MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

DEFINITE INTEGRAL

Example

1. Evaluate $\int_0^1 \frac{1}{3+4x}dx$

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2. Find the value of $\int_{-1}^{1} \frac{d}{dx}\left(\frac{\tan^{-1}1}{x}\right)dx$

A. $\pi/2$

B. $\pi/4$

C. $-\pi/2$

D. None of these

Answer: C

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3. If $I_n = \int_1^e (\log x)^n dx$, then $I_n + nI_{n-1}$ equal to

A. $\frac{1}{e}$

B. e

C. $e-1$

D. None of these

Answer: B

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4. All the values of a for which $\int_1^2 \left[a^2+(4-4a)x+4x^3\right]dx \leq 12$ are given by (A) $a=3$ (B) $a \leq 4$ (C) $0 \leq a < 3$ (D) none of these

A. $a=3$

B. $a \leq 4$

C. $0 \leq a < 3$

D. None of these

Answer: A

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5. Evaluate $\int_0^3 |(x-1)(x-2)|dx.$

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6. Prove that $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x} = \frac{\pi}{2ab}(a, b > 0)$

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7. Evaluate $\int_{-2}^{2} \frac{dx}{4+x^2}$ directly as well as by

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8. Evaluate: $\int_0^{\frac{1}{2}} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

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9. For any $n > 1$, evaluate the integral

$$\int_0^{\infty} \frac{1}{\left(x+\sqrt{x^2+1}\right)^n} dx$$

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10. Evaluate: $\int_0^{e-1} \frac{\frac{x^2+2x-1}{2}}{x+1} dx + \int_1^{e} x \log x e^{\frac{x^2-2}{2}} dx$

A. $\left(\sqrt{e}\right)^{\left(e^2+1\right)}$

B. $\left(\sqrt{e}\right)^{e^2-1}$

C. 0

D. $\left(\sqrt{e}\right)^{e^2-2}$

Answer: D

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11. Let $f(x)=\int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then , the value of g'(0) is

A. 1

B. 17

C. $\sqrt{17}$

D. None of these

Answer: C

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12. Let $a_n = \int_0^{\pi/2} (1 - \sin t)^n \sin 2t,$

then $\lim_{n \to \infty} \sum_{n=1}^{n} \frac{a_n}{n}$ is equal to

A. $1/2$

B. 1

C. $4/3$

D. $3/2$

Answer: A

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13. The value of $x > 1$ satisfying the equation

$\int_1^x t \ln t dt = \frac{1}{4}$ is

A. $\sqrt{e}$

B. e

C. e^2

D. $e - 1$

Answer: A

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14. If $\lim_{a \to \infty} \frac{1}{a} \int_0^{\infty} \frac{x^2 + ax + 1}{1 + x^4} . \tan^{-1}\left(\frac{1}{x}\right) dx$ is equal to $\frac{\pi^2}{K}$, where K in N, then K equals to

A. 4

B. 8

C. 16

D. 32

Answer: C

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15. If the value of definite integral $\int_1^a x \cdot a^{-[\log_a x]} dx$, where $a > 1$ and [x] denotes the greatest integer, is, $\frac{e-1}{2}$ then the value of equal to

A. $\sqrt{e}$

B. e

C. $\sqrt{e+1}$

D. $e - 1$

Answer: A

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16. Show that (i) $\int_0^{\pi/2} f(\sin x)dx = \int_0^{\pi/2} f(\cos x)dx$ (ii) $\int_0^{\pi/2} f(\tan x)dx = \int_0^{\pi/2} f(\cot x)dx$ (iii) $\int_0^{\pi/2} f(\sin 2x)\sin x dx = \int_o^{\pi/2} f(\sin 2x).\cos x dx$

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17. If $f and g$ are continuous function on $[0, a]$ satisfying $f(x) = f(a - x) and g(x)(a - x) = 2,$ then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx.$

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18. Evaluate

(i) $\int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$

(ii) $\int_0^{\pi/2} \log(\tan x)dx$

(iii) $\int_0^{\pi/4} \log(1 + \tan x)dx$

(iv) $\int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x}dx$

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19. The value of $\int_0^a \log(\cot a + \tan x)dx,$

where $a \in (0, \pi/2)$ is equal to

A. $a\log(\sin a)$

B. $-a\log(\sin a)$

C. $-a\log(\cos a)$

D. None of these

Answer: B

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20. Evaluate $\int_{\pi/6}^{\pi/3}\frac{dx}{1+\sqrt{\tan x}}$

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21. Prove that

$$\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx=\frac{b-a}{2}.$$

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22. Solve $l = \int_{\cos^4 t}^{-\sin^4 t} \frac{\sqrt{f(z)}dz}{\sqrt{f(\cos 2t - z)} + \sqrt{f(z)}}$

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23. The value of $f(x).\,f(-x)$ for all x is

A. 4

B. 9

C. 12

D. 16

Answer: B

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24. $\int_{-51}^{51} \frac{dx}{3 + f(x)}$ has the value equal to

A. 17

B. 34

C. 102

D. 0

Answer: A

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25. Number of roots $f(x) = 0 \ \ \text{in}[-2, 2]$ is

A. 0

B. 1

C. 2

D. 4

Answer: A

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26. Given fuction, $\begin{cases} x^2 & \text{for} 0 \leq x < 1 \\ \sqrt{x} & \text{for} 1 \leq x \leq 2 \end{cases}$

Evaluate $\int_0^2 f(x)dx$.

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27. Evaluate the integral $l = \int_0^2 |1 - x| dx$.

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28. Evaluate (i) $\int_0^{\pi} |\cos x| dx$ (ii) $\int_0^2 |x^2 + 2x - 3| dx$

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29. Evaluate $\int_{-1}^{1} (x - [x]) dx$, where $[.]$

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30. Evaluate $\int_0^2 \{x\}dx$, where $\{x\}$ denotes the fractional part of x.

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31. $\int_0^9 \{\sqrt{x}\}dx,$ where $\{x\}$ denotes the fractional part of $x,$ is 5 (b) 6 (c) 4 (d) 3

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32. If for all real numbers y, $[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2} [2s \in x]dx$ is $-\pi$ b. 0 c. $\pi/2$ d. $\pi/2$

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33. The value of $\int_0^{100} [\tan^{-1} x]dx$ is equal to (where $[.]$ denotes the greatest integer function)

A. $\tan 1 - 100$

B. $\pi/2 - \tan 1$

C. $100 - \tan 1$

D. None of these

Answer: C

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34. The value of

$\int_{-2}^{2} \min\{x - [x], -x - [-x]\} dx$ is equal to (where $[.]$denotes the greatest integer function)

A. $1/2$

B. 1

C. $3/2$

D. 2

Answer: B

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35. The value of $\int_1^2 \left(x^{[x^2]} + [x^2]^x\right)dx$ is equal to where $[.]$ denotes the greatest integer function

A. $\frac{5}{4}\sqrt{3} + \left(2^{\sqrt{3}} - 2^{\sqrt{2}}\right) + \frac{1}{\log 3}\left(9 - 3^{\sqrt{3}}\right)$

B. $\frac{5}{4} + \sqrt{3} + \frac{\sqrt{2}}{3} + \frac{1}{\log 2}\left(2^{\sqrt{3}} - 2^{\sqrt{2}}\right) + \frac{1}{\log 3}\left(9 - 3^{\sqrt{3}}\right)$

C. $\frac{5}{4} + \frac{\sqrt{2}}{2} + \frac{1}{\log 2}\left(2^{\sqrt{3}} - 2^{\sqrt{2}}\right) + \frac{1}{\log 3}\left(9 - 3^{\sqrt{3}}\right)$

D. None of the above

Answer: B

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36. The value of $\int_0^{2\pi} [|\sin x| + |\cos x|]dx$ is equal to

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. 2π

Answer: D

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37. The value of $\int_0^{\frac{\pi}{2}} \sin|2x - \alpha| dx$, where $\alpha \in [0, \pi]$, is

A. 1

B. $\cos\alpha$

C. $\frac{1 + \cos\alpha}{2}$

D. $\frac{1 - \cos\alpha}{2}$

Answer: A

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38. Let f be a continuous function satisfying $f'(Inx) = [1$ for $0 < x \leq 1, x$ for $x > 1$ and $f(0) = 0$ then $f(x)$ can be defined as

A. $f(x) = \begin{cases} 1 & \text{if} x \leq 0 \\ 1 - e^x & \text{if} x > 0 \end{cases}$

B. $f(x) = \begin{cases} 1 & \text{if} x \leq 0 \\ e^x - 1 & \text{if} x > 0 \end{cases}$

C. $f(x) = \begin{cases} x & \text{if} x < 0 \\ e^x & \text{if} x > 0 \end{cases}$

D. $f(x) = \begin{cases} x & \text{if} x \leq 0 \\ e^x - 1 & \text{if} x > 0 \end{cases}$

Answer: D

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39. The integral $\int_{\frac{\pi}{4}}^{5\frac{\pi}{4}} (|\cos t|\sin t + |\sin t|\cos t)$ has the value equal to

A. 0

B. $1/2$

C. $1/\sqrt{2}$

D. 1

Answer: A

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40. The value of $\int_0^2 f(x)dx$, where

$f(x) = \begin{cases} 0\text{,where}x = \frac{n}{n+1}\text{, n=1,2,3...} \\ 1\text{,elsewhere} \end{cases}$ 3 is equal to

A. 1

B. 2

C. 3

D. None of thses

Answer: B

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41. Evaluate $\int_{-1}^{1}(x^3+5x+\sin x)\,dx$

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42. Evaluate $\int_{-\pi/4}^{\pi/4} x^3 \sin^4 x\,dx$

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43. Evaluate $\int_{-\pi/2}^{\pi/4} \sin^2 x\,dx.$

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44. The value of $\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx$ is equal to

A. $\frac{1}{2}$

B. 1

C. -1

D. 0

Answer: D

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45. $\int_0^{\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x - \pi} dx$ is equal to

A. $\frac{8}{\pi}$

B. $\frac{\pi}{8}$

C. $\frac{8}{\pi^2}$

D. $\frac{\pi^2}{8}$

Answer: C

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46. Let $f(x) = \begin{vmatrix} \cos x & e^{x^2} & 2x\frac{\cos^2 x}{2} \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x + \tan x \end{vmatrix}$ then the value of

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^2+1)(f(x) + f''(x))dx$$

A. 1

B. -1

C. 2

D. None of these

Answer: D

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47. The value of $\int_{-1}^{1} \frac{x}{\sqrt{1-x^2}} . \sin^{-1}\left(2x\sqrt{1-x^2}\right)dx$ is equal to

A. $4\sqrt{2}$

B. $4\left(\sqrt{2}-1\right)$

C. $4(\sqrt{2}+1)$

D. `None of these

Answer: B

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48. Suppose the function $g_n(x) = x^{2n+1} + a_n x + b_n (N \in N)$ satisfes the equation $\int_{-1}^{1} (px+q) g_n(x) dx = 0$ for all linear functions $(px+q)$ then

A. $a_n = b_n = 0$

B. $b_n = 0, a_n = -\frac{3}{2n+3}$

C. $a_n = 0, b_n = -\frac{3}{2n+3}$

D. $a_n = \frac{3}{2n+3}, b_n = -\frac{3}{2n+3}$

Answer: B

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49. Evaluate $\int_0^{\pi} \frac{x}{1+\cos^2 x} dx.$

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50. Prove that $\int_0^{\pi/2} \log(\sin x) dx = \int_0^{\pi/2} \log(\cos x) dx = -\frac{\pi}{2} \log 2.$

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51. If $f(x) = -\int_0^x \log(\cos t) dt,$ then the value of $f(x) - 2f\left(\frac{\pi}{4} + \frac{x}{2}\right) + 2f\left(\frac{\pi}{4} - \frac{x}{2}\right)$ is equal to

A. $-x \log 2$

B. $\frac{x}{2} \log 2$

C. $\frac{x}{3} \log 2$

D. None of these

Answer: A

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52. If $\int_0^{\pi}\left(\frac{x}{1+\sin x}\right)^2 dx = A,$ then the value for $\int_0^{\pi}\frac{2x^2 . \cos^2 x/2}{(1+\sin x^2)}dx$ is equal to

A. $A + 2\pi - \pi^2$

B. $A - 2\pi + \pi^2$

C. $2\pi - A - \pi^2$

D. None of these

Answer: A

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53. The value of $\int_{-a}^{a}\left(\cos^{-1}x - \sin^{-1}\sqrt{1-x^2}\right)dx$ is $(a > 0)$ there $\left.\int_0^a \cos^{-1}x dx = A\right)$ is

A. $\pi a - A$

B. $\pi a + 2A$

C. $\pi a - 2A$

D. $\pi a + A$

Answer: C

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54. Evaluate $\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{1/3}^{2/3} e^{9\left(x - \frac{2}{3}\right)^2} dx$

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55. If $f(x) = \int_0^x \log\left(\frac{1-t}{1+t}\right) dt$, then discuss whether even or odd?

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56. Evaluate $\int_0^{4\pi} |\cos x| dx.$

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57. Prove that $\int_0^{25} e^{x - [x]} dx = 25(e - 1).$

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58. The value of $\int_0^{2n\pi} [\sin x \cos x] dx$ is equal to

A. $-n\pi$

B. $n\pi$

C. $-2n\pi$

D. None of these

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59. The value of $\int_{-5}^{5} f(x)dx,$ where

$f(x) = \text{minimum}(\{x+1\}, \{x-1\}), Aax \in R$ and $\{.\}$ denotes

fractional part of x, is equal to

A. 3

B. 4

C. 5

D. 6

Answer: C

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60. Show that $\int_{0}^{n\pi + v} |\sin x| dx = 2n + 1 - \cos v,$ where n is a positive

integer and `,lt=v

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61. The value of $\int_{-2\pi}^{5\pi} \cot^{-1}(\tan x)dx$ is equal to

A. $\left(\frac{7\pi^2}{2}\right)$

B. $-\left(7\pi^2\right)$

C. $\frac{3\pi}{2}$

D. π^2

Answer: B

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62. Let g(x) be a continuous and differentiable function such that $\int_0^2 \left\{\int_{\sqrt{2}}^{\frac{\sqrt{5}}{2}} \left[2x^2 - 3\right]dx\right\}. g(x)dx = 0,$ then g(x) = 0 when $x \in (0, 2)$ has (where [*] denote greatest integer function)

A. exactly one real root

B. atleat one real root

C. no real root None of these

D.

Answer: B

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63. The value of x satisfying $\int_0^{2[x+14]}\left\{\frac{x}{2}\right\}dx = \int_0^{\{x\}}[x+14]dx$ is equal to (where, $[.]$ and $\{.\}$ denotes the greates integer and fractional part of x)

A. $[-14, -13)$

B. $(0, 1)$

C. $(-15, -14]$

D. None of these

Answer: A

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64. Find the derivative of the following with respect to x.

(i) $\int_0^x \cos t dt$

(ii) $\int_0^{x^2} \cos^2 dt$

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65. Evaluate $\frac{d}{dx}\left(\int_{1/x}^{\sqrt{x}} \cos t^2 dt\right)$

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66. If $\frac{d}{dx}\left(\int_0^y e^{-t^2} dt + \int_0^{x^2} \sin^2 t dt\right) = 0, \text{ find} \frac{dy}{dx}.$

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67. The points of extremum of $\int\limits_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$ are

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68. Find $\frac{d}{dx}\left(\int_{x^2}^{x^3}\frac{1}{\log t}dt\right)$

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69. $Ify=\int_0^x f(t)\sin\left\{k(x-t)dt, thenprovethat\frac{dt^2y}{dx^2}+k^2y=kf(x)\cdot\right.$

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70. $If\int_{\frac{\pi}{3}}^{x}\sqrt{(3-\sin^2 t)}dt+\int_0^y \cos tdt=0, the\neq valuate\frac{dy}{dx}$

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71. Let $\frac{d}{dx}(F(x))=\frac{e^{\sin x}}{x}, x>0.$ If $\int_1^4 2\frac{e^{\sin(x^2)}}{x}dx=F(k)-F(1),$ then possible value of k is:

A. 10

B. 14

C. 16

D. 18

Answer: C

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72. The function $f(x) = \int_0^x \log_{|\sin t|}\left(\sin t + \frac{1}{2}\right) dt$, where $x \in (0, 2\pi)$, then $f(x)$ strictly increases in the interval

A. $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

B. $\left(\frac{5\pi}{6}, 2\pi\right)$

C. $\left(\frac{\pi}{6}, \frac{7\pi}{6}\right)$

D. $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

Answer: D

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73. $f:(0,\infty)\rightarrow R$ and $F(x)=\int_0^x tf(t)dt$

If $F(x^2)=x^4+x^5$, then$\Sigma_{r=1}^{12} f(r^2)$ is equal to S

A. 216

B. 219

C. 221

D. 223

Answer: B

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74. A function $f(x)$ satisfies $f(x)=\sin x+\int_0^x f'(t)(2\sin t-\sin^2 t)dt$

is

A. $\dfrac{x}{1-\sin x}$

B. $\dfrac{\sin x}{1-\sin x}$

C. $\dfrac{1-\cos x}{\cos x}$

D. $\dfrac{\tan x}{1-\sin x}$

Answer: B

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75. If $F(x)=\int_1^x f(t)dt$,where $f(t)=\int_1^{t^2}\frac{\sqrt{1+u^4}}{u}du$, then the value of F''(2) equals to

A. $\dfrac{7}{4\sqrt{17}}$

B. $\dfrac{15}{\sqrt{17}}$

C. $\sqrt{257}$

D. $\dfrac{15\sqrt{17}}{68}$

Answer: C

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76. Evaluate $I(b) = \int_0^1 (x^b)\,dx = \frac{x^b - 1}{\ln x}dx,\ b \geq 0.$

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77. Prove that $\int_0^{\pi/2} \frac{dx}{\left(a^2 \sin^2 x + b^2 \cos^2 x\right)^2} = \frac{\pi\left(a^2 + b^2\right)}{4a^3b^3}.$

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78. The value of

$\int_0^{\pi/2} \frac{\log\left(1 + x \sin^2 \theta\right)}{\sin^2 \theta} d\theta,\ x \geq 0$ is equal to

A. $\pi\left(\sqrt{1 + x - 1}\right)$

B. $\pi\left(\sqrt{1 + x - 2}\right)$

C. $\sqrt{\pi}\left(\sqrt{1 + x - 1}\right)$

D. None of these

Answer: A

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79. Let $f(x)$ be a continuous function for all x, which is not identically zero such that $\{f(x)\}^2 = \int_0^x f(t)\frac{2\sec^2 t}{4+\tan t}dt$ and $f(0)=\ln 4$, then

A. $f\left(\frac{\pi}{4}\right) = \frac{\log(5)}{4}$

B. $f\left(\frac{\pi}{4}\right) = \frac{3}{4}$

C. $f\left(\frac{\pi}{2}\right) = 2$

D. None of these

Answer: A

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80. Evaluate the following

(i) $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\frac{3}{n^2}....+\frac{n-1}{n^2}\right)$

(ii) $\lim_{n\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{2n}\right)$

(iii) $\lim_{n \to \infty} \left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \ldots + \frac{n}{2n^2} \right)$

(iv) $\lim_{n \to \infty} \frac{(1^p + 2^p + \ldots + n^p)}{n^{p+1}}, p > 0$

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81. Evaluate $S = \sum_{r=0}^{n-1} \frac{1}{\sqrt{4n^2 - r^2}} as n \to \infty.$

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82. Evaluate $\lim_{n \to \infty} \left(\frac{1}{2n+1} + \frac{1}{2n+2} + \ldots + \frac{1}{6n} \right).$

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83. Evaluate $\int_1^4 (ax^2 + bx + c)\,dx$.

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84. The value of $\lim_{n \to \infty} \left(\sin\frac{\pi}{2n}. \sin\frac{2\pi}{2n}. \sin\frac{3\pi}{2n}...\sin\frac{(n-1)\pi}{n}\right)^{1/n}$ is equal to

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. None of these

Answer: C

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85. The interval $[0, 4]$ isdivided into n equal sub-intervals by the points $x_0, x_1, x_2, \ldots, x_{n-1}, x_n \text{where} 0 = x_0 < x_1 < x_2 < x_3 < \ldots < x_n = 4$ If $\delta x = x_i - x_{i-1} \text{for} i = 1, 2, 3, \ldots, n, \text{ then } \lim_{\delta x \to 0} \sum_{i-1}^{n} x_i \delta x$ is equal to

A. 4

B. 8

C. $\frac{32}{3}$

D. 16

Answer: B

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86. The value of $\lim_{n \to \infty} \frac{1}{n}. \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ is equal to

A. $1 + \sqrt{5}$

B. $-1 + \sqrt{5}$

C. $-1 + \sqrt{2}$

D. $1 + \sqrt{2}$

Answer: B

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87. Estimate the absolute value of the integral $\int_{10}^{19} \frac{\sin x}{1+x^8} dx$

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88. The minimum odd value of 'a' $(a > 1)$ for which $\int_{10}^{19} \frac{\sin x}{1+x^a} dx < \frac{1}{9}$, is equal to

A. 1

B. 3

C. 5

D. 9

Answer: B

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89. Prove that $\int_0^1 \sqrt{(1+x)(1+x^3)} dx$ cannot exceed $\sqrt{\frac{15}{8}}$.

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90. If $f(x)$ is a continous function such that $f(x) \mid 0, \forall x \in [2, 10]$ and $\int_4^8 f(x) dx = 0$, then find f (6)

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91. Prove that `pi/6

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92. Prove that $4 \leq \int_1^3 \sqrt{3 + x^3} dx \leq 2\sqrt{30}.$

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93. Prove that 1 $\leq \int_0^1 e^{x^2} dx \leq e$

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94. Evaluate

(i) $\ulcorner 1$

(ii) $\ulcorner 2$

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95. Evaluate $\int_0^{\infty} e^{-x} x^3 dx.$

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96. Evaluate $\int_0^1 \left(\frac{\log 1}{x}\right)^{n-1} dx.$

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97. Evaluate $\int_0^1 x^6 \sqrt{1-x^2}$dx

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98. Evaluate $\int_0^{\pi/2} \sin^4 x. \cos^6 x dx.$

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99. The value of $\int_0^{\infty} e^{-a^2x^2} dx$ is equal to

A. $\frac{\sqrt{\pi}}{2a}$

B. $\frac{\pi}{2a}$

C. $\frac{\pi}{\sqrt{2a}}$

D. `None of these

Answer: A

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100. If $\int_0^{\infty} f(x)dx = \frac{\pi}{2}$ and $f(x)$ is an even function, then $\int_0^{\infty} f\left(x - \frac{1}{x}\right)dx$ is equal to

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. None of these

Answer: B

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101. The value of $\int_0^{1} (\pi_{r=1}^{n}(x+r))\left(\sum_{k=1}^{n} \frac{1}{x+k}\right)dx$

A. n

B. n!

C. $(n+1)!$

D. $n.\, n!$

Answer: D

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102. The true set values of 'a' for which the inequal- ity $\int_a^0 \left(3^{-2x} - 2.3^{-x}\right)dx \geq 0$ is true, is

A. $[0, 1]$

B. $(-\infty, -1]$

C. $[0, \infty)$

D. $(-\infty, -1] \cup [0, \infty)$

Answer: D

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103. The value of the definite integral $\int_0^{2n\pi} \max\left(\sin x, \sin^{-1}(\sin x)\right) dx$

equals to (where, n in I)

A. $\frac{n\left(\pi^2 - 4\right)}{2}$

B. $\frac{n\left(\pi^2 - 4\right)}{4}$

C. $\frac{n\left(\pi^2 - 8\right)}{4}$

D. $\frac{n\left(\pi^2 - 2\right)}{4}$

Answer: C

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104. $\int \frac{\sqrt{a^2 - x^2}}{x^2} dx$

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105. $\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t) dt}{x^2 - \frac{\pi^2}{16}}$ is equal to

A. $\frac{8}{\pi}f(2)$

B. $\frac{2}{\pi}f(2)$

C. $\frac{2}{\pi}f\left(\frac{1}{2}\right)$

D. $4f(2)$

Answer: A

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106. Let f be a non-negative function defined on the interval .[0,1].If $\int_0^x \sqrt{1-(f'(t))^2}.\,dt$=$\int_0^x f(t).\,dt, 0 \leq x \leq 1$ and f(0)=0,then

A. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

B. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

C. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

D. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Answer: C

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107. The value of $\lim_{x \to 0} \frac{1}{x^3} \int_0^x \frac{t \ln(1+t)}{t^4+4} dt$

A. 0

B. $\frac{1}{12}$

C. $\frac{1}{24}$

D. $\frac{1}{64}$

Answer: B

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108. The value(s) of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$ is (are)

A. $\frac{22}{7} - \pi$

B. $\frac{2}{105}$

C. 0

D. $\dfrac{71}{15} - \dfrac{3\pi}{2}$

Answer: A

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109. $\int_{\sin\theta}^{\cos\theta} f(x\tan\theta)dx$ (where $\theta \neq \frac{n\pi}{2}, n\varepsilon I$) is equal to

A. $-\cos\theta\int_1^{\tan\theta} f(x\sin\theta)dx$

B. $-\tan\theta\int_{\cos\theta}^{\sin\theta} f(x)dx$

C. $-\tan\theta\int_1^{\sin\theta} f(x\cos\theta)dx$

D. $\dfrac{1}{\tan\theta}\int_{\sin\theta}^{\sin\theta\tan\theta} f(x)dx$

Answer: A

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110. Evaluate $\int \frac{1}{x\sqrt{1+x^3}}dx$

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111. Let $A_1 = \int_n^{n+1} (\min\{|x-n|, |x-(n+1)|\})dx$,
$A_2 = \int_{n+1}^{n+2} (|x-n| - |x-(n+1)|)dx$,
$A_3 = \int_{n+2}^{n+3} (|x-(n+4)| - |x-(n+3)|)dx$ and
$g(x) = A_1 + A_2 + A_3$, then

A. $A_1 + A_2 + A_3 = 9$

B. $A_1 + A_2 + A_3 = \frac{9}{4}$

C. $\sum_{n=1}^{100} g(x) = \frac{900}{4}$

D. $\sum_{n=1}^{100} g(x) = 300$

Answer: B:: C

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112. Statement I $If f(x) = \int_0^1 (xf(t)+1)dt,$ then $\int_0^3 f(x)dx = 12$

Statement II $f(x) = 3x + 1$

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: c

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113. Statement I The function $f(x) = \int_0^x \sqrt{1+t^2dt}$ is an odd function and $g(x) = f'(x)$ is an even function , then $f(x)$ is an odd function.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: A

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114. Given, $f(x) = \sin^3 x$ and $P(x)$ is a quadratic polynomial with coefficient unity.

Statement I $\int_0^{2\pi} P(x).\, f''(x)dx$ vanishes.

Statement II $\int_0^{2\pi} f(x)dx$ vanishes.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement 1.

B. Statement I is true, Statement II is also true , Statement II is not the correct explanation of Statement II.

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: a

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115. Data could not be retrieved.

A. $\frac{\pi}{8}(1+\sqrt{2})$

B. $\frac{\pi}{4}(1+\sqrt{2})$

C. $\frac{\pi}{8\sqrt{2}}$

D. $\frac{\pi}{4\sqrt{2}}$

Answer: A

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116. Suppose we define integral using the following formula $\int_a^b f(x)dx = \frac{b-a}{2}(f(a)+f(b))$, for more accurate result for $c \in (a, b), F(c) = \frac{c-a}{2} f(a) + f(c)\Big) + \frac{b-c}{2}(f(b)+f(c))$.

When $c = \frac{a+b}{2}$, then $\int_a^b f(x)dx = \frac{b-a}{4}(f(a)+f(b)+2f(c))$.

$\lim_{t \to a} \frac{\int_a^t f(x)dx - \frac{(t-a)}{2}(f(t)+f(a))}{(t-a)^3} = 0$ for all a, then the degree of $f(x)$ can atmost be

A. 1

B. 2

C. 3

D. 4

Answer: A

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117. Data could not be retrieved.

A. $\dfrac{f(b)-f(a)}{a-b}$

B. $\dfrac{2(f(b)-f(a))}{b-a}$

C. $\dfrac{2f(b)-f(a)}{2b-a}$

D. 0

Answer: A

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118. Let $f(\alpha, \beta) = = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$

The value of $l = \int_0^{\pi/2} e^{\beta}\left(f(0,0)+f\left(\frac{\pi}{2},\beta\right)+f\left(\frac{3\pi}{2},\frac{\pi}{2}-\beta\right)\right)d\beta$ is

A. $e^{\pi/2}$

B. 0

C. $2\left(2e^{\pi/2}-2\right)$

D. None of these

Answer: C

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119. Let $f(\alpha, \beta) = = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$

$Ifl = \int_{-\pi/2}^{\pi/2} \cos^2\beta\left(f(0,\beta) + f\left(0, \frac{\pi}{2} - \beta\right)\right)d\beta$ then [I] is

A. $e^{\pi/2}$

B. 3

C. $2\left(2e^{\pi/2} - 1\right)$

D. `None of these

Answer: B

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120. Match the following

Column I	Column II
(A) The function $f(x)=\dfrac{e^{x\cos x}-1-x}{\sin x^2}$ is not defined at $x=0$. The value of $f(0)$, so that f is continuous at $x=0$, is	(p) -1
(B) The value of the definite integral $\int_0^1 \dfrac{dx}{\sqrt{x}+\sqrt[3]{x}}$ is equal to $a+b\ln 2$, where a and b are integers, then $(a+b)$ is equal to	(q) 0
(C) Given, $e^n\int_0^n \dfrac{\sec^2\theta-\tan\theta}{e^\theta}\,d\theta=1$, then the value of $\tan n$ is equal to	(r) $1/2$
(D) Let $a_n=\int_{\frac{1}{n+1}}^{\frac{1}{n}} \tan^{-1}(nx)\,dx$ and $b_n=\int_{\frac{1}{n+1}}^{\frac{1}{n}} \sin^{-1}(nx)\,dx$, then $\lim\limits_{n\to\infty}\dfrac{a_n}{b_n}$ has the value equal to	(s) 1

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121. Match the following

	Column I		Column II
(A)	$\int_{-1}^{1}\frac{dx}{1+x^2}$	(p)	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B)	$\int_{0}^{1}\frac{dx}{\sqrt{1-x^2}}$	(q)	$\frac{1}{2}\log\left(\frac{3}{2}\right)$
(C)	$\int_{2}^{3}\frac{dx}{1-x^2}$	(r)	$\frac{\pi}{3}$
(D)	$\int_{1}^{2}\frac{dx}{x\sqrt{x^2-1}}$	(s)	$\frac{\pi}{2}$

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122. Let $f: R \to R$ be a continous function which satisfies $f(x) = \int_0^x f(t)dt$. Then , the value of $f(\ln 5)$ is _____.

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123. $f(x) = \begin{cases} x - [x], & \text{if } [x] \text{is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}$

Then, the value of $\frac{\pi^2}{10} \int_{-10}^{10} f(x) \cos \pi \times dx$ is

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124. Let $f(x)$ be a differentiable function satisfying $f(x) + f\left(x + \frac{1}{2}\right) = f(x), \forall x \in R$ and $g(x) = \int_0^x f(t) dt$. If g(1)=1, then the value of $\sum_{n=2}^{\infty} \left(\frac{8}{\sum_{k=1}^{n} (g(x + k^2) - g(x + k))} \right)$ is

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125. Find the error in steps to evaluate the following integral $\int_0^{\pi} \frac{dx}{1 + 2\sin^2 x} = \int_0^{\pi} \frac{\sec^2 x dx}{\sec^2 x + 2\tan^2 x} = \int_0^{\pi} \frac{\sec^2 x dx}{1 + 3\tan^2 x}$ $= \frac{1}{\sqrt{3}} \left[\tan^{-1}(\sqrt{3} \tan x)\right]_0^{\pi} = 0$

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126. If $\int_a^b |\sin x| dx = 8$ and $\int_0^{a+b} |\cos x| dx = 9$, then find the value of $\int_a^b x \sin x dx$.

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127. Evaluate $\int_{\cos(\cos^{-1}\alpha)}^{\sin(\sin^{-1}\beta)} \left|\left(\frac{\cos(\cos^{-1}x)}{\sin(\sin^{-1}x)}\right)\right| dx$

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128. $\int_\alpha^\beta \sqrt{\frac{x-\alpha}{\beta-x}}$ dx is equal to

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129. Evaluate: $\int_{\sqrt{\frac{3a^2+b^2}{2}}}^{\sqrt{\frac{a^2+b^2}{2}}} \frac{x \cdot dx}{\sqrt{(x^2-a^2)(b^2-x^2)}}$

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130. Evaluate $\int_0^{\pi/4} \frac{e^{\sec x}\left[\sin\left(x + \frac{\pi}{4}\right)\right]}{\cos x(1 - \sin x)} dx$

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131. $\int_0^{\pi/4} \frac{x^2(\sin 2x - \cos 2x)}{(1 + \sin 2x)\cos^2 x} dx$

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132. Evaluate $\int_0^{\pi} x^2\left\{(1 + \sin x)^{-2} \cos x\right\} dx$

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133. Compute the following integrals.

(i) $\int_0^{\infty} f\left(x^n + x^{-n}\right)\ln \times \frac{dx}{x} = 0$

(ii) $\int_0^{\infty} f\left(x^n + x^{-n}\right)\ln \times \frac{dx}{1 + \times^2 = 0}$

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134. Show that

(a) $\int_0^{\infty} \sin x dx = 1$

(b) $\int_0^{\infty} \cos x dx = 0$

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135. Find a function $g: R \to R$, continuous in $[0, \infty)$ and positive in $(0, \infty)$ satisfying $g(1) = 1$ and $\frac{1}{2}\int_0^x g^2(t)dt = \frac{1}{x}\left(\int_0^x g(t)dt\right)^2$

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136. let $I_n = \int_0^{\frac{\pi}{4}} \tan^n x dx, n > 1$

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137. If $U_n = \int_0^{\pi} \frac{1 - \cos nx}{1 - \cos x} dx$, where n is positive integer or zero, then show that $U_{n+2} + U_n = 2U_{n+1}$. Hence, deduce that

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 n\theta}{\sin^2 \theta} = \frac{1}{2} n\pi.$$

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138. Prove that for any positive integer K, $\frac{\sin 2kx}{\sin x} = 2[\cos x + \cos 3x + ... + \cos(2k-1)x]$ Hence , proved that $\int_0^{\pi/2} \sin 2kx. \cot x dx = (\pi/2)$

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139. Evaluate: $\int_0^{\sqrt{3}} \frac{1}{1+x^2} \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

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140. Prove that $\int_0^x e^{xt}. e^{-t^2} dt = e^{x^2/4} \int_0^x e^{-t^2/4} dt.$

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141. $If f(x) = e^x + \int_0^1 (e^x + te^{-x}) f(t) dt, \text{find} f(x).$

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142. $If|a| < 1,$ show that $\int_0^{\pi} \frac{\log(1 + a\cos x)}{\cos x} dx = \pi \sin^{-1} a$

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143. Evaluate $\int_0^{\pi/2} \cos ec\theta \tan^{-1}(c\sin\theta) d\theta.$

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144. Evaluate $\int_0^{\pi/2} \cos ec\theta \tan^{-1}(c\sin\theta) d\theta.$

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145. Let f be a continous function on [a,b] . Prove that there exists a number $x \in [a, b]$ such that $\int_a^x f(t)dt = \int_x^b f(t)dt.$

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146. $Iff(x) = x + \int_0^1 (xy^2 + x^2y)(f(y))dy, \text{find} f(x)$ if x and y are independent.

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147. $\int_0^x \left\{\int_0^u f(t)dx\right\} duisequa < o$ $\int_0^x (x-u)f(u)du$ $\int_0^x uf(x-u)du$ $\xi nt_0^x f(u)du$ (d) $\xi nt_0^x uf(u-x)du$

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148. Evaluate: $\int_0^{\frac{3\pi}{2}} (\ln|\sin x|)\cos(2nx)dx, n \in N$

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149. Evaluate $\int_0^\infty e^{-x}\sin^n x dx, \quad \text{if} \quad n$ is an even integer.

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150. Evaluate $\int_0^1 (tx + 1 - x)^n dx$, where n is a positive integer and t is a parameter independent of x . Hence , show that

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151. Given a real valued fuction f(x) which is monotonic and differentiable , prove that for any real number a and b, $\int_a^b \{f^2(x) - f^2(a)\}dx = \int_{f(a)}^{f(b)} 2x\{b - f^{-1}(x)\}dx$

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152. $\int_0^1 \frac{\sin\theta\left(\cos^2\theta - \cos^2\pi/5\right)\left(\cos^2\theta^2 2\pi/5\right)}{\sin 5\theta} d\theta$

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153. Show that $\lim_{n \to \infty} \sum_{k=0}^{n} \frac{{}^{n}Ck}{n^k(k+3)} = e - 2$

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154. Let I= $\int_0^{\pi/2} \frac{\cos x}{a\cos x + b\sin x} dx$ and $J = \int_0^{\pi/2} \frac{\sin x}{a\cos x + b\sin x} dx$,

where a > 0 and $b > 0$ Compute the values of I and j

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155. Find a function f, continous for all x(and not zero everywhere) such that $f^2(x) = \int_0^x \frac{f(t)\sin t}{2 + \cos t} dt$

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156. Evaluate $\int_0^{\infty} \frac{\tan^{-1} ax - \tan^{-1} x}{x} dx$ where a is a parameter.

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157. Evaluate $\int_0^{\infty} \frac{\tan^{-1} ax - \tan^{-1} x}{x} dx$, where a is a parametar.

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158. Evaluate $\int_0^{\pi/2} \ln\left(\frac{1 + a\sin x}{1 - a\sin x}\right) \frac{dx}{\sin x} (|a| < 1)$

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Exercise For Session 1

1. $\int_0^{\pi/4} \cos^2 x dx$

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2. $\int_0^{\pi/2} \frac{dx}{1+\cos x}$

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3. $\int_0^{\pi/2} \sqrt{1+\cos x dx}$

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4. $\int_0^{\frac{\pi}{6}} \sin 2x . \cos x dx$

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5. $\int_1^2 \frac{dx}{\sqrt{x}-\sqrt{x-1}}$

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6. $\int_0^1 x dx$

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7. $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x}$

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8. $\int_a^b \frac{1}{\sqrt{(x-a)(b-x)dx, b > a}}$

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9. $\int_a^b \sqrt{\frac{x-a}{b-x}} dx$

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10. $\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\, dx$

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11. $\int_0^{\pi} \cos 2x . \log(\sin x) dx$

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12. $\int_0^{\pi/4} e^{\sin x}\left(\frac{(x\cos^3 x - \sin x)}{\cos^2 x}\right) dx$

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13. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right) + x^2 f(x) = 0$ for all nonzero x, then evaluate $\int_{\sin\theta}^{\cos ec\theta} f(x) dx$

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14. The value of $\int_0^1 \left(\pi_{r=1}^{n}(x+r)\right)\left(\sum_{k=1}^{n}\frac{1}{x+k}\right)dx$

A. n

B. $n!$

C. $(n+1)!$

D. $n.\ n!$

Answer: D

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15. The true set of values of 'a' for which the inequality $\int_x^0 \left(3^{-2x} - 2.3^{-x}\right)dx \geq 0$ is true , is

A. $[0, 1]$

B. $[-\infty, -1]$

C. $[0, \infty]$

D. $[-\infty, -1] \cup [1, \infty]$

Answer: D

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Exercise For Session 2

1. The value of $\int_0^{\pi/4} \log(1 + \tan\theta) d\theta$ is equal to

A. $\frac{\pi}{2} \log 2$

B. $-\frac{\pi}{4} \log 2$

C. $\frac{\pi}{8} \log 2$

D. None of these

Answer: C

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2. For any integer n, the integral $\int_0^{\pi} e^{\cos x} \cos^3(2n+1)x dx$ has the value

A. 0

B. 1

C. -1

D. None of these

Answer: A

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3. The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}} dx$ is equal

A. $1/2$

B. $1/3$

C. $1/4$

D. None of these

Answer: A

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4. Evaluate: $\int_0^2 \frac{dx}{(17+8x-4x^2)\left[e^{6(1-x)}+1\right)}$

A. $-\frac{1}{8\sqrt{21}}\log\left|\frac{2-\sqrt{21}}{2+\sqrt{21}}\right|$

B. $-\frac{1}{8\sqrt{21}}\log\left|\frac{2-\sqrt{21}}{\sqrt{21}-2}\right|$

C. $-\frac{1}{8\sqrt{21}}\left\{\log\left|\frac{2-\sqrt{21}}{2+\sqrt{21}}\right|-\log\left|\frac{2+\sqrt{21}}{\sqrt{21}-2}\right|\right\}$

D. None of these

Answer: C

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5. If f is an odd function, show that: $\int_{-a}^{a}\frac{f(\sin x)}{f(\cos x)+f\left(\sin^2 x\right)}dx=0$

A. 0

B. $f(\cos x) + f(\sin x)$

C. 1

D. None of these

Answer: A

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6. If [x] stands for the greatest integer function, the value of $\int_4^{10} \frac{[x^2]}{[x^2 - 28x + 196] + [x^2]} dx$, is

A. 1

B. 2

C. 3

D. 4

Answer: C

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7. $\int_0^{\pi} \frac{x dx}{1 + \cos \alpha \cdot \sin x} = \frac{\pi \alpha}{\sin \alpha}, 0 < \alpha < \pi$

A. $\frac{\pi}{\sin \alpha}$

B. $\frac{\pi \alpha}{\sin \alpha}$

C. $\frac{\alpha}{\sin \alpha}$

D. $\frac{\sin \alpha}{\alpha}$

Answer: B

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8. $f, g, h,$ are continuous in $[0, a], f(a - x) = f(x), g(a - x) = -g(x), 3h(x) - 4h(a - x) = 5.$

Then prove that $\int_0^a f(x) g(x) h(x) dx = 0$

A. 0

B. 1

C. a

D. 2a

Answer: A

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9. If $2f(x)+f(-x)=\frac{1}{x}\sin\left(x-\frac{1}{x}\right)$ then the value of $\int_{\frac{1}{e}}^{e} f(x)dx$ is

A. 0

B. e

C. $1/e$

D. $e+1/e$

Answer: A

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10. Show that $\int_0^\pi fx(\sin x)dx = \frac{\pi}{2}\int_0^\pi f(\sin x)dx.$

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11. Evaluate : $\int_0^\pi \frac{x^2 \sin 2x \cdot \sin\left(\frac{\pi}{2} \cdot \cos x\right)}{2x - \pi}dx$

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12. The numbers of possible continuous $f(x)$ defined in $[0, 1]$ for which $I_1 = \int_0^1 f(x)dx = 1, I_2 = \int_0^1 xf(x)dx - a, I_3 = \int_0^1 x^2 f(x)dx = a^2 is / a$

1 (b) ∞ (c) 2 (d) 0

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13. Let $l_1 = \int_0^1 \frac{e^x}{1+x}dx$ and $l_2 = \int_0^1 \frac{x^2}{e^{x^3}(2-x^3)}dx$. Then $\frac{l_1}{l_2}$ is equal to

A. $\frac{3}{e}$

B. $\frac{3}{e}$

C. 3e

D. $\frac{1}{3e}$

Answer: C

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14. If f(x) $= \frac{e^2}{1+e^x}, I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, where g is not identify function. Then the value of I_2 / I_1, is

A. 1

B. -3

C. -1

D. 2

Answer: D

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Exercise For Session 3

1. The value of $\int_{-1}^{3}\{|x-2|+[x]\}dx$, where [.] denotes the greatest integer function, is equal to

A. 5

B. 6

C. 3

D. None of these

Answer: D

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2. The value of $\int_{-1}^{3}(|x|+|x-1|)dx$ is equal to

A. 9

B. 6

C. 3

D. None of these

Answer: A

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3. Let $f(x)=x-[x]$, for every real number x, where [x] is integral part of x. Then $\int_{-1}^{1}f(x)dx$ is

A. 0

B. 1

C. 2

D. None of tjhese

Answer: B

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4. The value of $\int_0^2 [x + [x + [x]]]dx$ (where, [.] denotes the greatest integer function)is equal to

A. 2

B. 3

C. -3

D. None of these

Answer: B

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5. The value of $\int_0^{[x]} \frac{2^x}{2^{[x]}} dx$ is equal to (where, [.] denotes the greatest integer function)

A. $\frac{[x]}{\log 2}$

B. $\frac{[x]}{2\log 2}$

C. $\frac{[x]}{4\log 2}$

D. None of these

Answer: A

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6. The value of $\int_0^4 \{x\} dx$ (where , {.} denotes fractional part of x) is equal to

A. $\frac{4}{3}$

B. $\frac{5}{3}$

C. $\frac{7}{3}$

D. None of these

Answer: D

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7. The value of $\int_1^4 \{x\}^{[x]} dx$ (where , [.] and {.} denotes the greatest integer and fractional part of x) is equal to

A. $\frac{11}{12}$

B. $\frac{13}{12}$

C. $\left(\frac{70}{12}\right.$

D. $\frac{19}{12}$

Answer: B

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8. The value of $\int_0^x [t+1]^3 dt$ (where, [.] denotes the greatest integer function of x) is qeual to

A. $\left(\frac{[x]([x]+1)}{2}\right)^2 + ([x]+1)^3\{x\}$

B. $\left(\frac{[x]([x]+1)}{2}\right)^3 + ([x]+1)^3\{x\}$

C. $\left(\frac{[x]([x]+1)}{2}\right)^3 + ([x]+1)^2\{x\}$

D. None of these

Answer: D

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9. The value of $\int_0^{10\pi} \left[\tan^{-1} x\right] dx$ (where, [.] denotes the greatest integer functionof x) is equal to

A. $\tan 1$

B. 10π

C. $10\pi - \tan 1$

D. None of these

Answer: C

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10. If $f(x) = \min\{|x-1|, |x|, |x+1|,$ then the value of $\int_{-1}^{1} f(x)dx$ is equal to

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Answer: B

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11. The value of $\int_0^{\infty} \left[2e^{-x}\right] dx$ (where ,[.] denotes the greatest integer function of x) is equal to S

A. 1

B. $\log_e 2$

C. 0

D. $\frac{1}{e}$

Answer: B

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12. The value of $\int_1^{10\pi} \left(\left[\sec^{-1} x\right]\right) dx$ (where ,[.] denotes the greatest integer function) is equal to

A. $(\sec 1) - 10\pi$

B. $10\pi - \sec 1$

C. $\pi - \sec 1$

D. None of these

Answer: B

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13. The value of $\int_{-\pi/2}^{\pi/2}[\cot^{-1}x]dx$ (where ,[.] denotes greatest integer function) is equal to

A. $\pi + \cot 1$

B. $\pi + \cot 2$

C. $\pi + \cot 1 + \cot 2$

D. $\cot 1 + \cot 2$

Answer: C

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14. The value of $\int_0^{\frac{\pi}{4}} \left(\tan^n(x-[x]) + \tan^{n-2}(x-[x])\right) dx$ (where, [*]

denote(d) cot 1+ cot2X-X)))dx (where, - denotes greatest integer function)

is equal to

A. $\frac{1}{n}$

B. $\frac{1}{n-1}$

C. $\frac{1}{n(n-1)}$

D. $\frac{1}{n(+1)}$

Answer: B

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15. The value of $\int_0^2 \left[x^2 - x + 1\right] dx$ (where , [.] denotes the greatest integer function) is equal to

A. $\frac{5+\sqrt{5}}{2}$

B. $\frac{1+\sqrt{5}}{2}$

C. $\dfrac{1-\sqrt{5}}{2}$

D. $\dfrac{5-\sqrt{5}}{2}$

Answer: D

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16. Evaluate $\int_0^a [x^n]dx$, (where,[*] denotes the greatest integer function).

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17. Prove that $\int_0^x [x]dx = [x]\dfrac{[x]-1}{2} + [x](x-[x])$, where [.] denotes the greatest integer function

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18. If $f(n) = \frac{\int_0^n [x] dx}{\int_0^n \{x\} dx}$(where,[*] and {*} denotes greatest integer and tractional part of x and $n \in N)$. Then, the value of $f(4)$ is...

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19. If $f(n) = \int_0^x [\cos t] dt,$ where $x \in \left(2n\pi, 2n\pi + \frac{\pi}{2}\right); n \in N$ and $[\cdot]$ denotes the greatest integer function. Then, the value of $\left|f\left(\frac{1}{\pi}\right)\right|$ is ...

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20. If $\int_0^x [x] dx = \int_0^{[x]} x dx, x \notin$ integer (where, [*] and {*} denotes the greatest integer and fractional parts respectively,then the value of $4\{x\}$ is equal to ...

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Exercise For Session 4

1. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be continuous functions. Then the value of the integral $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) - g(-x)]dx$ is

A. -1

B. 0

C. 1

D. None of these

Answer: B

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2. The value of $\int_{-1}^{1} (x|x|)dx$ is equal to

A. 1

B. $\frac{1}{2}$

C. 0

D. None of these

Answer: C

Watch Video Solution

3. The value of $\int_{-1}^{1}\left(\frac{x^2+\sin x}{1+x^2}\right)dx$ is equal to

A. 2π

B. $\pi-2$

C. $2-\frac{\pi}{2}$

D. None of these

Answer: C

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4. If $f(x)$ is an odd function, then the value of $\int_{-a}^{a}\left(\frac{f(\sin x)}{f(\cos x)+f(\sin^2 x)}\right)dx$ is equal to

A. 0

B. $f(\cos x)+f(\sin)$

C. 1

D. None of these

Answer: A

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5. Evaluate: $\int_{-\left(\frac{1}{\sqrt{3}}\right)}^{\frac{1}{\sqrt{3}}}\frac{\cos^{-1}\left(\frac{2x}{1+x^2}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x+1}dx$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{\sqrt{3}}$

C. $\frac{\pi}{2\sqrt{3}}$

D. $\frac{\pi}{3\sqrt{3}}$

Answer: C

Watch Video Solution

6. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x dx}$ where agt 0, is

A. π

B. $a\pi$

C. 2π

D. $\frac{\pi}{2}$

Answer: A

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7. The integral $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left([x]+1n\left(\frac{1+x}{1-x}\right)\right)dx$ is equal to (where [.] represents the greatest integer function) $-\frac{1}{2}$ (b) 0 (c) 1 (c) $21n\left(\frac{1}{2}\right)$

A. $\frac{-1}{2}$

B. 0

C. 1

D. $2\frac{\log(1)}{2}$

Answer: A

Watch Video Solution

8. The value of $\int_{-\pi/2}^{\pi/2}\frac{1}{e^{\sin x}+1}dx$ is equal to

A. 0

B. 1

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{2}$

Answer: C

Watch Video Solution

9. If [*] denots the greatest integer function then the value of the integeral $\int_{-\pi/2}^{\pi/2}\left(\left[\frac{x}{\pi}\right]+0.5\right)dx$, is

A. π

B. $\frac{\pi}{2}$

C. 0

D. $-\frac{\pi}{2}$

Answer: C

Watch Video Solution

10. The equation $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left\{a|\sin x|+\frac{b\sin x}{1+\cos^2 x}+c\right\}dx=0$ where a, b, c are constants gives a relation between

A. a, b and c

B. a and c

C. a and b

D. b and c

Answer: B

Watch Video Solution

11. The value of $\int_{-2}^{2}\frac{\sin^2 x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}dx$ where [.] denotes greatest integer function , is

A. 1

B. 0

C. $4\sin 4$

D. None of these

Answer: B

Watch Video Solution

12. Let $f(x)$ be a contiuous function such a $\int_{n}^{n+1} f(x)dx = n^3, n \in Z$. Then, the value of the intergral $\int_{-3}^{3} f(x)dx$ (A) 9 (B) -27 (C) -9 (D) none of these

A. 9

B. -27

C. -9

D. 27

Answer: B

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13. Let $f(x)=\frac{e^x+1}{e^x-1}$ and $\int_0^1 x^3.\ \frac{e^x+1}{e^x-1}dx=\alpha \text{Then, } \int_{-1}^{1} t^3 f(t)dt$ is equal to

A. 0

B. α

C. 2α

D. None of these

Answer: C

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14. Let $f: R \to R$be a continuous function given by $f(x+y)=f(x)+f(y)$ for all $x, y, \in R$, if $\int_0^2 f(x)dx=\alpha$, then $\int_{-2}^{2} f(x)dx$ is equal to

A. 2α

B. α

C. 0

D. None of these

Answer: C

Watch Video Solution

15. The value of $\int_{-2}^{2} |[x]| dx$ is equal to

A. 1

B. 2

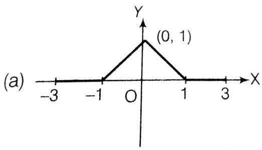
C. 3

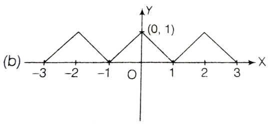
D. 4

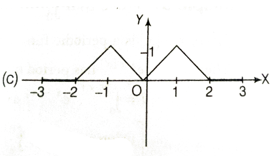
Answer: D

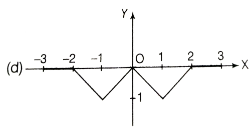
Watch Video Solution

16. Let $f(x) = \begin{cases} 1-|x|, |x| \leq 1 \\ 0, |x| > 1 \end{cases}$ and $g(x) = f(x-1)$ for all $x \in R$ The graph for g(x) is given by

A. 

B. 

C. 

D. 

Answer: C

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17. Let

$f(x) = \{1-|x|, |x| \leq 1$ and $0, |x| < 1$ and $g(x) = f(x-) + f(x+1)$

, for all $x \in R$.Then,the value of $\int_{-3}^{3} g(x)dx$ is

A. 2

B. 3

C. 4

D. 5

Answer: A

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18. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots\ldots$ then which one of the following is not true ?

A. $n\pi$

B. π

C. $-\pi$

D. 0

Answer: D

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19. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots,$ then (A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

A. 0

B. 5π

C. 10π

D. 0

Answer: C

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20. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx, n = 0, 1, 2, \ldots,$ then (A) $I_n = I_{n+2}$ (B) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (C) $\sum_{m=1}^{10} I_{2m} = 0$ (D) $I_n = I_{n+1}$

A. 0

B. 5π

C. 10π

D. None of these

Answer: A

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Exercise For Session 5

1. The value of $\int_{-1}^{10} sgn(x - [x]) dx$ is equal to (where, [:] denotes the greatest integer function

A. 9

B. 10

C. 11

D. 12

Answer: C

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2. the value of $\int_0^{[x]} dx$ (where , [.] denotes the greatest integer function)

A. [x]

B. $\frac{[x]}{2}$

C. $x[x]$

D. None of these

Answer: A

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3. Evaluate: $\int_{-\frac{\pi}{4}}^{n\pi-\frac{\pi}{4}} |\sin x+\cos x| dx$

A. $2\sqrt{2n}$

B. $\sqrt{2}n$

C. $\frac{1}{2\sqrt{2}}n$

D. None of these

Answer: A

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4. Let $f(x)=x-[x]$, for every real number x, where $[x]$ is the greatest integer less than or equal to x. Then, evaluate $\int_{-1}^{1} f(x)dx$.

A. 1

B. 2

C. 0

D. $\frac{1}{2}$

Answer: A

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5. $f(x) = \int_0^x f(t)dt = x + \int_x^1 tf(t)dt, thenthevalueofff(1)is \frac{1}{2}$ (b) 0 (c) 1 (d) $-\frac{1}{2}$

A. $\frac{1}{2}$

B. 0

C. 1

D. $-\frac{1}{2}$

Answer: A

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6. The least value of the function

$$\phi(x) = \int_{5\pi/4}^{x} (3\sin t + 4\cos t)dt$$

on the integral $[5\pi/4, 4\pi/3]$, is

A. $\sqrt{3} + \frac{3}{2}$

B. $1 - 2\sqrt{3} + \frac{3}{2} + \frac{1}{\sqrt{2}}$

C. $\frac{3}{2} + \frac{1}{\sqrt{2}}$

D. None of these

Answer: B

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7. The poins of extremum of $\phi(x) = \int_{1}^{x} e^{-t^2/2}(1 - t^2)dt$ are

A. $x = 1, -1$

B. $x = -1, 2$

C. $x = 2, 1$

D. $x = -2, 1$

Answer: A

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8. If f(x) is periodic function with period, T, then

A. $\int_a^b f(x)dx = \int_a^{b+T} f(x)dx$

B. $\int_a^b f(x) = \int_{a+T}^{b+T} f(x)dx$

C. $\int_a^b f(x)dx = \int_{a+T}^{b} f(x)dx$

D. $\int_a^b f(x)dx = \int_{a+T}^{b+2T} f(x)dx$

Answer: A

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9. Let $\frac{d}{dx}(F(x)) = \frac{e^{\sin x}}{x}, x > 0$. If $\int_1^4 2\frac{e^{\sin(x^2)}}{x}dx = F(k) - F(1)$, then possible value of k is:

A. 4

B. 8

C. 16

D. 32

Answer: C

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10. Let $f:(0, \infty) \to R$ and $F(x) = \int_0^x f(t)dt$. If $F(x^2) = x^2(1+x)$, then f(4) equals

A. $\frac{5}{4}$

B. 7

C. 4

D. 2

Answer: C

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11. Let $T > 0$ be a fixed real number. Suppose f is continuous function such that for all $x \in R$, $f(x+T) = f(x)\cdot$ If $I = \int_0^T f(x)dx$, then the value of $\int_3^{3+3T} f(2x)dx$ is $\frac{3}{2}I$ (b) $2I$ (c) $3I$ (d) $6I$

A. $\frac{3}{2}l$

B. $2l$

C. $3l$

D. $6l$

Answer: C

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12. Let $f(x) = \int_1^x \sqrt{2-t^2}dt$. Then the real roots of the equation , $x^2 - f'(x) = 0$ are: ± 1 b. $\pm \frac{1}{\sqrt{2}}$ c.$\pm \frac{1}{2}$ d. $0 \& 1$

A. ± 1

B. $\pm \frac{1}{\sqrt{2}}$

C. $\pm \frac{1}{2}$

D. $\pm \sqrt{2}$

Answer: A

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13. Let f(x) be an odd continuous function which is periodic with period 2.

if $g(x) = \int_0^x f(t)dt$, then

A. $g(x)$ is an odd function

B. $g(n) = 0 \text{for all} n \in N$

C. $g(2n) = 0 \text{for all} n \in N$

D. $g(x)$ is non-peridic

Answer: C

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14. Let f(x) be a function defined by $f(x) = \int_1^x t(t^2 - 3t + 2)dt, 1 \le x \le 3$ Then the range of f(x) is

A. $[0, 2]$

B. $\left[-\frac{1}{4}, 4\right]$

C. $\left[-\frac{1}{4}, 2\right]$

D. None of these

Answer: C

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15. The value of $\lim_{x \to 0} \dfrac{2\int_0^{\cos x} \cos^{-1}(t)}{2x - \sin 2x} dx$ is

A. 0

B. $\dfrac{1}{2}$

C. $-\dfrac{1}{2}$

D. $\dfrac{2}{3}$

Answer: C

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16. If $\int_0^x \dfrac{bt\cos 4t - a\sin 4t}{t^2} dt = \dfrac{a\sin 4x}{x}$ for all $x \neq 0$, then a and b are given by

A. $a = \dfrac{1}{4}, b = 1$

B. $a = 2, b = 2$

C. $a = -1b = 4$

D. $a = 2, b = 4$

Answer: A

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17. If $f(x) = \int_0^x \{f(t)\}^{-1} dt \text{ and } \int_0^1 \{f(t)\}^{-1} = \sqrt{2}$, then

A. $f(x) = \sqrt{2}x$

B. $f(x) = \sqrt{2 \log_e x}$

C. $f(x) = \sqrt{3x - 1}$

D. None of these

Answer: A

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18. Let f be a real-valued function defined on the inverval $(- 1, 1)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt,$ for all, $x \in (- 1, 1) and let f^{-1}$ be the inverse function of f. Then $\left(f^{-1}\right)'(2)$ is equal to 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

Answer: B

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19. Concider the function defined on $[0, 1] \to R f(x) = \frac{\sin x - \cos x}{x^2}, \quad \text{if} \quad x \neq 0 \text{ and } f(0) = 0$ $\int_0^1 f(x)$ is equal to

A. $1 - \sin(1)$

B. $\sin(1) - 1$

C. $\sin(1)$

D. $-\sin(1)$

Answer: A

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20. Concider the function defined on $[0, 1] \rightarrow Rf(x) = \frac{\sin x - \cos x}{x^2}, \quad \text{if} \quad x \neq 0 \text{ and } f(0) = 0$

$\lim_{t \to 00} \frac{1}{t^2 \int_0^t f(x) dx}$ is equal to

A. $1/3$

B. $1/6$

C. $1/12$

D. $1/24$

Answer: B

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Exercise For Session 6

1. The value of

$$\int_0^{\pi/2} \frac{\log(1 + x \sin^2 \theta)}{\sin^2 \theta} d\theta, x \geq 0$$ is equal to

A. $\frac{1}{\pi}(\sqrt{1 + x - 1})$

B. $\sqrt{\pi}(\sqrt{1 + x - 1})$

C. $\pi(\sqrt{1 + x - 1})$

D. None of these

Answer: C

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2. The value of $\lim_{n \to \infty} \frac{1}{2} \sum_{r-1}^{n} \left(\frac{r}{n + r}\right)$ is equal to

A. $1 - \log 2$

B. $\log 4 - 1$

C. $\log 2$

D. None of these

Answer: A

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3. The value of $\lim_{n \to \infty} \frac{1}{n}\{(n+\ )(n+2)(n+3)\ldots(n+n)\}^{1/n}$ is equal to

A. $4e$

B. $\frac{e}{4}$

C. $\frac{4}{e}$

D. None of these

Answer: C

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4. If $m, n \in N$, then the value of $\int_a^b (x-a)^m (b-x)^n dx$ is equal to

A. $\frac{(b-a)^{m+n}. M!n!}{(m+n)}!$

B. $\frac{(b-a)^{m+n+1}. m!n!}{(m+n+1)!}$

C. $\frac{(b-a)^m. m!}{m!}$

D. None of these

Answer: B

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5. The value of $\lim_{n \to \infty} \left(\frac{n!}{(n^n)^{2n^4+1}} / (5n^5+1)\right)$ is equal

A. e

B. $\frac{2}{e}$

C. $\dfrac{\left(\frac{1}{e}\right)^2}{5}$

D. None of these

Answer: C

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6. The value of $\lim_{n \to \infty} n\left\{\dfrac{1}{3n^2+8n+4}+\dfrac{1}{3n^2+16n+16}+\ldots+n\text{terms}\right\}$ is equal to

A. $\dfrac{1}{2}\log\left(\dfrac{9}{5}\right)$

B. $\dfrac{1}{3}\log\left(\dfrac{9}{5}\right)$

C. $\dfrac{1}{4}\log\left(\dfrac{9}{5}\right)$

D. None of these

Answer: C

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7. The value of $\lim_{x \to \infty} \frac{1}{2}\left\{\frac{\sin^3(\pi)}{4n} + 2\frac{\sin^3(2\pi)}{4n} + \dots n\frac{\sin^3(n\pi)}{4n}\right\}$ Is equal to

A. $\frac{\sqrt{2}}{9\pi^2(52 - 15n)}$

B. $\frac{2}{9\pi^2(52 - 15n)}$

C. $\frac{1}{9\pi^2(15n - 15)}$

D. None of these

Answer: A

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8. The value of $f(k) = \int_0^{\pi/2} \log\left(\sin^2\theta + k^2\cos^2\theta\right) d\theta$ is equal to

A. $\pi\log(1 + k) - \pi\log^2$

B. $\pi\log 2 - \log(1 + k)$

C. $\log(1+k) - \pi \log 2$

D. None of these

Answer: A

View Text Solution

9. $If m, n \in N$, then $l_{mn} = \int_0^1 x^m (1-x)^n dx$ is equal to

A. $\frac{m!n!}{(m+n+2)!}$

B. $\frac{2m!n!}{(m+n+1)!}$

C. $\frac{m!n!}{(m+n+1)!}$

D. None of tese

Answer: C

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10. The value of $I(n) = \int_0^{\pi} \frac{\sin^2 n\theta}{\sin^2 \theta} d\theta$ is $(\forall n \in N)$

A. $n\pi$

B. $\frac{n\pi}{2}$

C. $\frac{n\pi}{4}$

D. None of these

Answer: A

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Exercise (Single Option Correct Type Questions)

1. $\int_0^4 \frac{(y^2 - 4y + 5)\sin(y-2)dy}{[2y^2 - 8y + 1]}$ is equal to

A. 0

B. 2

C. -2

D. 2

Answer: A

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2. Let $f(x) = x^2 + ax + b$ and the only solution of the equation $f(x) = \min (f(x)) is x = 0$ and $f(x) = 0$has root α and β, then $\int_{\alpha}^{\beta} x^3 d$

is equal to

A. $\frac{1}{4}(\beta^4 + \alpha^4)$

B. $\frac{1}{4}(a^2 - b^2)$

C. 0

D. None of these

Answer: C

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3. $If\int_{\frac{\pi}{3}}^{x}\sqrt{\left(3-\sin^2 t\right)}dt+\int_0^y \cos t dt=0, the \ne valuate\frac{dy}{dx}$

A. $\sqrt{3}$

B. $-\sqrt{2}$

C. $-\sqrt{3}$

D. None of these

Answer: A

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4. $\int_{-4}^{4}\frac{\sin^{-1}(\sin x)+\cos^{-1}(\cos x)}{\left(1+x^2\right)\left(1+\left[\frac{x^2}{17}\right]\right)}dx=\log\left(\frac{\left(1+\pi^2\right)}{\sqrt{a}}\right)$

$b\pi \ \tan^{-1}\left(\frac{c-\pi}{1+c\pi}\right)$ (where, [.] denotes greatest inteher function), then the number of ways in which $a-(2b+c)$ distinct object can distributed among $\frac{a-5}{c}$ persons equally , is

A. $\frac{9!}{(3!)^3}$

B. $\frac{12!}{(140^3)}$

C. $\frac{15!}{(5!)^3}$

D. $\frac{10!}{(6!)^3}$

Answer: A

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5. The value of the definite integral $\int_0^\infty \frac{dx}{(1+x^a)(1+x^2)}(a > 0)$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. some function of a

Answer: A

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6. The value of the definite integral $\int_0^{3\pi/4} (1+x)\sin x + (1-x)\cos x \Big) dx$

is

A. $2\dfrac{\tan(3\pi)}{8}$

B. $2\dfrac{\tan(\pi)}{4}$

C. $2\dfrac{\tan(\pi)}{8}$

D. 0

Answer: A

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7. Let $C_n = \int_{1/n+1}^{1/n} \dfrac{\tan^{-1}(nx)}{\sin^{-1}(nx)} dx$, then $\lim_{n \to \infty} n^n . C_n$ is equal to

A. 1

B. 0

C. -1

D. $\frac{1}{2}$

Answer: D

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8. If

$$\left[\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right]x^2-\left[\int_{-3}^{3}\frac{t^2\sin 2t}{t^2+1}dt\right]x-2=0(0<\alpha<\pi)$$

then the value of x is (i)$\pm\sqrt{\frac{\alpha}{2\sin\alpha}}$ (ii)$\pm\sqrt{\frac{2\sin\alpha}{\alpha}}$ (iii) $\pm\sqrt{\frac{\alpha}{\sin\alpha}}$ (iv) $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

A. $\pm\left(\frac{\sqrt{\alpha}}{2\sin\alpha}\right)$

B. $\pm\sqrt{\frac{2\sin\alpha}{\alpha}}$

C. $\pm\sqrt{\frac{\alpha}{\sin\alpha}}$

D. $\pm\sqrt{\frac{\sin\alpha}{\alpha}}$

Answer: D

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9. If $f(x)=e^{g(x)}$ and $g(x)=\int_2^x \frac{dt}{1+t^4}$ then $f'(2)$ is equal to

A. $2/17$

B. 0

C. 1

D. Cannot be determined

Answer: A

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10. If a, b and c are real numbers, then the value of $\lim_{t\to 0} In\left(\frac{1}{t}\int_0^t (1+a\sin bx)^{c/x}dx\right)$ equals

A. abc

B. $\frac{ab}{c}$

C. $\frac{bc}{a}$

D. $\frac{ca}{b}$

Answer: A

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11. The value of $\lim_{n \to \infty} \sum_{r=1}^{r=4n} \frac{\sqrt{n}}{\sqrt{r\left(3\sqrt{r}+4\sqrt{n}\right)^2}}$ is equal to

A. $\frac{1}{35}$

B. $\frac{1}{14}$

C. $\frac{1}{10}$

D. $\frac{1}{5}$

Answer: C

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12. Let $f(x)=\int_{-1}^{x}e^{t^2}dt$ and $h(x)=f(1+g(x))$, where $g(x)$ is defined for all $x, g'(x)$ exists for all x, and $g(x)<0f$ or $x>0$. If $h'(1)=e$ and $g'(1)=1$, then the possible values which $g(1)$ can take

A. 0

B. -1

C. -2

D. -4

Answer: C

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13. Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ be the function satisfying $f(x)+g(x)=x^2$. Then the value of integral $\int_{0}^{1}f(x)g(x)dx$ is equal to (A) $\frac{e-2}{4}$ (B) $\frac{e-3}{2}$ (C) $\frac{e-4}{2}$ (D) none of these

A. $e-\frac{1}{2}e^2-\frac{5}{2}$

B. $e-e^2-3$

C. $\frac{1}{2}(e-3)$

D. $e-\frac{1}{2}e^2-\frac{3}{2}$

Answer: D

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14. Let $f(x)=\int_0^{g(x)}\frac{dx}{\sqrt{1+t^2}}$ where $g(x)=\int_0^{\cos x}(1+\sin t^2)dt.$ Also $h(x)=e^{-|x|}$ and $l(x)=x^2\sin\left(\frac{1}{x}\right)$ if $x\neq 0$ and $l(0)$ then $f'\left(\frac{\pi}{2}\right)$ equals

A. $l'(0)$

B. $h'(0^-)$

C. $h'(0^+)$

D. $\lim_{x\to 0}\frac{1-\cos x}{x\sin x}$

Answer: C

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15. For $f(x) = x^4 + |x|, \leq tI_1 = \int_0^{\pi} f(\cos x) dx$ and $I_2 = \int_0^{\pi/2} f(\sin x) dx \text{then} \frac{I_1}{I_2}$ has the value equal to

A. 1

B. $1/2$

C. 2

D. 4

Answer: C

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16. Let f be a positive function. If $I_1 = \int_{1-k}^{k} x f[x(1-x)]\, dx$ and $I_2 = \int_{1-k}^{k} f[x(1-x)]\, dx$, where $2k - 1 > 0$. Then $\frac{I_1}{I_2}$ is

A. k

B. $1/2$

C. 1

D. 2

Answer: D

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17. Suppose that the quadratic function $f(x)=ax^2+bx+c$ is non-negative on the interval [-1,1] . Then, the area under tha graph of f over the interval [-1,1] and the X - axis is given by the formula

A. $A=f(-1)+f(1)$

B. $A=f\left(-\frac{1}{2}\right)+f\left(\frac{1}{2}\right)$

C. $A=\frac{1}{2}[f(-1)+2f(0)+f(1)]$

D. $A=\frac{1}{3}[f(-1)+4f(0)+f(1)]$

Answer: D

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18. Let $I(a)=\int_0^{\pi}\left(\frac{x}{a}+a\sin x\right)^2 dx$, where 'a' is positive real .

The value of 'a' for which I(a) attains its minimum value, is

A. $\sqrt{\pi\sqrt{\frac{2}{3}}}$

B. $\sqrt{\pi\sqrt{\frac{3}{2}}}$

C. $\sqrt{\frac{\pi}{16}}$

D. $\sqrt{\frac{\pi}{13}}$

Answer: A

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19. The set of value of 'a' which satisfy the equation $\int_0^2 (t-\log_2 a)dt=\log_2\left(\frac{4}{a^2}\right)$ is

A. $a\in R$

B. $a \in R^{+}$

C. $a < 2$

D. $a > 2$

Answer: B

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20. $\lim_{x \to \infty} \left(x^3 \int_{-1/x}^{1/x} \frac{\text{In}(1+t^2)}{1+e^t} dt \right)$ is equal to `

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. 1

D. 0

Answer: A

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21. The value of $\sqrt{\pi\left(\int_0^{2008} x|\sin \pi x|dx\right)}$ is equal to

A. $\sqrt{2008}$

B. $\pi\sqrt{2008}$

C. 1004

D. 2008

Answer: D

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22. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{n}{n^2 + k^2x^2}, x > 0$ is equal to

A. $x\tan^{-1}(x)$

B. $\tan^{-1}(x)$

C. $\frac{\tan^{-1}(x)}{x}$

D. $\frac{\tan^{-1}(x)}{x}$

Answer: C

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23. Let $a > 0$ and $f(x)$ is monotonic increase such that $f(0) = 0$ and $f(a) = b$, then $\int_0^a f(x)dx + \int_0^b f^{-1}(x)dx$ is equal to

A. $a + b$

B. $ab + b$

C. $ab + a$

D. ab

Answer: D

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24. $\int_{-\frac{1}{\sqrt{3}}}^{-\frac{1}{\sqrt{3}}} \frac{x^4}{1 - x^4} \cos^{-1}\left(\frac{2x}{1 + x^2}\right) dx$

A. π

B. 2π

C. 2

D. 1

Answer: A

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25. $\int_0^\infty f\left(x+\frac{1}{x}\right).\frac{Inx}{x}dx$ is equal to

A. 0

B. 1

C. $\frac{1}{2}$

D. Cannot be evaluated

Answer: A

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26. $\lim_{\lambda \to 0} \left(\int_0^1 (1+x)^{\lambda} dx \right)^{1/\lambda}$ Is equal to

A. $2In2$

B. $\frac{4}{e}$

C. $In\frac{4}{e}$

D. 4

Answer: B

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27. If $g(x)$ is the inverse of $f(x)$ and $f(x)$ has domain $x \in [1, 5]$, where $f(1) = 2$ and $f(5) = 10$ then the values of $\int_1^5 f(x)dx + \int_2^{10} g(y)dy$ equals

A. 48

B. 64

C. 71

D. 52

Answer: A

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28. The value of the definite integral $\int_0^{\pi/2} \sin x \sin 2x \sin 3x dx$ is equal to

A. $\frac{1}{3}$

B. $-\frac{2}{3}$

C. $-\frac{1}{6}$

D. $\frac{1}{6}$

Answer: D

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29. If $f(x) = \int_0^x (f(t))^2 dt$, $f: R \to R$ be differentiable function and $f(g(x))$ is differentiable at $x = a$, then

A. g (x) must be differentiable at $x = a$

B. g(x) may be non-differentiable at $x = a$

C. g(x) may be discontinous at $x = a$

D. None of the above

Answer: A

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30. The number of integral solutions of the equation $4\int_0^\infty \frac{In\ \ tdt}{x^2 + t^2} - \pi In2 = 0, x > 0,$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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31. $\int_0^{16n^2/\pi} \cos\frac{\pi}{2}\left[\frac{x\pi}{n}\right]dx$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: A

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32. If $\int_{-2}^{-1} (ax^2 - 5)dx = 0$ and $5 + \int_1^2 (bx + c)dx = 0$, then

A. $ax^2 - bx + x = 0$ has aleast one root in $(1, 2)$

B. $ax^2 - bx + c = 0$ has aleast one root in $(-2, -1)$

C. $ax^2 + bx + c = 0$ has atleast one root in $(-2, -1)$

D. None of these

Answer: B

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33. The value of $\int_3^6 \left(\sqrt{x + \sqrt{12x - 36}} + \sqrt{x - \sqrt{12x - 36}}\right)dx$ is equal to

A. $6\sqrt{3}$

B. $4\sqrt{3}$

C. $12\sqrt{3}$

D. None of these

Answer: A

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34. Let $I_n = \int_{-n}^{n} \left(\{x+1\} \cdot \{x^2+2\} + \{x^2+2\}\{x^3+4\}\right) dx,$ where {*} denotet the fractional part of x. Find I_1.

A. $-\frac{1}{3}$

B. $-\frac{2}{3}$

C. $\frac{1}{3}$

D. None of these

Answer: B

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Exercise (More Than One Correct Option Type Questions)

1. If $f(x) = \left[\sin^{-1}(\sin 2x)\right]$ (where, [] denotes the greatest integer function), then

A. $\int_0^{\pi/2} f(x)dx = \frac{\pi}{2} - \sin^{-1}(\sin 1)$

B. $f(x)$ is periodic with period π

C. $\lim_{x \to \frac{\pi}{2}} f(x) = -1$

D. None of thses

Answer: A::B::C

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2. Which of the following definite integral (s) vanishes ?

A. $\int_0^{\pi/2} In(\cot x)dx$

B. $\int_0^{\pi 2} \sin^3 x dx$

C. $\int_{1/e}^{e} \frac{dx}{x(Inx)^{1/3}}$

D. $\int_0^{\pi} \sqrt{\frac{1+\cos 2x}{2}}dx$

Answer: A::C

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3. The equation $x^3 - 3x + 1 = 0$ has

A. atleast one root in $(-1, 0)$

B. alteast one root in $(0, 1)$

C. alteast two roots in $(-1, 1)$

D. no roots in $(-1, 1)$

Answer: A

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4. Suppose

$I_1 = \int_0^{\frac{\pi}{2}} \cos\left(\pi \sin^2 x\right) dx \text{ and } I_2 \int_0^3 \cos\left(2\pi \sin^2 x\right) dx \text{ and } I_3 = \int_0^{\frac{\pi}{2}} \cos(\pi s$

, then

A. $I_1 = 0$

B. $I_2 \_ I_3 = 0$

C. $I_1 + I_2 + I_3 = 0$

D. $I_2 = I_3$

Answer: A::B::C

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5. Let $f(x) = \int_{-1}^{1} (1 - |t|)\cos(xt)dt$, then which of the following holds true?

A. $f(0)$ is not defined

B. $\lim_{x \to 0} f(x)$ exists and is equal to 2

C. $\lim_{x \to 0} f(x)$exists and is equal to 1

D. $f(x) iscont \in uousat$x=0`

Answer: C::D

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6. The function f is continuous and has the property $f(f(x)) = 1 - x$ for all $x \in [0, 1]$ and $J = \int_0^1 f(x)dx$. Then which of the following is/are true?

(A) $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) = 1$

(B)$f\left(\frac{1}{3}\right) \cdot f\left(\frac{2}{3}\right) = 1$

(C) the value of J equals to $\frac{1}{2}$

(D) $\int_0^{\frac{\pi}{2}} \frac{\sin x dx}{(\sin x + \cos x)^3}$ has the same value as J

A. $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) = 1$

B. $f\left(\frac{1}{3}\right) \cdot f\left(\frac{2}{3}\right) = 1$

C. the value of J equals to $1/2$

D. $\int_0^{\pi/2} \frac{\sin x dx}{(\sin x + \cos x)^3}$ has the value of as J

Answer: A::C::D

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7. Let $f(x)$ is a real valued function defined by $f(x) = x^2 + x^2 \int_{-1}^{1} t f(t) dt + x^3 \int_{-1}^{1} f(t) dt$

then which of the following hold (s) good?

A. $\int_{-1}^{1} t f(t) dt = \frac{10}{11}$

B. $f(1) + f(-1) = \frac{30}{11}$

C. $\int_{-1}^{1} t f(t) dt > \int_{-1}^{1} f(t) dt$

D. $f(1) - f(-1) = \frac{20}{11}$

Answer: B::D

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8. Let $f(x)$ and $g(x)$ be differentiable functions such that $f(x)+\int_0^x g(t)dt=\sin x(\cos x-\sin x)$ and $(f'(x))^2+(g(x))^2=1$, then

respectively , can be

A. $\frac{1}{2}\sin 2x, \sin 2x$

B. $\frac{\cos 2x}{2}, \cos 2x$

C. $\frac{1}{2}\sin 2x, \ -\sin 2x$

D. $-\sin^2 x, \cos 2x$

Answer: C

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9. Let $f(x)=\int_{-x}^{x}(t\sin at+bt+c)dt$, where a,b,c are non-zero real numbers , then $\lim_{x\to 0}\frac{f(x)}{x}$ is

A. independent of a

B. independent of a and b , and has the value equals to c

C. independent a, b and c

D. dependent only on c

Answer: A,D

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10. Let $L = \lim_{n \to \infty} \int_a^{\infty} \frac{n dx}{1 + n^2 x^2}$, where $a \in R$, then L can be

A. π

B. $\pi/2$

C. 0

D. 1

Answer: C

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Exercise (Passage Based Questions)

1. $Suppose \lim_{x \to 0} \frac{\int_0^x \frac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > \ > 0, r > 0 \text{ and } b \neq 0$

If l exists and is non- zero, then

A. $b > 1$

B. $0 < b < 1$

C. $b < 0$

D. $b = 1$

Answer: D

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2. Suppose $\lim_{x \to 0} \frac{\int_0^x \frac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > \ > 0, r > 0 \text{ and } b \neq 0$

If $p = 3$ and $l = 1$, then the value of 'a' is equal to

A. 8

B. 3

C. 6

D. $3/2$

Answer: A

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3. Suppose $\sum_{x \to 0} \frac{\int_0^x \frac{t^2 dt}{(a+t^r)^{1/p}}}{bx - \sin x} = l,$

$p \in N, p \geq 2, a > \; > 0, r > 0$ and $b \neq 0$

If $p = 2$ and $a = 9$ and l exists , then the value of l is equal to

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{1}{3}$

D. $\frac{7}{9}$

Answer: B

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4. Suppose $f(x)$ and $g(x)$ are two continuous functions defined for $0 \leq x \leq 1$.Given,

$f(x) = \int_0^1 e^{x+1}. f(t)dt$ and $g(x) = \int_0^1 e^{x+1} \cdot g(t)dt + x$ The value of $f(1)$ equals

A. 0

B. 1

C. e^{-1}

D. e

Answer: A

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5. Suppose $f(x)$ and $g(x)$ are two continuous functions defined for $0 \leq x \leq 1$.Given,

$f(x) = \int_0^1 e^{x+1}. f(t)dt$ and $g(x) = \int_0^1 e^{x+1} \cdot g(t)dt + x$ The value of $f(1)$ equals

A. $\dfrac{2}{3 - e^2}$

B. $\dfrac{3}{e^2 - 2}$

C. $\dfrac{1}{e^2 - 1}$

D. 0

Answer: A

Watch Video Solution

6. Suppose $f(x)$ and $g(x)$ are two continuous functions defined for $0 \leq x \leq 1$.Given,

$f(x) = \int_0^1 e^{x+1}. f(t)dt$ and $g(x) = \int_0^1 e^{x+1} \cdot g(t)dt + x$ The value of $f(1)$ equals

A. 0

B. $\frac{1}{3}$

C. $\frac{1}{e^2}$

D. $\frac{2}{e^2}$

Answer: B

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7. We are given the curvers $y = \int_{-\infty}^{x} f(t)dt$ through the point $\left(0, \frac{1}{2}\right)$ any $y = f(x)$, where $f(x) > 0$ and $f(x)$ is differentiable ,$\forall x \in$ R through $(0, 1)$ Tangents drawn to both the curves at the points with equal abscissae intersect on the same point on the X- axists

The number of solutions $f(x) = 2ex$ is equal to

A. 0

B. 1

C. 2

D. None of these

Answer: B

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8. We are given the curves $y = \int_{-\infty}^{x} f(t)$ dt through the point $\left(0, \frac{1}{2}\right)$ and y=f(X), where $f(x) > 0$ and f(x) is differentiable, $\forall x \in R$ through (0,1). If tangents drawn to both the curves at the point wiht equal abscissae intersect on the point on the X-axis, then

$\lim_{x \to \infty} (f(x))^{f(-x)}$ is

A. 3

B. 6

C. 1

D. None of these

Answer: C

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9. We are given the curvers $y = \int_{-\infty}^{x} f(t)dt$ through the point $\left(0, \frac{1}{2}\right)$ any $y = f(x)$, where $f(x) > 0$ and $f(x)$ is differentiable ,$\forall x \in$ R through $(0, 1)$ Tangents drawn to both the curves at the points with equal abscissae intersect on the same point on the X- axists

The function $f(x)$ is

A. increasing for all x

B. non-monotonic

C. decreasing for all x

D. None of these

Answer: A

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10. $f(x)=\int_0^x\left(4t^4-at^3\right)dt$ and $g(x)$ is quadratic satifying $g(0)+6=g'(0)-c=g''(c)+2b=0$. $y=h(x)$ intersect in 4 distinct points with abscissae $x_i, i=1,2,3,4$ such that $\sum\frac{i}{x_i}=8, a, b, c\in R^+$ and $h(x)=f'(x)$.

$f(x)=\int_0^x\left(4t^4-at^3\right)dt$ and $g(x)$

A. AP

B. GP

C. HP

D. None of these

Answer: A

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11. $f(x)=\int_0^x\left(4t^4-at^3\right)dt$ and $g(x)$ is quadratic satifying $g(0)+6=g'(0)-c=g''(c)+2b=0$. $y=h(x)$ intersect in 4 distinct points with abscissae $x_i, i=1,2,3,4$ such that

$\sum \frac{i}{x_i} = 8, a, b, c \in R^+$ and h (x)=f' (x)`.

'a' is equal to

A. 6

B. 8

C. 20

D. 12

Answer: C

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12. $f(x) = \int_0^x (4t^4 - at^3) dt$ and $g(x)$ is quadratic satifying $g(0) + 6 = g'(0) - c = g''(c) + 2b = 0$. $y = h(x)$ intersect in 4 distinct points with abscissae $x_i, i = 1, 2, 3, 4$ such that $\sum \frac{i}{x_i} = 8, a, b, c \in R^+$ and h (x)=f' (x)`.

'c' is equal to

A. 25

B. $25/2$

C. $25/4$

D. $25/8$

Answer: A

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13. If $y = \int_{u(x)}^{v(x)} f(t)dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx} = v'(x) f^2(v(x)) - u'(x) f^2(u(x))$ alnd the equation of the tangent at (a, b) as $y - b = \left(\frac{dy}{dx}\right)_{a \quad b} (x - a)$

If $F(x) = \int_1^x e^{t^2/2} (1 - t^2) dt$, then $\frac{d}{dx} F(x)$ at $x = 1$ is

A. $x + y = 1$

B. $y = x - 1$

C. $y = x$

D. $y = x + 1$

Answer: B

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14. Let $y = \int_{u(x)}^{y(x)} f(t)dt$, let us define $\frac{dy}{dx} as \frac{dy}{dx} = v'(x) f^2(v(x)) - u'(x) f^2(u(x))$ and the equation of the tangent at (a, b) and $y - b = \left(\frac{dy}{dx}\right)(a, b)(x - a)$.

If $y = \int_{x^2}^{x^4} (Int)dt$, then $\lim_{x \to 0^+} \frac{dy}{dx}$ is equal to

A. 0

B. 1

C. 2

D. -1

Answer: A

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15. If $y = \int_{u(x)}^{v(x)} f(t)dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx} = v'(x)f^2(v(x)) - u'(x)f^2(u(x))$ alnd the equation of the tangent at (a, b) as $y - b = \left(\frac{dy}{dx}\right)_{a\ \ b}(x - a)$

If $F(x) = \int_1^x e^{t^2/2}(1 - t^2)dt$, then $\frac{d}{dx}F(x)$ at $x = 1$ is

A. 0

B. 1

C. 2

D. -1

Answer: A

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16. Consider $f:(0,\infty)\rightarrow\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, defined as $f(x)=\tan^{-1}\left(\frac{\log_e x}{(\log_e x)^2+1}\right)$. The about function can be classified as

A. Injective but nor surjective

B. Surjective but not bijective

C. Neither injective nor surjective

D. Both injective and surjective

Answer: C

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17. The value of $\int_0^\infty \left[\tan^{-1}x\right]dx$ is equal to (where ,[.] denotes the greatest integer function)

A. $-\frac{\pi}{2}$

B. $\frac{\pi}{2}$

C. ∞

D. 1

Answer: C

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Exercise (Matching Type Questions)

1. Let $\lim_{T \to \infty} \frac{1}{T} \int_0^T (\sin x + \sin ax)^2 dx = L$, then

	Column I		Column II
(A)	For $a = 0$, the value of L is	(p)	0
(B)	For $a = 1$, the value of L is	(q)	1/2
(C)	For $a = -1$, the value of L is	(r)	1
(D)	For $a \in R - \{-1, 0, 1\}$, the value of L is	(s)	2

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2. Let $f(\theta) = \int_0^1 (x + \sin\theta)^2 dx$ and $g(\theta) = \int_0^1 (x + \cos\theta)^2 dx$

where $\theta \in [0, 2\pi]$. The quantity $f(\theta) - g(\theta)$, $\forall\theta$ in the interval given in column I, is

Column I	Column II
(A) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$	(p) negative
(B) $\left(\frac{3\pi}{4}, \pi\right]$	(q) positive
(C) $\left[\frac{3\pi}{2}, \frac{7\pi}{4}\right)$	(r) non-negative
(D) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$	(s) non-positive

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3. Match the following

Column I	Column II
(A) $\int_0^1 (1 + 2008\,x^{2008})\, e^{x^{2008}}\, dx$ equals	(p) e^{-1}

Column I	Column II
(B) The value of the definite integral $\int_0^1 e^{-x^2}\,dx + \int_1^{1/e} \sqrt{-\ln x}\,dx$ is equal to	(q) $e^{-1/4}$
(C) $\lim_{n \to \infty} \left(\dfrac{1^1 \cdot 2^2 \cdot 3^3 \ldots (n-1)^{n-1} \cdot n^n}{n^{1+2+3+\ldots+n}} \right)^{\frac{1}{n^2}}$ equals	(r) $e^{1/2}$
	(s) e

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4. Match the following

Column I	Column II
(A) If $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$, where $g(x)=\int_0^{\cos x}(1+\sin t^2)\,dt$, then value of $f'\left(\frac{\pi}{2}\right)$ is	(p) -2
(B) If $f(x)$ is a non-zero differentiable function such that $\int_0^x f(t)\,dt=\{f(x)\}^2$, $\forall x\in R$, then $f(2)$ is equal to	(q) 2
(C) If $\int_a^b(2+x-x^2)\,dx$ is maximum, then $a+b$ is equal to	(r) 1
(D) If $\lim_{x\to 0}\left(\frac{\sin 2x}{x^3}+a+\frac{b}{x^2}\right)=0$, then $3a+b$ has the value	(s) -1

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Exercise (Single Integer Answer Type Questions)

1. If $f(x)=\sum_{n=1}\frac{\sin nx}{4^n}$ and $\int_0^{\pi} f(x)dx=\log\left(\frac{m}{n}\right)$, then the value of $(m+n)$ is ....

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2. The value of $I=\int_{-\pi/2}^{\pi/2}\frac{\cos x dx}{1+2\left[\sin^{-1}(\sin x)\right]}$ (where ,[.] denotes greatest integer function) is ...

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3. If $f(x)=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{\sin^2(n\theta)}{\sin^2\theta}d(\theta)$ then evaluate $\frac{f(15)+f(3)}{f(15)-f(9)}$

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4. Let $f(x)=\int_{-2}^{x} e^{(1+t)^2}dt$ and $g(x)=f(h(x))$,where h(x) is defined for all $x\in R$. If $g'(2)=e^4$ and $h'(2)=1$ then absolute value of sum of all possible values of h(2), is

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5. $If = \int_0^{\pi/2} \sin x . \log(\sin x) dx = \log\left(\frac{K}{e}\right)$. Then, the value of K is ....

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Exercise (Questions Asked In Previous 13 Years Exam)

1. The value of $\int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1+e^x} dx$ is equal to

A. $\frac{\pi^2}{4} - 2$

B. $\frac{\pi^2}{4} + 2$

C. $\pi^2 - e^{-\pi/2}$

D. $\pi^2 + e^{\pi/2}$

Answer: A

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2. The total number for distinct $x\varepsilon[0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4} dt = 2x - 1$ is ________.

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3. Let $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression (s) is are

A. $\int_0^{\pi/4} x f(x) dx = \frac{1}{12}$

B. $\int_0^{\pi/4} f(x) dx = 0$

C. $\int_0^{\pi/4} x f(x) dx = \frac{1}{6}$

D. $\int_0^{\pi/4} f(x) dx = 1$

Answer: A::B

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4. Let $f'(x) = \frac{192x^3}{2+\sin^4 \pi x}$ for all $x \in \mathbb{R}$ with $f\left(\frac{1}{2}\right) = 0.$ If $m \leq \int_{\frac{1}{2}}^{1} f(x)dx \leq M$ then the possible values of m and M are (i) $m = 13, M = 24$ (ii)$m = \frac{1}{4}, M = \frac{1}{2}$ (iii) $m = -11, M = 0$ (iv) $m = 1, M = 12$

A. $m = 13, M = 24$

B. $m = \frac{1}{4}, M = \frac{1}{2}$

C. $m = -11, M = 0$

D. $m = 1, M12$

Answer: B

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5. The option(s) with the values of $aandL$ that satisfy the following equation is (are) $\frac{\int 04\pi e^t \left(s \in^6 at + \cos^4 at\right) dt}{\int 0\pi e^t (s \in^6 at + \cos^4 at) dt} = L$ $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$ (b) $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$ $a = 4, L = \frac{e^{4\pi} - 1}{e^{\pi} - 1}$ (d) $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi} + 1}$

A. $a = 2, L = \frac{e^{4\pi} - 1}{e^{\pi}}$

B. $a = 2, L = \frac{e^{4\pi} + 1}{e^{\pi + 1}}$

C. $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi + 1}}$

D. $a = 4, L = \frac{e^{4\pi} + 1}{e^{\pi + 1}}$

Answer: A::C

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6. Let $F: R - R$ be a thrice differentiable function. Suppose that $F(1) = 0, F(3) = -4$ and $F(x) < 0$ for all $x \in \left(\frac{1}{2}, 3\right)$. Let $f(x) = xF(x)$ for all $x \in R$ The correct statement is

A. $f'(1) < 0$

B. $f(2) < 0$

C. $f'(x) \neq 0 \text{for any} x \in (1, 3)$

D. $f'(x) = 0 \text{for some } x \in (1, 3)$

Answer: A::B::C

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7. Let F : R $\rightarrow$ R be a thrice differentiable function . Suppose that $F(1) = 0, F(3) = -4$ and $F(x) < 0$ for all $x \in (1, 3)$. f(x) = x F(x) for all $x \in R$.

$If \int_1^3 x^2 F'(x) dx = -12$ and $\int_1^3 x^3 F''(x) dx = 40$, then the correct expression (s) is //are

A. $9f'(3) + f'(1) - 32 = 0$

B. $\int_1^3 f(x) dx = 12$

C. $9f'(3) - f'(1) + 32 = 0$

D. $\int_1^3 f(x) dx = -12$

Answer: C::D

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8. Let $f: R\overrightarrow{R}$ be a function defined by $f(x) = \{[x], x \leq 20, x > 2$ where $[x]$ is the greatest integer less than or equal to $x\cdot$ If $I = \int_{-1}^{2} \frac{xf(x^2)}{2 + f(x+1)} dx, thenthevalueof f(4I - 1)is$

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9. If $\alpha = \int_0^1 \left(e^9 x + 3\tan^{(-1)x}\right)\left(\frac{12 + 9x^2}{1 + x^2}\right) dxwhere m^{-1}$ takes only principal values, then the value of $\left((\log)_e |1 + \alpha| - \frac{3\pi}{4}\right) is$

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10. The integral $\int_{\pi/4}^{\pi/2} (2\cos ecx)^{17} dx$ is equal to

A. $\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u + e^{-u}\right)^{16} du$

B. $\int_0^{\log\left(1+\sqrt{2}\right)} \left(e^u + e^{-u}\right)^{17} du$

C. $\int_0^{\log\left(1+\sqrt{2}\right)} \left(e^u + e^{-u}\right)^{17} du$

D. $\int_0^{\log\left(1+\sqrt{2}\right)} 2\left(e^u + e^{-u}\right)^{16} du$

Answer: a

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11. Let $f:[0, 2]\overrightarrow{R}$ be a function which is continuous on [0,2] and is differentiable on (0,2) with $f(0) = 1$ $Let: F(x) = \int_0^{x^2} f\left(\sqrt{t}\right) dt f \text{ or } x \in [0, 2]\dot{I} f F'(x) = f'(x)$. for all $x \in (0, 2)$, then $F(2)$ equals $e^2 - 1$ (b) $e^4 - 1$ $e - 1$ (d) e^4

A. $e^2 - 1$

B. $e^4 - 1$

C. $e - 1$

D. e^4

Answer: B

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12. Match the conditions/ expressions in Column I with statement in Column II

	Column I		Column II
A.	$\int_{-1}^{1} \frac{dx}{1+x^2}$	p.	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
B.	$\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}}$	q.	$2\log\left(\frac{2}{3}\right)$
C.	$\int_{2}^{3} \frac{dx}{1-x^2}$	r.	$\frac{\pi}{3}$
D.	$\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	s.	$\frac{\pi}{2}$

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13. Match List I with List II and select the correct answer using codes given below the lists

	List I		List II
A.	The number of polynomials $f(x)$ with non-negative integer coefficients of degree ≤ 2, satisfying $f(0) = 0$ and $\int_0^1 f(x)\,dx = 1$, is	p.	8
B.	The number of points in the interval $[-\sqrt{13}, \sqrt{13}]$ at which $f(x) = \sin(x^2) + \cos(x^2)$ attains its maximum value, is	q.	2

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14. The value of $\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2} (1 - x^2)^5 \right\} dx is$

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15. The value of the integral $\int_{-\pi/2}^{\pi/2} \left(x^2 + \log \frac{\pi - x}{\pi + x} \right) \cos x dx$

A. 0

B. $\frac{\pi^2}{2} - 4$

C. $\frac{\pi^2}{2} + 4$

D. $\frac{\pi^2}{2}$

Answer: B

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16. The value of $\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx$ is

A. $\frac{1}{4} \frac{\log(3)}{2}$

B. $\frac{1}{2} \frac{\log(3)}{2}$

C. $\frac{\log(3)}{2}$

D. $\frac{1}{6} \frac{\log(3)}{2}$

Answer: B::D

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17. Let $f: [1, \infty]$ be a differentiable function such that $f(1) = 2$. If $\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $f(2)$ is

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18. The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is (are)

A. $\frac{22}{7} - \pi$

B. $\frac{2}{105}$

C. 0

D. $\frac{71}{15} - \frac{3\pi}{2}$

Answer: A

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19. For $a \in R$ (the set of all real number) , $a \neq -1$,

$\lim_{n \to \infty} \frac{(1^a + 2^a + ... + n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+...+(na+n)]} = \frac{1}{60}$ Then,

a is equal to

A. 5

B. 7

C. $\frac{-15}{2}$

D. $\frac{-17}{2}$

Answer: B::D

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20. Consider the statements : P : There exists some x IR such that f(x) + 2x = 2(1+x2) Q : There exists some x IR such that 2f(x) +1 = 2x(1+x) Then (A) both P and Q are true (B) P is true and Q is false (C) P is false and Q is true (D) both P and Q are false.

A. both P and Q are true

B. P is true and Q is false

C. P is false and Q is true

D. both P and Q are false

Answer: C

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21. Which of the following is true?

A. g is increasing on $(1, \infty)$

B. g is decreasing on $(1, \infty)$

C. g is increasing on $(1, 2)$ and decreasing on$(2, \infty)$

D. g is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

Answer: B

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22. For any real number $x, let[x]$ denote the largest integer less than or equal to $x, Let f$ be a real-valued function defined on the interval $[-10, 10]$ be

$f(x) = \{x - [x], \quad \text{if} \quad [x] is odd 1 + [x] - x, \quad \text{if} \quad [x] is even$ Then the value of $\frac{\pi^2}{10}\int_{-1}^{10} f(x)\cos \pi x dx is$ _ _

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23. Let f be a non-negative function defined on the interval .[0,1].If $\int_0^x \sqrt{1 - (f'(t))^2}. dt$=$\int_0^x f(t). dt, 0 \leq x \leq 1$ and f(0)=0,then

A. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

B. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

C. $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

D. $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Answer: C

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24. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}\,dx, n = 0, 1, 2, ...$ then which one of the following is not true ?

A. $I_n = I_{n+2}$

B. $\sum_{k=0}^{10} I_{2m+1} = 10\pi$

C. $\sum_{m=1}^{n} I_{2m} = 0$

D. $I_n = I_{n+1}$

Answer: A::B::C

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25. Let $S_n = \sum_{k=0}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$,for $n = 1, 2, 3, ..., $ then

A. $S_n < \frac{\pi}{3\sqrt{3}}$

B. $S_n > \frac{\pi}{3\sqrt{3}}$

C. $T_n < \frac{\pi}{3\sqrt{3}}$

D. $T_n > \frac{\pi}{3\sqrt{3}}$

Answer: D

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26. The Integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to: (2) (3) (4)

A. -1

B. -2

C. 2

D. 4

Answer: C

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27. Let $I_n = \int \tan^n x dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, Where C is a constant of integration, then the ordered pair (a, b) is equal to :

A. $\left(-\frac{1}{5}, 0\right)$

B. $\left(-\frac{1}{5}, 1\right)$

C. $\left(\frac{1}{5}, 0\right)$

D. $\left(\frac{1}{5}, -1\right)$

Answer: C

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28. $\lim_{n \to \infty} \left[\frac{(n+1)(n+2)...3n}{n^{2n}}\right]^{1/n}$ is equal to

A. $\frac{18}{e^4}$

B. $\frac{27}{e^2}$

C. $\frac{9}{e^2}$

D. None of these

Answer: B

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29. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)dx}$ is equal to: (1) 2 (2) 4 (3) 1 (4) 6

A. 2

B. 4

C. 1

D. 6

Answer: C

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30. The integral $\int_0^{\pi} \sqrt{1 + 4\sin\frac{x}{2}} - 4\sin\frac{x}{2}dx$ is equal to

A. $\pi - 4$

B. $\frac{2\pi}{3} - 4 - 4\sqrt{3}$

C. $4\sqrt{3} - 4$

D. $4\sqrt{3} - 4 - \pi/3$

Answer: D

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31. Statement I The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}} is \frac{\pi}{6}$

Statement II $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

A. Statement In is true, Statement II is true, Statement II is a true explanation for Statement I

B. Statement I is true , Statement II is true' Statement II is not a true explanation for Statement I

C. Statement I is true, Statement II is false

D. Statement I is false , Statement II is true

Answer: D

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32. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t| dt, x \in R,$ which are parallel to the line $y = 2x$, are equal to

(1) ± 2 (2) ± 3 (3) ± 4 (4) ± 1

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: A

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33. $I f g(x)=\int_0^x \cos^4 t dt, \text{ then } \ g(x+\pi)$ equals

A. $\dfrac{g(x)}{g(\pi)}$

B. $g(x)+g(\pi)$

C. $g(x)-g(\pi)$

D. $g(x).\ g(\pi)$

Answer: B

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34. The value of $\int_0^1 \dfrac{8\log(1+x)}{1+x^2}dx$ is

A. $\dfrac{\pi}{8}\log 2$

B. $\frac{\pi}{2}\log 2$

C. $\log 2$

D. $\pi \log 2$

Answer: D

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35. For $x\varepsilon\left(0, \frac{5\pi}{2}\right)$, definite $f(x) = \int_0^x \sqrt{t}\sin t dt$. Then f has

A. local minimum at π and 2π

B. local minimum at π and local minimum at 2π

C. local minimum at π and local minimum at 2π

D. local maximum at π and 2π

Answer: C

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36. Let $p(x)$ be a function defined on R such that $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1, p'(x) = p'(1 - x)$, for all$x \in [0, 1], p(0) = 1$ and $p(1)$ equals

A. $\sqrt{41}$

B. 21

C. 41

D. 42

Answer: B

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37. $\int_0^{\pi} [\cos x] dx$, [] denotes the greatest integer function , is equal to

A. $\frac{\pi}{2}$

B. 1

C. (-1)

D. $-\frac{\pi}{2}$

Answer: D

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38. Let $I=\int_0^1 \frac{\sin x}{\sqrt{x}}dx \text{ and } f=\int_0^1 \frac{\cos x}{\sqrt{x}}dx$

Then , which one of the following is true ?

A. $I>\frac{2}{3} \text{ and } f>2$

B. $I<\frac{2}{3} \text{ and } f<2$

C. $I<\frac{2}{3} \text{ and } f>2$

D. $I>\frac{2}{3} \text{ and } f<2$

Answer: B

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