MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

CONTINUITY AND DIFFERENTIABILITY

Examples

1. If $f(x) = \frac{|X|}{X}$. Discuss the continuity at $x \to 0$

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2. If $f(x) = \begin{cases} 2x+3, & \text{when } x < 0 \\ 0, & \text{when } x = 0 \\ x^2+3, & \text{when } x > 0 \end{cases}$ Discuss the continuity.

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3. If $f(x) = \frac{x^2 - 1}{x - 1}$ Discuss the continuity at $x \to 1$

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4. Show that the function $f(x) = \begin{cases} 2x + 3, & -3 \leq x < -2 \\ x + 1, & -2 \leq x < 0 \\ x + 2, & 0 \leq x \leq 1 \end{cases}$ is discontinuous at x = 0 and continuous at every point in interval $[-3, 1]$

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5. Examine the function, $f(x) = \begin{cases} \frac{\cos x}{\pi/2 - x}, & x \neq \pi/2 \\ 1, & x = \pi/2 \end{cases}$ for continuity at $x = \pi/2$

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6. Discuss the continuity of $f(x) = \tan^{-1} x$

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7. Let y = f(x) be defined parametrically as $y = t^2 + t|t|, x = 2t - |t|, t \in R$. Then, at x = find f(x) and discuss continuity.

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8. Let $f(x) = \frac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x}$ be a continuous function at x=0. The value f(0) equals

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 2

Answer: C

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9. If $f(x) = \sqrt{\dfrac{1}{\tan^{-1}(x^2 - 4x + 3)}}$, then f(x) is continuous for

A. (1, 3)

B. $(-\infty, 0)$

C. $(-\infty, 1) \cup (3, \infty)$

D. None of these

Answer: C

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10. If $f(x) = [x]$, where $[\cdot]$ denotes greatest integral function. Then, check the continuity on $(1, 2]$

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11. Examine the function, $f(x) = \begin{cases} x - 1, & x < 0 \\ 1/4, & x = 0 \\ x^2 - 1, & x > 0 \end{cases}$ Discuss the continuity and if discontinuous remove the discontinuity.

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12. Show the function, $f(x) = \begin{cases} \frac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ has non-removable discontinuity at x = 0

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13. Show $f(x) = \frac{1}{|x|}$ has discontinuity of second kind at x = 0.

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14. $f(x) = \begin{cases} \left(\tan\left(\frac{\pi}{4} + x\right)\right)^{1/x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ for what value of k, f(x) is continuous at x = 0 ?

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15. A function f(x) is defined by, $f(x) = \begin{cases} \frac{[x^2] - 1}{x^2 - 1}, & \text{for} \quad x^2 \neq 1 \\ 0, & \text{for} \quad x^2 = 1 \end{cases}$ Discuss the continuity of f(x) at x = 1.

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16. Discuss the continuity of the function $f(x) = \lim_{n \to \infty} \frac{\log(2 + x) - x^{2n}\sin x}{1 + x^{2n}}$ at x $= 1$

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17. Discuss the continuity of f(x), where $f(x) = \lim_{n \to \infty} \left(\sin\frac{\pi x}{2}\right)^{2n}$

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18. Let $f(x) = \begin{cases} \{1+|\sin x|\}^{a/|\sin x|}, & -\pi/6 < x < 0 \\ b, & x = 0 \\ e^{\tan 2x/\tan 3x}, & 0 < x < \pi/6 \end{cases}$ Determine a and b such that f(x) is continuous at x = 0

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19. Fill in the blanks so that the resulting statement is correct. Let $f(x) = [x+2]\sin\left(\frac{\pi}{[x+1]}\right)$, where $[\cdot]$ denotes greatest integral function. The domain of f is ....and the points of discontinuity of f in the domain are

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20. Let $f(x+y) = f(x) + f(y)$ for all $x and y$. If the function $f(x)$ is continuous at $x = 0$, show that $f(x)$ is continuous for all x.

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21. Let $f(x)$ be a continuous function defined for $1 \leq x \leq 3$. If $f(x)$ takes rational values for all x and $f(2) = 10$ then the value of $f(1.5)$ is :

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22. Discuss the continuity for $f(x) = \frac{1-u^2}{2+u^2}$, where u = tan x.

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23. Find the points of discontinuity of $y = \frac{1}{u^2 + u - 2}$, where $u = \frac{1}{x-1}$

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24. Show that the function $f(x) = (x-a)^2(x-b)^2 + x$ takes the value $\frac{a+b}{2}$ for some value of $x \in [a, b]$.

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25. Suppose that f(x) is continuous in [0, 1] and $f(0)=0, f(1)=0$. Prove $f(c)=1-2c^2$ for some $c \in (0, 1)$

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26. The left hand derivative of $f(x)=[x]\sin(\pi x)$ at $x=k$, k is an integer, is

A. $(-1)^k(k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1}k\pi$

Answer: A

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27. Which of the following functions is differentiable at x = 0 ?

A. $\cos(|x|) + |x|$

B. $\cos(|x|) - |x|$

C. $\sin(|x|) + |x|$

D. $\sin(|x|) - |x|$

Answer: D

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28. Show that $f(x) = \begin{cases} \text{x} \sin\frac{1}{x}, & \text{when} \quad x \neq 0 \\ 0, & \text{when} \quad x = 0 \end{cases}$ is continuous but not differentiable at x = 0

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29. Let $f(x) = (xe)^{\frac{1}{|x|}+\frac{1}{x}}; x \neq 0, f(0) = 0,$ test the continuity & differentiability at $x = 0$

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30. Let $f(x) = |x-1| + |x+1|$ Discuss the continuity and differentiability of the function.

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31. Discuss the continuity and differentiability for $f(x) = [\sin x]$ when $x \in [0, 2\pi]$, where $[\cdot]$ denotes the greatest integer function x.

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32. If $f(x) = \{|x| - |x-1|\}^2$, draw the graph of f(x) and discuss its continuity and differentiability of f(x)

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33. If $f(x) = \begin{cases} x-3, & x<0 \\ x^2-3x+2, & x \geq 0 \end{cases}$ and let $g(x) = f(|x|) + |f(x)|$.

Discuss the differentiability of g(x).

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34. Let f(x) = [n + p sin x], $x \in (0, \pi), n \in Z$, p a prime number and [x] = the greatest integer less than or equal to x. The number of points at which f(x) is not not differentiable is :

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35. If $f(x) = ||x| - 1|$, then draw the graph of f(x) and fof(x) and also discuss their continuity and differentiability. Also, find derivative of $(fof)^2$ at x $= \frac{3}{2}$

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36. Draw the graph of the function $g(x) = f(x+I) + f(x-I)$, where $f(x) = \begin{cases} k\left\{1 - \frac{|x|}{I}\right\}, & \text{for } |x| \leq I \\ 0, & |x| > I \end{cases}$ Also, discuss the continuity and differentiability of the function g(x).

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37. Let $f(x) = \begin{cases} \int_0^x \{5 + |1-t|\}dt, & \text{if } x > 2 \\ 5x+1, & \text{if } x \leq 2 \end{cases}$ Test f(x) for continuity and differentiability for all real x.

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38. Draw the graph of the function and discuss the continuity and differentiability at x = 1 for, $f(x) = \begin{cases} 3^x, & \text{when } -1 \leq x \leq 1 \\ 4-x, & \text{when } 1 < x < 4 \end{cases}$

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39. Match the conditions/expressions in Column I with statement in Column II. (A). $\sin(\pi[x])$ (B) $\sin\{\pi(x-[x])\}$

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40. The set of points where ,$f(x) = x|x|$ is twice differentiable is

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41. is The function $f(x) = (x^2-1)\left|x^2-3x+2\right| + \cos(|x|)$ is differentiable not differentiable at (a)-1 (b)0 (c)1 (d)2

A. -1

B. 0

C. 1

D. 2

Answer: D

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42. If $f(x) = \sum_{r=1}^{n} a_r |x|^r$, where a_i s are real constants, then f(x) is

A. continuous at x = 0, for all a_i

B. differentiable at x = 0, for all $a_i \in R$

C. differentiable at x = 0, for all $a_{2k+1} = 0$

D. None of the above

Answer: A::C

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43. Let f and g be differentiable functions satisfying $g(a) = b, g'(a) = 2$ and fog =I (identity function). then f' (b) is equal to

A. 2

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. None of these

Answer: C

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44. If $f(x) = \frac{x}{1 + (\log x)(\log x).... \infty}, \forall x \in [1, 3]$ is non-differentiable at x = k. Then, the value of $[k^2]$, is (where $[\cdot]$ denotes greatest integer function).

A. 5

B. 6

C. 7

D. 8

Answer: C

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45. If $f(x) = |1 - x|$, then the points where $\sin^{-1}(f|x|)$ is non-differentiable are

A. {0, 1}

B. {0, -1}

C. {0, 1, -1}

D. None of these

Answer: C

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46. Discuss the differentiability of $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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47. Let $[.]$ represent the greatest integer function and $f(x) = [\tan^2 x]$ then :

A. $\lim_{x \to 0} f(x)$ doesn't exist

B. f(x) is continuous at x = 0

C. f(x) is not differentiable at x = 0

D. $f'(0) = 1$

Answer: B

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48. Let $h(x) = \min\{x, x^2\}$, for every real number of X. Then (A) h is continuous for all x (B) h is differentiable for all x (C) $h'(x)$ = 1, for all x > 1 (D) h is not differentiable at two values of x

A. h is not continuous for all x

B. h is differentiable for all x

C. $h'(x) = 1$ for all x

D. h is not differentiable at two values of x

Answer: D

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49. let $f: R \to R$ be a function defined by $f(x) = \max\{x, x^3\}$. The set of values where f(x) is differentiable is:

A. {-1, 1}

B. {-1, 0}

C. {0, 1}

D. {-1, 0, 1}

Answer: D

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50. Let f(x) be a continuous function, $\forall x \in R, f(0) = 1$ and $f(x) \neq x$ for any $x \in R$, then show $f(f(x)) > x, \forall x \in R^+$

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51. The total number of points of non-differentiability of $f(x) = \max\left\{\sin^2 x, \cos^2 x, \frac{3}{4}\right\}$ in $[0, 10\pi]$, is

A. 40

B. 30

C. 20

D. 10

Answer: A

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52. If $f(x) = |x+1|\{|x|+|x-1|\}$, then draw the graph of f(x) in the interval [-2, 2] and discuss the continuity and differentiability in [-2, 2].

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53. If the function $f(x) = \left[\frac{(x-2)^3}{a}\right]\sin(x-2) + a\cos(x-2)$, $[.]$ denotes the greatest integer function, is continuous in $[4, 6]$, then find the values of a.

A. $a \in [8, 64]$

B. $a \in (0, 8]$

C. $a \in [64, \infty)$

D. None of these

Answer: C

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54. If

$$f(x) = x^2 - 2|x| \text{ and } g(x) = \begin{cases} \min\{\mathrm{f(t)}: -2 \le t \le x, & -2 \le x \le \} \\ \max\ \{\mathrm{f(t)}:0 \le t \le x, & 0 \le x \le 3\} \end{cases}$$

(i) Draw the graph of f(x) and discuss its continuity and differentiablity.

(ii) Find and draw the graph of g(x Also, discuss the continuity.

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55. Let $f(x) = \phi(x) + \Psi(x)$ and $\Psi'(a)$ are finite and definite. Then,

A. f(x) is continuous at x = a

B. f(x) is differentiable at x = a

C. f'(x) is continuous at x = a

D. f'(x) is differentiable at x = a

Answer: A::B

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56. If $f(x) = x + \tan x$ and $g(x)$ is the inverse of f(x), then g'(x) is equal to

A. $\dfrac{1}{1 + (g(x) - x)^2}$

B. $\dfrac{1}{2 + (g(x) + x)^2}$

C. $\dfrac{1}{2 + (g(x) - x)^2}$

D. None of these

Answer: C

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57. If $f(x) = \int_0^x (f(t))^2 dt$, $f: R \to R$ be differentiable function and $f(g(x))$ is differentiable at $x = a$, then

A. g(x) must be differentiable at x = a

B. g(x) is discontinuous, then f(a) = 0

C. $f(a) \neq 0$, then g(x) must be differentiable

D. None of these

Answer: B::C

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58. If $f(x) = [x^{-2}[x^2]]$, (where $[\cdot]$ denotes the greatest integer function) $x \neq 0$, then incorrect statement

A. f(x) is continuous everywhere

B. f(x) is discontinuous at $x = \sqrt{2}$

C. f(x) is non-differentiable at x = 1

D. f(x) is discontinuous at infinitely many points

Answer: A

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59. If

$f(x) = \{x^2(sgn[x]) + \{x\}, 0 \leq x \leq 2 \sin x + |x-3|, 2 < x < 4,$

(where[.] & {.} greatest integer function & fractional part functiopn respectively), then -

A. f(x) is differentiable at x = 1

B. f(x) is continuous but non-differentiable at x

C. f(x) is non-differentiable at x = 2

D. f(x) is discontinuous at x = 2

Answer: C::D

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60. A real valued function f(x) is given as

$$f(x) = \begin{cases} \int_0^x 2\{x\}dx, & x + \{x\} \in I \\ x^2 - x + \frac{1}{2}, & \frac{1}{2} < x < \frac{3}{2} \text{ and } x \neq 1, \text{where}[] \\ x^2 - x + \frac{1}{6}, & \text{otherwise} \end{cases}$$ denotes

greatest integer less than or equals to x and {} denotes fractional part function of x. Then,

A. f(x) is continuous and differentiable in $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

B. f(x) is continuous and differentiable in $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

C. f(x) is continuous and differentiable in $x \in \left[\frac{1}{2}, \frac{3}{2}\right]$

D. f(x) is continuous but not differentiable in $x \in (0, 1)$

Answer: D

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61. The values of a and b so that the function $f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \le x < \pi/4 \\ 2x\cot x + b, & \pi/4 \le x \le \pi/2 \\ a\cos 2x - b\sin x, & \pi/2 < x \le \pi \end{cases}$ is continuous for $x \in [0, \pi]$, are

A. $a = \frac{\pi}{6}, b = -\frac{\pi}{6}$

B. $a = -\frac{\pi}{6}, b = \frac{\pi}{12}$

C. $a = \frac{\pi}{6}, b = -\frac{\pi}{12}$

D. None of these

Answer: C

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62. Let f be an even function and f'(0) exists, then f'(0) is

A. 1

B. 0

C. -1

D. -2

Answer: B

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63. The set of points where $x^2|x|$ is thrice differentiable, is

A. R

B. R - {0, 1}

C. $[0, \infty)$

D. R - {0}

Answer: D

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64. The function $f(x) = \frac{|x+2|}{\tan^{-1}(x+2)}$, is continuous for

A. $x \in R$

B. $x \in R - \{0\}$

C. $x \in R - \{-2\}$

D. None of these

Answer: C

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65. If $f(x) = \begin{cases} \frac{\sin[x^2]\pi}{x^2 - 3x + 8} + ax^3 + b & 0 \le x \le 1 \\ 2\cos \pi x + \tan^{-1} x & 1 < x \le 2 \end{cases}$ is differentiable in $[0, 2]$ then: ([.] denotes greatest integer function)

A. $a = \frac{1}{6}, b = \frac{\pi}{4} - \frac{13}{6}$

B. $a = -\frac{1}{6}, b = \frac{\pi}{4}$

C. $a = -\frac{1}{6}, b = \frac{\pi}{4} - \frac{13}{6}$

D. None of these

Answer: A

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66. If $g(x) = \lim_{m \to \infty} \frac{x^m f(1) + h(x) + 1}{2x^m + 3x + 3}$ is continuous at x = 1 and $g(1) = \lim_{x \to 1} \{\log_e(ex)\}^{2/\log_e x}$, then the value of $2g(1) + 2f(1) - h(1)$ when f(x) and h(x) are continuous at x = 1, is

A. 0

B. 1

C. 2

D. 3

Answer: B

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67. Let $g(x) = \ln f(x)$ where f(x) is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = x f(x)$. Then for N = 1,2,3

$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$

A. $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + ... + \frac{1}{(2N-1)^2}\right\}$

B. $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots.+\frac{1}{(2N-1)^2}\right\}$

C. $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots.+\frac{1}{(2N-1)^2}\right\}$

D. $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots.+\frac{1}{(2N-1)^2}\right\}$

Answer: A

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68. Let y = f(x) be a differentiable function, $\forall x \in R$ and satisfies,

$f(x) = x + \int_0^1 x^2 z f(z) dz + \int_0^1 x z^2 f(z) dz$, then

A. $f(x) = \frac{20x}{119}(2+9x)$

B. $f(x) = \frac{20x}{119}(4+9x)$

C. $f(x) = \frac{10x}{119}(4+9x)$

D. $f(x) = \frac{5x}{119}(4+9x)$

Answer: B

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69. A function $f: R \rightarrow R$ satisfies the equation $f(x+y) = f(x).\ f(y)$ for all, $f(x) \neq 0$. Suppose that the function is differentiable at x = 0 and f'(0) = 2. Then,

A. f'(x) = 2f(x)

B. f'(x) = f(x)

C. f'(x) = f(x) + 2

D. f'(x) = 2f(x) + x

Answer: A

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70. Let f be a function such that $f(x + f(y)) = f(x) + y, \forall x, y \in R$, then find f(0). If it is given that there exists a positive real δ such that f(h) = h for $0 < h < \delta$, then find f'(x)

A. 0, 1

B. $-1, 0$

C. 2, 1

D. $-2, 0$

Answer: A

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71. If the function of $f(x) = \left[\frac{(x-5)^2}{A}\right]\sin(x-5) + a\cos(x-2)$, where$[\cdot]$ denotes the greatest integer function, is continuous and differentiable in (7, 9), then find the value of A

A. $A \in [8, 64]$

B. $A \in [0, 8)$

C. $A \in [16, \infty)$

D. $A \in [8, 16]$

Answer: C

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72. If $f(x) = [2 + 5|n|\sin x]$, where $n \in I$ has exactly 9 points of non-derivability in $(0, \pi)$, then possible values of n are (where [x] dentoes greatest integer function)

A. ± 3

B. ± 2

C. ± 1

D. None of these

Answer: C

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73. The number of points of discontinuity of $f(x) = [2x]^2 - \{2x\}^2$ (where [ ] denotes the greatest integer function and { } is fractional part of x) in the interval $(-2, 2)$, is

A. 6

B. 8

C. 4

D. 3

Answer: A

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74. If $x \in R^+$ and $n \in N$, we can uniquely write $x = mn + r$, where $m \in W$ and $0 \leq r < n$. We define $x \text{ mod } n = r$. The number of points of discontinuity of the function, $f(x) = (x \text{ mod } 2)^2 + (x \text{ mod } 4)$ in the interval$0 < x < 9$ is

A. 0

B. 2

C. 4

D. None of these

Answer: C

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75. Let $f : R \rightarrow R$ be a differentiable function at x = 0 satisfying f(0) = 0 and f'(0) = 1, then the value of $\lim_{x \rightarrow 0} \frac{1}{x} . \sum_{n=1}^{\infty} (-1)^n . f\left(\frac{x}{n}\right)$, is

A. 0

B. $-\log 2$

C. 1

D. e

Answer: B

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76. Let f(x) is a function continuous for all $x \in R$ except at x = 0 such that $f'(x) < 0, \forall x \in (-\infty, 0)$ and $f'(x) > 0, \forall x \in (0, \infty)$. If $\lim_{x \to 0^+} f(x) = 3$, $\lim_{x \to 0^-} f(x) = 4$ and $f(0) = 5$, then the image of the point (0, 1) about the line, $y. \lim_{x \to 0} f(\cos^3 x - \cos^2 x) = x. \lim_{x \to 0} f(\sin^2 x - \sin^3 x)$, is

A. $\left(\frac{12}{25}, \frac{-9}{25}\right)$

B. $\left(\frac{12}{25}, \frac{9}{25}\right)$

C. $\left(\frac{16}{25}, \frac{-8}{25}\right)$

D. $\left(\frac{24}{25}, \frac{-7}{25}\right)$

Answer: D

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77. If f(x) be such that $f(x) = \max(|3 - x|, 3 - x^3)$, then

A. f(x) is continuous $\forall x \in R$

B. f(x) is differentiable $\forall x \in R$

C. f(x) is non-differentiable at three points only

D. f(x) is non-differentiable at four points only

Answer: A::D

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78. Let $f(x) = |x-1|([x]-[-x])$, then which of the following statement(s) is/are correct. (where [.] denotes greatest integer function.)

A. f(x) is continuous at x = 1

B. f(x) is derivable at x = 1

C. f(X) is non-derivable at x = 1

D. f(x) is discontinuous at x = 1

Answer: A::C

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79. If y = f(x) defined parametrically by $x = 2t - |t-1|$ and $y = 2t^2 + t|t|$, then

A. f(x) is continuous for all $x \in R$

B. f(x) is continuous for all $x \in R - \{2\}$

C. f(x) is differentiable for all $x \in R$

D. f(x) is differentiable for all $x \in R - \{2\}$

Answer: A::D

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80. $f(x) = \sin^{-1}[e^x] + \sin^{-1}[e^{-x}]$ where [.] greatest integer function then

A. domain of $f(x) = (-\text{In } 2, \text{In } 2)$

B. range of f(x) = $\{\pi\}$

C. f(x) has removable discontinuity at x = 0

D. $f(x) = \cos^{-1} x$ has only solution

Answer: A::C

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81. $f: R \rightarrow R$ is one-one, onto and differentiable and graph of y = f (x) is symmetrical about the point (4, 0), then

A. $f^{-1}(2010) + f^{-1}(-2010) = 8$

B. $\int_{-2010}^{2018} f(x)dx = 0$

C. if $f'(-100) > 0$, then roots of $x^2 - f'(10)x - f'(10) = 0$ may be non-real

D. if $f'(10) = 20$, then f'(-2) = 20

Answer: A::B::D

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82. Let f be a real-valued function defined on interval $(0, \infty)$,by $f(x) = \ln x + \int_0^x \sqrt{1 + \sin t}.\, dt.$ Then which of the following statement(s) is (are) true? (A). f"(x) exists for all $\in (0, \infty)$. (B). f'(x) exists for all x $\in (0, \infty)$ and f' is continuous on $(0, \infty)$, but not differentiable on $(0, \infty)$. (C). there exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all x $\in (\alpha, \infty)$. (D). there exists $\beta > 1$ such that $|f(x)| + |f'(x)| \leq \beta$ for all x $\in (0, \infty)$.

A. f''(x) exists for all $x \in (0, \infty)$

B. f'(x) exists for all $x \in (0, \infty)$ and f' is continuous on $(0, \infty)$ but not differentiable on $(0, \infty)$

C. There exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (0, \infty)$

D. There exists $\beta > 0$ such that $|f(x)| + |f'(x)| \leq \beta$ from all $x \in (0, \infty)$

Answer: B::C

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83. If $f(x)+f(y)=f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R(xy \neq 1)$ and $\lim_{x \to 0} \frac{f(x)}{x}=2$, then

A. $f\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{3}$

B. $f\left(\frac{1}{\sqrt{3}}\right)=-\frac{\pi}{3}$

C. $f'(1)=1$

D. $f'(1)=-1$

Answer: A::C

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84. Let $f : R\vec{R}$ be a function satisfying condition $f(x+y^3)=f(x)+[f(y)]^3 f \text{ or } all x, y \in R$. If $f'(0) \geq 0$, find $f(10)$.

A. f(x) = 0 only

B. f(x) = x only

C. f(x) = 0 or x only

D. f(10) = 10

Answer: C::D

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85. Let

$f(x) = x^3 - x^2 + x + 1$ and $g(x) = \begin{cases} \max f(t), & 0 \le t \le x \quad \text{for} \quad 0 \le \\ 3 - x, & 1 < x \le 2 \end{cases}$

Then, g(x) in [0, 2] is

A. continuous for $x \in [0, 2] - \{1\}$

B. continuous for $x \in [0, 2]$

C. differentiable for all $x \in [0, 2]$

D. differentiable for all $x \in [0, 2] - \{1\}$

Answer: B::D

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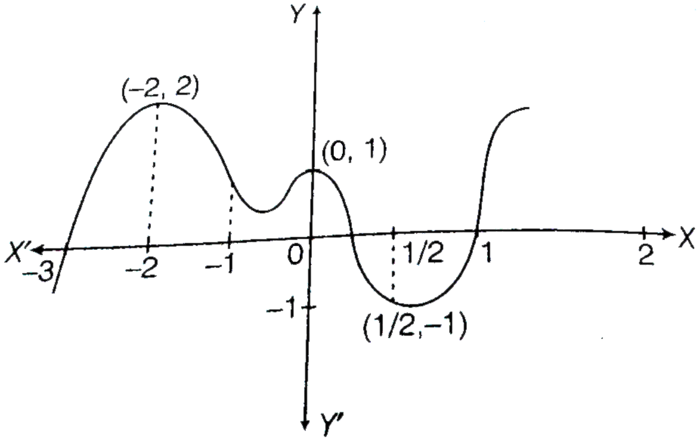
86. If $p''(x)$ has real roots α, β, γ. Then , $[\alpha]+[\beta]+[\gamma]$ is



A. -2

B. -3

C. -1

D. 0

Answer: B

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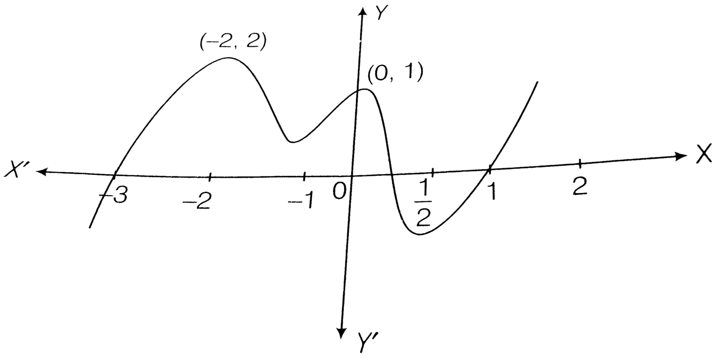
87. In the given figure graph of $y = P(x) = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$, is given.



The minimum number of real roots of equation $(P''(x))^2 + P'(x). P'''(x) = 0$, is

A. 5

B. 7

C. 6

D. 4

Answer: C

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88. If α, β (where $\alpha < \beta$) are the points of discontinuity of the function g(x) = f(f(f(x))), where `f(x) = (1)/(1-x). Then, The points of discontinuity of g(x) is

A. x = 0, -1

B. x = 1 only

C. x = 0 only

D. x = 0, 1

Answer: D

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89. If α, β (where $\alpha < \beta$) are the points of discontinuity of the function g(x) = f(f(f(x))), where $f(x) = \frac{1}{1-x}$, and $P(a, a^2)$ is any point on XY - plane. Then,

The domain of f(g(x)), is

A. $x \in R$

B. $x \in R - \{1\}$

C. $x \in R - \{0, 1\}$

D. $x \in R - \{0, 1, -1\}$

Answer: C

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90. If α, β (where $\alpha < \beta$) are the points of discontinuity of the function $g(x) = f(f(x))$, where $f(x) = \frac{1}{1-x}$, and $P(a, a^2)$ is any point on XY - plane. Then, If point $P(a, a^2)$ lies on the same side as that of (α, β) with respect to line x + 2y - 3 = 0, then

A. $a \in \left(-\frac{3}{2}, 1\right)$

B. $a \in R$

C. $a \in \left(-\frac{3}{2}, 0\right)$

D. $a \in (0, 1)$

Answer: A

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91. In the following, [x] denotes the greatest integer less than or equal to x. Match the functions in Column I with the properties Column II.

	Column I		Column II
(A)	$x\|x\|$	(p)	continuous in (-1, 1)
(B)	$\sqrt{\|x\|}$	(q)	differentiable in (-1, 1)
(C)	$x+[x]$	(r)	strictly increasing (-1, 1)
(D)	$\|x-1\|+\|x+1\|$	(s)	not differentiable at least at one point in (-1,

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92. Let $f(X)=\begin{cases}[x], & -2\leq x<0\\ |x|, & 0\leq x\leq 2\end{cases}$ (where [.] denotes the greatest integer function) g(x) = sec x, $x\in R-(2n+1)\pi/2$.

Match the following statements in Column I with their values in Column II in the interval $\left(-\frac{3\pi}{2}, \frac{3\pi}{2}\right)$.

	Column I		Column II
(A)	Lemit of fog exist at	(p)	-1
(B)	Limit of gof doesn't exist at	(q)	π
(C)	Points of discontinuity of fog is/are	(r)	$\frac{5\pi}{6}$
(C)	Points of differentiability of fog is/are	(s)	$-\pi$

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93. Suppose a function f(x) satisfies the following conditions

$$f(x+y) = \frac{f(x)+f(y)}{1+f(x)f(y)}, \forall x, y \text{ and } f'(0) = 1. \text{ Also, } -1 < f(x) < 1, \forall$$

.

Match the entries of the following two columns.

	Column I		Column II
(A)	f(x) is differentiable over the set	(p)	$R-(-1, 0, 1)$
(B)	f(x) increases in the interval	(q)	R
(C)	Number of the solutions of f(x) = 0 is	(r)	0
(D)	The value of the limit $\lim_{x \to \infty} [f(x)]^x$ is	(s)	1

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94. Let

$$f(x) = \left\{\left(\frac{1-\cos 4x}{x^2},, x<0\right), (a,, x=0), \left(\frac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}\right),, x>0\right.$$

Then, the value of a if possible, so that the function is continuous at x = 0, is..........

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95. $f(x) =$ maximum $\left\{4, 1+x^2, x^2-1\right) \forall x \in R.$ Total number of points, where $f(x)$ is non-differentiable,is equal to

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96. Let $f(x) = x^n$ n being a non negative integer. The value of n for which the equality$f'(a+b) = f'(a) + f'(b)$ is valid for all $a. b > 0$ is

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97. The number of points where $f(x) = [\sin x + \cos x]$ (where [.] denotes the greatest integer function) $x \in (0, 2\pi)$ is not continuous is (A) 3 (B) 4 (C) 5 (D) 6

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98. The number of points where $|xf(x)| + |x - 2| - 1|$ is non-differentiable in $x \in (0, 3\pi)$, where $f(x) = \prod_{k=1}^{\infty} \left(\frac{1 + 2\cos\left(\frac{2x}{3^k}\right)}{3} \right)$, is..........

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99. If $f\left(\frac{xy}{2}\right) = \frac{f(x).\,f(y)}{2}, x, y \in R, f(1) = f'(1). \text{Then}, \frac{f(3)}{f'(3)}$ is.........

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100. Let $f: R \to R$ be a differentiable function satisfying $f(x) = f(y)f(x-y), \forall x, y \in R$ and $f'(0) = \int_0^4 \{2x\} dx$, where {.} denotes the fractional part function and $f'(-3) - \alpha e^{\beta}$. Then, $|\alpha + \beta|$ is equal to.......

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101. Let f(x) is a polynomial function and $f(\alpha))^2 + f'(\alpha))^2 = 0$, then find $\lim_{x \to \alpha} \frac{f(x)}{f'(x)} \left[\frac{f'(x)}{f(x)}\right]$, where [.] denotes greatest integer function, is........

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102. Let $f: R \to R$ is a function satisfying $f(2-x) = f(2+x)$ and $f(20-x) = f(x), \forall x \in R$. On the basis of above information, answer the following questions If $f(0) = 5$, then minimum possible number of values of x satisfying $f(x) = 5$, for $x \in [10, 170]$ is

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103. If $f(x)$ is a differentiable function for all $x \in R$ such that $f(x)$ has fundamental period 2.$f(x) = 0$ has exactly two solutions in $[0, 2]$, also $f(0) \neq 0$ If minimum number of zeros of $h(x) = f'(x)\cos x - f(x)\sin x$ in $(0, 99)$ is $120 + k$, then k is ......

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104. Discuss the differentiability of $f(x) = ma\xi\mu m\{2\sin x, 1 - \cos x\} \forall x \in (0, \pi)$.

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105. Discuss the continuity of the function g(x) = [x] + [-x] at integral values of x.

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106. If $f(x) = \frac{\sin 2x + A\sin x + B\cos x}{x^3}$ is continuous at x = 0. Find the values of A and B. Also, find f(0)

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107. Let $f: R \to R$ satisfying $|f(x)| \le x^2, \forall x \in R$, then show that f(x) is differentiable at x = 0.

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108. Show that the function defined by $f(x) = \begin{cases} x^2 \sin 1/x, & x \ne 0 \\ 0, & x = 0 \end{cases}$ is differentiable for every value of x, but the derivative is not continuous for x =

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109. If $f(x)=\begin{cases} x-[x], & x \notin I \\ 1, & x \in I \end{cases}$ where I is an integer and [.] represents the greatest integer function and

$$g(x)=\lim_{n \to \infty} \frac{\{f(x)\}^{2n}-1}{\{f(x)\}^{2n}+1}, \text{ then}$$

(a) Draw graphs of f(2x), g(x) and g{g(x)} and discuss their continuity.

(b) Find the domain and range of these functions.

(c) Are these functions periodic ? If yes, find their periods.

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110. Prove that $f(x)=[\tan x]+\sqrt{\tan x-[\tan x]}$. (where [.] denotes greatest integer function) is continuous in $\left[0, \frac{\pi}{2}\right)$.

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111. Determine the values of x for which the following functions fails to be continuous or differentiable $f(x)=\begin{cases} (1-x), & x<1 \\ (1-x)(2-x), & 1 \leq x \leq 2 \\ (3-x), & x>2 \end{cases}$

justify your answer.

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112. If g(x) is continuous function in $[0, \infty)$ satisfying $g(1) = 1. If \int_0^x 2x.\ g^2(t)dt = \left(\int_0^x 2g(x-t)dt\right)^2$, find g(x).

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113. Q. f={$(x + a \quad \text{if} \quad x < 0)$, $(x - 11 \quad \text{if} \quad x \geq 0)$ $g(x) = \left\{(x + 1 \quad \text{if} \quad x < 0), (x - 1)^2 \quad \text{if} \quad x < 0\right)$ where a and b are non-negative real numbers. Determine the composite function gof. If $(gof)(x)$ is continuous for all real x, determine the values of a and b, Further for these values of a and b, is gof differentiable at x=0? Justify your answer.

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114. If a function $f:[-2a, 2a] \rightarrow R$ is an odd function such that, $f(x) = f(2a - x)$ for $x \in [a, 2a]$ and the left-hand derivative at $x = a$ is 0, then find the left-hand derivative at $x = -a$.

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115. Discuss the continuity of f(x) in [0, 2], where $f(x) = \begin{cases} [\cos \pi x], & x \leq 1 \\ |2x - 3|[x - 2], & x > 1 \end{cases}$ where [.] denotes the greatest integral function.

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116. Let $f: R \rightarrow R$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x - t) dt.$

$f(x)$ increases for

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117. Let $f: R^{+} \to R$ satisfies the functional equation $f(xy) = e^{xy-x-y}\{e^{y}f(x) + e^{x}f(y)\}, \forall x, y \in R^{+}$. If f'(1) = e, determine f(x).

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118. Let f is a differentiable function such that $f'(x) = f(x) + \int_0^2 f(x)dx,\ f(0) = \frac{4-e^2}{3}$, find f(x).

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119. A function $f(x)$ satisfies the following property: $f(x+y) = f(x)f(y)$. Show that the function is continuous for all values of x if its is continuous at $x = 1$.

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120. Let $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ for all real x and y. If f'(0) exists and equals-1 and f(0)=1, find f(2)

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121. Let $f(x)=1+4x-x^2, \forall x \in R$

$g(x)=\max\{f(t), x \leq t \leq (x+1), 0 \leq x < 3 \min\{(x+3), 3 \leq x \leq 5$

Verify conntinuity of g(x), for all $x \in [0, 5]$

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122. Let

$f(x)=x^3-8x^3+22x^2-24x \text{ and } g(x)=\begin{cases}\min f(x), & x \leq t \leq x+1 \\ x-10, & x \geq 1\end{cases}$

Discuss the continuity and differentiability of g(x) in $[-1, \infty)$

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123. Let $g(x)=\int_0^x f(t).\,dt$,where f is such that $\frac{1}{2}\leq f(t)\leq 1$ for $t\in[0,1]$ and $0\leq f(t)\leq\frac{1}{2}$ for $t\in[1,2]$.Then g(2) satisfies the inequality

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124. Let f be a one-one function such that $f(x).\,f(y)+2=f(x)+f(y)+f(xy),\ \forall x,y\in R-\{0\}$ and $f(0)=1, f$

. Prove that $3\left(\int f(x)dx\right)-x(f(x)+2)$ is constant.

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125. Let $f:R\rightarrow R,$ such that f'(0) = 1 and $f(x+2y)=f(x)+f(2y)+e^{x+2y}(x+2y)-x.\,e^{x}-2y.\,e^{2y}+4xy,\ \forall x,y$

. Find f(x).

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126. Let f be a function such that $f(xy) = f(x).\, f(y), \forall y \in R$ and $R(1+x) = 1 + x(1 + g(x))$. where $\lim_{x \to 0} g(x) = 0$. Find the value of $\int_1^2 \frac{f(x)}{f'(x)} \cdot \frac{1}{1+x^2} dx$

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127. If $f(x) = ax^2 + bx + c$ is such that $|f(0)| \leq 1, |f(1)| \leq 1$ and $|f(-1)| \leq 1,$ prove that $|f(x)| \leq 5/4, \forall x \in [-1, 1]$

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128. Let $\alpha + \beta = 1, 2\alpha^2 + 2\beta^2 = 1$ and $f(x)$ be a continuous function such that $f(2+x) + f(x) = 2$ for all $x \in [0, 2]$ and $p = \int_0^4 f(x)dx - 4, q = \frac{\alpha}{\beta}$. Then, find the least positive integral value of 'a' for which the equation $ax^2 - bx + c = 0$ has both roots lying between p and q, where $a, b, c \in N$.

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129. Prove that the function $f(x) = a\sqrt{x-1} + b\sqrt{2x-1} - \sqrt{2x^2 - 3x + 1}$, where a + 2b = 2 and $a, b \in R$ always has a root in $(1, 5)\ \forall b \in R$

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130. Let $\alpha \in R$. prove that a function $f: R - R$ is differentiable at α if and only if there is a function $g: R - R$ which is continuous at α and satisfies $f(x) - f(\alpha) = g(x)(x - \alpha),\ \forall x \in R$.

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Exercise For Session 1

1. If function $f(x) = \dfrac{\sqrt{1+x} - \sqrt[3]{1+x}}{x}$ is continuous function at x = 0, then f(0) is equal to

A. 2

B. $\frac{1}{4}$

C. $\frac{1}{6}$

D. $\frac{1}{3}$

Answer: C

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2. If $f(x) = \begin{cases} \frac{1}{e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

A. $\lim_{x \to 0^-} f(x) = 0$

B. $\lim_{x \to 0^+} f(x) = 1$

C. f(x) is discontinuous at x = 0

D. f(x) is continuous at x = 0

Answer: C

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3. If $f(x)=\begin{cases}\frac{x^2-(a+2)x+2a}{x-2}, & x\neq 2\\ 2, & x=2\end{cases}$ is continuous at x = 2, then a is equal to

A. 0

B. 1

C. -1

D. 2

Answer: A

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4. If $f(x)=\begin{cases}\frac{\log(1+2ax)-\log(1-bx)}{x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at x = 0, then k is equal to

A. 2a + b

B. 2a - b

C. b - 2a

D. a + b

Answer: A

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5. If $f(x) = \begin{cases} [x] + [-x], & x \neq 2 \\ \lambda, & x = 2 \end{cases}$ and f is continuous at x = 2, where $[\cdot]$ denotes greatest integer function, then λ is

A. -1

B. 0

C. 1

D. 2

Answer: A

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Exercise For Session 2

1. Let $f(x)=\begin{cases}-2\sin x & \text{for } -\pi\le x\le -\frac{\pi}{2}\\ a\sin x+b & \text{for } -\frac{\pi}{2}<x<\frac{\pi}{2}\\ \cos x & \text{for } \frac{\pi}{2}\le x\le \pi\end{cases}$. If f is continuous on $[-\pi,\pi)$, then find the values of a and b.

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2. Draw the graph of the function $f(x)=x-\left|x-x^2\right|,\ -1\le x\le 1$ and discuss the continuity or discontinuity of f in the interval $-1\le x\le 1$

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3. Discuss the continuity of 'f' in [0, 2], where $f(x)=\begin{cases}|4x-5|[x] & \text{for } x>1\\ [\cos \pi x] & \text{for } x\le 1\end{cases}$, where [x] is greastest integer not greater than x.

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4. Let $f(x) = \begin{cases} Ax - B & x \le 1 \\ 2x^2 + 3Ax + B & x \in (-1, 1] \\ 4 & x > 1 \end{cases}$

Statement I f(x) is continuous at all x if $A = \frac{3}{4}, B = -\frac{1}{4}$. Because

Statement II Polynomial function is always continuous.

A. Both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect

Answer: D

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Exercise For Session 3

1. which of the following function(s) not defined at $x = 0$ has/have removable discontinuity at $x = 0$.

A. $f(x) = \frac{1}{1 + 2^{\cot x}}$

B. $f(x) = \cos\left(\frac{(|\sin x|)}{x}\right)$

C. $f(x) = \text{x} \sin\frac{\pi}{x}$

D. $f(x) = \frac{1}{\text{In}|x|}$

Answer: B::C::D

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2. Function whose jump (non-negative difference of LHL and RHL) of discontinuity is greater than or equal to one. is/are

A. $f(x) = \begin{cases} \frac{(e^{1/x}+1)}{e^{1/x}-1}, & x < 0 \\ \frac{(1-\cos x)}{x}, & x > 0 \end{cases}$

B. $g(x) = \begin{cases} \frac{(x^{1/3}-1)}{x^{1/2}-1}, & x > 0 \\ \frac{\text{In x}}{(x-1)}, & \frac{1}{2} < x < 1 \end{cases}$

C. $u(x) = \begin{cases} \frac{\sin^{-1} 2x}{\tan^{-1} 3x}, & x \in \left(0, \frac{1}{2}\right] \\ \frac{|\sin x|}{x}, & x < 0 \end{cases}$

D. $v(x) = \begin{cases} \log_3(x+2), & x > 2 \\ \log_{1/2}(x^2+5), & x < 2 \end{cases}$

Answer: A::C::D

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3. Consider the piecewise defined function $f(x) = \begin{cases} \sqrt{-x} & \text{if} \quad x < 0 \\ 0 & \text{if} \quad 0 \leq x \leq 4 \\ x - 4 & \text{if} \quad x > 4 \end{cases}$ choose the answer which best describes the continuity of this function.

A. the function is unbounded and therefore cannot be continuous

B. the function is right continuous at x = 0

C. the function has a removable discontinuity at 0 and 4, but is continuous on the rest of the real line.

D. the function is continuous on the entire real line

Answer: D

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4. If $f(x) = sgn(\cos 2x - 2\sin x + 3)$, where sgn () is the signum function, then f(x)

A. is continuous over its domain

B. has a missing point discontinuity

C. has isolated point discontinuity

D. has irremovable discontinuity

Answer: C

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5. If $f(x) = \begin{cases} \dfrac{2\cos x, s \in 2x}{(\pi - 2x)^2}, x \leq \dfrac{\pi}{2} \dfrac{e^{-\cot x} - 1}{8x - 4\pi}, x > \dfrac{\pi}{2} \end{cases}$, then which of the following holds? f is continuous at $x = \pi/2$ f has an irremovable discontinuity at $x = \pi/2$ f has a removable discontinuity at $x = \pi/2$

None of these

A. h is continuous at $x = \pi/2$

B. h has an irremovable discontinuity at $x = \pi/2$

C. h has a removable discontinuity at $x = \pi/2$

D. $f\left(\dfrac{\pi^+}{2}\right) = g\left(\dfrac{\pi^-}{2}\right)$

Answer: A::C::D

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Exercise For Session 4

1. If $f(x) = \frac{1}{x^2 - 17x + 66}$, then $f\left(\frac{2}{x-2}\right)$ is discontinuous at x =

A. 2

B. $\frac{7}{3}$

C. $\frac{24}{11}$

D. 6, 11

Answer: A::B::C

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2. Let f be a continuous function on R such that $f\left(\frac{1}{4n}\right) = \frac{\sin e^n}{e^{n^2}} + \frac{n^2}{n^2+1}$ Then the value of $f(0)$ is

A. not unique

B. 1

C. data sufficient to find f(0)

D. data insufficient to find f(0)

Answer: B::C

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3. $f(x)$ is continuous at $x = 0$ then which of the following are always true ?

A. $\lim_{x \to 0} f(x) = 0$

B. f(x) is non coninuous at x = 1

C. $g(x) = x^2 f(x)$ is continuous x = 0

D. $\lim_{x \to 0^+} (f(x) - f(0)) = 0$

Answer: C::D

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4. If $f(x) = \cos\left[\frac{\pi}{x}\right]\cos\left(\frac{\pi}{2}(x-1)\right)$; where [x] is the greatest integer function of x,then $f(x)$ is continuous at :

A. x = 0

B. x = 1

C. x = 2

D. None of these

Answer: B::C

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5. Let $f(x) = [x]$ and $g(x) = \{0, x \in Zx^2, x \in R - Z$ then (where [.]denotest greatest integer funtion)

A. $\lim_{x \to 1} g(x)$ exists, but g(x) is not continuous at x = 1

B. $\lim_{x \to 1} f(x)$ does not exist and f(x) is not continuous at x = 1

C. gof is continuous for all x.

D. fog is continuous for all x.

Answer: A::B::C

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6. Let $f(x)=\begin{cases} a\sin^{2n}x & \text{for } x\geq 0 \text{ and } n\to\infty \\ b\cos^{2m}x-1 & \text{for } x<0 \text{ and } m\to\infty \end{cases}$ then

A. $f(0^-)\neq f(0^+)$

B. $f(0^+)\neq f(0)$

C. $f(0^-)=f(0)$

D. f is continuous at x = 0

Answer: A

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7. Consider $f(x)=\lim_{n\to\infty}\dfrac{x^n-\sin x^n}{x^n+\sin x^n}$ for x $>0, x\neq 1, f(1)=0$ then

A. f is continuous at x = 1

B. f has a finite discontinuity at x = 1

C. f has an infinite or oscillatory discontinuity at x = 1

D. f has a removal type of discontinuity at x = 1

Answer: B

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Exercise For Session 5

1. Examine the continuity at $x = 0$ of the sum function of the infinite series:

$$\frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \dots\dots\dots\dots\infty$$

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2. Let

$$y_n(x) = x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots\ldots \frac{x^2}{(1+x^2)^{n-1}} \text{ and } y(x) = \lim_{n \to \infty}$$

. Discuss the continuity of $y_n(x)(n = 1, 2, 3.... n)$ and $y(x)$at x $= 0$

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Exercise For Session 6

1. If a function f(x) is defined as $f(x) = \begin{cases} -x, & x < 0 \\ x^2, & 0 \leq x \leq 1 \\ x^2 - x + 1, & x > 1 \end{cases}$ then

A. f(x) is differentiable at x = 0 and x = 1

B. f(x) is differentiable at x = 0 but not at x = 1

C. f(x) is not differentiable at x = 1 but not at x = 0

D. f(x) is not differentiable at x = 0 and x = 1

Answer: D

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2. If $f(x) = x^3$sgn (x), then

A. f is differentiable at x = 0

B. f is continuous but not differentiable at x = 0

C. $f'(0^-) = 1$

D. None of these

Answer: A

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3. Which one of the following functions is continuous everywhere in its domain but has atleast one point where it is not differentiable ?

A. $f(x) = x^{1/3}$

B. $f(x) = \frac{|x|}{x}$

C. $f(x) = e^{-x}$

D. f(x) = tan x

Answer: A

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4. If $f(x) = \begin{cases} x + \{x\} + x\sin\{x\}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$, where {x} denotes the fractional part function, then

A. f is continuous and differentiable at x = 0

B. f is continuous but not differentiable at x = 0

C. f is continuous and differentiable at x = 2

D. None of these

Answer: D

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5. If $f(x) = \begin{cases} x\left(\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{1/x}}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then at x = 0 f(x) is

A. differentiable

B. not differentiable

C. $f'(0^+) = -1$

D. $f'(0^-) = 1$

Answer: B

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Exercise For Session 7

1. Number of points of non-differerentiable of $f(x) = \sin \pi(x - [x])\text{in } (-\pi/2, [\pi/2)$. Where [.] denotes the greatest integer function is

A. f(x) is discontinuous at $x = \{-1, 0, 1\}$

B. f(x) is differentiable for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{0\}$

C. f(x) is differentiable for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \{-1, 0, 1\}$

D. None of these

Answer: C

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2. $f(x) = \begin{cases} x-1, & -1 \leq x0 \\ x^2, & 0 < x \leq 1 \end{cases}$ and g(x)=sinx. Find $h(x) = f(|g(x)|) + |f(g(x))|$.

A. h(x) is continuous for $x \in [-1, 1]$

B. h(x) is differentiable for $x \in [-1, 1]$

C. h(x) is differentiable for $x \in [-1, 1] - \{0\}$

D. h(x) is differentiable for $x \in (-1,$)-{0}`

Answer: C

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3. If $f(x) = \begin{cases} \left|1 - 4x^2\right|, & 0 \le x < 1 \\ \left[x^2 - 2x\right], & 1 \le x < 2 \end{cases}$, where [] denotes the greatest integer function, then

A. f(x) is continuous for all $x \in [0, 2)$

B. f(x) is differentiable for all $x \in [0, 2) - \{1\}$

C. f(X) is differentiable for all $x \in [0, 2) - \left\{\frac{1}{2}, 1\right\}$

D. None of these

Answer: C

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4. Let $f(x) = \int_0^1 |x - t| dt$, then

A. f(x) is continuous but not differentiable for all $x \in R$

B. f(x) is continuous and differentiable for all $x \in R$

C. f(x) is continuous for $x \in R - \left\{\frac{1}{2}\right\}$ and $f(x)$ is differentiable for $x \in R - \left\{\frac{1}{4}, \frac{1}{2}\right\}$

D. None of these

Answer: B

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5. Let f be a function such that $f(x+y) = f(x) + f(y)$ for all $x and y and f(x) = (2x^2 + 3x)g(x)$ for all x, where $g(x)$ is continuous and $g(0) = 3$. Then find $f'(x)$.

A. 6

B. 9

C. 8

D. None of these

Answer: B

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6. If a function g(x) which has derivaties g'(x) for every real x and which satisfies the following equation $g(x+y)=e^y g(x)+e^x g(x)$ for all x and y and g'(0) = 2, then the value of $\{g'(x)-g(x)\}$ is equal to

A. e^x

B. $\frac{2}{3}e^x$

C. $\frac{1}{2}e^x$

D. $2e^x$

Answer: D

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7. Let $f: R \to R$ be a function satisfying $f\left(\frac{xy}{2}\right)=\frac{f(x)\cdot f(y)}{2}, \forall x, y \in R \text{ and } f(1)=f'(1)= \neq 0.$ Then,

$f(x) + f(1 - x)$ is (for all non-zero real values of x) a.) constant b.) can't be discussed c.) $xd.$ $\Big)\frac{1}{x}$

A. constant

B. can't be discussed

C. x

D. $\frac{1}{x}$

Answer: A

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8. Let $f: R \to R$ satisfying $f\left(\frac{x+y}{k}\right) = \frac{f(x) + f(y)}{k}(k \neq 0, 2)$.Let $f(x)$ be differentiable on R and $f'(0) = a$, then determine $f(x)$.

A. even function

B. neither even nor odd function

C. either zero or odd function

D. either zero or even function

Answer: C

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9. If $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ for all $x, y \in R (xy \neq 1)$ and $\lim_{x \to 0} \frac{f(x)}{x} = 2$, then

A. $2\tan^{-1}x$

B. $\frac{1}{2}\tan^{-1}x$

C. $\frac{\pi}{2}\tan^{-1}x$

D. $2\pi\tan^{-1}x$

Answer: A

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10. Let

$$f(x) = \sin x \text{ and c g(x)} = \begin{cases} \max\{f(t), 0 \le x \le \pi\} & \text{for} \quad 0 \le x \le \pi \\ \frac{1-\cos x}{2}, & \text{for} \quad x > \pi \end{cases}$$

Then, g(x) is

A. differentiable for all $x \in R$

B. differentiable for all $x \in R - \{\pi\}$

C. differentiable for all $x \in (0, \infty)$

D. differentiable for all $x \in (0, \infty) - \{\pi\}$

Answer: C

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Exercise Single Option Correct Type Questions

1. If $f(x) = \begin{cases} \sin\frac{\pi x}{2}, & x < 1 \\ [x], & x \ge 1 \end{cases}$, where [x] denotes the greatest integer function, then

A. f(x) is continuous at x = 1

B. f(x) is discontinuous at x = 1

C. $f(1^+) = 0$

D. $f(1^-) = -1$

Answer: A

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2. Consider $f(x) = \begin{cases} \frac{8^x - 4^x - 2^x + 1}{x^2}, & x > 0 \\ e^x \sin x + \pi x + k \log 4, & x < 0 \end{cases}$ Then, f(0) so that f(x) is continuous at x = 0, is

A. log 4

B. log 2

C. (log 4) (log 2)

D. None of these

Answer: C

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3. Let $f(x) = \begin{cases} \frac{a(1 - x\sin x) + b\cos x + 5}{x^2}, & x < 0 \\ 3, & x = 0 \\ \left[1 + \left(\frac{cx + dx^3}{x^2}\right)\right]^{1/x}, & x > 0 \end{cases}$ If f is continuous at x = 0,

then (a + b + c + d) is

A. 5

B. -5

C. log 3 - 5

D. 5 - log 3

Answer: C

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4. `f(x)={cos^(-1){cotx),xpi/2$where$[dot] $representsthegreatestfunction$ and {dot}` represents the fractional part function. Find the jump of discontinuity.

A. 1

B. $\pi / 2$

C. $\frac{\pi}{2} - 1$

D. 2

Answer: C

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5. Let $f: [0, 1] \overrightarrow{0, 1}$ be a continuous function. Then prove that $f(x) = x$ for at least one $0 \leq x \leq 1$.

A. atleast one $x \in [0, 1]$

B. atleast one $x \in [1, 2]$

C. atleast one $x \in [-1, 0]$

D. can't be discussed

Answer: A

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6. If $f(x) = \frac{x+1}{x-1}$ and $g(x) = \frac{1}{x-2}$, then (fog)(x) is discontinuous at

A. x = 3 only

B. x = 2 only

C. x = 2 and 3 only

D. x = 1 only

Answer: C

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7. Let

$y_n(x) = x^2 + \frac{x^2}{(1+x^2)} + \frac{x^2}{(1+x^2)^2} + \dots + \frac{x^2}{(1+x^2)^{n-1}}$ and $g(x) = \lim_{n \to}$

, then $y_n(x), n = 1, 2, 3, \dots, n$ and $y(x)$ is

A. continuous for $x \in R$

B. continuous for $x \in R - \{0\}$

C. continuous for $x \in R - \{1\}$

D. data unsufficient

Answer: B

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8. If $g(x) = \dfrac{1 - a^x + xa^x \log a}{x^2 \cdot a^x}, x < 0 \ \dfrac{(2a)^x - x\log(2a) - 1}{x^2}, x > 0$ (where a > 0) then find a and $g(0)$ so that $g(x)$ is continuous at $x = 0$.

A. $\dfrac{-1}{\sqrt{2}}$

B. $\dfrac{1}{\sqrt{2}}$

C. 2

D. -2

Answer: B

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9. Let $f(x) = \begin{cases} \dfrac{\frac{\pi}{2} - \sin^{-1}\left(1 - \{x\}^2\right) . \sin^{-1}(1 - \{x\})}{\sqrt{2}\left(\{x\} + \{x\}^3\right)}, & x \neq 0 \end{cases}$, where {.} is fractional part of x, then

A. $f(0^+) = -\dfrac{\pi}{2}$

B. $f(0^-) = \dfrac{\pi}{4\sqrt{2}}$

C. f(x) is continuous at x = 0

D. None of the above

Answer: B

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10. Let $f(x) = \begin{cases} sgn(x) + x, & -\infty < x < 0 \\ -1 + \sin x, & 0 \leq x \leq \pi/2 \\ \cos x, & \pi/2 \leq x < \infty \end{cases}$, then number of points, where f(x) is not differentiable, is/are

A. 0

B. 1

C. 2

D. 3

Answer: B

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11. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| > 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$ If f(x) is continuous and differentiable at any point, then values of a and b are

A. $\frac{-1}{2}, \frac{3}{2}$

B. $\frac{1}{2}, \frac{-3}{2}$

C. $\frac{1}{2}, \frac{3}{2}$

D. None of these

Answer: A

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12. If $f(x) = \begin{cases} A + Bx^2, & x < 1 \\ 3Ax - B + 2, & x \geq 1 \end{cases}$, then A and B, so that f(x) is differentiabl at x = 1, are

A. $-2, 3$

B. $2, \ -3$

C. 2, 3

D. $-2, \ -3$

Answer: C

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13. If $f(x) = \begin{cases} |x-1|([x]-x), & x \neq 1 \\ 0, & x = 1 \end{cases}$, then

A. $f'(1^+) = 0$

B. $f'(1^-) = 0$

C. $f'(1^-) = -1$

D. f(x) is differentiable at x = 1

Answer: A

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14. If $f(x) = \begin{cases} [\cos \pi x], & x \leq 1 \\ 2\{x\} - 1, & x > 1 \end{cases}$, where [.] and {.} denotes greatest integer and fractional part of x, then

A. $f'(1^-) = 2$

B. $f'(1^+) = 2$

C. $f'(1^-) = -2$

D. $f'(1^+) = 0$

Answer: B

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15. If $f(x) = \begin{cases} x - 3, & x < 0 \\ x^2 - 3x + 2, & x \geq 0 \end{cases}$, then $g(x) = f(|x|)$ is

A. $g'(0^+) = -3$

B. $g'(0^-) = -3$

C. $g'(0^+) = g'(0^-)$

D. g(x) is not continuous at x = 0

Answer: A

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16. If $f(x) = \begin{cases} \left\{x + \frac{1}{3}\right\}[\sin \pi x], & 0 \leq x < 1 \\ [2x] sgn\left(x - \frac{4}{3}\right), & 1 \leq x \leq 2 \end{cases}$, where [.] and {.} denotes greatest integerd and fractional part of x respectively, then the number of points, which is not differentiable, is

A. 3

B. 4

C. 5

D. 6

Answer: C

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17. Let f be differentiable function satisfying $f\left(\frac{x}{y}\right) = f(x) - f(y)\text{for all}x, y > 0$. If f'(1) = 1, then f(x) is

A. $2\log_e x$

B. $3\log_e x$

C. $\log_e x$

D. $\frac{1}{2}\log_e x$

Answer: C

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18. Let $f(x+y) = f(x) + f(y) - 2xy - 1$ for all x and y. If f'(0) exists and $f'(0) = -\sin\alpha$, then $f\{f'(0)\}$ is

A. -1

B. 0

C. 1

D. 2

Answer: C

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19. A derivable function $f: R^{+} \rightarrow R$ satisfies the condition $f(x) - f(y) \geq \log\left(\frac{x}{y}\right) + x - y, \forall x, y \in R^{+}$. If g denotes the derivative of f, then the value of the sum $\sum_{n=1}^{100} g\left(\frac{1}{n}\right)$ is

A. 5050

B. 5510

C. 5150

D. 1550

Answer: C

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20. If $\frac{d(f(x))}{dx} = e^{-x}f(x) + e^{x}f(-x)$, then f(x) is, (given f(0) = 0)

A. an even function

B. an odd function

C. neither even nor odd function

D. can't say

Answer: B

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21. Let $f:(0,\infty)\to R$ be a continuous function such that $f(x)=\int_0^x tf(t)dt$. If $f\left(x^2\right)=x^4+x^5$, then $\sum_{r=1}^{12} f\left(r^2\right)$, is equal to

A. 216

B. 219

C. 222

D. 225

Answer: B

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22. For let $h(x)=\left\{\frac{1}{q}\right.$ if $x=\frac{p}{q}$ and 0 if x is irrational where $p\&q>0$ are relatively prime integers 0 then which one does not hold good?

A. h(x) is discontinuous for all x in $(0,\infty)$

B. h(x) is continuous for each irrational in $(0,\infty)$

C. h(x) is discontinuous for each rational in $(0, \infty)$

D. h(x) is not derivable for all x in $(0, \infty)$

Answer: B

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23. Let $f(x) = \frac{g(x)}{h(x)}$, where g and h are continuous functions on the open interval (a, b). Which of the following statements is true for $a < x < b$?

A. f is continuous at all x for which $x \neq 0$

B. f is continuous at all x for which g(x) = 0

C. f is continuous at all x for which $g(x) \neq 0$

D. f is continuous at all x for which $h(x) \neq 0$

Answer: D

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24. $f(x) = \frac{\cos x - \sin 2x}{(\pi - 2x)^2}; g(x) = \frac{e^{-\cos x} - 1}{8x - 4\pi}$

A. h is continuous at $x = \pi/2$

B. h has an irremovable discontinuity at $x = \pi/2$

C. h has a removable discontinuity at $x = \pi/2$

D. $f\left(\frac{\pi^+}{2}\right) = g\left(\frac{\pi^-}{2}\right)$

Answer: B

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25. If $f(x) = \frac{x - e^x + \cos 2x}{x^2}, x \neq 0$ is continuous at x = 0, then

A. $f(0) = \frac{5}{2}$

B. $[f(0)] = -2$

C. $\{f(0)\} = -0.5$

D. $[f(0)].\{f(0)\} = -1.5$

Answer: D

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26. Consider the function $f(x)=\begin{bmatrix} x\{x\}+1, & \text{if } \ 0 \leq x < 1 \\ 2-\{x\}, & \text{if } \ 1 \leq x \leq 2 \end{bmatrix}$, where {x} denotes the fractional part function. Which one of the following statements is not correct ?

A. $\lim_{x \to 1} f(x)$ exists

B. $f(0) \neq f(2)$

C. f(x) is continuous in [0, 2]

D. Rolle's theorem is not applicable to f(x) in [0, 2]

Answer: C

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27. Let $f(x)=\begin{cases}\dfrac{2^x+2^{3-x}-6}{\sqrt{2^{-x}}-2^{1-x}}, & \text{if } x>2\\[2mm] \dfrac{x^2-4}{x-\sqrt{3x-2}}, & \text{if } x<2\end{cases}$,then

A. $f(2)=8 \Rightarrow f$ is continuous at x = 2

B. $f(2)=16 \Rightarrow f$ is continuous at x = 2

C. $f(2^-)\neq f(2^+) \Rightarrow f$ is discontinuous

D. f has a removable discontinuity at x = 2

Answer: C

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28. Let [x] denote the integral part of $x\in R$ and $g(x)=x-[x]$. Let $f(x)$ be any continuous function with $f(0)=f(1)$ then the function $h(x)=f(g(x))$:

A. has finitely many discontinuities

B. is discontinuous at some x = c

C. is continuous on R

D. is a constant function

Answer: C

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29. Let f be a differentiable function on the open interval(a, b). Which of the following statements must be true? (i) f is continuous on the closed interval [a,b],(ii) f is bounded on the open interval (a,b)

A. Only I and II

B. Only I and III

C. Only II and III

D. Only III

Answer: D

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30. Number of points where the function $f(x) = (x^2 - 1)\left|x^2 - x - 2\right| + \sin(|x|)$ is not differentiable, is:

A. 0

B. 1

C. 2

D. 3

Answer: C

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31. Consider function $f: R - \{-1, 1\} \to R.\ f(x) = \dfrac{x}{1 - |x|}$ Then the incorrect statement is

A. it is continuous at the origin

B. it is not derivable at the origin

C. the range of the function is R

D. f is continuous and derivable in its domain

Answer: B

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32. If the functions $f: R \rightarrow R$ and $g: R \rightarrow R$ are such that $f(x)$ is continuous at $x = \alpha$ and $f(\alpha) = a$ and $g(x)$ is discontinuous at $x = a$ but $g(f(x))$ is continuous at $x = \alpha$. where, $f(x)$ and $g(x)$ are non-constant functions (a) $x = \alpha$ extremum of $f(x)$ and $x = \alpha$ is an extremum $g(x)$ (b) $x = \alpha$ may not be extremum $f(x)$ and $x = \alpha$ is an extermum of $g(x)$ (c) $x = \alpha$ is an extremum of $f(x)$and $x = \alpha$ may not be an extremum $g(x)$ (d) not of the above

A. $x = \alpha$ is a extremum of f(x) and x = a is an extremum of g(x)

B. $x = \alpha$ may not be an extremum of f(x) and x = a is an extremum of g(x)

C. $x = \alpha$ is an extremum of f(x) and x = a may not be an extremum of g(x)

D. None of the above

Answer: C

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33. The total number of points of non-differentiability of $f(x) = \min\left[|\sin x|, |\cos x|, \frac{1}{4}\right] \text{in}(0, 2\pi)$ is

A. 8

B. 9

C. 10

D. 11

Answer: D

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34. The function $f(x) = [x]^2 - [x^2]$ is discontinuous at (where $[\gamma]$ is the greatest integer less than or equal to γ), is discontinuous at

A. all integers

B. all integers except 0 and 1

C. all integers except 0

D. all integers except 1

Answer: D

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35. The function $f(x) = (x^2 - 1)|x^2 - 6x + 5| + \cos|x|$ is not differentiable at

A. -1

B. 0

C. 1

D. 5

Answer: D

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36. If $f(x) = \begin{cases} \frac{1}{e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then

A. 0

B. 1

C. -1

D. desn't exist

Answer: A

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37. Given

$$f(x)=\frac{e^{x}-\cos 2x-x}{x^{2}}, \text{ for x} \in R-\{0\} g(x)=\begin{cases} f(\{x\}), & \text{for } n < \\ f(1-\{x\}), & \text{for } n+ \\ & \begin{cases} \text{w} \\ \text{fr} \end{cases} \\ \frac{5}{2}\text{otherwise}, & \end{cases}$$

then g(x) is

A. discontinuous at all integral values of x only

B. continuous everywhere except for x = 0

C. discontinuous at $x=n+\frac{1}{2}, n \in I$ and at some $x \in I$

D. continuous everywhere

Answer: D

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38. The function $g(x)=\begin{cases} x+b, & x<0 \\ \cos x, & x \geq 0 \end{cases}$ cannot be made differentiable at x = 0.

A. if b is equal to zero

B. if b is not equal to zero

C. if b takes any real value

D. for no value of b

Answer: D

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39. The graph of function f contains the point $P(1, 2)$ and $Q(s, r)$. The equation of the secant line through P and Q is $y = \left(\frac{s^2 + 2s - 3}{s - 1}\right)x - 1 - s$. The value of $f'(1)$, is

A. 2

B. 3

C. 4

D. non-existent

Answer: C

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40. Consider

$$f(x) = \frac{(2(\sin x - \sin x - \sin^3 x)) + |\sin x - \sin^3 x|}{2(\sin x - \sin^3 x) - |\sin x - \sin^3 x|}, x \neq \frac{\pi}{2}$$ for

$x \in (0, \pi), f\left(\frac{\pi}{2}\right) = 3$ where [ ] denotes the greatest integer function then,

A. f is continuous and differentiable at $x = \pi/2$

B. f is continuous but not differentiable at $x = \pi/2$

C. f is neither continuous nor differentiable at $x = \pi/2$

D. None of the above

Answer: A

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41. If $f(x+y) = f(x) + f(y) + |x|y + xy^2$, $\forall x, y \in R$ and $f'(0) = 0$, then

A. f need not be differentiable at every non-zero x

B. f is differentiable for all $x \in R$

C. f is twice differentiable at x = 0

D. None of the above

Answer: B

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42. Let f(x) = max. {|x^2 - 2 |x||,|x|} and g(x) = min. {|x^2 - 2|x||, |x|} then

A. both f(x) and g(x) are non-differentiable at 5 points

B. f(x) is not differentiable at 5 points whether g(x) is non-differentiable at 7 points

C. number of points of non-differentiability for f(x) and g(x) are 7 and 5 points, respectively

D. both f(x) and g(x) are non-differentiable at 3 and 5 points, respectively

Answer: B

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43. Let $g(x) = \left[\begin{matrix} 3x^2 - 4\sqrt{x} + 1 & x < 1 \\ ax + b & x \geq 1 \end{matrix}\right)$ If $g(x)$ is continuous and differentiable for all numbers in its domain then (A) a=b=-4 (B) a=b=4 (C) a=4 and b =-4 (D) a=-4 and b=4

A. a = b = 4

B. a = b = -4

C. a = 4 and b = - 4

D. a = - 4 and b = 4

Answer: C

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44. Let f(x) be continuous and differentiable function for all reals and f(x + y) = f(x) - 3xy + f(y). If $\lim_{h \to 0} \frac{f(h)}{h} = 7$, then the value of f'(x) is

A. $-3x$

B. 7

C. $-3x+7$

D. $2f(x)+7$

Answer: C

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45. Let [x] be the greatest integer function $f(x) = \left(\frac{\sin\left(\frac{1}{4}(\pi[x])\right)}{[x]}\right)$ is

A. Not continuous at any point

B. Continuous at 3/2

C. Discontinuous at 2

D. Differentiable at 4/3

Answer: C

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46. If $f(x) = \begin{cases} b\left([x]^2 + [x]\right) + 1, & \text{for } \quad x \geq -1 \\ \sin(\pi(x+a)), & \text{for } \quad x < -1 \end{cases}$ where [x] denotes the integral part of x, then for what values of a, b the function is continuous at x = - 1?

A. $a = 2n + (3/2), b \in R, n \in I$

B. $a = 4n + 2, b \in R, n \in I$

C. $a = 4n + (3/2), b \in R^+, n \in I$

D. $a = 4n + 1, b \in R^+, n \in I$

Answer: A

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47. If both $f(x) \& g(x)$ are differentiable functions at $x = x_0$then the function defiend as $h(x)$=Maximum$\{f(x), g(x)\}$

A. is always differentiable at $x = x_0$

B. is never differentiable at $x = x_0$

C. is differentiable at $x = x_0$ when $f(x_0) \neq g(x_0)$

D. cannot be differentiable at $x = x_0$, if $f(x_0) = g(x_0)$

Answer: C

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48. Number of points of non-differentiability of the function

$g(x) = [x^2]\{\cos^2 4x\} + \{x^2\}[\cos^2 4x] + x^2 \sin^2 4x + [x^2][\cos^2 4x] + \{x^2 in $(-50, 50)$ where $[x]$ and $\{x\}$ denotes the greatest integer function and fractional part function of x respectively, is equal to :

A. 98

B. 99

C. 100

D. 0

Answer: D

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49. If $f(x) = \dfrac{\{x\}g(x)}{\{x\}g(x)}$ is a periodic function with period $\dfrac{1}{4}$, where g(x) is differentiable function, then (where {.} denotes fractional part of x).

A. g'(x) has exactly three roots in $\left(\dfrac{1}{4}, \dfrac{5}{4}\right)$

B. g(x) = 0 at $x = \dfrac{k}{4}$, where $k \in I$

C. g(x) must be non-zero function

D. g(x) must be periodic function

Answer: B

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50. If $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ for all x, y $\in R, y \neq 0$ and $f'(x)$ exists for all x, f(2) = 4. Then, f(5) is

A. 3

B. 5

C. 25

D. None of the above

Answer: C

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Exercise More Than One Correct Option Type Questions

1. Function whose jump (non-negative difference of LHL and RHL) of discontinuity is greater than or equal to one. Is/are

A. $f(x) = \begin{cases} \frac{e^{1/x}+1}{e^{1/x}-1}, & x < 0 \\ \frac{1-\cos x}{x}, & x > 0 \end{cases}$

B. $g(x) = \begin{cases} \frac{x^{1/3}-1}{x^{1/2}-1}, & x > 1 \\ \frac{\log x}{x-1}, & \frac{1}{2} < x < 1 \end{cases}$

C. $u(x) = \begin{cases} \frac{\sin^{-1} 2x}{\tan^{-1} 3x}, & x \in \left[0, \frac{1}{2}\right] \\ \frac{|\sin x|}{x}, & x < 0 \end{cases}$

D. $v(x) = \begin{cases} \log_3(x+2), & x > 2 \\ \log_{1/2}(x^2+5), & x < 2 \end{cases}$

Answer: A::C

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2. Indicate all correct alternatives if, $f(x) = \frac{x}{2} - 1$, then on the interval $[0, \pi]$

A. $\tan(f(x))$ and $\frac{1}{f(x)}$ are both continuous

B. $\tan(f(x))$ and $\frac{1}{f(x)}$ are both discontinuous

C. $\tan(f(x))$ and $f^{-1}(x)$ are both continuous

D. $\tan(f(x))$ is continuous but $\frac{1}{f(x)}$ is not continuous

Answer: C::D

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3. On the interval $I = [-2, 2]$, the function

$$f(x) = \begin{cases} (x+1)e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

A. f(x) is continuous for all values of $x \in I$

B. f(x) is continuous for $x \in I - \{0\}$

C. f(x) assumes all intermediate values from f(-2) to f(2)

D. f(x) has a maximum value equal to 3/e

Answer: B::C::D

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4. Given $f(x)=\left\{3-\left[\cot^{-1}\left(\frac{2x^3-3}{x^2}\right)\right]f \text{ or } x>0\{x^2\}\cos\left(e^{\frac{1}{x}}\right)f \text{ or } x<0\right.$ (where {} and [] denotes the fractional part and the integral part functions respectively). Then which of the following statements do/does not hold good? $f(0^-)=0$ b. $f(0^+)=3$ c. if $f(0)=0$, then $f(x)$ is continuous at $x=0$ d. irremovable discontinuity of f at $x=0$

A. $f(0^{0-})=0$

B. $f(0^+)=0$

C. $f(0)=0 \Rightarrow$ Continuous at x = 0

D. Irremovable discontinuity at x = 0

Answer: A::B::C

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5. If $f(x) = \begin{cases} b\left([x]^2 + [x]\right) + 1, & \text{for} \quad x > -1 \\ \sin(\pi(x+a)), & \text{for} \quad x < -1 \end{cases}$, where [x] denotes the integral part of x, then for what values of a, b, the function is continuous at x = - 1?

A. $a = 2n + \frac{3}{2}, b \in R, n \in I$

B. $a = 4n + 2, b \in R, n \in I$

C. $a = 4n + \frac{3}{2}, b \in R^+, n \in I$

D. $a = 4n + 1, b \in R^+, n \in I$

Answer: A::C

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6. Q. For every integer n, let an and bn be real numbers. Let function $f: R \to R$ be given by a $f(x) = \{a_n + \sin \pi x, f \text{ or } x \in [2n, 2n+1],$

$\_ n + \cos \pi x, f \text{ or } x \in (2n+1, 2n)$ for all integers n.

A. $a_{n-1} - b_{n-1} = 0$

B. $a_n - b_n = 1$

C. $a_n = b_{n+1} = 1$

D. $a_{n-1} - b_n = -1$

Answer: B::D

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7. If $f(x) = |x+1|(|x| + |x-1|)$, then at what points the function is/are not differentiable at the interval [-2, 2] ?

A. -1

B. 0

C. 1

D. $\frac{1}{2}$

Answer: A::B::C

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8. Let [x] be the greatest integer function $f(x) = \left(\frac{\sin\left(\frac{1}{4}(\pi[x])\right)}{[x]}\right)$ is

A. Not continuous at any point

B. continuous at $x = \frac{3}{2}$

C. discontinuous at x = 2

D. differentiable at $x = \frac{4}{3}$

Answer: B::C::D

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9. If $f(x) = \begin{cases} (\sin^{-1}x)^2 \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ then f(x) is

A. continuous nowhere in $-1 \leq x \leq 1$

B. continuous everywhere in $-1 \leq x \leq 1$

C. differentiable nowhere in $-1 \leq x \leq 1$

D. differentiable everywhere in $-1 \leq x \leq 1$

Answer: B::D

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10. Q. Let $f(x) = \cos x$ \$ $H(x) = [\ \min [f(t) 0 \leq t \leq x$ for $0 \leq x \leq \frac{\pi}{2}$, $\frac{\pi}{2} - x$ for $\frac{\pi}{2} < x \leq 3$

A. H(x) is continuous and derivable in [0, 3]

B. H(x) is continuous but not derivable at $x = \frac{\pi}{2}$

C. H(x) is neither continuous nor derivable at $x = \frac{\pi}{2}$

D. maximum value of H(x) in [0, 3] is 1

Answer: A::D

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11. If $f(x) = 3(2x+3)^{2/3} + 2x + 3$, then

A. f(x) is continuous but not differentiable at $x = -\frac{3}{2}$

B. f(x) is differentiable at x = 0

C. f(x) is continuous at x = 0

D. f(x) is differentiable but not continuous at $x = -\frac{3}{2}$

Answer: A::B::C

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12. If $f(x) = \begin{cases} -x - \frac{\pi}{2} & x \leq -\frac{\pi}{2} \\ -\cos x & -\frac{\pi}{2} < x \leq 0 \\ x - 1 & 0 < x \leq 1 \\ \text{In } x & x > 1 \end{cases}$ then which one of the following is not correct?

A. f(x) is continuous at $x = -\frac{\pi}{2}$

B. f(x) is not differentiable at x = 0

C. f(x) is differentiable at x = 1

D. f(x) is differentiable at $x = -\frac{\pi}{2}$

Answer: A::B::C::D

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13. If f(x)=$\begin{cases} \frac{x \log \cos x}{\log(1+x^2)} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then

A. f is continuous at x = 0

B. f is continuous at x = 0 but not differentiable at x = 0

C. f is differentiable at x = 0

D. f is not continuous at x = 0

Answer: A::C

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14. Let [x] denotes the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then f(x) is

A. continuous at x = 0

B. continuous in (-1, 0)

C. differentiable at x = 1

D. differentiable in (-1, 1)

Answer: A::B::C

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15. The function, $f(x) = [|x|] - |[x]|$ where [] denotes greatest integer function:

A. is continuous for all positive integers

B. is discontinuous for all non-positive integers

C. has finite number of elements in its range

D. is such that its graph does not lie above the X-axis

Answer: A::B::C::D

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16. The function $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

A. has its domain $-1 \leq x \leq 1$

B. has finite one sided derivates at the point x = 0

C. is continuous and differentiable at x = 0

D. is continuous but not differentiable at x = 0

Answer: A::B::D

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17. Consider the function $f(x) = \left|x^3 + 1\right|$. Then,

A. domain of f $x \in R$

B. range of f is R^+

C. f has no inverse

D. f is continuous and differentiable for every $x \in R$

Answer: A::B::C

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18. f is a continous function in $[a, b]$; g is a continuous function in [b,c]. A function h(x) is defined as $h(x) = f(x) f \text{ or } x \in [a, b), g(x) f \text{ or } x \in (b, c]$ if f(b) =g(b) then

A. h(x) has a removable discontinuity at x = b

B. h(x) may or may not be continuous in [a, c]

C. $h(b^-) = g(b^+) \text{ and } h(b^+) = f(b^-)$

D. $g(b^+) = g(b^-) \text{ and } h(b^-) = f(b^+)$

Answer: A::B

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19. Which of the following function(s) has/have the same range ?

A. $f(x)=\frac{1}{1+x}$

B. $f(x)=\frac{1}{1+x^2}$

C. $f(x)=\frac{1}{1+\sqrt{x}}$

D. $f(x)=\frac{1}{\sqrt{3-x}}$

Answer: B::C

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20. If f(x) = sec 2x + cosec 2x, then f(x) is discontinuous at all points in

A. $\{n\pi, n \in N\}$

B. $\left\{(2n \pm 1)\frac{\pi}{4}, n \in I\right\}$

C. $\left\{\frac{n\pi}{4}, n \in I\right\}$

D. $\left\{(2n \pm 1)\frac{\pi}{8}, n \in I\right\}$

Answer: A::B::C

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21. Let $f(x) = \begin{cases} x^n \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ Then f(x) is continuous but not differentiable at x=0. If

A. $\lim_{x \to 0} f(x)$ exists for every $n > 1$

B. f is continuous at x = 0 for $n > 1$

C. f is differentiable at x= 0 for every $n > 1$

D. None of the above

Answer: A::B::C

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22. A function is defined as $f(x) = \begin{cases} e^x, & x \le 0 \\ |x-1|, & x > 0 \end{cases}$, then f(x) is

A. continuous at x = 0

B. continuous at x = 1

C. differentiable at x = 0

D. differentiable at x = 1

Answer: A::B

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23. Let $f(x) = \int_{-2}^{x} |t+1| dt$, then

A. f(x) is continuous in [-1, 1]

B. f(x) is differentiable in [-1, 1]

C. f'(x) is continuous in [-1, 1]

D. f'(x) is differentiable in [-1, 1]

Answer: A::B::C::D

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24. A function f(x) satisfies the relation $f(x+y) = f(x) + f(y) + xy(x+y), \forall x, y \in R$. If f'(0) = - 1, then

A. f(x) is a polynomial function

B. f(x) is an exponential function

C. f(x) is twice differentiable for all $x \in R$

D. f'(3) = 8

Answer: A::C::D

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25. If $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \leq x \leq 2 \\ 37 - x, & 2 < x \leq 3 \end{cases}$, then

A. f(x) is increasing on [-1, 2]

B. f(x) is continuous on [-1, 3]

C. f'(2) doesn't exist

D. f(x) has the maximum value at x = 2

Answer: A::B::D

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26. If f(x) = 0 for $x < 0$ and $f(x)$ is differentiable at x = 0, then for $x > 0$, $f(x)$ may be

A. x^2

B. x

C. $-x$

D. $-x^{3/2}$

Answer: A::D

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Exercise Statement I And Ii Type Questions

1. Statement I $f(x) = \sin x + [x]$ is discontinuous at x = 0.

Statement II If g(x) is continuous and f(x) is discontinuous, then g(x) + f(x) will necessarily be discontinuous at x = a.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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2. Consider $f(x) = \begin{cases} 2\sin\left(a\cos^{-1}x\right), & \text{if } \ x \in (0,1) \\ \sqrt{3}, & \text{if } \ x = 0 \\ ax + b, & \text{if } \ x < 0 \end{cases}$

Statement I If b = $\sqrt{3}$ and $a = \frac{2}{3}$, then f(x) is continuous in $(-\infty, 1)$.

Statement II If a function is defined on an interval I and limit exists at every point of interval I, then function is continuou in I.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: C

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3. Let $f(x) = \begin{cases} \frac{\cos x - e^{x^2/2}}{x^3}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

Statement I f(x) is continuous at x = 0.

Statement II $\lim_{x \to 0} \frac{\cos x - e^{-x^2/2}}{x^3} = -\frac{1}{12}$

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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4. Statement I The equation $\frac{x^3}{4} - \sin \pi x + \frac{2}{3} = 0$ has atleast one solution in [-2, 2].

Statement II Let $f : [a, b] \rightarrow R$ be a function and c be a number such that $f(a) < c < f(b)$, then there is atleast one number $n \in (a, b)$ such that f(n) = c.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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5. Statement I Range of $f(x) = x\left(\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}\right) + x^2 + x^4$ is not R.

Statement II Range of a continuous evern function cannot be R.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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6. Let $f(x) = \begin{cases} Ax - B & x \leq 1 \\ 2x^2 + 3Ax + B & x \in (-1, 1] \\ 4 & x > 1 \end{cases}$

Statement I f(x) is continuous at all x if $A = \frac{3}{4}, B = -\frac{1}{4}$. Because

Statement II Polynomial function is always continuous.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: B

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7. Let $h(x) = f_1(x) + f_2(x) + f_3(x) + \ldots + f(n)(x)$, where $f_1(x), f_2(x), f_3(x), \ldots., f_n(x)$ are real valued functions of x.

Statement I $f(x) = |\cos|x|| + \cos^{-1}(\text{sgn x}) + |\text{In x}|$ is not differentiable at 3 points in $(0, 2\pi)$

Statement II Exactly one function, is $f_i(x), i = 1, 2, \ldots, n$ is not differentiable and the rest of the function is differentiable at x = a makes h(x) not differentiable at x = a.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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8. Statement I $f(x) = |x| \sin x$ is differentiable at $x = 0$.

Statement II If $g(x)$ is not differentiable at $x = a$ and $h(x)$ is differentiable at $x = a$, then $g(x).h(x)$ cannot be differentiable at $x = a$

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: C

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9. Statement I f(x) = |cos x| is not derivable at $x = \frac{\pi}{2}$.

Statement II If g(x) is differentiable at x = a and g(a) = 0, then |g|(x)| is non-derivable at x = a.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: C

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10. Let f(x) = $x - x^2$ and $g(x) = \{x\}, \forall x \in R$ where denotes fractional part function.

Statement I f(g(x)) will be continuous, $\forall x \in R$.

Statement II $f(0) = f(1)$ and $g(x)$ is periodic with period 1.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: A

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11. Let

$f(x) = -ax^2 - b|x| - c, \ -\alpha \le x < 0, ax^2 + b|x| + c0 \le x \le \alpha$

where a,b,c are positive and $\alpha > 0$, then- Statement-1 : The equation f(x)=0has atleast one real root for $x \in [-\alpha, \alpha]$ Statement-2: Values of $f(-\alpha)$ and $f(\alpha)$ are opposite in sign.

A. Statement I is correct, Statement II is also correct, Statement II is the correct explanation of Statement I

B. Statement I is correct, Statement II is also correct, Statement II is not the correct explanation of Statement I

C. Statement I is correct, Statement II is incorrect

D. Statement I is incorrect, Statement II is correct.

Answer: D

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Exercise Passage Based Questions

1. Let f be a function that is differentiable everywhere and that has the follwong properties :

(i) $f(x) > 0$

(ii) $f'(0) = -1$

(iii) $f(-x) = \frac{1}{f(x)}$ and $f(x+h) = f(x).\, f(h)$

A standard result is $\frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

Range of f(x) is

A. R

B. $R - \{0\}$

C. R^{+}

D. (0, e)

Answer: C

2. Let f be a function that is differentiable everywhere and that has the follwong properties :

(i) $f(x) > 0$

(ii) $f'(0) = -1$

(iii) $f(-x) = \frac{1}{f(x)}$ and $f(x+h) = f(x).\,f(h)$

A standard result is $\frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

Range of f(x) is

A. [0, 1]

B. [0, 1)

C. (0, 1]

D. None of these

Answer: A

3. Let f be a function that is differentiable everywhere and that has the follwong properties :

(i) $f(x) > 0$

(ii) $f'(0) = -1$

(iii) $f(-x) = \frac{1}{f(x)}$ and $f(x+h) = f(x).\,f(h)$

A standard result is $\frac{f'(x)}{f(x)}dx = \log|f(x)| + C$

The function y = f(x) is

A. odd

B. even

C. increasing

D. decreasing

Answer: D

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4. Let f be a function that is differentiable everywhere and that has the follwong properties :

(i) $f(x) > 0$

(ii) $f'(0) = -1$

(iii) $f(-x) = \frac{1}{f(x)}$ and $f(x+h) = f(x).\, f(h)$

A standard result is $\frac{f'(x)}{f(x)} dx = \log|f(x)| + C$

If h(x) = f'(x), then h(x) is given by

A. $-f(x)$

B. $\frac{1}{f(x)}$

C. f(x)

D. $e^{f(x)}$

Answer: A

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5. Let y = f(x) be defined in [a, b], then

(i) Test of continuity at $x = c$, $a < c < b$

(ii) Test of continuity at x = a

(iii) Test of continuity at x = b

Case I Test of continuity at $x = c$, $a < c < b$

If y = f(x) be defined at x = c and its value f(c) be equal to limit of f(x) as $x \to c$ i.e. f(c) = $\lim_{x \to c} f(x)$

or $\lim_{x \to c^-} f(x) = f(c) = \lim_{x \to c^+} f(x)$

or LHL = f(c) = RHL

then, y = f(x) is continuous at x = c.

Case II Test of continuity at x = a

If RHL = f(a)

Then, f(x) is said to be continuous at the end point x = a

Case III Test of continuity at x = b, if LHL = f(b)

Then, f(x) is continuous at right end x = b.

If $f(x) = \begin{cases} \sin x, & x \leq 0 \\ \tan x, & 0 < x < 2\pi \\ \cos x, & 2\pi \leq x < 3\pi \\ 3\pi, & x = 3\pi \end{cases}$, then f(x) is discontinuous at

A. $\frac{\pi}{2}, \frac{3\pi}{2}, 2\pi, 3\pi$

B. $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 3\pi$

C. $\frac{\pi}{2}, 2\pi$

D. None of these

Answer: A

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6. Let y = f(x) be defined in [a, b], then

(i) Test of continuity at $x = c, a < c < b$

(ii) Test of continuity at x = a

(iii) Test of continuity at x = b

Case I Test of continuity at $x = c, a < c < b$

If y = f(x) be defined at x = c and its value f(c) be equal to limit of f(x) as

$x \to c$ i.e. f(c) = $\lim_{x \to c} f(x)$

or $\lim_{x \to c^-} f(x) = f(c) = \lim_{x \to c^+} f(x)$

or LHL = f(c) = RHL

then, y = f(x) is continuous at x = c.

Case II Test of continuity at x = a

If RHL = f(a)

Then, f(x) is said to be continuous at the end point x = a

Case III Test of continuity at x = b, if LHL = f(b)

Then, f(x) is continuous at right end x = b.

Number of points of discontinuity of $[2x^3 - 5]$ in [1, 2) is (where [.] denotes the greatest integral function.)

A. 14

B. 13

C. 10

D. None of these

Answer: B

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7. Let y = f(x) be defined in [a, b], then

(i) Test of continuity at $x = c, a < c < b$

(ii) Test of continuity at x = a

(iii) Test of continuity at x = b

Case I Test of continuity at $x = c, a < c < b$

If y = f(x) be defined at x = c and its value f(c) be equal to limit of f(x) as

$x \rightarrow c$ i.e. f(c) = $\lim_{x \rightarrow c} f(x)$

or $\lim_{x \rightarrow c^-} f(x) = f(c) = \lim_{x \rightarrow c^+} f(x)$

or LHL = f(c) = RHL

then, y = f(x) is continuous at x = c.

Case II Test of continuity at x = a

If RHL = f(a)

Then, f(x) is said to be continuous at the end point x = a

Case III Test of continuity at x = b, if LHL = f(b)

Then, f(x) is continuous at right end x = b.

Max ([x],|x|) is discontinuous at

A. x = 0

B. ϕ

C. $x = n, n \in I$

D. None of these

Answer: B

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8.

$(f(x) = \cos x \text{ and } H_1(x) = \min\{f(t), 0 \le t < x\},), \left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2}\right.$
$\left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2} - x, \frac{\pi}{2} < x \le \pi\right), (g(x) = \sin x \text{ and } H_3(x) = \min\{$
$(g(x) = \sin x \text{ and } H_4(x) = \max\{g(t), 0 \le t \le x\},), \left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2}\right.$

Which of the following is true for $H_2(x)$?

A. Continuous and derivable in $[0, \pi]$

B. Continuous but not derivable at $x = \frac{\pi}{2}$

C. Neither continuous nor derivable at $x = \frac{\pi}{2}$

D. None of the above

Answer: C

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9.

$\left(f(x) = \cos x \text{ and } H_1(x) = \min\{f(t), 0 \le t < x\},\right), \left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2}\right.$

$\left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2} - x, \frac{\pi}{2} < x \le \pi\right), \left(g(x) = \sin x \text{ and } H_3(x) = \min\{\right.$

$\left(g(x) = \sin x \text{ and } H_4(x) = \max\{g(t), 0 \le t \le x\},\right), \left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2}\right.$

Which of the following is true for $H_3(x)$?

A. Continuous and derivable in $[0, \pi]$

B. Continuous but not derivable at $x = \frac{\pi}{2}$

C. Neither continuous nor derivable at $x = \frac{\pi}{2}$

D. None of the above

Answer: B

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10.

$(f(x) = \cos x$ and $H_1(x) = \min\{f(t), 0 \le t < x\},), \left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2}\right.$

$\left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2} - x, \frac{\pi}{2} < x \le \pi\right), (g(x) = \sin x$ and $H_3(x) = \min\{$

$(g(x) = \sin x$ and $H_4(x) = \max\{g(t), 0 \le t \le x\},), \left(0 \le x \le \frac{\pi}{2} = \frac{\pi}{2}\right.$

Which of the following is true for $H_4(x)$?

A. Continuous and derivable in $[0, \pi]$

B. Continuous but not derivable at $x = \frac{\pi}{2}$

C. Neither continuous nor derivable at $x = \frac{\pi}{2}$

D. None of the above

Answer: C

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11. Let f(x) be a real valued function not identically zero, which satisfied the following conditions

I. $f\left(x+y^{2n+1}\right)=f(x)+(f(y))^{2n+1}, n \in N, x, y$ are any real numbers.

II. $f'(0) \geq 0$

The value of f(1), is

A. 0

B. 1

C. 2

D. Not defined

Answer: B

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12. Let f(x) be a real valued function not identically zero, which satisfied the following conditions

I. $f\left(x+y^{2n+1}\right)=f(x)+(f(y))^{2n+1}, n \in N, x, y$ are any real numbers.

II. $f'(0) \geq 0$

The value of f(x), is

A. 2x

B. $x^2 + x + 1$

C. x

D. None of these

Answer: C

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13. Let f(x) be a real valued function not identically zero, which satisfied the following conditions

I. $f\left(x + y^{2n+1}\right) = f(x) + (f(y))^{2n+1}, n \in N, x, y$ are any real numbers.

II. $f'(0) \geq 0$

The value of f'(10), is

A. 10

B. 0

C. 2n + 1

D. 1

Answer:

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14. Let f(x) be a real valued function not identically zero, which satisfied the following conditions

I. $f\left(x+y^{2n+1}\right)=f(x)+(f(y))^{2n+1}, n \in N, x, y$ are any real numbers.

II. $f'(0) \geq 0$

The function f(x) is

A. odd

B. even

C. neither even nor odd

D. both even as well as odd

Answer: A

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15. If $f: R \to (0, \infty)$be a differentiable function $f(x)$ satisfying

$f(x+y) - f(x-y) = f(x) \cdot \{f(y) - f(y) - y\}, \forall x, y \in R, (f(y) \neq f(-$

and $f'(0) = 2010.$

Now, answer the following questions.

Which of the following is true for $f(x)$

A. f(x) is one-one and into

B. {f(x)} is non-periodic, where {.} denotes fractional part of x

C. f(x) = 4 has only two solutions

D. f(x) = f'(x) has only one solution

Answer: B

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16. If $f: R \to (0, \infty)$ is a differentiable function f(x) satisfying $f(x+y) - f(x-y) = f(x).\{f(y) - f(-y)\}, \forall x, y \in R, (f(y) \neq f(-$

. Now, answer the following questions :

The value of $\frac{f'(x)}{f(x)}$ is

A. 2016

B. 2014

C. 2012

D. 2010

Answer: D

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Exercise Matching Type Questions

1. Match the column.

Column I	Column II
(A) $f(x)=\begin{cases} x+1, & \text{if } x<0 \\ \cos x, & \text{if } x\geq 0 \end{cases}$ at $x=0$ is	(p) continuous
(B) For every $x \in R$, the function $g(x)=\dfrac{\sin(\pi[x-\pi])}{1+[x]^2}$, where $[x]$ denotes the greatest integer function, is	(q) differentiability
(C) $h(x)=\sqrt{\{x\}^2}$ where $\{x\}$ denotes fractional part function for all $x \in I$, is	(r) discontinuous
(D) $k(x)=\begin{cases} x^{\frac{1}{\ln x}}, & \text{if } x\neq 1 \\ e, & \text{if } x=1 \end{cases}$ at $x=1$ is	(s) non-derivable

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Exercise Single Integer Answer Type Questions

1. Number points of discontinuity of $f(x)=\tan^2 x-\sec^2 x$ in $(0, 2\pi)$ is

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2. Number, of pointis) of discontinuity of the function $f(x) = \left[x^{\frac{1}{x}}\right], x > 0$, where [.] represents GIF is

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3. Let $f(x) = x + \cos x + 2$ and $g(x)$ be the inverse function of f(x), then g'(3) equals to ....

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4. Let $f(x) = x \tan^{-1}(x^2) + x^4$ Let $f^k(x)$ denotes k^{th} derivative of $f(x)$ w.r.t. $x, k \in N$. If

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5. Let $f_1(x)$ and $f_2(x)$ be twice differentiable functions where $F(x) = f_1(x) + f_2(x)$ and $G(x) = f_1(x) - f_2(x), \forall x \in R, f_1(0) = 2$ an

. then the number of solutions of the equation $(F(x))^2 = \frac{9x^4}{G(x)}$ is...... .

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6. Suppose, the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative 7 at $x = 2$. The derivative of the function $f(x) - f(4x)$ at $x = 1$, has the value $10 + \lambda$, then the value of λ is equal to........

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7. Let $f(x) = \begin{cases} \frac{x\left(\frac{3}{4}\right)^{1/x} - \left(\frac{3}{4}\right)^{-1/x}}{\left(\frac{3}{4}\right)^{1/x} + \left(\frac{3}{4}\right)^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. If P = $f'(0^-) - f'(0^+)$, then $4\left(\lim_{x \to p^-} \frac{(\exp((x+2)\log 4))\left[\frac{x+1}{4}\right] - 16}{4^x - 16}\right)$, is...... (where [x] denotes greatest integer function.)

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8. Let

$$f(x) = -x^3 + x^2 - x + 1 \text{ and } g(x) = \begin{cases} \min(f(t)), & 0 \le t \le x \text{ and } 0 \\ x - 1, & 1 < x \le 2 \end{cases}$$

Then, the value of $\lim_{x \to 1} g(g(x))$, is........ .

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9. If $f(x) = \begin{cases} \dfrac{\frac{\pi}{2} - \sin^{-1}\left(1 - \{x\}^2\right)\sin^{-1}(1 - \{x\})}{\sqrt{2}\left(\{x\} - \{x\}^3\right)}, & x > 0 \\ k, & x = 0 \\ \dfrac{A\sin^{-1}(1 - \{x\})\cos^{-1}(1 - \{x\})}{\sqrt{2\{x\}}(1 - \{x\})}, & x < 0 \end{cases}$ is continuous at x = 0, then the value of $\sin^2 k + \cos^2\left(\dfrac{A\pi}{\sqrt{2}}\right)$, is..... (where {.} denotes fractional part of x).

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Exercise Subjective Type Questions

1. Check continuity and differentibilty of $f(x) = [x] + |1 - x|$, [ ] denotes the greatest integer function

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2. If $f(x) = \begin{cases} x[x] & 0 \leq x < 2 \\ (x-1)[x] & 2 \leq x < 3 \end{cases}$ where [.] denotes the greatest integer function, then

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3. Let f be a twice differentiable function such that $f^x = -f(x), and f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2.$ Find $h(10)$ if $h(5) = 11$

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4. A function $f: R \rightarrow R$ satisfies the equation $f(x+y) = f(x)f(y), \forall x, y$ in R and $f(x) \neq 0$ for any x in R . Let the function be differentiable at x = 0 and f'(0) = 2. Show that $f'(x) = 2f(x), \forall x$ in R. Hence, determine f(x)

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5. A function $f: R \rightarrow R$ satisfies the relation $f\left(\frac{x+y}{3}\right) = \frac{1}{3}|f(x) + f(y) + f(0)|$ for all $x, y \in R$. If $f'(0)$ exists, prove that $f'(x)$ exists for all $x, \in R$.

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6. Let $f(x+y) = f(x) + f(y) + 2xy - 1$ for all real $x and y$ and $f(x)$ be a differentiable function. If $f'(0) = \cos\alpha,$ the prove that $f(x) > 0 \forall x \in R\cdot$

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Exercise Questions Asked In Previous 13 Years Exam

1. For every pair of continuous functions $f, g: [0, 1] \to R$ such that $\max\{f(x): x \in [0, 1]\} = \max\{g(x): x \in [0, 1]\}$ then which are the correct statements

A. $[f(c)]^2 + 3f(c) = [g(c)]^2 + 3g(c) \text{for some c} \in [0, 1]1$

B. $[f(c)]^2 + f(c) = [g(c)]^2 + 3g(c) \text{for some c} \in [0, 1]$

C. $[f(c)]^2 + 3f(c) = [g(c)]^2 + g(c) \text{for some c} \in [0, 1]$

D. $[f(c)]^2 = [g(c)]^2 \text{for some c} \in [0, 1]$

Answer: A::D

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2. Let $f: R \to R$ and $g: R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1.$ Define $h: R \to R$ by

$h(x) = \{ \max \{f(x), g(x)\}, \quad \text{if} \quad x \leq 0 \text{ and } \quad \min \{f(x), g(x)\}, \quad \text{if} \quad x >$

.The number of points at which $h(x)$ is not differentiable is

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3. Let $f(x) = \left\{x^2 \left|(\cos)\frac{\pi}{x}\right|, x \neq 0 \text{ and } 0, x = 0, x \in \mathbb{R},\right.$ then f is

A. differentiable both at x = 0 and at x = 2

B. differentiable at x = 0 but not differentiable at x = 2

C. not differentiable at x = 0 but differentiable at x = 2

D. differentiable neither at x = 0 nor at x = 2

Answer: B

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4. Q. For every integer n, let an and bn be real numbers. Let function

$f: R \rightarrow R$ be given by a $f(x) = \{a_n + \sin \pi x, f \text{ or } x \in [2n, 2n+1],$

_ $n + \cos \pi x, f$ or $x \in (2n+1, 2n)$ for all integers n.

A. $a_{n-1} - b_{n-1} = 0$

B. $a_n - b_n = 1$

C. $a_n - b_{n+1} = 1$

D. $a_{n-1} - b_n = -1$

Answer: D

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5. Let $f: R \to R$ be a function such that $f(x+y) = f(x) + f(y), \forall x, y \in R.$

A. f(x) is differentiable only in a finite interval containing zero

B. f(x) is continuous for all $x \in R$

C. f'(x) is constant for all $x \in R$

D. f(x) is differentiable except at finitely many points

Answer: B::C

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6. If $f(x) = \begin{cases} -x - \frac{\pi}{2} & x \leq -\frac{\pi}{2} \\ -\cos x & -\frac{\pi}{2} < x \leq 0 \\ x - 1 & 0 < x \leq 1 \\ \text{In} \quad x & x > 1 \end{cases}$ then which one of the following is not correct?

A. f(x) is continuous at $x = -\frac{\pi}{2}$

B. f(x) is not differentiable at x = 0

C. f(x) is differentiable x = 1

D. f(x) is differentiable at $x = -\frac{3}{2}$

Answer: D

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7. For the fucntion $f(x) = x\cos\frac{1}{x}, x \geq 1$which one of the following is incorrect ?

A. for atleast one x in the interval $[1, \infty)$, $f(x+2) - f(x) < 2$

B. $\lim_{x \to \infty} f'(x) = 1$

C. for all x in the interval $[1, \infty)$, $f(x+2) - f(x) > 2$

D. f'(x) is strictly decreasing in the interval $[1, \infty)$

Answer: C

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8. Let $g(x) = \frac{(x-1)^n}{\log\cos^m(x-1)}, 0 < x < 2$, m and n are integers, $m \neq 0, n > 0$ and let p be the left hand derivative of $|x-1| \text{at x} = 1|$. If $\lim_{x \to 1^+} g(x) = p$, then

A. n = 1, m = 1

B. n = 1, m = - 1

C. n = 2, m = 2

D. $n > 2, m = n$

Answer: C

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9. Let f and g be real valued functions defined on interval (-1, 1) such that g''(x) is continuous, $g(0) \neq 0, g'(0) = 0, g''(0) \neq 0,\ \text{ and } f(x) = g(x)\sin x.$

Statement I $\lim_{x \to 0} [g(x)\cos x - g(0)\text{cosec x}] = f''(0)$. and

Statement II f'(0) = g(0).

For the following questions, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: B

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10. In the following, [x] denotes the greatest integer less than or equal to x.

	Column I		Column II
A	$x\|x\|$	p	continuous in (-1, 1)
B	$\sqrt{\|x\|}$	q	differentiable in (-1, 1)
C	$x+[x]$	r	strictly increasing (-1, 1)
D	$\|x-1\|+\|x+1\|$, in$(-1, 1)$	s	not differentiable atleast at one point

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11. If $f(x) = \min .\left(1, x^2, x^3\right)$, then

A. f(x) is continuous everywhere

B. f(x) is continuous and differentiable everywhere

C. f(x) is not differentiable at two points

D. f(x) is not differentiable at one point

Answer: A::D

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12. Let $f(x) = ||x| - 1|$, then points where, $f(x)$ is not differentiable is/are

A. 0 ± 1

B. ± 1

C. 0

D. 1

Answer: A

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13. Iff is a differentiable function satisfying $f\left(\frac{1}{n}\right)=0, \forall n \geq 1, n \in I,$ then

A. $f(x)=0, x \in (0, 1]$

B. $f'(0)=0=f(0)$

C. f(0) = 0 but f'(0) not necessarily zero

D. $|f(x)| \leq 1, x \in (0, 1]$

Answer: B

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14. The domain of the derivative of the function $f(x)=\left\{tan^{-1}x, \quad \text{if} \quad |x| \leq 1\frac{1}{2}(|x|-1), \quad \text{if} \quad |x|>1 \quad R-\{0\}\right.$ b. $R-\{1\}$ c. $-\{-1\}$ d. $R-\{-1, 1\}$

A. $R - \{0\}$

B. $R - \{1\}$

C. $R - \{-1\}$

D. $R - \{-1, 1\}$

Answer: D

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15. The left hand derivative of $f(x) = [x]\sin(\pi x)$ at $x = k$, k is an integer, is

A. $(-1)^k (k-1)\pi$

B. $(-1)^{k-1}(k-1)\pi$

C. $(-1)^k k\pi$

D. $(-1)^{k-1} k\pi$

Answer: A

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16. Which of the following functions is differentiable at $x = 0$?

$\cos(|x|) + |x|$

A. $\cos(|x|) + |x|$

B. $\cos(|x|) - |x|$

C. $\sin(|x|) + |x|$

D. $\sin(|x|) - |x|$

Answer: D

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17. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g is not differentiable at x = 0

B. g'(0) = cos (log 2)

C. g'(0) = - cos (log 2)

D. g is differentiable at x = 0 and g'(0) = - sin(log 2)

Answer: B

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18. If the function

$g(x) = \begin{cases} k\sqrt{x+1} & 0 \leq x \leq 3 \\ mx + 2 & 3 < x \leq 5 \end{cases}$ is differentiable, then the value of k+m is

A. 2

B. $\frac{16}{5}$

C. $\frac{10}{3}$

D. 4

Answer: A

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19. If f and g are differentiable functions in [0, 1] satisfying $f(0) = 2 = g(1), g(0) = 0$ and $f(1) = 6$, then for some $c \in]0, 1[$ (1) $2f'(c) = g'(c)$ (2) $2f'(c) = 3g'(c)$ (3) $f'(c) = g'(c)$ (4) $f'(c) = 2g'(c)$

A. $2f'(c) = g'(c)$

B. $2f'(c) = 3g'(c)$

C. $f'(c) = g'(c)$

D. $f'(c) = 2g'(c)$

Answer: D

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20. The function $f(x) = [x]\cos\left(\frac{2x-1}{2}\right)\pi$ where [ ] denotes the greatest integer function, is discontinuous

A. continuous for every real x

B. discontinuous only at $x = 0$

C. discontinuous only at non-zero integral values of x

D. continuous only at $x = 0$

Answer: D

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