MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

DETERMINANTS

Examples

1. Find the value of the determinant

$$\begin{vmatrix} 1 & 2 & 4 \\ 3 & 4 & 9 \\ 2 & -1 & 6 \end{vmatrix}$$

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2. If $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} \text{prove that} 2 \leq \Delta \leq 4.$

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3. Expand $\begin{vmatrix} 3 & 2 & 5 \\ 9 & -1 & 4 \\ 2 & 3 & -5 \end{vmatrix}$ by Sarrus rule.

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4. If a,b,c, $\in$ R, find the number of real root of the equation $\begin{vmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{vmatrix}$=0

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5. Expand $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 6 & 2 \\ 5 & 9 & 4 \end{vmatrix}$

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6. Find the value of the determinant

$$\begin{vmatrix} -1 & 2 & 1 \\ 3+2\sqrt{2} & 2+2\sqrt{2} & 1 \\ 3-2\sqrt{2} & 2-2\sqrt{2} & 1 \end{vmatrix}$$

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7. Find the determinants of minors and cofactors of the determinant

$$\begin{vmatrix} 2 & 3 & 4 \\ 7 & 2 & -5 \\ 8 & -1 & 3 \end{vmatrix}$$

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8. Find the determinants of minors of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$

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9. If the value of a third order determinant is 11, find the value of the square of the determinat formed by the cofactors.

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10. Evaluate $\begin{vmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{vmatrix}$.

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11. Prove that $\begin{vmatrix} \alpha & \beta & \gamma \\ \theta & \phi & \psi \\ \lambda & \mu & \nu \end{vmatrix} = \begin{vmatrix} \beta & \mu & \phi \\ \alpha & \lambda & \theta \\ \gamma & \nu & \psi \end{vmatrix}$.

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12. Use the properties of determinant and without expanding prove that

$$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}.$$

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13. Without expanding as far as possible, prove that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix}$ = $(x-y)(y-z)(z-x)(x+y+z)$.

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14. Solve for x,

$\begin{vmatrix} 4x & 6x+2 & 8x+1 \\ 6x+2 & 9x+3 & 12x \\ 8x+1 & 12x & 16x+2 \end{vmatrix}$=0

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15. Prove that

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2.$$

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16. If a,b, and c are all different and if

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}$$ =0 Prove that abc =-1.

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17. The largest value of a third order determinant whose elements are equal to 1 or 0 is

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18. Find the largest value of a third order determinant whose elements are 0 or -1.

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19. Show that the value of a third order determinant whose all elements are 1 or -1 is an even number.

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20. Evaluate $\begin{vmatrix} 1 & 2 & 3 \\ -2 & 3 & 2 \\ 3 & 4 & -4 \end{vmatrix} \times \begin{vmatrix} -2 & 1 & 3 \\ 3 & -2 & 1 \\ 2 & 1 & -2 \end{vmatrix}$. Using the concept of multiplication of determinants.

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21. If

$ax12 + by12 + cz12 = ax22 + by22 + cz22 = ax32 + by32 + cz32 = d, ax$

then prove that $|x_1 y_1 z_1 x_2 y_2 z_2 x_3 y_3 z_3| = (d - f)\left\{\frac{(d + 2f)}{abc}\right\}^{1/2}$

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22. Prove that $\begin{vmatrix} a_1\alpha_1+b_1\beta_1 & a_1\alpha_2+b_1\beta_2 & a_1\alpha_3+b_1\beta_3 \\ a_2\alpha_1+b_2\beta_1 & a_2\alpha_2+b_2\beta_2 & a_2\alpha_3+b_2\beta_3 \\ a_3\alpha_1+b_3\beta_1 & a_3\alpha_2+b_3\beta_2 & a_3\alpha_3+b_3\beta_3 \end{vmatrix}$=0.

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23. Prove that

$$\begin{vmatrix} 2 & \alpha+\beta+\gamma+\delta & \alpha\beta+\gamma\delta \\ \alpha+\beta+\gamma+\delta & 2(\alpha+\beta)(\gamma+\delta) & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) \\ \alpha\beta+\gamma\delta & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) & 2\alpha\beta\gamma\delta \end{vmatrix}=0$$

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24. Prove that

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix}=0.$$

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25. If $\alpha, \beta \text{and} \gamma$ are real number without expanding at any stage prove that

$$\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix} = 0.$$

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26. Solve the following system of equation by Cramer's rule.

x+y=4 and 3x-2y=9

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27. Solve the following system of equation by Cramer's rule.

x+y+z=9

2x+5y+7z=52

2x+y-z=0

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28. For what values of p and q the system od equations

x+y+z=6

x+2y+3z=10

x+2y+pz=q has

(i) unique sollution ?

(ii) an infinitely many solutions ?

(iii) no solution ?

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29. If the following equations

$x+y-3=0, (1+\lambda)x+(2+\lambda)y-8=0, x-(1+\lambda)y+(2+\lambda)=0$

are consistent then the value of λ is

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30. Find all values of λ for which the

$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=02x$

possess non-trivial solution and find the ratios $x : y : z$, where λ has the smallest of these value.

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31. Given x=cy+bz,y=az+cx and z=bx+ay, then prove $a^2 + b^2 + c^2$+2abc =1.

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32. If f(x) = $\begin{vmatrix} \sin x & \cos x & \sin x \\ \cos x & -\sin x & \cos x \\ x & 1 & 1 \end{vmatrix}$ find the value of $2(f'(o)) + \{f'(1)\}^2$

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33. Let f(x)=$\begin{vmatrix} \cos x & \sin x & \cos x \\ \cos 2x & \sin 2x & 2\cos 2x \\ \cos 3x & \sin 3x & 3\cos 3x \end{vmatrix}$ then find the value of $f'\left(\frac{\pi}{2}\right)$.

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34. Let α be a repeated root of a quadratic equation $f(x) = 0 and A(x), B(x), C(x)$ be polynomials of degrees 3, 4, and 5, respectively, then show that $|A(x)B(x)C(x)A(\alpha)B(\alpha)C(\alpha)A'(\alpha)B'(\alpha)C'(\alpha)|$ is divisible by $f(x)$, where prime ($'$) denotes the derivatives.

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35. If $(x) = |\alpha + x\theta + x\lambda + x\beta + x\varphi + x\mu + x\gamma + x\psi + xv + x|$ show that $\text{Delta}\hat{}(x) = 0 and \text{Delta}(0) + Sx, where S$ denotes he sum of all the cofactors of all elements in $\text{Delta}(0) and$ dash denotes the derivative with respect of $x\cdot$

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36. If $f(x) = \begin{vmatrix} x^n & \sin x & \cos x \\ n! & \sin\frac{n\pi}{2} & \cos\frac{n\pi}{2} \\ a & a^2 & a^3 \end{vmatrix}$, then the value of $\frac{d^n}{dx^n}(f(x))$ at $x = 0$ for $n = 2m + 1$ is

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37. If $\Delta(x) = \begin{vmatrix} a & b & c \\ 6 & 4 & 3 \\ x & x^2 & x^3 \end{vmatrix}$ then find $\int_0^1 \Delta(x)$dx.

38. Let f(x) = $\begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \cos ecx \\ \cos^2 x & \cos^2 x & \cos ec^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$ then find the value of $f_0^{\pi/2}$ f(x) dx.

39. If for any +ve integer n

$\Delta_n = [[2r - 1,^n C_r, 1], [n^2 - 1, 2^n, n + 1], [\cos^2(n^2), \cos^2(n), \cos^2(n + 1)$

then $\sum_{r=0}^{n} \Delta_r, n \in N$, is equal to

40. Let n be a positive integer and

$$\Delta_r = \begin{vmatrix} r^2 + r & r+1 & r-2 \\ 2r^2 + 3r - 1 & 3r & 3r - 3 \\ r^2 + 2r + 3 & 2r - 1 & 2r - 1 \end{vmatrix}$$ and

$\sum_{r=1}^{n} \Delta_r = an^2 + bn + c$ find the value of a+b+c.

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41. if

$(x_1, x_2)^2 + (y_1 - y_2)^2 = a^2, (x_2 - x_3)^2 + (y_2 - y_3)^2 = b^2(x_3 - x_1)^2 + (y_3$

where a,b,c are positive then prove that

$$4\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = (a+b+c)(b+c-a)(c+a-b)(a+b-c)$$

A. 1

B. 2

C. 4

D. 8

Answer:

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42. If a,b and c are complex numbers the determinant

$$\Delta\begin{vmatrix}0 & -b & -c \\ \bar{b} & 0 & -a \\ \bar{c} & \bar{a} & 0\end{vmatrix}$$ is

A. a non-zero real number

B. purely imaginary

C. 0

D. None of these

Answer:

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43. The equation

$$\begin{vmatrix}(1+x)^2 & (1-x)^2 & -(2+x^2) \\ 2x+1 & 3x & 1-5x \\ x+1 & 2x & 2-3x\end{vmatrix}+\begin{vmatrix}(1+x)^2 & 2x+1 & x+1 \\ (1-x)^2 & 3x & 2x \\ 1-2x & 3x-2 & 2x-3\end{vmatrix}=0$$

has has (a) no real solution (b) 4 real solutions (c) two real and two non-real solutions (d) infinite number of solutions, real or non-real

A. no real solution

B. 4 real solution

C. two real and two non-real solutions

D. infinite number of solution real or non-real

Answer:

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44. If X, Y and Z are positive numbers such that Y and Z have respectively 1 and 0 at their unit's place and Δ is the determinant

$$\begin{vmatrix} X & 4 & 1 \\ Y & 0 & 1 \\ Z & 1 & 0 \end{vmatrix}$$

If $(\Delta + 1)$ is divisible by 10, then x has at its unit's place

A. 0

B. 1

C. 2

D. 3

Answer:

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45. The number of distinct values of $a 2 \times 2$ determinant whose entries are from the set {-1,0,1}, is

A. 3

B. 4

C. 5

D. 6

Answer:

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46. If f(x)= $\begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$, a,b being positive integers then

A. constant term in f(x) is 4

B. constant term in f(x) is 0

C. constant term in f(x) is (a-b)

D. constant term in f(x) is (a+b)

Answer:

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47. Let $\Delta_a = \begin{vmatrix} (a-1) & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$ the value of $\sum_{a=1}^{n} \Delta_a$ is

A. 0

B. $\dfrac{(n-1)n}{2}$

C. $\dfrac{(n-1)n^2}{2}$

D. $\dfrac{(n-1)n(2n-1)}{3}$

Answer:

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48. If $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x-\cos x \\ \sin x & \sin x & 1 \end{vmatrix}$ then $\int_0^{\pi/2} \Delta(x)\, dx$ is equal to

A. $-\dfrac{1}{2}$

B. 0

C. $\dfrac{1}{4}$

D. $\dfrac{1}{2}$

Answer: A

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49. Number of values of a for which the system of equations $a^2 + (2-a)y = 4 + a^2$ and ax +(2a-1)y=a^5-2 possess no solution is

A. 0

B. 1

C. 2

D. infinite

Answer:

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50. The value of determinant $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ is

A. a+b+c

B. (a+b)(b+c)(c+a)

C. $a^2 + b^2 + c^2$

D. (a-b) (b-c) (c-a)

Answer:

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51. The value of θ lying between $-\frac{\pi}{4}$and $\frac{\pi}{2}$ and $0 \leq A \leq \frac{\pi}{2}$ and satisfying the equation

$\begin{vmatrix} 1+\sin^2 A & \cos^2 A & 2\sin 4\theta \\ \sin^2 A & 1+\cos^2 A & 2\sin 4\theta \\ \sin^2 A & \cos^2 A & 1+2\sin 4\theta \end{vmatrix}$ =0 are

A. $A = \frac{\pi}{4}, \theta = -\frac{\pi}{8}$

B. $A = \frac{3\pi}{8} = \theta$

C. $A = \frac{\pi}{5}, \theta = -\frac{\pi}{8}$

D. $A = \frac{\pi}{6}, \theta = -\frac{3\pi}{8}$

Answer:

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52. The digits A,B,C are such that the three digit numbers A88, 6B8, 86 C are divisible by 72 the determinant

$\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by

A. 72

B. 144

C. 288

D. 216

Answer:

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53. If p,q,r and s are in AP and f(x) = $\begin{vmatrix} p+\sin x & q+\sin x & p-r+\sin x \\ q+\sin x & r+\sin x & -1+\sin x \\ r+\sin x & s+\sin x & s-q+\sin x \end{vmatrix}$

such that $\int_0^1$f(x) dx =-2, the common difference of the AP can be

A. -1

B. $1/2$

C. 1

D. 2

Answer:

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54. If the system of equation $a^2x - by = a^2 - b$ and $bx - b^2y = 2 + 4b$

possess an infinite number of solution , the possible values of a and are

A. a=1,b=-1

B. a=1,b=-2

C. a=-1,b=-1

D. a=-1,b=-2

Answer:

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55. If $\Delta_r = \begin{vmatrix} r & r-1 \\ r-1 & r \end{vmatrix}$ where is a natural number, the value of $\sqrt[10]{\sum_{r=1}^{1024} \Delta_r}$ is

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56. If P,Q and R are the angles of a triangle the value of $\begin{vmatrix} \tan P & 1 & 1 \\ 1 & \tan Q & 1 \\ 1 & 1 & \tan R \end{vmatrix}$ is

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57. Let f(x) denotes the determinant

$$f(x)\begin{vmatrix} x^2 & 2x & 1+x^2 \\ x^2+1 & x+1 & 1 \\ x & -1 & x-1 \end{vmatrix}$$

On expansion g(x) is seen to be a 4th degree polynomial given by f(X) =

$a_0x^4 + a_1x^3 + a_2x^2 + a_3xa_4.$

Using differentiation of determinant or otherwise match the entries in Column I with one or more entries of the elements of Column II.

	Column I		Column II
(A)	$a_0^2 + a_1$ is divisible by	(p)	2
(B)	$a_2^2 + a_4$ is divisible by	(q)	3
(C)	$a_0^2 + a_2$ is divisible by	(r)	4
(D)	$a_4^2 + a_3^2 + a_1^2$ is divisible by	(s)	5

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58. Suppose a, b and c are distinct and x,y and z are connected by the system of eqations

$x + ay + a^2 z = a^3, x + by + b^2 z = b^3 \text{and} x + cy + c^2 z = c^3.$

	Column I	Column II	
(A)	For $x = 1, y = 2$ and $z = 3, (a + b + c)^{-(ab + bc + ca)}$ is divisible by	(p)	3
(B)	For $x = 4, y = 3$ and $z = 2, (ab + bc + ca)^{abc}$ is divisible by	(q)	6
(C)	For $x = 6, y = 4$ and $z = 2, (abc)^{a + b + c}$ is divisible by	(r)	9
		(s)	12

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59. Consider the determinant

$$f(x) = \begin{vmatrix} 0 & x^2 - a & x^3 - b \\ x^2 + a & 0 & x^2 + c \\ x^4 + b & x - c & 0 \end{vmatrix}$$

Statement -1 f(x) =0 has one root x =0.

Statement -2 The value of skew -symmetric determinant of odd order is always zero.

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60. A determinant fo second order is made with the elements 0 and 1. Find the number of determinants with non-negative values.

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61. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ =abc $\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ hence find the value of the determinant, if a,b, and c are the roots of the equation $px^3+qx^2+rx+s=0$.

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62. If a,b,and c are positive and are the pth, qth and rth terms respectively of a GP. Show without expanding that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

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63. Prove that:

$$| - 2aa + ba + cb + a - 2 + \; + ac + b - 2c| = 4(a + b)(b + c)(c + a)$$

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64. If bc +qr=ca+rp=ab+pq=-1,

show that $\begin{vmatrix} ap & bp & cr \\ a & b & c \\ p & q & r \end{vmatrix} = 0$

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65. If $\alpha \text{and} \beta$ are the roots of the equations

$x^2 - 2x + 4 = 0$,find the value of $\begin{vmatrix} \sum \alpha & \sum \alpha^2 & \sum \alpha^3 \\ \sum \alpha^2 & \sum \alpha^3 & \sum \alpha^4 \\ \sum \alpha^3 & \sum \alpha^4 & \sum \alpha^5 \end{vmatrix}$.

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66. If $a^2+b^2+c^2=1$, then

$$\begin{vmatrix} a^2+(b^2+c^2)\cos\phi & ab(1-\cos\phi) & ac(1-\cos\phi) \\ ba(1-\cos\phi) & b^2+(c^2+a^2)\cos\phi & bc(1-\cos\phi) \\ ca(1-\cos\phi) & cb(1-\cos\phi) & c^2+(a^2+b^2)\cos\phi \end{vmatrix}$$ is

independent of

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67. If a $\neq 0$ and a $\neq 1$, show that

$$\begin{vmatrix} x+1 & x & x \\ x & x+a & x \\ x & x & x+a^2 \end{vmatrix} = a^3\left[1+x\frac{(a^3-1)}{a^2(a-1)}\right].$$

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68. If a $\neq 0$ and a $\neq 1$, show that

$$\begin{vmatrix} x+1 & x & x \\ x & x+a & x \\ x & x & x+a^2 \end{vmatrix} = a^3\left[1+x\frac{(a^3-1)}{a^2(a-1)}\right].$$

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69. If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$ then evaluate $\begin{vmatrix} 3 & 1+s_1 & +s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix}$

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70. Without expanding at any stage, evaluate the value of the determinant

$$\begin{vmatrix} 2 & \tan A \cot B + \cot A \tan B & \tan A \cot C + \cot \\ \tan B \cot A + \cot B \tan A & 2 & \tan B \cot C + \cot \\ \tan C \cot A + \cot C \tan A & \tan B \cot C + \cot B \tan C & 2 \end{vmatrix}$$

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71. Suppose that digit numbers A28,3B9 and 62 C, where A,B and C are integers between 0 and 9 are divisible by a fixed integer k, prove that the determinant $\begin{vmatrix} A & 3 & 6 \\ 8 & 9 & C \\ 2 & B & 2 \end{vmatrix}$ is also divisible by k.

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72. If $\Delta = \begin{vmatrix} \sin x & \sin(x+h) & \sin(x+2h) \\ \sin(x+2h) & \sin x & \sin(x+h) \\ \sin(x+h) & \sin(x+2h) & \sin x \end{vmatrix}$

find $\lim_{h \to 0} \left(\frac{\Delta}{h^2}\right)$.

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73. If $f(x) = \begin{vmatrix} x+c_1 & x+a & x+a \\ x+b & x+c_2 & x+a \\ x+b & x+b & x+c_3 \end{vmatrix}$ then show that f(x) is linear in x.

Hence deduce $f(0) = \frac{bg(a) - ag(b)}{(b-a)}$ where

$g(x) = (c_1 - x)(c_2 - x)(c_3 - x)$

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74. If f(a,b) $= \frac{f(b) - f(a)}{b - a}$ and

$f(a, b, c) = \frac{f(b, c) - f(a, b)}{c - a}$ prove that

$f(a, b, c) = \begin{vmatrix} f(a) & f(b) & f(c) \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix} \div \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$.

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75. the value of the determinant

$$\begin{vmatrix} (a_1 - b_1)^2 & (a_1 - b_2)^2 & (a_1 - b_3)^2 & (a_1 - b_4)^2 \\ (a_2 - b_1)^2 & (a_2 - b_2)^2 & (a_2 - b_3)^2 & (a_3 - b_4)^2 \\ (a_3 - b_1)^2 & (a_3 - b_2)^2 & (a_3 - b_3)^2 & (a_3 - b_4)^2 \\ (a_4 - b_1)^2 & (a_4 - b_2)^2 & (a_4 - b_3)^2 & (a_4 - b_4)^2 \end{vmatrix}$$ is

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Single Option Correct Type Questions

1. If f(x) is a polynomial of degree $n(\,>2)$ and $f(x) = f(\alpha - x)$,

(where α is a fixed real number), then the degree of f'(x) is

A. 6

B. 10

C. 14

D. 18

Answer:

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2. If $g(x) - \dfrac{f(x)}{(x-a)(x-b)(x-c)}, where f(x)$ is a polynomial of degree $< 3,$ then prove that $\dfrac{dg(x)}{dx} = \left|1af(a)(x-a)^{-2}1bf(b)(x-b)^{-2}1cf(c)(x-c)^{-2}\right| + \left|a^2a1b^2b1c\right.$

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Exercise For Session 1

1. Sum of real roots of the euation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ is

A. -2

B. -1

C. 0

D. 1

Answer: D

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2. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy, i = \sqrt{-1}$ then

A. x=3, y=1

B. x=1, y=3

C. x=0,y=3

D. x=0, y=0

Answer: D

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3. If $p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t = \begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda + 1 & 2 - \lambda & \lambda - 4 \\ \lambda - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$, then value of t is

A. 7

B. 14

C. 21

D. 28

Answer: C

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4. If one root of the equation $\begin{vmatrix} 7 & 6 & x^2 - 13 \\ 2 & x^2 - 13 & 2 \\ x^2 - 13 & 3 & 7 \end{vmatrix}$ =0 is x=2 the sum of all other five roots is

A. $2\sqrt{15}$

B. -2

C. $\sqrt{20}+\sqrt{15}-2$

D. None of these

Answer: B

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5. If A,B and C are the angle of a non-right angled ΔABC the value of $\begin{vmatrix} \tan A & 1 & 1 \\ 1 & \tan B & 1 \\ 1 & 1 & \tan C \end{vmatrix}$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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6. If $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, the maximum value of Δ is

A. -10

B. $-\sqrt{10}$

C. $\sqrt{10}$

D. 10

Answer: D

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7. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive then

A. abc $>$ 1

B. abc $>$ -8

C. abc $<$ -8

D. abc $>$ -2

Answer: B

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Exercise For Session 2

1. If $\lambda \text{and} \mu$ are the cofactors of 3 and -2 respectively in the determinant $\begin{vmatrix} 1 & 0 & -2 \\ 3 & -1 & 2 \\ 4 & 5 & 6 \end{vmatrix}$ the value of $\lambda + \mu$ is

A. 5

B. 7

C. 9

D. 11

Answer: C

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2. If a, b and c are distinct and $D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$. then the square of the determinant of its cofactors is divisible by

A. $\left(a^2 + b^2 + c^2\right)^2$

B. $(ab + bc + ca)^2$

C. $(a + b + c)^2$

D. $(a + b + c)^4$

Answer: D

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3. An equilateral triangle has each of its sides of length 4 cm. If (x_r, y_r) (r=1,2,3) are its vertices the value of $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

A. 192

B. 768

C. 1024

D. 128

Answer: A

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4. If the lines ax+y+1=0, x+by+1=0 and x+y+c=0 (a,b and c being distinct and different from 1) are concurrent the value of $\frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1}$ is

A. 0

B. 1

C. 2

D. 3

Answer: C

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5. If p + q + r = a + b + c = 0, then the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ equals

A. 0

B. pa+qb+rc

C. 1

D. None of these

Answer: A

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6. If p,q and r are in AP the value of determinant $\begin{vmatrix} a^2 + 2^{n+1} + 2p & b^2 + 2^{n+2} + 2q & c^2 + p \\ 2^n + p & 2^{n+1} + q & 2q \\ a^2 + 2^n + p & b^2 + 2^{n+1} + q & c^2 - r \end{vmatrix}$ is

A. 1

B. 0

C. $a^2 + b^2 + c^2 - 2^n$

D. $\left(a^2 + b^2 + c^2\right) - 2^n$q

Answer: B

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7. Let $\{D_1, D_2, D_3, \cdots, D_n\}$ be the set of third order determinants that can be made with the distinct non-zero real numbers $a_1, a_2, \cdots, a_9$. Then ,

A. $\sum_{i=1}^{n} D_i = 1$

B. $\sum_{i=1}^{n} D_i = 0$

C. $D_i = D_j, \forall i, j$

D. None of these

Answer: B

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8. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$ then x is equal to

A. 0

B. -9

C. 3

D. None of these

Answer: B

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9. If a+b+c=0 the one root of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}$ =0 is

A. 1

B. 2

C. $a^2+b^2+c^2$

D. 0

Answer: D

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10. If $a^2+b^2+c^2$=-2 and f(x)= $\begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$ the f(x) is a polynomial of degree

A. 0

B. 1

C. 2

D. 3

Answer: C

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11. If a,b,c,d,e and f are in GP the value of $\begin{vmatrix} a^2 & d^2 & x \\ b^2 & e^2 & y \\ c^2 & f^2 & z \end{vmatrix}$ is

A. depends on x and y

B. depends on x and z

C. dependes on y and z

D. independent of x,y and z

Answer: D

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Exercise For Session 3

1. Number of second order determiants which have maximum values whose each entry is either -1 or 1 is equal to

A. 2

B. 4

C. 6

D. 8

Answer: B

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2. Minimum value of a second order determinant whose each is either 1 or 2 is equal to

A. 0

B. -1

C. -2

D. -3

Answer: C

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3. If $l_i^2 + m_i^2 + n_i^2 = 1$, (i=1,2,3) and $l_i l_j + m_i m_j + n_i n_j = 0, (i \neq j, i, j = 1, 2, 3)$ and $\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$ then

A. $|\Delta|$=3

B. $|\Delta|$=2

C. $|\Delta|$=1

D. $|\Delta|$=0

Answer: C

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4. Let $A = [a_{ij}]$ be a 3×3 matrix and let A_1 denote the matrix of the cofactors of elements of matrix A and A_2 be the matrix of cofactors of elements of matrix A_1 and so on. If A_n denote the matrix of cofactros of elements of matrix A_{n-1}, then $|A_n|$ equals

A. Δ_0^{2n}

B. Δ_0^{2n}

C. $\Delta_0^{n^2}$

D. Δ_0^2

Answer: B

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5. If $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}$ =3 then the value of $\begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$ is

A. 6

B. 9

C. 18

D. 27

Answer: B

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6. the value of the determinant

$$\begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 & (a_1-b_4)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 & (a_3-b_4)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 & (a_3-b_4)^2 \\ (a_4-b_1)^2 & (a_4-b_2)^2 & (a_4-b_3)^2 & (a_4-b_4)^2 \end{vmatrix}$$ is

A. depends on a_i, i=1,2,3,4

B. depends on b_i,i=1,2,3,4

C. dependes on c_i,i=1,2,3,4

D. 0

Answer: D

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7. Value of $\begin{vmatrix} 1+x_1 & 1+x_1x & 1+x_1x^2 \\ 1+x_2 & 1+x_2x & 1+x_2x^2 \\ 1+x_3 & 1+x_3x & 1+x_3x^2 \end{vmatrix}$ depends upon

A. only x

B. only x_1

C. only x_2

D. None of these

Answer: D

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8. If the system of linear equations x+y+z=6, x+2y+3z=14 and 2x +5y+ $\lambda z = \mu(\lambda, \mu \neq$ R) has a unique solution if λ is

A. $\lambda \neq 8$

B. $\lambda = 8 \text{and} \mu \neq 36$

C. $\lambda = 8 \text{and} \mu = 36$

D. None of these

Answer: A

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9. The system of equations ax-y- z=a-1 ,x-ay-z=a-1,x-y-az=a-1 has no solution if a is

A. either -2 or 1

B. -2

C. 1

D. not(-2)

Answer: B::C

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10. The system of equations x+2y-4z=3,2x-3y+2z=5 and x -12y +16z =1 has

A. inconsistent solution

B. unique solution

C. infinitely many solutions

D. None of these

Answer: C

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11. If c $<$ 1 and the system of equations x+y-1=0 2x-y-c=0 and -bx+3by-c =0 is consistent then the possible real values of b are

A. $b \in \left(-3, \frac{3}{4}\right)$

B. $b \in \left(-\frac{3}{2}, 1\right)$

C. $b \in \left(-\frac{3}{4}, 3\right)$

D. None of these

Answer: B

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12. The equation $x+2y=3, y-2x=1$ and $7x-6y+a=0$ are consisten for

A. $a=7$

B. $a=1$

C. $a=11$

D. None of these

Answer: A

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13. Values of k for which the system of equation $x+ky+3z=0$, $kx+2y+2z=0$ and $2x+3y+4z=0$ possesses non-trivial solution

A. $\left\{2, \frac{5}{4}\right\}$

B. $\left\{2, \frac{5}{4}\right\}$

C. $\left\{2, -\frac{5}{9}\right\}$

D. $\left\{-2, -\frac{5}{4}\right\}$

Answer: A

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Exercise For Session 4

1. If f(x)=$\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2 & 3 \\ 0 & 1 & x \end{vmatrix}$ $\lim_{x \to 1} f(x)$ is equal to

A. -1

B. 0

C. 1

D. 2

Answer: A

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2. Let f(x) $= \begin{vmatrix} \sec x & x^2 & x \\ 2\sin x & x^3 & 2x^2 \\ \tan 3x & x^2 & x \end{vmatrix}$. find $\lim_{x \to 0} \frac{f(x)}{x^4}$ is equal to

A. 0

B. -1

C. 2

D. 3

Answer: B

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3. Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E$ the value of 5A+4B+3C+2D+E is equal to

A. -16

B. -11

C. 0

D. 16

Answer: B

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4. Let f(x) = $\begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$ where p is a constant. Then $\frac{d^3}{dx^3}\{f(x)\}$ at x=0 is

A. p

B. p+p^2

C. p+p^3

D. independent of p

Answer: D

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5. If y=sin mx the value of the determinant $\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix}$ where $y_n = \frac{d^n y}{dx^n}$ is

A. m^2

B. m^3

C. m^9

D. None of these

Answer: D

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6. Let $f(x) = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, then the value of $\int_0^{\pi/2} \{f(x) + f'(x)\} dx$ is

A. $\frac{\pi}{2}$

B. π

C. $\frac{2\pi}{2}$

D. 2π

Answer: B

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7. If f(x) = $\begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2\left(\frac{x}{2}\right) \\ x^2 & \sec x & \sin x + x \\ 1 & 2 & x + \tan x \end{vmatrix}$ the value of $f_{-\pi/2}^{\pi/2}(x^2+1)$ [f(x)+f''(x)]dx is

A. -1

B. 0

C. 1

D. 2

Answer: B

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8. If f(x) = $\begin{vmatrix} \sin^2 x + & \cos^4 x In \cos x & \frac{1}{1+(\tan x)^{\sqrt{2}}} \\ \pi & \pi^2 & \pi^4 \\ \frac{7}{16} & -\frac{1}{2} In 2 & \frac{1}{4} \end{vmatrix}$ the value of $\int_0^{\pi/2}$f(x) dx is

A. 2

B. -1

C. 0

D. None of these

Answer: C

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9. if $D_k = \begin{vmatrix} 1 & n & n \\ 2k & n^2+n+1 & n^2+n \\ 2k-1 & n^2 & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 56$

then n equals

A. 4

B. 6

C. 8

D. None of these

Answer: D

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10. The value of $\sum_{r=2}^{n} (-2)^r \begin{vmatrix} n-2C_{r-2} & n-2C_{r-1} & n-2C_r \\ -3 & 1 & 1 \\ 2 & -1 & 0 \end{vmatrix} (n > 2)$

A. $2n-1+(-1)^n$

B. $2n + 1 + (-1)^n$

C. $2n - 3 + (-1)^n$

D. None of these

Answer: A

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Exercise Single Option Correct Type Questions

1. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $|31 + f(1)1 + f(2)1 + f(1)1 + f(2)1 + f(3)1 + f(2)1 + f(3)1 + f(4)| =$, then K is equal to (1) $\alpha\beta$ (2) $\frac{1}{\alpha\beta}$ (3) 1 (4) -1

A. 1

B. -1

C. $\alpha\beta$

D. $\alpha\beta\gamma$

Answer: A

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2. Let $\Delta(x) = \begin{vmatrix} x+a & x+b & x+a-c \\ x+b & x+c & x-1 \\ x+c & x+d & x-b+d \end{vmatrix}$ and $\int_0^2 \Delta(x)dx = -16,$ where a, b, c, d are in A.P. then the common difference (i) 1 (ii)2 (iii)3 (iv)4

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: B

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3. If $\Delta(x) = \begin{vmatrix} x & 1+x^2 & x^3 \\ \log(1+x^2) & e^x & \sin x \\ \cos x & \tan x & \sin^2 x \end{vmatrix}$ then

A. Δ(x) is divisible by x

B. Δ (x)=0

C. Δ'(x)=0

D. None of these

Answer: A

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4. If a, b, c are sides of a triangle and $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} =$ then

A. Δ ABC is an equilateral triangle

B. Δ ABC is a right angled isosceles triangle

C. Δ ABC is an isosceles triangle

D. None of the above

Answer: C

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5. If $\begin{vmatrix} \alpha & x & x & x \\ x & \beta & x & x \\ x & x & \gamma & x \\ x & x & x & \delta \end{vmatrix}$ =f(x)-xf'(x) then f(x) is equal to

A. $(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

B. $(x+\alpha)(x+\beta)(x+\gamma)(x+\delta)$

C. $2(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)$

D. None of the above

Answer: A

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6. If $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = 0$ the line ax +by +c=0 passes through the fixed point which is

A. (1,2)

B. (1,1)

C. (-2,1)

D. (1,0)

Answer: B

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7. If f(x) $= a + bx + cx^2$ and $\alpha, \beta \text{and} \gamma$ are the roots of the equation $x^3 = 1$, then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is equal to

A. $f(\alpha) + f(\beta) + f(\gamma)$

B. $f(\alpha)f(\beta) + f(\beta)f(\gamma) + f(\gamma)f(\alpha)$

C. $f(\alpha)f(\beta)f(\gamma)$

D. $-f(\alpha)f(\beta)f(\gamma)$

Answer: D

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8. When the determinant $\begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix}$ is expanded in powers of sin x , the constant term in than or equal to expression is

A. 1

B. 0

C. -1

D. 2

Answer: C

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9. If [] denotes the greatest integer less than or equal to the real number under consideration and $-1 \leq x < 0, 0 \leq y < 1, 1 \leq z < 2,$ the value of the determinant $\begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is

A. [x]

B. [y]

C. [z]

D. None of these

Answer: C

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10. The determinant $\begin{vmatrix} y^2 & -xy & x^2 \\ a & b & c \\ a' & b' & c' \end{vmatrix}$ is equal to

A. $\begin{vmatrix} bx+ay & cx+by \\ b'x+a'y & c'x+b'y \end{vmatrix}$

B. $\begin{vmatrix} a'x+b'y & bx+cy \\ ax+by & b'x+c'y \end{vmatrix}$

C. $\begin{vmatrix} bx+cy & ax+by \\ b'x+c'y & a'x+b'y \end{vmatrix}$

D. $\begin{vmatrix} ax+by & bx+cy \\ a'x+b'y & b'x+c'y \end{vmatrix}$

Answer: D

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11. If A,B and C are angle of a triangle of a triangle ,the value of $\begin{vmatrix} e^{2iA} & e^{-iC} & e^{-iB} \\ e^{-iC} & e^{2iB} & e^{-iA} \\ e^{-iB} & e^{-iA} & e^{2iC} \end{vmatrix}$ is (where i=$\sqrt{-1}$)

A. 1

B. -1

C. -2

D. -4

Answer: D

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12. If $\begin{vmatrix} x^n & x^{n+2} & x^{2n} \\ 1 & x^a & a \\ x^{n+5} & x^{a+6} & x^{2n+5} \end{vmatrix}$ =0 $\forall x \in \mathrm{R}$ where $\mathrm{n} \in$ N, the value of a is

A. n

B. n-1

C. n+1

D. None of these

Answer: C

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13. If x,y and z are the integers in AP lying between 1 and 9 and x 51, y 41 and z 31 are three digits number the value of $\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$ is

A. x+y+z

B. x-y+z

C. 0

D. None of these

Answer: C

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14. If $a_1, b_1, c_1, a_2, b_2, c_2 \text{and} a_3, b_3, c_3$ are three digit even natural numbers and $\Delta = \begin{vmatrix} c_1 & a_1 & b_1 \\ c_2 & a_2 & b_2 \\ c_3 & a_3 & b_3 \end{vmatrix}$, then Δ is

A. divisible by 2 but not necessarily by 4

B. divisible by 4 but not necessarily by 8

C. divisible by 8

D. None of these

Answer: A

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15. If a,b,and c are sides of Δ ABC such that

$$\begin{vmatrix} c & b\cos B+\alpha\beta & a\cos A+b\alpha+c\gamma \\ a & c\cos B+a\beta & b\cos A+c\alpha+a\gamma \\ b & a\cos B+b\beta & c\cos A+a\alpha+b\gamma \end{vmatrix}=0$$

(where $\alpha, \beta, \gamma, \in R^{+}\text{and}\angle A, \angle B, \angle C \neq \frac{\pi}{2}\Big)$, Δ ABC is

A. an isosceles

B. an equilateral

C. can 't say

D. None of these

Answer: B

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16. If $x_1, x_2\text{and}y_1, y_2$ are the roots of the equations

$3x^2-18x+9=0\text{and}y^2-4y+2=0$ the value of the determinant

$$\begin{vmatrix} x_1x_2 & y_1y_2 & 1 \\ x_1+x_2 & y_1+y_2 & 2 \\ \sin(\pi x_1x_2) & \cos(\pi/2y_1y_2) & 1 \end{vmatrix}$$ is

A. 0

B. 1

C. 2

D. None of these

Answer: A

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17. The value of $\begin{vmatrix} .^{10}C_4 . ^{10}C_5 . ^{11}C_m \\ .^{11}C_6 . ^{11}C_7 . ^{12}C_{m+2} \\ .^{12}C_8 . ^{12}C_9 . ^{13}C_{m+4} \end{vmatrix}$ is equal to zero when m is

A. 6

B. 4

C. 5

D. None of these

Answer: C

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18. The value of the determinant $\begin{vmatrix} 1 & \sin(\alpha-\beta)\theta & \cos(\alpha-\beta)\theta \\ a & \sin\alpha\theta & \cos\alpha\theta \\ a^2 & \sin(\alpha-\beta)\theta & \cos(\alpha-\beta)\theta \end{vmatrix}$ is independent of

A. α

B. β

C. θ

D. a

Answer: A

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19. If $f(x), h(x)$ are polynomials of degree 4 and $\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix}$ $= mx^4 + nx^3 + rx^2 + sx + r$ be an identity in x, then

$$\begin{vmatrix} f''(0)-f''(0) & g''(0)-g''(0) & h''(0)-h''(0) \\ a & b & c \\ p & q & r \end{vmatrix}$$ is

A. $2(3n+r)$

B. $3(2n-r)$

C. $3(2n+r)$

D. $2n(3n-r)$

Answer: D

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20. If f(x) $=\begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta+\gamma) & \sin(\gamma+\alpha) & \sin(\alpha+\beta) \end{vmatrix}$ then $f(\theta)-2f(\phi)+f(\psi)$ is equal to

A. 0

B. $\alpha-\beta$

C. $\alpha+\beta+\gamma$

D. $\alpha + \beta \pm \gamma$

Answer: A

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21. If $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$

where a, b, c are all different, then the determinant

$\begin{vmatrix} 1 & 1 & 1 \\ (x-a)^2 & (x-b)^2 & (x-c)^2 \\ (x-b)(x-c) & (x-c)(x-a) & (x-a)(x-b) \end{vmatrix}$ vanishes when

A. a+b+c=0

B. $x = \frac{1}{3}$(a+b+c)

C. $x = \frac{1}{2}$(a+b+c)

D. x=a+b+c

Answer: B

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22. Let $a, b, c \in R$ such that no two of them are equal and satisfy $|2abcbc2ac2ab| = 0$, then equation $24ax^2 + 4bx + c = 0$ has at last one root in $[0, 1]$ at last one root in $\left[-\frac{1}{2}, \frac{1}{2}\right]$ at last one root in $[-1, 0]$ at last two roots in $[0, 2]$

A. atleast one root in $\left[0, \frac{1}{2}\right]$

B. atleast one root in $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. atleast one root in $[-1, 0]$

D. atleast one root in [0,2]

Answer: A

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23. The number of positive integral solutions of the equation $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & z^2y & z^3+1 \end{vmatrix} = 11$ is

A. 0

B. 3

C. 6

D. 12

Answer: B

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24. If f(x)=$ax^2+bx+c, a, b, c \in$ R and eqation f(x)-x=0 has imaginary roots α, β, γ and δ be the roots of f(x) -x=0 then $\begin{vmatrix} 1 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$ is

A. 0

B. purely real

C. purely imaginary

D. None of these

Answer: B

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25. If the system of equtions 2x-y+z=0,x-2y+z=0, tx-y+2z=0 has infinitely many solution and f(x) be a continuous function such that f(5+x)+f(x)=2, then $\int_0^{-2t}$ f(x) dx is equal to

A. 0

B. $-2t$

C. 5

D. t

Answer: B

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26. if $\left(1+ax+bx^2\right)^4 = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_8x^8$, where $a, b, a_0, a_1\ldots\ldots, a_8 \in R$ such that $a_0 + a_1 + a_2 \neq 0$ and

$$\begin{vmatrix} a_0 & a_1 & a_2 \\ a_1 & a_2 & a_0 \\ a_2 & a_0 & a_1 \end{vmatrix} = 0$$ then the value of $5.\dfrac{a}{b}$ is ____

A. $a = \dfrac{3}{4}, b = \dfrac{5}{8}$

B. $a = \dfrac{1}{4}, b = \dfrac{5}{32}$

C. $a = 1, b = \dfrac{2}{3}$

D. None of these

Answer: B

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27. Given (f(x) $= \log_{10}$ x and g(x) = $e^{\pi ix}$.

$$\phi(x) = \begin{vmatrix} f(x).\,g(x) & (f(x))^{g(x)} & 1 \\ f(x^2).\,g(x^2) & (f(x^2))^{g(x^2)} & 0 \\ f(x^3).\,g(x^3) & (f(x^3))^{g(x^3)} & 1 \end{vmatrix}$$ the value of ϕ (10), is

A. 1

B. 2

C. 0

D. None of these

Answer: C

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28. The value of the determinant $\begin{vmatrix} 1 & \left(\alpha^{2x}-\alpha^{-2x}\right)^2 & \left(\alpha^{2x}+\alpha^{-2x}\right)^2 \\ 1 & \left(\beta^{2x}-\beta^{-2x}\right)^2 & \left(\beta^{2x}+\beta^{-2x}\right)^2 \\ 1 & \left(\gamma^{2x}-\gamma^{-2x}\right)^2 & \left(\gamma^{2x}+\gamma^{-2x}\right)^2 \end{vmatrix}$ is (a) 0 (b) $(\alpha\beta\gamma)^{2x}$ (c) $(\alpha\beta\gamma)^{-2x}$ (d) None of these

A. 0

B. $(\alpha\beta)^{2x}$

C. $(\alpha\beta)^{-2x}$

D. None of these

Answer: A

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29. If a,b and c are non-zero real numbers and if the equation $(a-1)x=y+z$,

$(b-1)y=z+x$,

$(c-1)z=x+y$ has a non-trival solution then $ab+bc+ca$ equals to

A. $a+b+c=0$

B. abc

C. 1

D. None of these

Answer: B

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30. the set of equations $\lambda x - y + (\cos 0)z = 0, 3x + y + 2z = 0$

(cos 0) x+y+2z=0 ,$0 \leq 0 < 2\pi$ has non-trivial solution (s)

A. for no value of λ and θ

B. for all value of $\lambda$and θ

C. for all value of λ and only two values of θ

D. for only one value of λ and all values of θ

Answer: A

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Exercise More Than One Correct Option Type Questions

1. The determinant $\Delta = \begin{vmatrix} a^2 + x^2 & ab & ac \\ ab & b^2 + x^2 & bc \\ ac & bc & c^2 + x^2 \end{vmatrix}$ is divisible by

A. x

B. x^2

C. x^3

D. x^4

Answer: A::B::C::D

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2. The value of the determinant $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6}i \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$ is (where i=$\sqrt{-1}$

A. complex

B. real

C. irrational

D. rational

Answer: B::D

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3. If $\Delta_r = \begin{vmatrix} 2^{r-1} & \frac{1}{r(r+1)} & \sin r\theta \\ x & y & z \\ 2^n - 1 & \frac{n}{n+1} & \frac{(\sin)\frac{n+1}{2}\theta(\sin)\frac{n}{2}\theta}{\frac{\sin\theta}{2}} \end{vmatrix}$, $then \sum_{r=1}^{n} \Delta_r$ is equal to

A. independent of x,y,z

B. independent of n

C. independent of θ

D. All of above

Answer: D

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4. the determinant $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$ is equal to zero if

A. a,b and c are in AP

B. a,b,c, are in GP

C. a,b, and c are in HP

D. $(x - \alpha)$ is a factor of $ax^2 + 2bx + c$

Answer: B::D

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5. Let f(x) = $\begin{vmatrix} \cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then

A. $f\left(\frac{\pi}{3}\right) = -1$

B. $f'\left(\frac{\pi}{3}\right) = \sqrt{3}$

C. $\int_0^{\pi} f(x)dx = 0$

D. $\int_{-\pi}^{\pi} f(x)dx = 0$

Answer: A::C::D

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6. If $\Delta(x) = \begin{vmatrix} x^2 - 5x + 3 & 2x - 5 & 3 \\ 3x^2 + x + 4 & 6x + 1 & 9 \\ 7x^2 - 6x + 9 & 14x - 6 & 21 \end{vmatrix}$ $= ax^3 + bx^2 + cx + d$, then

A. a=0

B. b=0

C. c=0

D. d=141

Answer: A::B::C::D

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7. If a,b,and c are the side of a triangle and A,B and C are the angles opposite to a,b,and c respectively, then

$\Delta = \begin{vmatrix} a^2 & b\sin A & C\sin A \\ b\sin A & 1 & \cos A \\ C\sin A & \cos A & 1 \end{vmatrix}$ is independent of

A. a

B. b=0

C. c

D. A,B,C

Answer: A::B::C::D

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8. $= \left|aa^2 012a + b(a+b)012a + 3b\right|$ is divisible by $a+b$ b. $a+2b$ c. $2a+3b$ d. a^2

A. (a+b) is a factor of f'(a,b)

B. (a+2b) is a factor of f'(a,b)

C. (2a+b) is a factor of f'(a,b)

D. a is a factor of f(a,b)

Answer: A::B::C::D

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9. Let $f(x) = \begin{vmatrix} \sec^2 x & 1 & 1 \\ \cos^2 x & \cos^2 x & \cos ec^2 x \\ 1 & \cos^2 x & \cot^2 x \end{vmatrix}$, then

A. $f_{-\pi/4}^{\pi/4} f(x)dx = \frac{1}{16}(3\pi + 8)$

B. $f' \frac{\pi}{2} = 0$

C. maximum value of f(x) is 1

D. minimum value of f(x) is 0

Answer: A::B::C::D

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10. If $\begin{vmatrix} a & a+x^2 & a+x^2+x^4 \\ 2a & 3a+2x^2 & 4a+3x^3+2x^4 \\ 3a & 6a+3x^2 & 10a+6x^2+3x^4 \end{vmatrix}$

$=a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7$ and f(x) =+

$a_0x^2 + a_3x + a_6$ then

A. $f(x) \geq 0, \forall \in R \;\; \text{if} \;\; al > 0$

B. f(x)=0, only if a=0

C. f(x)=0, has of a=0 f(x) =0 has equal roots

D. f(x)=0, has more than two root if a=0

Answer: A::B::C::D

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11. If $\Delta(x) = \begin{vmatrix} 4x-4 & (x-2)^2 & x^3 \\ 8x-4\sqrt{2} & \left(x-2\sqrt{2}^2\right) & (x+1)^3 \\ 12x-4\sqrt{3} & \left(x-2\sqrt{3}\right)^2 & (x-1)^3 \end{vmatrix}$ then

A. term independent of x in $\Delta(x) = 16\left(5-\sqrt{2}-\sqrt{3}\right)$

B. coefficient of x in $\Delta(x) = 48\left(1+\sqrt{2}-\sqrt{3}\right)$

C. coefficient of x in $\Delta(x) = 16\left(5+\sqrt{2}-\sqrt{3}\right)$

D. coefficient of x in $\Delta(x)$ is divisible by 16

Answer: A::B

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12. If $f(x)=\begin{vmatrix}3 & 3x & 3x^2+2a^2\\ 3x & 3x^2+2a^2 & 3x^3+6a^2x\\ 3x^2+2a^2 & 3x^3++6a^2x & 3x^4+12a^2x^2+2a^4\end{vmatrix}$

then

A. f'(x)=0

B. y=f(x) is a straight line parallel to X-axis

C. $\int_0^2 f(x)dx=32a^4$

D. None of these

Answer: A::B

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13. If $a>b>c$ and the system of equtions ax +by +cz =0,n bx +cy+az=0,cx+ay+bz=0 has a non-trivial solution then both the roots of the equadratic equation at^2+bt+c are

A. real

B. of opposite sign

C. positive

D. complex

Answer: A::B

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14. The values of λ and b for which the equations x+y+z=3, x+3y+2z=6, and x+λy+3z=b have

A. a uniqe solution if $\lambda \neq 5, b \in R$

B. no solution if $\lambda \neq 5, b = 9$

C. infinite many solution $\lambda = 5, b = 9$

D. None of the above

Answer: A::C

15. Let $\lambda \text{and} \alpha$ be real. Let S denote the set of all values of λ for which the system of linear equations

$\lambda x + (\sin \alpha) y + (\cos \alpha) z = 0$

$x + (\cos \alpha) y + (\sin \alpha) z = 0$

-x+$(\sin \alpha) y - (\cos \alpha) z = 0$

has a non- trivial solution then S contains

A. (-1,1)

B. $[-\sqrt{2}, -1]$

C. $[1, \sqrt{2}]$

D. (-2,2)

Answer: A::B::C

Exercise Passage Based Questions

1. Consider the system of equations

x+y+z=5, x+2y+3z=9, x+3y+$\lambda z = \mu$

The sustem is called smart brilliant good and lazy according as it has solution unique solution infinitely many solution respectively .

The system is smart if

A. $\lambda \ne 5 \text{or} \lambda = 5 \text{and} \mu = 13$

B. $\lambda \ne 5 \quad \text{and} \quad \mu = 13$

C. $\lambda \ne 5 \quad \text{and} \quad \mu \ne 13$

D. $\lambda \ne 5 \text{ or } \lambda \quad \text{and} \quad \mu \ne 13$

Answer: A

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2. Consider the system of equations

x+y+z=5, x+2y+3z=9, x+3y+$\lambda z = \mu$

The sustem is called smart brilliant good and lazy according as it has solution unique solution infinitely many solution respectively .

The system is good if

A. $\lambda \ne 5$ or $\lambda \quad$ and $\quad \mu \ne 13$

B. $\lambda = 5 \quad$ and $\quad \mu = 13$

C. $\lambda = 5 \quad$ and $\quad \mu \ne 13$

D. $\lambda \ne 5$and $\quad \mu$ is any real number

Answer: B

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3. Consider the system of equations

x+y+z=5, x+2y+3z=9, x+3y+$\lambda z = \mu$

The sustem is called smart brilliant good and lazy according as it has solution unique solution infinitely many solution respectively .

The system is lazy if

A. $\lambda \neq 5$or' lambda $=5 \quad$ and $\mu = 13$

B. $\lambda = 5 \quad$ and $\quad \mu = 13$

C. $\lambda = 5 \quad$ and $\quad \mu \neq 13$

D. $\lambda \neq 5$or $\quad \lambda = 5$ and $\mu \neq 13$

Answer: C

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4. $If \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and $C_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is a determinant obtained by deleting ith row and jth column then then $\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} = \Delta^2$.

If $\begin{vmatrix} 1 & x & x^2 \\ x & x^2 & 1 \\ x^2 & 1 & x \end{vmatrix}$ =5 and $\Delta = \begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \end{vmatrix}$ then sum of digits of Δ^2 is

A. 7

B. 8

C. 13

D. 11

Answer: C

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5. $If\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and $C_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is a determinant obtained by deleting ith row and jth column then then $\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} = \Delta^2$.

Suppose a,b,c, $\in R, a + b + c > 0, A = bc - a^2, B = ca - b^2$ and $c = ab - c^2$ and $\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix}$ =49 then the valu of $a^3 + b^3 + c^3$-3abc is

A. -7

B. 7

C. -2401

D. 2401

Answer: B

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6. $If \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and $C_{ij} = (-1)^{i+j} M_{ij}, \text{where } M_{ij}$ is a determinant obtained by deleting ith row and jth column then then $\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} = \Delta^2.$

Suppose a,b,c, $\in R, a+b+c>0, A=bc-a^2, B=ca-b^2$ and $c=ab-c^2$ and $\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix}$ =49 then the valu of $a^3+b^3+c^3$-3abc is

A. -3

B. 3

C. -9

D. 9

Answer: B

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7. If α, β, γ are the roots of $x^3 + 2x^2 - x - 3 = 0$ The value of $\begin{vmatrix} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{vmatrix}$ is equal to

A. 14

B. -2

C. 10

D. -14

Answer: D

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8. If α, β, γ are the roots of $x^3 + 2x^2 - x - 3 = 0$. If the absolute value of the expression $\frac{\alpha - 1}{\alpha + 2} + \frac{\beta - 1}{\beta + 2} + \frac{\gamma - 1}{\gamma + 2}$ can be expressed as $\frac{m}{n}$ where m and n are co-prime the value of $\begin{vmatrix} m & n^2 \\ m - n & m + n \end{vmatrix}$ is

A. 17

B. 27

C. 37

D. 47

Answer: C

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9. If α, β, γ are the roots of $x^3 + 2x^2 - x - 3 = 0$. If $a = \alpha^2 + \beta^2 + \gamma^2, b = \alpha\beta + \beta\gamma + \gamma\alpha$ the value of $\begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix}$ is

A. 14

B. 49

C. 98

D. 196

Answer: D

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10. Suppose f(x) is a function satisfying the folowing conditions:

(i)f(0)=2,f(1)=1

(ii) f(x) has a minimum value at $x = \frac{5}{2}$

(iii) for all xf'(x)= $\begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

The value of f(2)+f(3) is

A. 1

B. $\frac{3}{2}$

C. 2

D. $\frac{5}{2}$

Answer: A

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11. Suppose f(x) is a function satisfying the folowing conditions:

(i)f(0)=2,f(1)=1

(ii) f(x) has a minimum value at $x=\frac{5}{2}$

(iii) for all xf'(x)= $\begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

The number of solutions of the equation f(x) +1=0 is

A. 0

B. 1

C. 2

D. infinite

Answer: A

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12. Suppose f(x) is a function satisfying the folowing conditions:

(i)f(0)=2,f(1)=1

(ii) f(x) has a minimum value at $x=\frac{5}{2}$

(iii) for all xf'(x)= $\begin{vmatrix} 2ax & 2ax-1 & 2ax+b+1 \\ b & b+1 & -1 \\ 2(ax+b) & 2ax+2b+1 & 2ax+b \end{vmatrix}$

Range of f(x) is

A. $\left(-\infty, \frac{7}{16}\right]$

B. $\left[\frac{3}{4}, \infty\right)$

C. $\left[\frac{7}{16}, \infty\right)$

D. $\left(-\infty, \frac{3}{4}\right]$

Answer: C

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13. If

$$\begin{vmatrix} x & e^{x-1} & (x-1)^3 \\ x-lx & \cos(x-1) & (x-1)^2 \\ \tan x & \sin^2 x & \cos^2 x \end{vmatrix} = a_0 + a_1(x-1) + a_2(x-1)^2 \cdots$$

The value of $\cos^{-1}(a_1)$ is

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

Answer: C

Watch Video Solution

14. If

$$\begin{vmatrix} x & e^{x-1} & (x-1)^3 \\ x-lx & \cos(x-1) & (x-1)^2 \\ \tan x & \sin^2 x & \cos^2 x \end{vmatrix} = a_0 + a_1(x-1) + a_2(x-1)^2 \cdots$$

The value of $\lim_{x \to 0} (\sin x)^x$ is

A. 1

B. e

C. e-1

D. None of these

Answer: A

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15. If

$$\begin{vmatrix} x & e^{x-1} & (x-1)^3 \\ x-lx & \cos(x-1) & (x-1)^2 \\ \tan x & \sin^2 x & \cos^2 x \end{vmatrix} = a_0 + a_1(x-1) + a_2(x-1)^2 \cdots$$

The eqution whose roots are $a_0$and a_1 is

A. $x^2 - x = 0$

B. $x^2 - 2x = 0$

C. $x^2 - 3x = 0$

D. None of these

Answer: D

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16. Let $\Delta = \begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ and the equation $x^3 - px^2 + qx - r = 0$ has roots a,b,c, where a,b,c $\in R^+$

The value of $D \leq ta$ is

A. $\leq 9r^3$

B. $\geq 27r^2$

C. $\leq 27r^2$

D. $\geq 81r^3$

Answer: B

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17. Let $\Delta = \begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ and the equation

$x^3 - px^2 + qx - r = 0$ has roots a,b,c, where a,b,c $\in R^+$

If a,b,c are in GP then

A. $r^3 = p^3 q$

B. $p^3 = r^3 q$

C. $p^3 = q^3 r$

D. $q^{3 = p^3 r}$

Answer: D

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18. Let $\Delta = \begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ and the equation

$x^3 - px^2 + qx - r = 0$ has roots a,b,c, where a,b,c $\in R^+$

If Δ =27 and $a^2 + b^2 + c^2 = 2$ then the value of $\sum a^2 b$ is

A. $3(2\sqrt{2}-p)$

B. $3(2\sqrt{2}-r)$

C. $3(2\sqrt{2}-q)$

D. $3(2\sqrt{2}-p-q)$

Answer: B

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19. If $\Delta n = \begin{vmatrix} a^2+n & ab & ac \\ ab & b^2+n & bc \\ ac & bc & c^2+n \end{vmatrix}, n \in$ N and the equation $x^3 - \lambda x^2 + 11x - 6 = 0$ has roots a,b,c and a,b,c are in AP.

The value of $\sum_{r=1}^{7} \Delta_n$ is

A. $(12)^3$

B. $(14)^3$

C. $(26)^3$

D. $(28)^3$

Answer: B

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20. If $\Delta n = \begin{vmatrix} a^2+n & ab & ac \\ ab & b^2+n & bc \\ ac & bc & c^2+n \end{vmatrix}, n \in$ N and the equation $x^3 - \lambda x^2 + 11x - 6 = 0$ has roots a,b,c and a,b,c are in AP.

The value of $\frac{\Delta 2_n}{\Delta_n}$ is

A. < 8

B. $= 8$

C. > 8

D. None of these

Answer: A

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21. If $\Delta n = \begin{vmatrix} a^2+n & ab & ac \\ ab & b^2+n & bc \\ ac & bc & c^2+n \end{vmatrix}, n \in$ N and the equation $x^3 - \lambda x^2 + 11x - 6 = 0$ has roots a,b,c and a,b,c are in AP.

The value of $\sum_{r=1}^{30} \left(\frac{27\Delta_r - \Delta 3_r}{27r^2} \right)$ is

A. 130

B. 190

C. 280

D. 340

Answer: C

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Exercise Single Integer Answer Type Questions

1. If $\begin{vmatrix} 3^2+k & 4^2 & 3^2+3+k \\ 4^2+k & 5^2 & 4^2+4+k \\ 5^2+k & 6^2 & 5^2+5+k \end{vmatrix}$ =0

$\sqrt{2^k\sqrt{2^k\sqrt{2^k\cdots\infty}}}$ is

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2. Let $\alpha, \beta$and γ are three distinct roots of

$\begin{vmatrix} x-1 & -6 & 2 \\ -6 & x-2 & -4 \\ 2 & -4 & x-6 \end{vmatrix}$ =0 the value of $\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)^{-1}$ is

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3. If $\begin{vmatrix} x & e^{x-1} & (x-1)^3 \\ x-lx & \cos(x-1) & (x-1)^2 \\ \tan x & \sin^2 x & \cos^2 x \end{vmatrix} = a_0 + a_1(x-1) + a_2(x-1)^2\cdots$

The value of $\cos^{-1}(a_1)$ is

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4.

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ is equal to

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5. Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

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6. If $0 \leq \theta \leq \pi$ and the system of equations

x=$(\sin\theta)y + (\cos\theta)z$

y=z+$(\cos\theta)$ x

z=(sin θ) x+y

has a non-trivial solution then $\frac{8\theta}{\pi}$ is equal to

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7. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$ is

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8. If a,b,c and d are the roots of the equation

$x^4 + 2x^3 + 4x^2 + 8x + 16 = 0$ the value of the determinant

$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix}$ is

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9. If $a \neq 0, b \neq 0, c \neq 0$ and $\begin{vmatrix} 1+a & 1 & 1 \\ 1+b & 1+2b & 1 \\ 1+c & 1+c & 1+3c \end{vmatrix}$=0

the value of $|a^{-1} + b^{-1} + c^{-1}|$ is equal to

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10. If the system of equations

ax+hy+g=0 ....(i)

hx+by+f=0...(ii)

and $ax^2+2hxy+by^2+2gx+2fy+c+t=0$....(iii)

has a unique solution and $\frac{abc+2fgh-af^2-bg^2-ch^2}{h^2-ab}=8$, find the value of 't'.

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Exercise Statement I And Ii Type Questions

1. Statement -1 If $\Delta(r)=\begin{vmatrix} r & r+1 \\ r+3 & r+4 \end{vmatrix}$ then $\sum_{r=1}^{n}\Delta(r)=-3n$

Satement-2 If $\Delta(r)\begin{vmatrix} f_1(r) & f_2(r) \\ f_3(r) & f_4(r) \end{vmatrix}$

$\Sigma_{r=1}^{n}\Delta(r)=\begin{vmatrix} \Sigma_{r=1}^{n}f_1(r) & \Sigma_{r=1}^{n}f_2(r) \\ \Sigma_{r=1}^{n}f_3(r) & \Sigma_{r=1}^{n}f_4(r) \end{vmatrix}$

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2. Statement -1 Consider the determinant

$$\Delta = \begin{vmatrix} a_1 + b_1x^2 & a_1x^2 + b_1 & c_1 \\ a_2 + b_2x^2 & a_2x^2 + b_2 & c_2 \\ a_3 + b_3x^2 & a_3x^2 + b_3 & c_3 \end{vmatrix} = 0,$$

where $a_i, b_i, c_i \in$ R (i=1,2,3) and x $\in$ R

Stement -2 If $\begin{vmatrix} a_1 & b_2 & c_3 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ =0, then Δ =0

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3. Statement -1 The value of determinant

$$\begin{vmatrix} \sin\pi & \cos\left(x + \frac{\pi}{4}\right) & \tan\left(-\frac{\pi}{4}\right) \\ \sin\left(x - \frac{\pi}{4}\right) & -\cos\left(\frac{\pi}{2}\right) & In\left(\frac{x}{y}\right) \\ \cot\left(\frac{\pi}{4} + x\right) & In\left(\frac{y}{x}\right) & \tan(\pi) \end{vmatrix}$$ is zero

Statement -2 The value of skew -symetric determinat of odd order equals zero.

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4. Statement-1 f(x) = $\begin{vmatrix} (1+x)^{11} & (1+x)^{12} & (1+x)^{13} \\ (1+x)^{21} & (1+x)^{22} & (1+x)^{23} \\ (1+x)^{31} & (1+x)^{32} & (1+x)^{33} \end{vmatrix}$

the cofferent of x in f(x)=0

Statement -2 If P(x)=$a_0 + a_1x + a_2x^2 + a_2x_3 + \cdots + a_ns^n$ then $a_1 = P'(0)$, where dash denotes the differential coefficient.

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5. Statement -1 If system of equations 2x+3y=a and bx +4y=5 has infinite solution, the $a = \frac{15}{4}, b = \frac{8}{5}$

Statement-2 Straight lines $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = \; = 0$ are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \ne \frac{c_1}{c_2}$

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6. Statement -1 The value of the determent $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} \neq 0$

Statement -2 Neither of two rows rows or columns of $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix}$ is identical.

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7. The digits A,B,C are such that the three digit numbers A88, 6B8, 86 C are divisible by 72 the determinant

$\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by

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Exercise Subjective Type Questions

1. Prove that $\begin{vmatrix} b+c & c & b \\ c & c+a & a \\ b & a & a+b \end{vmatrix}$ =4 abc.

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2. Prove that: $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

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3. Find the value of determinat $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$.

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4. The value of the determinant $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$, where a, b and c respectively the pth,qth and rth terms of a H.P., is

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5. Whitout expanding the determinat at any stage prove that $\begin{vmatrix} -5 & 3+5i & \frac{3}{2}-4i \\ 3-5i & 8 & 4+5i \\ \frac{3}{2}+4i & 4-5i & 9 \end{vmatrix}$ has a purely real value.

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6. Prove without expansion that $|ah+bggab+chbf+bafhb+bcaf+bbg+fc|=a|ah+bgahbf+bahb$

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7. If A,B and C are the angles of a triangle and $\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix}$ =0 then prove that Δ ABC must be isoceles.

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8. The value of $\begin{vmatrix} \beta\gamma & \beta\gamma' + \beta'\gamma & \beta'\gamma' \\ \gamma\alpha & \gamma\alpha' + \gamma'\alpha & \gamma'\alpha' \\ \alpha\beta & \alpha\beta' + \alpha'\beta & \alpha'\beta' \end{vmatrix}$ is

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9. If $y = \frac{u}{v}$, where u and v are functions of x, show that

$$v^3 \frac{d^2y}{dx^2} = \begin{vmatrix} u & v & 0 \\ u' & v' & v \\ u'' & v'' & 2v' \end{vmatrix}.$$

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10. Show that the determinant Δ (x) is given by Δ(x) =

$\begin{vmatrix} \sin(x+\alpha) & \cos(x+\alpha) & a + x\sin\alpha \\ \sin(x+\beta) & \cos(x+\beta) & b + x\sin\beta \\ \sin(x+\gamma) & \cos(x+\gamma) & c + x\sin\gamma \end{vmatrix}$ is independent of x.

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11. Evaluate $\begin{vmatrix} .^{x}C_1 & .^{x}C_2 & .^{x}C_3 \\ .^{y}C_1 & .^{y}C_2 & .^{y}C_3 \\ .^{x}C_1 & .^{z}C_2 & .^{z}C_3 \end{vmatrix}$

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12. (i) Find maximum value of

$$f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & 4\sin 2x \\ \sin^2 x & 1+\cos^2 x & 4\sin 2x \\ \sin^2 x & \cos^2 x & 1+4\sin 2x \end{vmatrix}.$$

(ii) Let A,B and C be the angles of triangle such that $A \geq B \geq C$.

Find the minimum value of Δ where

$$\Delta = \begin{vmatrix} \sin^2 A & \sin A\cos A & \cos^2 A \\ \sin^2 B & \sin B\cos B & \cos^2 B \\ \sin^2 C & \sin C\cos C & \cos^2 C \end{vmatrix}.$$

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13. If f(x) = $\begin{vmatrix} x^2-4x+6 & 2x^2+4x+10 & 3x^2-2x+16 \\ x-2 & 2x+2 & 3x-1 \\ 1 & 2 & 3 \end{vmatrix}$ then find the value of $\int_{-3}^{3} \frac{x^2\sin x}{1+x^6} f(x)$ dx.

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14. prove that

$$\begin{vmatrix} \frac{1}{(a-a_1)^2} & \frac{1}{a-a_1} & \frac{1}{a_1} \\ \frac{1}{(a-a_2)^2} & \frac{1}{a-a_2} & \frac{1}{a_2} \\ \frac{1}{(a-a_3)^2} & \frac{1}{a-a_3} & \frac{1}{a_3} \end{vmatrix} = \frac{-a^2(a_1-a_2)(a_2-a_3)(a_3-a_1)}{a_1a_2a_3(a-a_1)^2(a-a_2)^2(a-a_3)^2}.$$

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15. Show that in general there are three values of t for which the following system of equations has a non- trival solution (a-t)x+by+cz=0, bx+(c-t)y+az=0 and cx+ay+(b-t)z=0.

Express the product of these values of t in the form of a determinant.

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16. Eliminates

(i) a,b, and c

(ii) x,y,z from the equations

$$-a+\frac{by}{z}+\frac{cz}{y}=0, \quad -b+\frac{cz}{x}+\frac{ax}{z}=0$$

and $-c+\frac{ax}{y}+\frac{by}{x}=0.$

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17. If x, y, z are not all zero & if $ax+by+cz=0, bx+cy+az=0\&cx+ay+bz=0,$ then prove that $x:y:z=1:1:1$ OR $1:\omega:\omega^2 OR 1:\omega^2:\omega$, where ω is one ofthe complex cube root of unity.

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Exercise Questions Asked In Previous 13 Years Exam

1. If $a^2+b^2+c^2$=-2 and f(x)= $\begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$ then f(x) is a polynomial of degree

A. 3

B. 2

C. 1

D. 0

Answer: B

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2. The system of equations

$\alpha x + y + z = \alpha - 1,$

$x + \alpha y + z = \alpha - 1$

$x + y + \alpha z = \alpha - 1$

and has no solution if α is

A. not -2

B. 1

C. -2

D. Either -2 or 1

Answer: C

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3. If $a_1, a_2, a_3, \ldots . a_n \ldots$ are in G.P. then the determinant $\Delta = \begin{vmatrix} \log a_n, \log a_{n+1}, \log a_{n+2} \\ \log a_{n+3}, \log a_{n+4}, \log a_{n+5} \\ \log a_{n+6}, \log a_{n+7}, \log a_{n+8} \end{vmatrix}$ is equal to-

A. 1

B. 0

C. 4

D. 2

Answer: B

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4. If D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$ then D is

A. divisible by neither x nor y

B. divisible by both x and y

C. divisible by x but not y

D. divisible by y but not x

Answer: B

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5. Consider the system of equations

x-2y+3z=-1

-x+y-2z=k

x-3y+4z=1

Statement -1 The system of equation has no solutions for $k \neq 3.$

statement -2 The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0, \text{ for } \quad k \neq 3.$

A. Satement -1 is true Statement -2 is true and Statement -2 is correct explanation for Statement -1.

B. Satement -1 is true Statement -2 is true and Statement -2 is not a correct explanation for Statement -1.

C. Statement -1 is true Statement -2 is false.

D. Statement-1 is false, Statement -2 is true.

Answer: A

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6. Let a,b,c, be any real number. Suppose that there are real numbers x,y,z not all zero such that x=cy+bz,y=az+cx and z=bx+ay. Then $a^2 + b^2 + c^2$ +2abc is equal to

A. -1

B. 0

C. 1

D. 2

Answer: C

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7. Let a,b,c be such that b(a+c) $\neq 0$. If

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0,$$

Then the value of 'n' is:

A. any integer

B. zero

C. an even integer

D. any odd integer

Answer: D

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8. If $f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$ then the set $\left\{ f(\theta) : 0 \le \theta < \frac{\pi}{2} \right\}$ is

A. $(-\infty, -1) \cup (1, \infty)$

B. $[2, \infty)$

C. $(-\infty, 0] \cup [2, \infty)$

D. $(-\infty, -1] \cup [1, \infty)$

Answer: B

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9. The number of values of k which the linear equations

4x+ky+2z=0

kx+4y+z=0

2x+2y+z=0

Possess a non-zero solution is

A. zero

B. 3

C. 2

D. 1

Answer: C

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10. If the trivial solution is the only solution of the system of equations

$x - ky + z = 0,$

$kx + 3y - kz = 0,$

$3x + y - z = 0$

Then the set of all values of k is:

A. {2,-3}

B. R-{2,-3}

C. R-{2}

D. R-{-3}

Answer: B

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11. The number of values of k, for which the system of equations $(k+1)x+8y=4k\ kx+(k+3)y=3k-1$ has no solution, is (1) 1 (2) 2 (3) 3 (4) infinite

A. 1

B. 2

C. 3

D. infinite

Answer: A

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12. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $|31 + f(1)1 + f(2)1 + f(1)1 + f(2)1 + f(3)1 + f(2)1 + f(3)1 + f(4)| =$

, then K is equal to (1) $\alpha\beta$ (2) $\frac{1}{\alpha\beta}$ (3) 1 (4) -1

A. 1

B. -1

C. $\alpha\beta$

D. $1/\alpha\beta$

Answer: A

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13. The set of the all values of λ for which the system of linear equations

$2x_1 - 2x_2 + x_3 = \lambda x_1$

$2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$-x_1 + 2x_2 = \lambda x_3$ has a non-trivial solution,

A. contains two elements

B. contains more than two elements

C. is an empty set

D. is a singleton

Answer: A

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14. Which of the following values of α satisfying the equation

$|(1+\alpha)^2(1+2\alpha)^2(1+3\alpha)^2(2+\alpha)^2(2+2\alpha)^2(2+3\alpha)^2(3+\alpha)^2(3+2\alpha)$

-4 b. 9 c. -9 d. 4

A. -4

B. 9

C. -9

D. 4

Answer: B::C

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15. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for

A. exactly one-value of λ

B. exactly two values of λ

C. exactly three values of λ

D. infinitely many values of λ

Answer: C

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16. The total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is (A) 0 (B) 1 (C) 2 (D) 3

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17. Let $a, \lambda, \mu \in R,$ Consider the system of linear equations $ax + 2y = \lambda 3x - 2y = \mu$ Which of the flollowing statement (s) is (are) correct?

A. If a=-3, then the system has infinitely many solutions for all values of λ and μ

B. If $a \neq -3$, then the system has a unique solution for all values of $\lambda \text{and} \mu$

C. If $\lambda + \mu$ =0 , then the system has infinitely many solutions for a=-3

D. If $\lambda + \mu \neq$ 0 then the system has no solution for a =-3

Answer: B::C::D

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18. If S is the set of distinct values of $'b$ for which the following system of linear equations $x+y+z=1$ $x+ay+z=1$ $ax+by+z=0$ has no solution, then S is : a finite set containing two or more elements (2) a singleton an empty set (4) an infinite set

A. an infinite set

B. a finite set containing two or more elements

C. a singleton

D. an empty set

Answer: C

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