in R \quad \text{and} \quad i = \sqrt{-1}.$$

If $z = CiS\theta$, which of the following does hold good?

A. $\sin\theta = \dfrac{9b}{1-4a}$

B. $\cos\theta = \dfrac{1-5a}{1+4a}$

C. $(1+5a)^2 + (3b)^2 = (1-4a)^2$

D. All of these

Answer: C

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11. Consider the two complex numbers z and w, such that

$$w = \frac{z-1}{z+2} = a + ib, \quad \text{where} \quad a, b \in R \quad \text{and} \quad i = \sqrt{-1}.$$

Which of the following is the value of $-\dfrac{b}{a}$, whenever it exists?

A. $3\tan\left(\frac{\theta}{2}\right)$

B. $\frac{1}{3}\tan\left(\frac{\theta}{2}\right)$

C. $-\frac{1}{3}\cot\theta$

D. $3\cot\frac{\theta}{2}$

Answer: D

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12. Consider the two complex numbers z and w, such that

$$w = \frac{z-1}{z+2} = a + ib, \quad \text{where} \quad a, b \in R \quad \text{and} \quad i = \sqrt{-1}.$$

Which of the following equals to $|z|$?

A. $|w|$

B. $(a+1)^2 + b^2$

C. $a^2 + (b+2)^2$

D. $(a+1)^2 + (b+1)^2$

Answer: B

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Exercise Single Integer Answer Type Questions

1. The number of values of z (real or complex) e simultaneously satisfying the system of equations $1+z+z^2+z^3+\ldots z^{17}=0$ and $1+z+z^2+z^3+..+z^{13}=0$ is

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2. Number of complex numbers satisfying $z^3=\bar{z}$ is

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3. Let z=9+ai, where $i=\sqrt{-1}$ and a be non-zero real.

If $Im(z^2)=Im(z^3)$, sum of the digits of a^2 is

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4. Numbers of complex numbers z, such that $|z| = 1$

and $\left|\frac{z}{\bar{z}} + \frac{\bar{z}}{z}\right| = 1$ is

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5. If $x = a + bi$ is a complex number such that $x^2 = 3 + 4iadnx^3 = 2 + 1i, wherei = \sqrt{-1}, then(a + b)$ equal to

______.

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6. If $z = \frac{\pi}{4}(1 + i)^4\left(\frac{1 - \sqrt{\pi}i}{\sqrt{\pi} + i} + \frac{\sqrt{\pi} - i}{1 + \sqrt{\pi}i}\right), then\left(\frac{|z|}{amp(z)}\right)$ equal

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7. Suppose A is a complex number and $n \in N,$ such that $A^n = (A+1)^n = 1,$ then the least value of n is 3 b. 6 c. 9 d. 12

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8. Let $z_r, r = 1, 2, 3, \ldots, 50$ be the roots of the equation $\sum_{r=0}^{50} (z)^r = 0$. If $\sum_{r=1}^{50} \frac{1}{z_r - 1} = -5\lambda$, then λ equals to

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9. If $p = \sum_{p=1}^{32} (3p+2)\left(\sum_{q=1}^{10}\left(\sin\frac{2q\pi}{11} - i\cos\frac{2q\pi}{11}\right)\right)^p$, where $i = \sqrt{-1}$ and if (1+i)P=n(n!),n $\in$ N, then the value of n is

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10. The least positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\sin^{-1}\left(\frac{1+x^2}{2x}\right)$, where $x > 0$ and $i = \sqrt{-1}$ is

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Exercise Statement I And Ii Type Questions

1. **Statement-1** $3 + 7i > 2 + 4i, \quad \text{where} \quad i = \sqrt{-1}.$

Statement-2 $3 > 2 \quad \text{and} \quad 7 > 4$

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2. **statement-1** $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin\theta, i = \sqrt{-1}$

statement-2 $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^2 = i$

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3. **statement-1** Locus of z satisfying the equation $|z-1|+|z-8|=5$ is an ellipse.

statement-2 Sum of focal distances of any point on ellipse is constant for an ellipse.

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4. Let z_1, z_2 and z_3 be three complex numbers in AP.

Statement-1 Points representing z_1, z_2 and z_3 are collinear

Statement-2 Three numbers a,b and c are in AP, if b-a=c-b

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5. **Statement-1** If the principal argument of a complex numbere z is 0, the principal argument of z^2 is 2θ.

Statement-2 $arg(z^2)=2arg(z)$

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Exercise Subjective Type Questions

1. For any complex numbers z_1, z_2 and z_3, $z_3 Im(\overline{z_2} z_3) + z_2 Im(\overline{z_3} z_1) + z_1 Im(\overline{z_1} z_2)$ is

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2. The roots z_1, z_2, z_3 of the equation $x^3 + 3ax^2 + 3bx + c = 0$ in which a, b, c are complex numbers correspond to points A, B, C. Show triangle will be an equilateral triangle if $a^2 = b$.

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3. If $1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots $x^5 - 1 = 0$, then value of $\frac{\omega - \alpha_1}{\omega^2 - \alpha_1} \cdot \frac{\omega - \alpha_2}{\omega^2 - \alpha_2} \cdot \frac{\omega - \alpha_3}{\omega^2 - \alpha_3} \cdot \frac{\omega - \alpha_4}{\omega^2 - \alpha_4}$ is (where ω is imaginary cube root of unity)

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4. If z_1 and z_2 both satisfy the relation $z + \bar{z} = 2|z - 1|$ and $arg(z_1 - z_2) = \frac{\pi}{4}$, then $Im(z_1 + z_2)$ equals

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5. For every real number c $\geq$ 0, find all complex numbers z which satisfy the equation $|z|^2 - 2iz + 2c(1+i) = 0$, where $i = \sqrt{-1}$ and passing through (-1,4).

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6. Show that the product $\left[1 + \frac{1+i}{2}\right]\left[1 + \left(\frac{1+i}{2}\right)^2\right]\left[1 + \left(\frac{1+i}{2}\right)^{2^2}\right] \dots \left[1 + \left(\frac{1+i}{2}\right)^{2^n}\right]$ is equal to $\left(1 - \frac{1}{2^{2^n}}\right)(1+i)$

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7. Find the point of intersection of the curves $arg(z - 3i) = \frac{3\pi}{4} and arg(2z + 1 - 2i) = \pi/4.$

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8. Show that if a and b are real, the principal value of arg a is 0 or ti according as a is positive or negative and that of bi is $\frac{\pi}{2}$ or $-\frac{\pi}{2}$ according as b is positive or negative.

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9. If $|z| \leq 1 \text{ and } |\omega| \leq 1,$ show that $|z - \omega|^2 \leq (|z| - |\omega|)^2 + (argz - arg\omega)^2$

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10. If $z_1 and z_2$ are two complex numbers and $c > 0$, then prove that $|z_1 + z_2|^2 \leq (1 + c)|z_1|^2 + (1 + c^{-1})|z_2|^2$.

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11. Find the circumstance of the triangle whose vertices are given by the complex numbers z_1, z_2 and z_3.

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12. Find the circumstance of the triangle whose vertices are given by the complex numbers z_1, z_2 and z_3.

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Exercise Questions Asked In Previous 13 Years Exam

1. If ω is a cube root of unity but not equal to 1, then minimum value of $|a + b\omega + c\omega^2|$, (where a,b and c are integers but not all equal), is

A. 0

B. $\frac{\sqrt{3}}{2}$

C. 1

D. 2

Answer: C

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2. If one of the vertices of the square circumscribing the circle $|z - 1| = \sqrt{2}$ is $2 + \sqrt{3}\iota$. Find the other vertices of square

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3. If z_1 and z_2, are two non-zero complex numbers such tha $|z_1 + z_2| = |z_1| + |z_2|$ then $arg(z_1) - arg(z_2)$ is equal to

A. $-\pi$

B. $-\pi/2$

C. $\pi/2$

D. 0

Answer: D

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4. If $1, \omega, \omega^2$ are the cube roots of unity, then the roots of the equation $(x-1)^3 + 8 = 0$ are

A. $-1, 1+2\omega, 1+2\omega^2$

B. $-1, 1-2\omega, 1-2\omega^2$

C. $-1-1-1$

D. None of these

Answer: B

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5. If $\omega = z/[z - (1/3)i]$ and $|\omega| = 1$, then find the locus of z.

A. a straight line

B. a parabola

C. an ellipse

D. a circle

Answer: A

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6. If $w = \alpha + i\beta$ where $B\eta 0$ and $z \neq 1$ satisfies the condition that $\left(\frac{w - \overline{w}z}{1 - z}\right)$ is purely real then the set of values of z is

A. $\{z : |z| = 1\}$

B. $\{z : z = \bar{z}\}$

C. $\{z : z \neq 1\}$

D. $\{z : |z| = 1, z \neq 1\}$

Answer: D

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7. Find the value of $\sum_{k=1}^{10}\left[\sin\left(\frac{2\pi k}{11}\right) - i\cos\left(\frac{2\pi k}{11}\right)\right], wherei = \sqrt{-1}.$

A. i

B. 1

C. -1

D. $-i$

Answer: D

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8. If $z^2+z+1=0$ where z is a complex number, then the value of $\left(z+\frac{1}{z}\right)^2+\left(z^2+\frac{1}{z^2}\right)^2+\ldots.+\left(z^6+\frac{1}{z^6}\right)^2$ is

A. 18

B. 54

C. 6

D. 12

Answer: D

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9. A man walks a distance of 3 units from the origin towards north-east to reach position A. from there he walks a distance of 4 units towards North west to reach position P. Then position of P in argand plane is

A. $3e^{i\pi/4}+4i$

B. $(3-4i)e^{i\pi/4}$

C. $(4+3i)e^{i\pi/4}$

D. $(3+4i)e^{i\pi/4}$

Answer: D

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10. If $|z|=1$ and $z\ne\pm1$, then all the values of $\dfrac{z}{1-z^2}$ lie on

A. a line not passing through the origin

B. $|z|=\sqrt{2}$

C. the X-axis

D. the Y-axis

Answer: D

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11. If $|z+4| \leq 3$, the maximum value of $|z+1|$ is

A. 4

B. 10

C. 6

D. 0

Answer: C

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12. Let A, B, C be three sets of complex number as defined below:

$A = \{z : Im \geq 1\}, B = \{z : |z - 2 - i| = 3\}, C : \{z : Re((1 - i)z) = \sqrt{2}\}$

The number of elements in the set $A \cap B \cap C$ is

A. 0

B. 1

C. 2

D. ∞

Answer: B

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13. Let A,B and C be three sets of complex numbers as defined below:

$$A = \{z : Im(z) \geq 1\}$$
$$B = \{z : |z - 2 - i| = 3\}$$
$$C = \{z : Re(1 - i)z) = 3\sqrt{2} \text{where} i = \sqrt{-1}$$

Let z be any point in $A \cap B \cap C$. Then, $|z + 1 - i|^2 + |z - 5 - i|^2$ lies between

A. 25 and 29

B. 30 and 34

C. 35 and 39

D. 40 and 44

Answer: C

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14. Let A,B and C be three sets of complex numbers as defined below:

$A = \{z : Im(z) \geq 1\}$
$B = \{z : |z - 2 - i| = 3\}$
$C = \{z : Re(1 - i)z) = 3\sqrt{2} \text{where} i = \sqrt{-1}$

Let z be any point in $A \cap B \cap C$ and ω be any point satisfying $|\omega - 2 - i| < 3$. Then, $|z| - |\omega| + 3$ lies between

A. -6 and 3

B. -3 and 6

C. -6 and 6

D. -3 and 3

Answer: D

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15. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1 the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{i} + \hat{j}$ and then it moves through an angle $\frac{\pi}{2}$ in anticlockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by $6 + 7i$ (b) $-7 + 6i$ $7 + 6i$ (d) $-6 + 7i$

A. 6+7i

B. $-7 + 6i$

C. 7+6i

D. $-6 + 7i$

Answer: D

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16. If the conjugate of a complex numbers is $\frac{1}{i-1}$, where $i=\sqrt{-1}$. Then, the complex number is

A. $\frac{-1}{i-1}$

B. $\frac{1}{i+1}$

C. $\frac{-1}{i+1}$

D. $\frac{1}{i-1}$

Answer: C

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17. Let $z=x+iy$ be a complex number where $x and y$ are integers. Then, the area of the rectangle whose vertices are the roots of the equation

$zz^3 + zz^3 = 350$ is 48 (b) 32 (c) 40 (d) 80

A. 48

B. 32

C. 40

D. 80

Answer: A

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18. Let $z = \cos\theta + i\sin\theta$. Then the value of $\sum_{m \to 1-15} Img\left(z^{2m-1}\right)$ at $\theta = 2^\circ$ is:

A. $\dfrac{1}{\sin 2^\circ}$

B. $\dfrac{1}{3\sin 2^\circ}$

C. $\dfrac{1}{2\sin 2^\circ}$

D. $\dfrac{1}{4\sin 2^\circ}$

Answer: D

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19. If $\left|Z - \frac{4}{Z}\right| = 2$ then maximum value of $|Z|$ is equal to

A. $2 + \sqrt{2}$

B. $\sqrt{3} + 1$

C. $\sqrt{5} + 1$

D. 2

Answer: C

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20. Let z_1 and z_2 be two distinct complex numbers and $z = (1 - t)z_1 + iz_2$, for some real number t with $0 < t < 1$ and

$i = \sqrt{-1}$. If arg(w) denotes the principal argument of a non-zero compolex number w, then

A. $|z - z_1| + |z - z_2| = |z_1 - z_2|$

B. $arg(z - z_1) = arg(z - z_2)$

C. $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

D. $arg(z - z_1) = arg(z_2 - z_1)$

Answer: A:B:C::D

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21. Let ω be the complex number $\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$. Then the number of distinct complex cos numbers z satisfying $\Delta = \begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is

A. 0

B. 1

C. 2

D. 3

Answer: B

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22. If α and β are the roots of the equation $x^2 - x + 1 = 0$, $\alpha^{2009} + \beta^{2009}$ is equal to

A. -1

B. 1

C. 2

D. -2

Answer: B

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23. The number of complex numbers z such that $|z-1|=|z+1|=|z-i|$ is

A. 1

B. 2

C. ∞

D. 0

Answer: A

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24. If z is any complex number satisfying $|z-3-2i| \leq 2$, where $i=\sqrt{-1}$, then the minimum value of $|2z-6+5i|$, is

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25. The set $\left\{Re\left(\frac{2iz}{1-z^2}\right): zisacomplex\nu mber, |z| = 1, z = \pm 1\right\}$ is________.

A. $(-\infty, -1] \cap [1, \infty)$

B. $(-\infty, 0) \cup (0, \infty)$

C. $(-\infty, -1] \cup [1, \infty)$

D. $[2, \infty)$

Answer: A

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26. The maximum value of $\left|arg\left(\frac{1}{1-z}\right)\right|f$ or $|z| = 1, z \neq 1$ is given by.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer: C

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27. Let $\omega = e^{\frac{i\pi}{3}}$ and a, b, c, x, y, z be non-zero complex numbers such that $a+b+c=x$, $a+b\omega+c\omega^2=y$, $a+b\omega^2+c\omega=z$.Then, the value of $\frac{|x|^2+|y|^2|+|y|^2}{|a|^2+|b|^2+|c|^2}$

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28. Let α and β be real numbers and z be a complex number. If $z^2+\alpha z+\beta=0$ has two distinct non-real roots with Re(z)=1, then it is necessary that

A. $\beta \in (-1, 0)$

B. $|\beta| = 1$

C. $\beta \in (1, \infty)$

D. $\beta \in (0, 1)$

Answer: C

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29. If $\omega(\neq 1)$ be a cube root of unity and $(1+\omega)^7 = A + B\omega$, then A and B are respectively the numbers.

A. (1,1)

B. (1,0)

C. (-1,1)

D. (0,1)

Answer: A

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30. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a cannot take the value

A. -1

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: D

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31. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, the point represented by the complex numbers z lies

A. on a circle with centre at the origin

B. either on the real axis or on a circle not passing through the origin

C. on the imaginary axis

D. either on the real axis or on a circle passing through the origin

Answer: D

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32. If z is a complex number of unit modulus and argument θ, then $arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals to

A. $\frac{\pi}{2}-\theta$

B. θ

C. $\pi-\theta$

D. $-\theta$

Answer: B

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33. Let complex numbers α and $\frac{1}{\alpha}$ lies on circle $(x-x_0)^2(y-y_0)^2=r^2$ and $(x-x_0)^2+(y-y_0)^2=4r^2$ respectively. If $z_0=x_0+iy_0$ satisfies the equation $2|z_0|^2=r^2+2$ then $|\alpha|$ is equal to (a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{7}}$ (d) $\frac{1}{3}$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{\sqrt{7}}$

D. $\frac{1}{3}$

Answer: C

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34. Let w = $\left(\sqrt{3}+\frac{\iota}{2}\right)$ and $P=\{w^n : n=1,2,3,\ldots..\}$, Further $H_1=\left\{z\in C : Re(z)>\frac{1}{2}\right\}$ and $H_2=\left\{z\in c : Re(z)< -\frac{1}{2}\right\}$ Where C is set of all complex numbers. If $z_1\in P\cap H_1$, $z_2\in P\cap H_2$ and O represent the origin, then $\angle Z_1OZ_2$ =

A. $\frac{\pi}{2}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: C

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35. Let $S = S_1 \cap S_2 \cap S_3,$ where $s_1 = \{z \in C : |z| < 4\}, S_2 = \left\{z \in C : \ln\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{31}}\right] > 0\right\}$ and $S_3 =$

A. $\frac{2 - \sqrt{3}}{2}$

B. $\frac{2 + \sqrt{3}}{2}$

C. $\frac{3 - \sqrt{3}}{2}$

D. $\frac{3 + \sqrt{3}}{2}$

Answer: C

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36. Let $S = S_1 \cap S_2 \cap S_3$, where $s_1 = \{z \in C : |z| < 4\}$, $S_2 = \left\{z \in C : \ln\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{31}}\right] > 0\right\}$ and $S_3 =$

A. $\frac{10\pi}{3}$

B. $\frac{20\pi}{3}$

C. $\frac{16\pi}{3}$

D. $\frac{32\pi}{3}$

Answer: B

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37. If z is a complex number such that $|z| \geq 2$ then the minimum value of $\left|z + \frac{1}{2}\right|$ is

A. is strictly greater than $\frac{5}{2}$

B. is equal to $\frac{5}{2}$

C. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

D. lies in the interval (1,2)

Answer: D

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38. A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 z_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a

A. circle of radius z

B. circle of radius $\sqrt{2}$

C. straight line parallel to X-axis

D. straight line parallel to y-axis

Answer: A

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39. Let $\omega \neq 1$ be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4n+3}+(2+3\omega-3\omega^2)^{4n+3}+(-3+2\omega+2\omega^2)^{4n+3}=0$, then the set of possible value(s) of n is are

A. 1

B. 2

C. 3

D. 4

Answer: A::B::D

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40. For any integer k, let $\alpha_k=\frac{\cos(k\pi)}{7}+i\frac{\sin(k\pi)}{7}, wherei=\sqrt{-1}$. Value of the expression $\frac{\sum k=112|\alpha_{k+1}-\alpha_k|}{\sum k=13|\alpha_{4k-1}-\alpha_{4k-2}|}$ is

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41. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is

A. $\frac{\pi}{6}$

B. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

C. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

D. $\frac{\pi}{3}$

Answer: C

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42. Let $0 \neq a, 0 \neq b \in R$. Suppose

$S = \left\{z \in C, z = \frac{1}{a+ibt} t \in R, t \neq 0\right\}$, where $i = \sqrt{-1}$. If $z = x + iy$ and $z \in S$, then (x, y) lies on

A. the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for $a > 0, b \neq 0$

B. the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for $a < 0, b \neq 0$

C. the X-axis for $a \neq 0, b = 0$

D. the Y-axis for $a = 0, b \neq 0$

Answer: A::C::D

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43. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\left|(1, 1, 1), \left(1, -\omega^2 - 1, \omega^2\right), \left(1, \omega^2, \omega^7\right)\right| = 3k$, then k is equal to

A. 1

B. $-z$

C. z

D. -1

Answer: B

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