MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

DIFFERENTIATION

Examples

1. Differentiate the following functions w.r.t x using first principle.

$f(x) = \tan x$

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2. Differentiate the following functions w.r.t x using first principle.

e^{x^2}

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3. Differentiate the following functions w.r.t x using first principle.

$\sqrt{\sin x}$

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4. If $y = \sin x + e^x$, Then find $\frac{dy}{dx}$.

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5. If $Y = x^2 + \sin^{-1} x + \log_e x$, then find $\frac{dy}{dx}$

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6. If $y = \log_{10} x + \log_e x + \log_{10} 10$, then find $\frac{dy}{dx}$

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7. If $y = x^{-1/2} + \log_5 x + \frac{\sin x}{\cos x} + 2^x$, then find $\frac{dy}{dx}$

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8. If $y = m^2 \sec^{-1} x$, then find $\frac{dy}{dx}$.

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9. If $y = \log_e x^3 + 3\sin^{-1} x + kx^2$, then find $\frac{dy}{dx}$

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10. If $y = e^x \sin x$, then find $\frac{dy}{dx}$

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11. If $y = e^x \tan x + x.\log_e x$, then find $\frac{dy}{dx}$

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12. Let f, g and h are differentiable functions. If $f(0) = 1; g(0) = 2; h(0) = 3$ and the derivatives of theirpair wise products at $x = 0$ are $(fg)'(0) = 6; (gh)'(0) = 4$ and $(hf)'(0) = 5$ then compute the value of $(fgh)'(0)$.

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13. If $y = (1 + x)(1 + x^2)(1 + x^4)(1 + x^{2n})$, then find $\frac{dy}{dx} atx = 0.$

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14. If $y = \frac{x}{x^2 + 1}$, then find $\frac{dy}{dx}$

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15. if $y = \dfrac{x \sin x}{\log_e x}$, then find $\dfrac{dy}{dx}$

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16. If $y = \dfrac{3a^2x - x^3}{a^3 - 3ax^2}$ then find $\dfrac{dy}{dx}$

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17. If $y = \dfrac{e^x - \tan x}{x^n + \cot x}$, then find $\dfrac{dy}{dx}$

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18. If $y = \dfrac{\log_e x}{x} + e^x \sin x + \log_5 x$ then find $\dfrac{dy}{dx}$.

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19. If $y = \dfrac{x^4 + x^2 + 1}{x^2 + x + 1}$ then $\dfrac{dy}{dx} = ax + b$, find a and b

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20. Let $y = \frac{\sec x + \tan x - 1}{\tan x - \sec x + 1}$.If $\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = a + \sqrt{b}$, then value of $a + b$ is equal to

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21. If $y = \frac{\tan^{-1} x - \cot^{-1} x}{\tan^{-1} x + \cot^{-1} x}$, find $\left(\frac{dy}{dx}\right]\Big)_{x=-1}$ 0 (b) 1 (c) $2/\pi$ (d) -1

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22. If $y = e^{(\tan^{-1} x)^3}$ then find $\frac{dy}{dx}$

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23. $y = \log_e\left(\tan^{-1}\sqrt{1 + x^2}\right)$ then $\frac{dy}{dx}$ is

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24. Find $\frac{dy}{dx}$, when $y = e^{ax}\cos(bx + c)$

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25. Differentiate the following w. r. t. x. $\sqrt{\log_e\left\{\sin\left(\frac{x^2}{3} - 1\right)\right\}}$

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26. If $y = x\cos y + y\cos x$, then find $\frac{dy}{dx}$

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27. If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1, then \frac{dy}{dx} =$

A. $\sqrt{\frac{1-y^2}{1-x^2}}$

B. $-\sqrt{\frac{1-y^2}{1-x^2}}$

C. $\sqrt{\frac{1-x^2}{1-y^2}}$

D. $-\sqrt{\frac{1-x^2}{1-y^2}}$

Answer: B

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28. If $\sqrt{1-x^6}+\sqrt{1-x^6}=a\left(x^3-y^3\right)$, then prove that $\frac{dy}{dx}=\frac{x^2}{y^2}\sqrt{\frac{1-y^6}{1-x^6}}$

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29. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\ldots\infty}}}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{2y-1}{\cos x}$

B. $\frac{\cos x}{2y-1}$

C. $\frac{2x-1}{\cos y}$

D. $\frac{\cos y}{2x-1}$

Answer: B

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30. If $y = \sec^{-1}\left(\sqrt{1+x^2}\right)$, when $-1 < x < 1$, then find $\frac{dy}{dx}$

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31. $y = \tan^{-1}\sqrt{\frac{1-x}{1+x}} find \frac{dy}{dx}$

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32. Prove that $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{x}{2}, x < \pi.\right.$

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33. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then find $\frac{dy}{dx}$ when $-1 \leq x \leq 1$.

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34. Differentiate the following functions with respect to x : $\tan^{-1}\left(\frac{a+x}{1-ax}\right)$ (ii) `tan^(-1)((acosx-bsinx)/(bcosx+asinx)),\ -pi/2-1`

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35. यदि $y = \sin^{-1}\left\{\frac{5x+12\sqrt{1-x^2}}{13}\right\}$ तब $\frac{dy}{dx}$ का मान ज्ञात कीजिए|

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36. Find $\frac{dy}{dx}$ for `y=tan^(-1)sqrt((a-x)/(a+x),)-a

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37. Sketch the graph for $y = \sin^{-1}(\sin x)$.

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38. Sketch the graph for $y = \cos^{-1}(\cos x)$.

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39. Sketch the graphs for $y = \sin(\sin^{-1} x)$

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40. Sketch the graphs for $y = \cos(\cos^{-1} x)$

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41. Sketch the graphs for $y = \tan(\tan^{-1} x)$

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42. Draw the graph of $y = \text{cosec}\left(\text{cosec}^{-1}x\right)$ or $y = \sec\left(\sec^{-1}x\right)$.

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43. Sketch the graphs for $y = \sec\left(\sec^{-1}x\right)$

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44. Sketch the graphs for $y = \cot\left(\cot^{-1}x\right)$ and hence find $\frac{dy}{dx}$

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45. If $x = e^{-t^2}, y = \tan^{-1}(2t+1)$, then $\frac{dy}{dx} =$

A. $\frac{e^{(t^2)}}{(2t)(2t^2-2t+1)}$

B. $\frac{e^{-(t^2)}}{(2t)(2t^2+2t+1)}$

C. $-\frac{e^{-(t^2)}}{(2t)(2t^2-2t+1)}$

D. $-\frac{e^{(t^2)}}{(2t)(2t^2+2t+1)}$

Answer: D

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46. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta)$ and $y = a(1-\cos\theta)$.

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47. If $x = \sqrt{a^{\sin\hat{}((-1)t)}}, y = \sqrt{a^{\cos\hat{}((-1)t)}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

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48. If $x = a\left(\cos t + \frac{1}{2}\log \tan^2 t\right)$ and $y = a \sin t$ then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

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49. If $y = x^{\sin x}$, then find $\frac{dy}{dx}$

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50. $x^y \cdot y^x = 1$. Find $\frac{dy}{dx}$

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51. If $\left(\tan^{-1} x\right)^y + y^{\cot x} = 1$, then find $\frac{dy}{dx}$.

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52. If $f(x) = \prod_{n=1}^{100} (x-n)^{n(101-n)}$ then find $\frac{f(101)}{f,(101)}$

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53. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x} + \ldots\ldots\infty}}$, then $\frac{dy}{dx}$ is

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54. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ w. r. t. $\tan^{-1} x$, $-1 < x < 1$.

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55. If $x \in \left(\frac{1}{\sqrt{2}}1\right)$. Differentiate $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) w.\ r.\ t.\ \cos^{-1}\left(2x\sqrt{1-x^2}\right)$.

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56. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1-x^2}$ at $x-\frac{1}{2}$.

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57. If $y=f\left(x^3\right), z=g\left(x^5\right), f'(x)=\tan x$ and $g'(x)=\sec x$ then find the value of $\lim_{x\to 0}\frac{dy/dz}{x}$.

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58. Find the derivative of $f(\tan x)$ w.r.t. $g(\sec x)$ at $x=\frac{\pi}{4}$, where $f'(1)=2$ and $g'\left(\sqrt{2}\right)=4$.

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59. If $y=x^3\log_e x$, then find y'' and y'''.

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60. If $y = \sin(\sin x)$ then prove that

$$\frac{d^2y}{dx^2} + \tan x . \frac{dy}{dx} + y\cos^2 x = 0$$

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61. If $x = a(t \sin t)$ and $y = (1 - \cos t)$, then find $\frac{d^2y}{dx^2}$.

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62. 112. If f(x)= $\begin{bmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{bmatrix}$ find $f'(x)$

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63. $\begin{vmatrix} x^2 + a^2 & ab & ac \\ ab & x^2 + b^2 & bc \\ ac & bc & x^2 + c^2 \end{vmatrix} =$

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64. Let $f(x) = \left|x^3 \sin x \cos x 6 - 10pp^2p^3\right|$, $wherep$ is a constant. Then $\frac{d^3}{dx^3}(f(x))$ at $x = 0$ is p (b) $p - p^3$ $p + p^3$ (d) independent of p

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65. if y= sin mx, them the value of the determinant

$$\begin{vmatrix} y & y_1 & y_2 \\ y_3 & y_4 & y_5 \\ y_6 & y_7 & y_8 \end{vmatrix} \text{ Where } y_n = \frac{d^n y}{dx^n} \text{ is}$$

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66. If $f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$ then find constant term and coefficient of x

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67. If $a_i, b_i \in N$ for i 1,2,3, then coefficient of x in the determinant;

$$\begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_3} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$$

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68. If $y = f(x) = x^3 + x^5$ and g is the inverse of f find $g'(2)$

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69. Let $f(x) = \exp\left(x^3 + x^2 + x\right)$ for any real number and let $g(x)$ be the inverse function of $f(x)$ then $g'\left(e^3\right)$

A. $\frac{1}{6e^3}$

B. $\frac{1}{6}$

C. $\frac{1}{34e^{19}}$

D. 6

Answer:

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70. Let f be a twice differentiable function such that $f^x = -f(x), and f'(x) = g(x), h(x) = [f(x)]^2 + [g(x)]^2.$ Find $h(10)$ if $h(5) = 11$

A. 0

B. 9

C. 11

D. None of these

Answer:

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71. if $y = \frac{(a-x)\sqrt{a-x} - (b-x)\sqrt{x-b}}{\sqrt{a-x} + \sqrt{x-b}}$ then $\frac{dy}{dx}$ wherever it is defined is equal to :

A. $\frac{x-(a-b)}{\sqrt{(a-x)(x-b)}}$

B. $\frac{2x+(a+b)}{\sqrt{(a-x)(x-b)}}$

C. $\frac{2x-(a+b)}{\sqrt{(a-x)(x-b)}}$

D. None of these

Answer:

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72. If $x^2 + y^2 = R^2 (where R > 0) and k = \frac{y^n}{(1+y^2)^3}$ then find k in terms of R alone.

A. $\frac{1}{R}$

B. $-\frac{1}{R}$

C. R

D. $\frac{1}{2R}$

Answer:

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73. Let $f(x) = x + \sin x$. Suppose g denotes the inverse function of f. The value of $g'\left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$ has the value equal to

A. $2 + \sqrt{2}$

B. $\sqrt{2} - 2$

C. $2 - \sqrt{2}$

D. $2\sqrt{2}$

Answer: C

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74. Let $e^{f(x)} = \ln x$. If $g(x)$ is the inverse function of $f(x)$, then $g'(x)$ equal to: e^x (b) $e^x + x \,.\, e^x + e^x$ (d)

A. e^{e^x}

B. e^x

C. $e^{e^x + x}$

D. None of these

Answer:

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75. Given: $f(x) = 4x^3 - 6x^2 \cos 2a + 3x \sin 2a \sin 6a + \sqrt{In\left(2a - a^2\right)}$ then

A. $af'\left(\frac{1}{2}\right) < 0$

B. $af'\left(\frac{1}{2}\right) \leq 0$

C. $af'\left(\frac{1}{2}\right) > 0$

D. None of these

Answer:

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76. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$, and that $f''(x) - 2f'(x) - 15f(x) = 0$ for all x. Then the value of ab is equal to:

A. 15

B. -15

C. 10

D. 16

Answer:

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77. Which of the following is not true $\log(1+x) < x$ for $x > 0$

A. $e^y = xy' + 1$

B. $y' = -\dfrac{1}{(x-1)}$

C. $y' + e^y = 0$

D. $y' = e^y$

Answer:

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78. If $x^p y^q = (x+y)^{(p+q)}$ then $\dfrac{dy}{dx} = ?$

A. independent of p

B. independent of q

C. dependent both p and q

D. $\dfrac{y}{x}$

Answer:

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79. Two functions f & g have first & second derivatives at x=0 & satisfy the relations,

$f(0) = \frac{2}{g(0)}, f'(0) = 2g'(0) = 4g(0), g(0) = 5\,f(0) = 6f(0) = 3$ then-

A. If $h(x) = \frac{f(x)}{g(x)}$, then $h'(0) = 15$

B. If $k(x) = f(x).\, g(x).\, \sin x$ then $k'(0)d = 2$

C. $\lim_{x \to 0} \frac{g'(x)}{f'(x)} = \frac{1}{2}$

D. None of above

Answer:

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80. $f(x) = \left|x^2 - 3x|x| + 2\right|$. Then which of the following is/are true ?

A. $f'(x) = 2x - 3$ for $x \in (0, 1) \cup (2, \infty)$

B. $f'(x) = 2x + 3$ for $x \in (-\infty, -2) \cup (-10)$

C. $f'(x) = -2x - 3$ for $x \in (-2, -1)$

D. None of the above

Answer:

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81. Consider $f(x) = \frac{x}{x^2 - 1}$ and $(x) = f''(x)$ Statement I Graph of $g(x)$ is cancave up for $x > 1$. Statement II $\frac{d^n}{dx^n} f(x) = \frac{(-1)^n n!}{2} \left\{ \frac{1}{(x-1)^{n+1}} + \frac{1}{(x+1)^{n+1}} \right\} n \in N$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is correct.

Answer:

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82. Statement 1: If differentiable function $f(x)$ satisfies the relation $f(x)+f(x-2)=0 \forall x \in R,$ and if $\left(\left(\frac{d}{dx}f(x)\right)\right)_{x=a}=b, then\left(\left(\frac{d}{dx}f(x)\right)\right)_{a+4000}=b.$ Statement 2: $f(x)$ is a periodic function with period 4.

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is correct.

Answer:

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83. Let $f(x) = x[x]$, where [*] denotes the greatest integer function, when x is not an integer then find the value of $f'(x)$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer:

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84. If f(x) is a polynomial of degree $n(>2)$ and $f(x)=f(\alpha-x)$, (where α is a fixed real number), then the degree of f'(x) is

A. 2

B. 3

C. 4

D. 5

Answer:

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85. The moment when A is at $(0,0)$ and B is at $(1,2)$. The derivative $\frac{dy}{dx}$ of line AB is

A. $\frac{1}{9}$

B. $\frac{1}{7}$

C. $\frac{1}{5}$

D. None of these

Answer:

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86. The moment when A is at $(1, 2)$ and $Bisat(0, 0)$. The derivative $\frac{dx_B}{dx_A}$, is

A. 16

B. 8

C. 9

D. 2

Answer:

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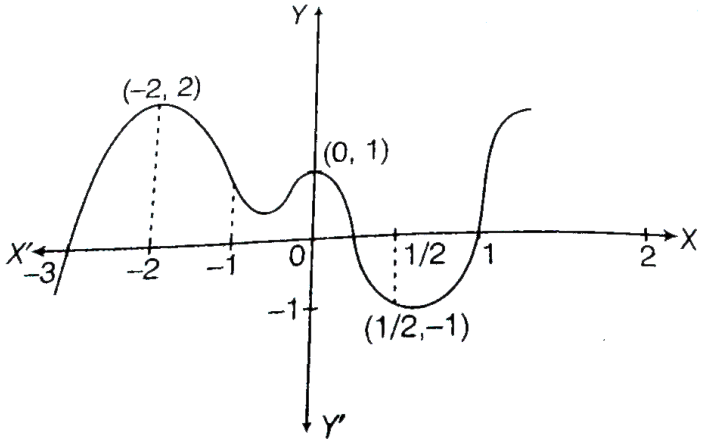
87. If $p''(x)$ has real roots α, β, γ. Then , $[\alpha] + [\beta] + [\gamma]$ is



A. -2

B. -3

C. -1

D. 0

Answer: -2

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88. Match the Column I with Column II and mark the correct option from the given below.

	Column I	Column II	
(i)	If $f'(x) = \sqrt{3x^2 + 6}$ and $y = f(x^3)$ then at $x = 1$, $\frac{dy}{dx}$ is	p.	-2
(ii)	If f is a differentiable function such that $f(xy) = f(x) + f(y) : x, y \in R$, then $f(e) + f(1/e)$ is	q.	-1
(iii)	If f is a twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h(x) = [f(x)]^2 + [g(x)]^2$ and $h(5) = 9$, then $h(10)$ is	r.	0
(iv)	$y = \tan^{-1}(\cot x) + \cot^{-1}(\tan x)$, $\frac{\pi}{2} < x < \pi$, then $\frac{dy}{dx}$ is	s.	9

A. i ii iii iv
s r s p

B. i ii iii iv
p q r s

C. i ii iii iv
() q p r r

D. i ii iii iv
s p q q

Answer:

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89. Match the Column I with Column II and mark the correct option from the given below.

	Column I	Column II	
(i)	If $y = \cos^{-1}(\cos x)$, then y' at $x = 5$ is equal to	p.	-1
(ii)	For the function $f(x) = \log_e \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, if $\frac{dy}{dx} = \sec x + p$, then p is equal to	q.	0
(iii)	The derivative of $\tan^{-1}\left(\frac{1+x}{1-x}\right)$ at $x = -1$ is	r.	1/2
(iv)	The derivative of $\frac{\log \lvert x \rvert}{x}$ at $x = -1$ is	s.	1

A. i ii iii iv
p q r s

B. i ii iii iv
q p r s

C. i ii iii iv
s r q p

D. i ii iii iv
r s p q

Answer:

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90. If $y = \sqrt{(x - \sin x) + \sqrt{x - \sin x} + \ldots,}$ then $\left| \left| \frac{dx}{dy} \right|_{x=\frac{\pi}{2}}^{2} - 2\pi \right| = \ldots\ldots\ldots$

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91. Let $f(x) = \int_{-2}^{x} e^{(1+t)^2} dt$ and $g(x) = f(h(x))$,where h(x) is defined for all $x \in R$. If $g'(2) = e^4$ and $h'(2) = 1$ then absolute value of sum of all possible values of h(2), is

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92. If $f(x) = \cos\left\{\frac{\pi}{2}[x] - x^3\right\}, 1 < x < 2$ and $[x] =$ the greatest integer $\leq x$, then find $f'\left(\sqrt[3]{\frac{\pi}{2}}\right)$

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93. If $u = f(x^3), v = g(x^2), f'(x) = \cos x, and g'(x) = \sin x, then \frac{du}{dv}$ is $\frac{3}{2}x\cos x^3 \cos ecx^2$ $\frac{2}{3}\sin x^3 \sec x^2 \tan x$ (d) none of these

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94.

$f: R \to R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R.$

The value of f(1) is

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95. Let

$f(x) = x^2 + x g^2(1) + g''(2)$ and $g(x) = f(1).\, x^2 + x f'(x) + f''(x),$

then find $f(x)$ and $g(x).$

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96. If $y = \dfrac{\sin x}{1 + \dfrac{\cos x}{1 + \dfrac{\sin x}{1 + \dfrac{\cos x}{1 + \ddots \to \infty}}}}$, prove that $\dfrac{dy}{dx} = \dfrac{(1+y)\cos x + y \sin x}{1 + 2y + \cos x - \sin x}$

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97. If $3\sqrt{3\sqrt{x - \dfrac{1}{3\sqrt{x}}}} = 2, then 3\sqrt{x}\left(\dfrac{1}{3\sqrt{x}}\right)$=_____

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98. If

$y = \tan^{-1}\left(\dfrac{1}{x^2 + x + 1}\right) + \tan^{-1}\left(\dfrac{1}{x^2 + 3x + 3}\right) + \tan^{-1}\left(\dfrac{1}{x^2 + 5x +$

to nterms, show that $\dfrac{dy}{dx} = \dfrac{1}{(x+n)^2 + 1} - \dfrac{1}{x^2 + 1}$

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99. if $f(\theta) = \cos\theta_1 . \cos\theta_2 . \cos\theta_3 . \ldots . \cos\theta_n$, show that

$\{\tan\theta_1 + \tan\theta_2 + \tan\theta_3 + \ldots . + \tan\theta_n\} = -\left\{\dfrac{f'(\theta)}{f(\theta)}\right\}$,

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100. Find the sum of

$\sin x + 3\sin 3x + 5\sin 5x + \ldots\ldots\ldots . + (2k-1)\sin(2k-1)x.$

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101. Find the sum of the series $\sin gma_{r=1}^{n} r x^{r-1}$ using calculus .

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102. Show that 1/(x+1)+5/(x^2+1)+4/x^4+1)+.....+2^n/(x^2^n+1)= /(x-1)-2^(n+1)/(x^2(n+1)−1)

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103.

$f_n(x) = e^{f_{n-1}(x)}$ for all $n \in N$ and $f_0(x) = x$, then $\frac{d}{dx}\{f_n(x)\}$ is

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104. If $y^3 - y = 2x$, prove that $\frac{d^2y}{dx^2} = -\frac{24y}{(3y^2-1)^3}$.

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105. If $2x = y^{\frac{1}{5}} + y^{-\frac{1}{5}}$ then $(x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = ky$, then find the value of k.

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106. Let

$$y = 1 + \frac{a_1}{x-a_1} + \frac{a_2 x}{(x-a_1)(x-a_2)} + \frac{a_3 x^2}{(x-a_1)(x-a_2)(x-a_3)} + \ldots$$

Find $\frac{dy}{dx}$

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107. If $y = f\left(\frac{2x-1}{x^2+1}\right)$ and $f'(x) = \sin x^2$, then $\frac{dy}{dx}$ is equal to

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108. let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a_1, a_2, a_3 are in AP, then show that $f'(a_1), f'(a_2), f'(a_3)$ are in AP.

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109. If $5f(x) + 3f\left(\frac{1}{x}\right) = x + 2$ and $y = xf(x)$, then find $\frac{dy}{dx}$ at $x = 1$

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Exercise More Than One Correct Option Type Questions

1. If $y = \frac{\sec x - \tan x}{\sec x + \tan x}$, then $\frac{dy}{dx}$ equals.

A. $2\sec x(\sec x - \tan x)$

B. $-2\sec x(\sec x - \tan x)^2$

C. $2\sec x(\sec x - \tan x)^2$

D. $-2\sec x(\sec x + \tan x)^2$

Answer: B

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2. If $y = \frac{x^4 + x^2 + 1}{x^2 + x + 1}$ then $\frac{dy}{dx} = ax + b$, find a and b

A. $a = 2, b = 1$

B. $a = -2, b = 1$

C. $a = 2, b = -1$

D. $a = -2, b = -1$

Answer: C

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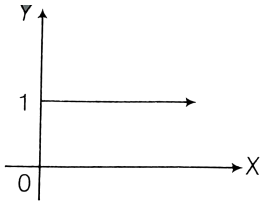
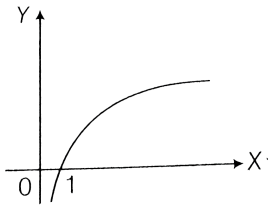
3. Which of the following could be the sketch graph of $y = \frac{d(x \ln x)}{dx}$

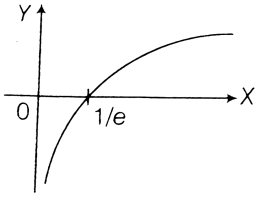
A.



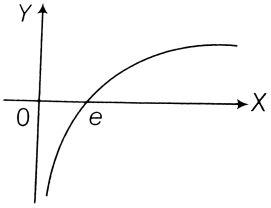
B.



C.



D.



Answer: C

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4. Let $f(x) = x + 3\ln(x-2) \& g(x) = x + 5\ln(x-1)$, then the set of x satisfying the inequality `f^(prime)(x)

A. $(2, 7/2)$

B. $(1, 2) \cup (7/2, \infty)$

C. $(2, \infty)$

D. $7/2, \infty$

Answer: D

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5. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = a, then \frac{dy}{dx} =$

A. $-\frac{x}{y}$

B. $-\frac{y}{x}$

C. $\frac{y}{x}$

D. $\frac{x}{y}$

Answer: C

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6. If $f(x) = |x|^{|\sin x|}$, then $f'\left(\frac{\pi}{4}\right)$ equals

A. $\left(\frac{\pi}{4}\right)^{1\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

B. $\left(\frac{\pi}{4}\right)^{1\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

C. $\left(\frac{\pi}{4}\right)^{1\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

D. $\left(\frac{\pi}{4}\right)^{1\sqrt{2}} \cdot \left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

Answer: A

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7. $y = \dfrac{x}{a + \dfrac{x}{b + \dfrac{x}{a + \dfrac{x}{b + \ldots \infty}}}}, \dfrac{dy}{dx} = \dfrac{b}{a(b + 2y)}$

A. $\dfrac{a}{ab + 2ay}$

B. $\dfrac{b}{ab + 2ay}$

C. $\dfrac{a}{ab + 2by}$

D. $\dfrac{b}{ab + 2ay}$

Answer: D

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8. If $y = x^{x^2}$, then $\dfrac{dy}{dx}$ equals

A. $2 \log x . x^2$

B. $(2\log x + 1).\, x^{x^2}$

C. $(2\log x + 1).\, x^{x^2} + 1$

D. $x^{x^2+1}.\, (\log(ex^2))$

Answer: D

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9. $x\sqrt{1+y} + y\sqrt{1+x} = 0$ then $\frac{dy}{dx} =$

A. $\frac{1}{(1+x)^2}$

B. $\frac{-1}{(1+x)^2}$

C. $\frac{-1}{(1+x)} + \frac{1}{(1+x)^2}$

D. None of these

Answer: B

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10. If $x^2e^y + 2xye^x + 13 = 0$ then $\frac{dy}{dx} =$

A. $\frac{-2xe^{y-x} + 2y(x-1)}{x(xe^{y-x} + 2)}$

B. $\left[\frac{2xe^{y-x} + 2y(x-1)}{x(xe^{y-x} + 2)}\right]$

C. $\frac{2xe^{y-x} + 2y(x-1)}{x(xe^{y-x} + 2)}$

D. None of these

Answer: B

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11. If $x = e^{y+e^{y+\ldots to\infty}}, x > 0,$ then $\frac{dy}{dx}$

A. $\frac{x}{1+x}$

B. $\frac{1+x}{x}$

C. $\frac{1-x}{x}$

D. $\frac{1}{x}$

Answer: C

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12. Let g be the inverse function of f and $f'(x) = \frac{x^{10}}{1+x^2}$. If $g(2) = a$ then $g'(2)$ is equal to

A. $\frac{5}{2^{10}}$

B. $\frac{1+a^2}{a^{10}}$

C. $\frac{a^{10}}{1+a^2}$

D. $\frac{1+a^{10}}{a^2}$

Answer: B

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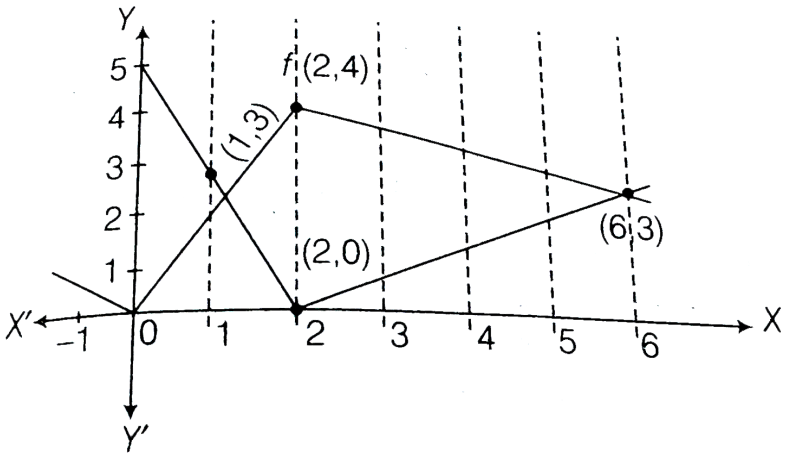
13. If f and g are the function whose graphs are as shown, let $u(x) = f(g(x)), w(x) = g(g(x))$



Then the value of $u'(1) + w'(1)$ is

A. $-\frac{1}{2}$

B. $-\frac{3}{2}$

C. $-\frac{5}{4}$

D. does not exist

Answer: B

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14. $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all real x and $f(5) = 2 = f'(5) then f^2(10) + g^2(10)$ is

A. 2

B. 4

C. 8

D. None of these

Answer: C

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15. if

$$f(x) = x + \frac{x^2}{1}! + \frac{x^3}{3}! \pm - - + \frac{x^n}{n}! then f(0) + f'(0) + f''(0) + -$$

is equal to

A. $\frac{n(n+1)}{2}$

B. $\frac{(n^2+1)}{2}$

C. $\left(\frac{n(n+1)}{2}\right)^2$

D. $\frac{n(n+1)(2n+1)}{6}$

Answer: A

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16. If $y = (f_0 f_0 f)(x)$ and $f(0) = 0, f'(0) = 2$ then $y'(0)$ is equal to

A. 6

B. 7

C. 8

D. 9

Answer: C

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17. If $y^2 = P(x)$ is a polynomial of degree $3,$ then $2\left(\frac{d}{dx}\right)\left(y^2 \frac{d^2 y}{dx^2}\right)$ is equal to $P^x + P'(x)$ (b) $P^x \dot{P}^x$ $P(x)\dot{P}^x$ (d) a constant

A. $p'''(x).p'(x)$

B. $p''(x).p'''(x)$

C. $p(x).p'''(x)$

D. None of these

Answer: C

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18. If $y = f(x)$ and $x = g(y)$ are inverse functions of each other, then

A. $g''(y) = \frac{1}{f''(x)}$

B. $g''(y) = \frac{f''(x)}{(f''(x))^3}$

C. $g''(y) = \frac{f'(x)}{(f'(x))^3}$

D. None of these

Answer: B

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19. If y is a function of x then $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0$. If x is a function of y then the equation becomes

A. $\frac{d^2x}{dy^2} - x\frac{dx}{dy} = 0$

B. $\frac{d^2x}{dy^2} + y\left(\frac{dx}{dy}\right)^2 = 0$

C. $\frac{d^2x}{dy^2} - y\left(\frac{dx}{dy}\right)^2 = 0$

D. $\frac{d^2x}{dy^2} - x\left(\frac{dx}{dy}\right)^2 = 0$

Answer: C

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20. Leg $g(x) = \ln(f(x))$, whre $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = xf(x)$. Then, for $N = 1, 2, 3,$ $g^{N+\frac{1}{2}} - g^{\frac{1}{2}} =$

A. $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\right\}$

B. $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N-1)^2}\right\}$

C. $-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N+1)^2}\right\}$

D. $4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots+\frac{1}{(2N+1)^2}\right\}$

Answer: A

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21. If the function $f(x)=x^3+e^{x/2}$ and $g(x)=f^{-1}(x)$, then the value of $g'(1)$ is

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22. If $f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4}<\theta<\frac{\pi}{4}$, then the value of $\frac{d}{d(\tan\theta)}f(\theta)$ is

A. 1

B. 2

C. 3

D. 4

Answer: A

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23. If $y = \log_{\sin x}(\tan x)$, then $\frac{dy}{dx}$ at $x = \frac{1}{4}$ is equal to

A. $\frac{4}{\log 2}$

B. $-4\log 2$

C. $\frac{-4}{\log 2}$

D. None of these

Answer: C

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24. If $y = \sum_{r=1}^{x} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)$, then $\frac{dy}{dx}$ is equal to

A. $\frac{1}{1+x^2}$

B. $\frac{1}{1+(1+x)^2}$

C. 0

D. None of these

Answer: B

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25. If $y = \left(\frac{\sin^{-1}(\sin\alpha \sin x)}{1-\cos\alpha \sin x}\right)$, then $y'(0)$ is equal to

A. 1

B. $2\tan\alpha$

C. $\left(\frac{1}{2}\right)\tan\alpha$

D. $\sin\alpha$

Answer: D

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26. If $f(x) = \cot^{-1}\left(\frac{x^x - x^{-x}}{2}\right)$ then f'(1) equals

A. -1

B. 1

C. $\log_e 2$

D. $-\log_e 2$

Answer: A

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27. The function $f(x) = e^x + x,$ being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)\cdot$ The value of $\frac{d}{dx}\left(f^{-1}\right)$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

A. $\frac{1}{1n2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. None of these

Answer: B

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28. If $f^x = -f(x)$ and $g(x) = f'(x)$ and $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5,$ then $F(10)$ is equal to 5 (b) 10 (c) 0 (d) 15

A. 5

B. 10

C. 0

D. 15

Answer: A

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29. If $x = \sec\theta - \cos\theta$ and $y = \sec^n\theta - \cos^n\theta$, then $(x^2+4)\left(\frac{dy}{dx}\right)^2$ is equal to

A. $n^2(y^2-4)$

B. $n^2(4-y^2)$

C. $n^2(y^2+4)$

D. None of these

Answer: C

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30. If $x = f(t)\cos t - f'(t)\sin t$ and $y = f(t)\sin t + f'(t)\cos t$, then $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = f(t) - f(t)$ (b) $\{f(t) - f(t)\}^2$ (c) $\{f(t) + f(t)\}^2$

(d) none of these

A. $f(t) - f''(t)$

B. $[f(t) - f''(t)]^2$

C. $[f(t) + f''(t)]^2$

D. None of these

Answer: C

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31. If $y = at^2 + 2bt - c$ and $t = ax^2 + 2bx + c$, then $\frac{d^3y}{dx^3}$ equals

A. $24a^2(at + b)$

B. $24a(ax + b)^2$

C. $24a(at + b)^2$

D. $24a^2(ax + 0b)$

Answer: D

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32. Differential coefficient of $\left(x^{\frac{l+m}{m-n}}\right)^{1/(n-l)} \cdot \left(x^{\frac{m+n}{n-l}}\right)^{1/(l-m)} \cdot \left(x^{\frac{n+l}{l-m}}\right)^{1/(m-n)}$ wrt x is

A. 1

B. 0

C. -1

D. x^{lmn}

Answer: B

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33. if $y = (A + Bx)e^{mx} + (m-1)^{-1}e^{x}$ then $\frac{d^2y}{dx^2} - 2m\frac{dy}{dx} + m^2y$ is equal to:

A. e^x

B. e^{mx}

C. e^{-mx}

D. $e^{(1-m)x}$

Answer: A

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34. If $f(x) = -\frac{x^3}{3} + x^2 \sin a . \sin 2a - 5\sin^{-1}\left(a^2 - 8a + 17\right)$,then

A. $f(x)$ is not defined at $x = \sin 8$

B. $f'(\sin 8) > 0$

C. $f'(x)$ is not defined at $x = \sin 8$

D. $f'(\sin 8), < 0$

Answer: D

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35. Let f and g be differentiable functions satisfying $g(a) = b, g'(a) = 2$ and fog =I (identity function). then f' (b) is equal to

A. $2/3$

B. 1

C. 0

D. $1/2$

Answer: D

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36. The derivative of the function, $f(x) = \cos^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x - 3\sin x)\right\} + \sin^{-1}\left\{\frac{1}{\sqrt{13}}(2\cos x + 3\sin x)\right\}$ at $x = \frac{3}{4}$ is

A. $\frac{3}{2}$

B. $\frac{5}{2}$

C. $\frac{10}{3}$

D. 0

Answer: C

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37. If $f(x) = \sqrt{x + 2\sqrt{2x - 4}} + \sqrt{x - 2\sqrt{2x - 4}}$ then the value of 10 $f'\left(102^{+}\right)$, is

A. -1

B. 0

C. 1

D. does not exist

Answer: C

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38. Let $y = \ln(1 + \cos x)^2$, then the value of $\frac{d^2y}{dx^2} + \frac{2}{e^{y/2}}$ equals

A. 0

B. $\frac{2}{1 + \cos x}$

C. $\frac{4}{(1 + \cos x)}$

D. $\frac{-4}{(1 + \cos x)^2}$

Answer: A

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39. If $f(x) = \frac{a + \sqrt{a^2 - x^2} + x}{\sqrt{a^2 - x^2} + a - x}$ where $a > 0$ and `x

A. $\sqrt{a}$

B. a

C. $\frac{1}{\sqrt{a}}$

D. $\frac{1}{a}$

Answer: D

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40. Let u(x) and v(x) be differentiable functions such that $\frac{u(x)}{v(x)} = 7.$ if $\frac{u'(x)}{v'(x)} = p$ and $\left(\frac{u(x)}{v(x)}\right)' = q,$ then $\frac{p+q}{p-q}$ has the value equal to

A. 1

B. 0

C. 7

D. -7

Answer: A

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41. If $f(x) = |\log_e|x||, \quad \text{then} \quad f'(x)$ equals

A. $\frac{1}{|x|}, x \neq 0$

B. $\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

C. $-\frac{1}{x}$ for $|x| > 1$ and $-\frac{1}{x}$ for $|x| < 1$

D. $\frac{1}{x}$ for $x > 0$ and $-\frac{1}{x}$ for $x < 0$

Answer: B

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42. If $f(x)$ is given by

$f(x) = (\cos x + i \sin x)(\cos 3x + i \sin 3x).....$

$.....[\cos(2n-1)x + i \sin(2n-1)x],$

then f''(x) is equal to

A. $n^3 f(x)$

B. $-n^4 f(x)$

C. $-n^2 f(x)$

D. $n^4 f(x)$

Answer: B

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43. Let $f(x) = x^n$, n being a non-negative integer, The value of n for which the equality $f'(x+y) = f'(x) + f'(y)$ is valid for all $x, y > 0$, is

A. 0,1

B. 1,2

C. 2,4

D. None of these

Answer: B

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44. If $f(x) = \sin\left\{\frac{\pi}{3}[x] - x^2\right\}$ for $2 < x < 3$ and [x] denotes the greatest integer less than or equal to x, then $f'(\sqrt{\pi/3})$ is equal to

A. $\sqrt{\pi/3}$

B. $-\sqrt{\pi/3}$

C. $-\sqrt{\pi}$

D. None of these

Answer: B

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45. The function $u = e^x s \in ; v = e^x \cos x$ satisfy the equation $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$ b. $\frac{d^2u}{dx^2} = 2v$ c. $\frac{d^2}{dx^2} = -2u$ d. $\frac{du}{dx} + \frac{dv}{dx} = 2v$

A. $v\frac{du}{dx} - u\frac{dv}{dx} = u^2 + v^2$

B. $v\frac{d^2u}{dx^2} = 2v$

C. $\frac{d^2v}{dx^2} = -2u$

D. All of these

Answer: D

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46. If $f(x) = \log_x(\ln x)$ then $f'(x)$ at x=e is

A. e

B. $-e$

C. e^2

D. e^{-1}

Answer: D

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47. Let f be a differentiable function satisfying

$[f(x)]^n = f(nx)$ for all $x \in R$.

Then, $f'(x)f(nx)$

A. $f(x)$

B. 0

C. $f(x)f'(nx)$

D. None of these

Answer: C

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48. If $y = f(x)$ is an odd differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2, then|f'(-3)|$ equals_____.

A. 4

B. 2

C. -2

D. 0

Answer: C

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49. If $y = \sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \ldots\infty}}}}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{y + x}{y^2 - 2x}$

B. $\frac{y^3 - x}{2y^2 - 2xy - 1}$

C. $\frac{y^3 + x}{2y^2 - x}$

D. None of these

Answer: D

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50. If $f(x) = |\cos x - \sin x|$, then $f'\left(\frac{\pi}{4}\right)$ is equal to

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. 0

D. None of these

Answer: D

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51. Let

$f(x) = x^2 + xg^2(1) + g''(2)$ and $g(x) = f(1).\, x^2 + xf'(x) + f''(x)$,

then find $f(x)$ and $g(x)$.

A. $f'(1) = 4 + f'(2)$

B. $g'(2) = 8 + g'(10$

C. $g''(2) + f''(3) = 4$

D. All of the above

Answer: D

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52. if $f(x) = x^n$ then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + - - - + \frac{(-1)^n f''^{--n} \times (1)}{n!}$

A. 1

B. 2^n

C. 2^{n-1}

D. 0

Answer: D

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53. If $y + \log(1 + x) = 0$ which of the following is true?

A. $e^y = xy' - 1$

B. $y' = -\frac{1}{(x+1)}$

C. $y' + e^y = 0$

D. $y' = e^y$

Answer: B::C

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54. If $y = 2^{3^x}$, then y' equals

A. $3^x . \log 3 . \log 2$

B. $y . (. \log 2y) . \log 3 . \log 2$

C. $2^{3^x} . 3^x . \log 6$

D. $2^{3^x} . 3^x . \log 3 . \log 2$

Answer: B::D

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55. If g is the inverse of f and $f(x) = x^2 + 3x - 3, (x > 0)$. then $g'(1)$ equals

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56. If $x^3 - 2x^2y^2 + 5x + y - 5 = 0 and y(1) = 1, then\ y'(1) = \frac{4}{3}$ (b) $y^1 = -\frac{4}{3}\ y^1 = -8\frac{22}{27}$ (d) $y'(1) = \frac{2}{3}$

A. $y'(1) = \frac{4}{3}$

B. $y''(1) = -\frac{1}{3}$

C. $y''(1) = -8\frac{22}{27}$

D. $y'(1) = \frac{2}{3}$

Answer: A::C

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57. let $y = \sqrt{x + \sqrt{x + \sqrt{x}...\infty}}$ then dy/dx equals

A. $\frac{1}{2y - 1}$

B. $\frac{x}{x + 2y}$

C. $\frac{1}{\sqrt{1 - 4x}}$

D. $\frac{y}{2x + y}$

Answer: A::D

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58. If $y = x^{(\ln x)^{\ln(\ln x)}}$, then $\frac{dy}{dx}$ is equal to

A. $\frac{y}{x}\left(\ln x^{\ln x - 1} + 2\ln + (\ln x)\right)$

B. $\frac{y}{x}(\ln x)^{\ln(\ln x)}(2\ln(\ln x) + 1)$

C. $\frac{y}{x \ln x}\left((nx)^2 + 2\ln x(\ln x)\right)$

D. $\frac{y}{x}.\frac{\ln y}{\ln x}(2\ln(\ln x)+1)$

Answer: B::D

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59. Which of the following functions are not derivable at $x=0$?

A. $f(x)=\sin^{-1}2x\sqrt{1-x^2}$

B. $g(x)=\sin^{-1}\left(\frac{2^x+1}{1+4^x}\right)$

C. $h(x)=\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

D. $k(x)=\sin^{-1}(\cos x)$

Answer: B::C::D

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60. Let $f(x)=\frac{\sqrt{x-2\sqrt{x-1}}}{\sqrt{x-1}-1}.x$ then

A. $f'(10) = 1$

B. $f'(3/2) = -1$

C. domain of $f(x)$ is $x \leq 1$

D. None of these

Answer: A::B

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61. If $2^x + 2^y = 2^{x+y}$ then $\frac{dy}{dx}$ is equal to

A. $-\frac{2^y}{2^x}$

B. $\frac{1}{1-2^x}$

C. $1-2^y$

D. $\frac{2^x(1-2^y)}{2^y(2^x-1)}$

Answer: A::B::C::D

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62. For the function $y = f(x) = \left(x^2 + bx + c\right)e^x$, which of the following holds?

A. If $f(x) > 0$ for all real $x \not\Rightarrow f'(x) > 0$

B. If $f(x) > 0$ for all real $x \Rightarrow f'(x) > 0$

C. If $f'(x) > 0$ for all real $x \Rightarrow f(x) > 0$

D. If $f'(x) > 0$ for all real $x \not\Rightarrow f(x) > 0$

Answer: A::C

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63. If $\sqrt{y+x} + \sqrt{y-x} = c,$ where $c \neq 0$, then $\frac{dy}{dx}$ has the value equal to

A. $\frac{2x}{e^2}$

B. $\frac{x}{y + \sqrt{y^2 - x^2}}$

C. $\frac{y-\sqrt{y^2-x^2}}{x}$

D. $\frac{c^2}{2y}$

Answer: C

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64. If $y=\tan x\tan 2x\tan 3x, (\sin 12x\neq 0)$ then $\frac{dy}{dx}$ has the value equal to

A.

$$3\sec^2 3x\tan x\tan 2x+\sec^2 x\tan 2x\tan 3x+2\sec^2 2x\tan 3x\tan x$$

B. $2y(\cos ec2x+2\cos ec4x+3\cos ec6x)$

C. $3\sec^2 3x-2\sec^2 2x-\sec^2 x$

D. $\sec^2 x+2\sec^2 2x+3\sec^2 3x$

Answer: A::B::C

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Exercise Statement I And Ii Type Questions

1. Consider $f(x) = \frac{x}{x^2 - 1}$

Statement I Graph of $f(x)$ is concave up for $x > 1$.

Statement II If $f(x)$ is concave up then $f''(x) > 0$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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2. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then Statement I The value of $f(2) = \sin^{-1}\left(\frac{4}{5}\right)$. Statement II $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = -2$, for $x < 1$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: C

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3. Statement I if $f(0) = a, f'(0) = b, g(0) = 0, (fog)'(0) = c$ then $g'(0) = \frac{c}{b}$. Statement II $(f(g(x))' = f'(g(x)).g'(x)$, for all n

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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4. Let f and g be real valued functions defined on interval $(-1, 1)$ such that $g''(x)$ is constinous, $g(0) = 0$, $g'(0) = 0, g''(0) = 0$ and $f(x) = g(x)\sin x$.

Statement I $\lim_{x \to 0} (g(x)\cot x - g(0)\operatorname{cosec} x) = f''(0)$

Statement II $f'(0) = g'(0)$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: B

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5. Statement I If $y = \sin^{-1}(3x - 4x^3)$, then $\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$ only when $\frac{-1}{2} \leq x < \frac{1}{2}$/.

Statement II $\sin^{-1}(3x - 4x^3)$

$$= \begin{cases} -\pi - 3\sin^{-1} x & -1 \leq x \leq -\frac{1}{2} \\ 3\sin^{-1} x & -\frac{1}{2} \leq x \leq \frac{1}{2} \\ \pi - 3\sin^{-1} x & \frac{1}{2} \leq x \leq 1 \end{cases}$$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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6. If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then

Statement I $\frac{dy}{dx} = \frac{2}{1+x^2}$ for $x \in R$

Statement II $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \begin{cases} 2\tan^{-1}x & x \leq 0 \\ -2\tan^{-1}x & x < 0 \end{cases}$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: D

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7. If $y = x + [x],$ then

Statement I $\frac{dy}{dx} = 1$ for all $x \in R$

Statement II

$\frac{d([x])}{dx} = \{(\quad 0, \quad x \not\in \text{Integer}), (\text{does not exist} \quad x. \in \text{integer}$

):}

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: D

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8. Statement I If $f(x)$ is a continuous function defined from R to Q and $f(5) = 3$, then differential coefficient of $f(x)w.\ r.\ t.\ x$ will be 0

Statement II Differentation of constant functions is always zero.

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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9. Statement I Derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right) w.r.t. \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ is 1 for $0 < x < 1$.

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ for $-1 \leq x \leq 1$

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: C

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10. Consider function $f(x)$ satisfies the relation $f\left(x+y^3\right)=f(x)+f\left(y^3\right)\forall x, y \in R$ and differentiable for all x.

Statement I If $f'(2)=a$ then $f'(-2)=a$

$f(x)$ is an odd function.

A. Both statement I and Statement II are correct and Statement II is the correct explanation of Statement I

B. Both Statement I and Statement II are correct but Statement II is not the correct explanation of Statement I

C. Statement I is correct but Statement II is incorrect

D. Statement II is correct but Statement I is incorrect.

Answer: A

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Exercise Passage Based Questions

1. Let $\frac{f(x+y)-f(x)}{2}=\frac{f(y)-1}{2}+xy$, for all $x, y \in R$, $f(x)$ is differentiable and $f'(0)=1$. Domain of $\log(f(x))$, is

A. R^+

B. $R-\{0\}$

C. R

D. R^-

Answer: C

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2. Let $\frac{f(x+y)-f(x)}{2}=\frac{f(y)-1}{2}+xy$, for all $x, y \in R$, $f(x)$ is differentiable and $f'(0)=1$.

Range of $y=\log_{3/4}(f(x))$ is

A. $(-\infty, 1]$

B. $[3/4, \infty$

C. $(-\infty, \infty)$

D. R

Answer: A

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3. Let $\frac{f(x+y)-f(x)}{2} = \frac{f(y)-1}{2} + xy$, for all $x, y \in R$, $f(x)$ is differentiable and $f'(0) = 1$. Let $g(x)$ be a derivable function at $x = 0$ and follows the function rule $g\left(\frac{x+y}{k}\right) = \frac{g(x)+g(y)}{k}$, $k \in R, k \neq 0, 2$ and $g'(0) - \lambda g'(0) \neq 0$. If the graphs of $y = f(x)$ and $y = g(x)$ intersect in coincident points then λ can take values

A. -3

B. 1

C. -1

D. 4

Answer: C

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4. Find the derivative of log(sin(logx)).

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5. Left hand derivative and right hand derivative of a function $f(x)$ at a point $x = a$ are defined as

$$f'(a^-) = \lim_{(h \to 0^+)} \frac{f(a) - f(a-h)}{h}$$

$$= \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

and $f'(a^+) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$

$$= \lim_{h \to 0^+} \frac{f(a) - f(a+h)}{h}$$

$$= \lim_{h \to 0^+} \frac{f(a) - f(x)}{a - x}$$ respectively.

Let f be a twice differentiable function. We also know that derivative of a even function is odd function and derivative of an odd function is even function.

If f is even function, which of the following is right hand derivative of f' at $x = a$?

A. $\lim_{h \to 0^-} \dfrac{f'(a) + f'(-a+h)}{-h}$

B. $\lim_{h \to 0^+} \dfrac{f'(a) + f'(-a-h)}{h}$

C. $\lim_{h \to 0^-} \dfrac{-f'(-a) + f'(-a-h)}{-h}$

D. $\lim_{h \to 0^+} \dfrac{f'(a) + f'(-a+h)}{-h}$

Answer: A

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6. Left hand derivative and right hand derivative of a function $f(x)$ at a point $x = a$ are defined as

$$f'(a^-) = \lim_{h \to 0^+} \frac{f(a) - f(a-h)}{h}$$
$$= \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$

and $f'(a^+) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$

$= \lim_{h \to 0^+} \frac{f(a) - f(a+h)}{h}$

$= \lim_{h \to 0^+} \frac{f(a) - f(x)}{a - x}$ respectively.

Let f be a twice differentiable function. We also know that derivative of a even function is odd function and derivative of an odd function is even function.

The statement $\lim_{h \to 0} \frac{f(-x) - f(-x-h)}{h} = \lim_{h \to 0} \frac{f(x) - f(x-h)}{-h}$ implies that for all $x \in R$,

A. f is odd

B. f is even

C. f is neither even nor odd

D. Nothing can be concluded

Answer: B

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7. If $f(x) = \sin^{-1}\left(3x - 4x^3\right)$. Then answer the following

The value of f'(0), is

A. -3

B. 3

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: B

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8. If $f(x) = \sin^{-1}\left(3x - 4x^3\right)$. Then answer the following

The value of $f'\left(\frac{1}{\sqrt{2}}\right)$, is

A. -3

B. 3

C. $-3\sqrt{2}$

D. $3\sqrt{2}$

Answer: C

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9. Let the derivative of $f(x)$ be defined as $D^* f(x) = \lim_{h \to 0} \frac{f^2 x + h - f^2(x)}{h}$, where $f^2(x) = \{f(x)\}^2$.

If $u = f(x), v = g(x)$, then the value of $D^*(u.\ v)$ is

A. $(D^* u)v + (D^* v)u$

B. $u^2(D^* v) + v^2(D^* u)$

C. $D^* u + D^* v$

D. $uvD^*(u + v)$

Answer: B

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10. Let the derivative of $f(x)$ be defined as $D^*f(x) = \lim_{h \to 0} \frac{f^2x + h - f^2(x)}{h}$, where $f^2(x) = \{f(x)\}^2$.

If $u = f(x), v = g(x)$, then the value of $D^*\left(\frac{u}{v}\right)$ is.

A. $\frac{u^2(D^*v) - v^2(D^*u)}{v^4}$

B. $\frac{u(D^*v) - v(D^*u)}{v^2}$

C. $\frac{v^2(D^*u) - u^2(D^*v)}{v^4}$

D. $\frac{v(D^*u) - u(D^*v)}{v^2}$

Answer: C

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11. A curve is represented parametrically by the equations $x = e^t \cos t$ and $y = e^t \sin t$ where t is a parameter. Then The relation between the parameter 't' and the angle a between the tangent to the given curve andthe x-axis is given by, 't' equals

A. $\frac{\pi}{2} - \alpha$

B. $\frac{\pi}{4} + \alpha$

C. $\alpha - \frac{\pi}{4}$

D. $\frac{\pi}{4} - \alpha$

Answer: C

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12. A curve is represented paramtrically by the equations $x = e^t \cos t$ and $y = e^t \sin t$ where t is a parameter. Then

The value of $\frac{d^2y}{dx^2}$ at the point where $t = 0$ is

A. 1

B. 2

C. -2

D. 3

Answer: B

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13. A curve is represented parametrically by the equations $x = e^1 \cos t$ and $y = e^1 \sin t$, where t is a parameter. Then, If $F(t) = \int (x + y)dt$, then the value of $F\left(\frac{\pi}{2}\right) - F(0)$ is

A. 1

B. -1

C. $e^{\pi/2}$

D. 0

Answer: C

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14. Equation $x^n - 1 = 0, n > 1, n \in N$, has roots $1, a_1, a_2, ..., a_{n-1}$, .

The value of $(1 - a_1)(1 - a_2)...(1 - a_{n-1})$, is

A. $\frac{n^2}{2}$

B. n

C. $(-1)^n n$

D. None of the above

Answer: B

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15. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, \ldots, a_n, .$

The value of $\sum_{r=2}^{n} \frac{1}{2 - a_r},$ is

A. $\frac{2^{n-1}(n-2)+1}{2^n - 1}$

B. $\frac{2^n(n-2)+1}{2^n - 1}$

C. $\frac{2^{n-1}(n-1)-1}{2^n - 1}$

D. None of the above

Answer: A

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16. Equation $x^n - 1 = 0, n > 1, n \in N,$ has roots $1, a_1, a_2, ..., a_n, .$

The value of $\sum_{r=2}^{n} \frac{1}{2 - a_r},$ is

A. $\frac{n}{4}$

B. $\frac{n(n-1)}{2}$

C. $\frac{n-1}{2}$

D. None of these

Answer: D

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Exercise Subjective Type Questions

1.

$f: R \to R, f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$ for all $x \in R.$

The value of f(1) is

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2. Let $f(x) = \sin^{-1}\left(\frac{2x+2}{\sqrt{4x^2+8x+13}}\right)$, then the value of $\frac{d(\tan f(x))}{d\left(\tan^{-1} x\right)}$, when $x = \frac{1}{2}$, is.....

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3. Let $x, x_1, x_2, x_3, x_4, \ldots, x_8,$ be 10 real zeros, of the polynomial $P(x) = x^{10} + ax^2 + bx + c$ where $a, b, c, \in R.$ If the value of $Q(x_1) = \frac{p}{q}$, where p and q are coprime to each other. If $Q(x_1) = (x - x_2)(x - x_3)\ldots(x - x_8)$ and $x_1 = \frac{1}{2}$, then the value of $q - p$ is.....

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4. If $f(x)=\begin{vmatrix}(x-a)^4 & (x-a)^3 & 1\\ (x-b)^4 & (x-b)^3 & 1\\ (x-c)^4 & (x-c)^3 & 1\end{vmatrix}$ then $f'(x)=\lambda\begin{vmatrix}(x-a)^4 & (x-a)^3 & 1\\ (x-b)^4 & (x-b)^3 & 1\\ (x-c)^4 & (x-c)^3 & 1\end{vmatrix}$. Find the value of λ

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5. Let $p(x)$ be a polynomial of degree 4 such that $P(1)=P(3)=P(5)=P'(7)=0$. If the real number $a\neq 1,3,5$ is such that $P(a)=0$ can be expressed as $a=\frac{p}{q}$, where p and q are relatively prime, then `(p-8q) is.....

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6. If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t_2}$, then $\left(\frac{dy}{dx}\right)_{(1.1)}$ is.....

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7. If $x^2+y^2+z^2-2xyz=1$, then the value of $\frac{dx}{\sqrt{1-x^2}}+\frac{dy}{\sqrt{1-y^2}}+\frac{dz}{\sqrt{1-z^2}}$ is equal to.....

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8. If y is twice differentiable function of x, then the expression $\left(1-x^2\right).\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y$ by means of the transformation $x=\sin t$ in terms of t is $\frac{d^2y}{dt^2}+\lambda y$. Thus λ is....

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9. The derivative of $f(x)=\cos^{-1}\left(\frac{1}{\sqrt{3}}(2\cos x-3\sin x)\right)+\left\{\sin^{-1}\left(\frac{1}{\sqrt{3}}(2\cos x+3\sin x)\right)\right.$ w.r.t. $\sqrt{1+x^2}$ at $x=\frac{1}{\sqrt{3}}$ is.....

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10. Suppose $f(x) = e^{ax} + e^{bx}$, where $a \neq b$ and $f''(x) - 2f'(x) - 15f(x) = 0$ for all x, then the value of $|a + b|$ is equal to.....

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11. Suppose $A = \frac{dy}{dx}$ of $x^2 + y^2 = 4$ at $(\sqrt{2}, \sqrt{2}), B = \frac{dy}{dx}$ of $\sin y + \sin x = \sin x . \sin y$ at (π, π) and $C = \frac{dy}{dx}$ of $2e^{xy} + e^x - e^x - e^y = e^{xy+1}$ at $(1, 1)$, then $(A - B - C)$ has the value equal to.....

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12. A function is represented parametrically by the equations $x = \frac{1+t}{t^3}; \ y = \frac{3}{2t^2} + \frac{2}{t}$ then $\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^3$ has the value equal to 2

(b) 0 (c) -1 (d) -2

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13. Suppose, the function $f(x) - f(2x)$ has the derivative 5 at $x = 1$ and derivative 7 at $x = 2$. The derivative of the function $f(x) - f(4x)$ at $x = 1$, has the value $10 + \lambda$, then the value of λ is equal to.....

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14. If $x + y = 3e^2$ then $D(x^y)$ vanishes when x equals to

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15. Let $h(x)$ be differentiable for all x and let $f(x) = (kx + e^x)h(x)$ where k is some constant. If $h(0) = 5, h'(0) = -2$ and $f'(0) = 18$, then the value of k is equal to.....

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Exercise Questions Asked In Previous 13 Years Exam

1. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then

A. g is not differentiable at $x = 0$

B. $g'(0) = \cos(\log 2)$

C. $g'(0) = -\cos(\log 2)$

D. g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

Answer: b

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2. Let $f: R \to R$ and $h: R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2, g(f(x)) = x$ and $h(g(x)) = x$ for all $x \in R$. Then, h'(1) equals.

A. $g'(2) = \frac{1}{15}$

B. $h'(1) = 666$

C. $h(0) = 16$

D. $h(g(3)) = 36$

Answer: b,c

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3. If for $x\left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}g(x)$, then $g(x)$ equals: $\frac{3x}{1-9x^3}$ (2) $\frac{3}{1+9x^3}$ (3) $\frac{9}{1+9x^3}$ (4) $\frac{3x\sqrt{x}}{1-9x^3}$

A. $\frac{3}{1+9x^3}$

B. $\frac{9}{1+9x^3}$

C. $\frac{3x\sqrt{x}}{1-9x^3}$

D. $\frac{3x}{1-9x^3}$

Answer: c

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4. Leg $g(x) = \ln(f(x))$, whre $f(x)$ is a twice differentiable positive function on $(0, \infty)$ such that $f(x+1) = xf(x)$. Then, for $N = 1, 2, 3,$ $g^{N+\frac{1}{2}} - g^{\frac{1}{2}} =$

A. $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\right\}$

B. $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N-1)^2}\right\}$

C. $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N+1)^2}\right\}$

D. $4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots + \frac{1}{(2N+1)^2}\right\}$

Answer: a

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5. $\frac{d^2x}{dy^2}$ equals

A. $\left(\frac{d^2y}{dx^2}\right)^{-1}$

B. $-\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$

C. $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$

D. $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

Answer: d

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6. If $f^x = -f(x) and g(x) = f'(x) and F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and given that $F(5) = 5,$ then $F(10)$ is 5 (b) 10 (c) 0 (d) 15

A. 0

B. 5

C. 10

D. 25

Answer: b

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7. If y is a function of x and $\log(x+y) = 2xy$, then the value of $y'(0)$ is

A. 1

B. -1

C. 2

D. 0

Answer: a

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8. If $x^2 + y^2 = 1$then

A. $yy'' - 2(y')^2 + 1 = 0$

B. $yy'' + (y')^2 + 1 = 0$

C. $yy'' + (y')^2 - 1 = 0$

D.

Answer: b

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Exercise For Session 1

1. Differentiate : $e^{x \log a} + e^{a \log x} + e^{a \log a}$

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2. Differentate : $\sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$

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3. $\log_3 x + 3 \log_e x + 2 \tan x$

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4. Differentiate $|x| + a_0x^n + a_1x^{n-1} + a_2x^{n-1} + \ldots + a_{n-1}x + a_n$

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5. $\sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

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6. Differentiate w.r.t x : $x^n \log_a xe^x$

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7. Differentiate: $\frac{2^x \cot x}{\sqrt{x}}$

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8. Differentiate $\frac{\sin x - x\cos x}{x\sin x + \cos x}$

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9. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\infty,$ then find the value of $\frac{dy}{dx}$.

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10. Find the values of 'x' for which the rate of change of $\frac{x^4}{4} + \frac{x^3}{3} - x$ is more than $\frac{x^4}{4}$

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Exercise For Session 2

1. Differentiate the following w.r.t.x. $\left(x^2 + x + 1\right)^4$

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2. Differentiate the following w.r.t.x. $\sqrt{x^2+x+1}$

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3. Differentiate the following w.r.t.x. $\sin^3 x$

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4. Differentiate the following w.r.t.x. $\frac{1}{\sqrt{a^2-x^2}}$

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5. Differentiate the following w.r.t.x. $e^x \sin x$

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6. Differentiate the following w.r.t.x. $\sin^{-1}\left(\frac{a+b\cos x}{b+a\cos x}\right), b > a > 1$

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7. Differentiate the following w.r.t.x. e^{e^x}

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8. Differentiate the following w.r.t.x. $\log\left(x+\sqrt{a^2+x^2}\right)$

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9. Differentiate the following w.r.t.x. $\log\left(\frac{a+b\sin x}{a-b\sin x}\right)$

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10. Differentiate the following w.r.t.x. $\log\sqrt{\frac{1+\sin x}{1-\sin x}}$

Watch Video Solution

11. Differentiate the following w.r.t.x. $\frac{e^x + \log x}{\sin 3x}$

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12. Differentiate the following w.r.t.x. $\sin\left(m \sin^{-1} x\right), |x| < 1$

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13. Differentiate the following w.r.t.x. $a^{\left(\sin^{-1} x\right)^2}, |x| < 1$

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14. Differentiate the following w.r.t.x. $e^{\cos^{-1}\left(\sqrt{1-x^2}\right)}, |x| < 1$

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15. Differentiate the following w.r.t.x. $\frac{x \sin^{-1} x}{\sqrt{1-x^2}} + \log\sqrt{1-x^2}, |x| < 1$

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16. Differentiate the following w.r.t.x. $\log_{10} x + \log_x 10 + \log_x x + \log_{10} 10$

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17. Differentiate the following w.r.t.x. $5^{3-x^2} + \left(3 - x^2\right)^5$

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18. Differentiate the following w.r.t.x. $\frac{\sqrt{a^2 + x^2} + \sqrt{a^2 - x^2}}{\sqrt{a^2 - x^2} - \sqrt{a^2 - x^2}}$

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19. Differentiate the following w.r.t.x. $\sqrt{4 + \sqrt{4 + \sqrt{4 + x^2}}}$

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20. Differentiate the following w.r.t.x. The differentiation coneffiecient of $f(\log_e x) w.r.t.x$, where $f(x) = \log_e x$, is

A. $\frac{x}{\log_e x}$

B. $\frac{1}{x}\log_e x$

C. $\frac{1}{x \log_e x}$

D. None of these

Answer: C

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21. $f(x) = \log_e |x|, x \neq 0$, then $f'(x)$ is equal to

A. $\frac{1}{|x|}$

B. $\frac{1}{x}$

C. $-\frac{1}{x}$

D. None of these

Answer: B

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22. If $f(x) = \sin x, g(x) = x^2$ and $h(x) = \log x.$ IF $F(x) = h(f(g(x))),$ then $F'(x)$ is

A. $2x\cot x^2$

B. $2\cos ec^3 x$

C. $-2\cos ec^2 x$

D. None of these

Answer: A

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23. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x,$ then $f'\left(\frac{\pi}{4}\right)$ is

A. $\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. 1

D. None of these

Answer: A

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24. If $y = f\left(\frac{3x+4}{5x+6}\right)$ and $f'(x) = \tan x^2$, then $\frac{dy}{dx}$ is equal to

A. $-2\tan\left(\frac{3x+4}{5x+6}\right)^2 . \frac{1}{(5x+6)^2}$

B. $f\left(\frac{3\tan x^2+3}{5\tan x^2+6}\right)\tan x^2$

C. $2x\tan\left(\frac{3x-4}{5x-6}\right)$

D. $\tan x^2$

Answer: A

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25. If $y = |\cos x| + |\sin x|$, then $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$ is

A. $\frac{1}{2}(\sqrt{3}+1)$

B. $2(\sqrt{3}-1)$

C. $\frac{1}{2}(\sqrt{3}-1)$

D. None of these

Answer: C

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26. If $f'(x) = \sin x + \sin 4x.\cos x$, then $f'(2x^2)$ is

A. $4x\{\cos(2x^2) - \sin 8x^2.\sin 2x^2\}$

B. $4x\{\cos(2x^2) + \sin 8x^2.\cos 2x^2\}$

C. $\{\cos(2x^2) - \sin 8x^2.\sin 2x^2\}$

D. None of these

Answer: D

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27. if $f'(x) = \sqrt{2x^2 - 1}$ and $y = f(x^2)$ then $\frac{dy}{dx}$ at $x = 1$ is:

A. 1

B. -1

C. -2

D. 2

Answer: D

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Exercise For Session 3

1. If $\log\left(x^2+y^2\right)=tan^{-1}\left(\frac{y}{x}\right)$, then show that $\frac{dy}{dx}=\frac{x+y}{x-y}$

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2. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\tan^{-1}a$ then prove that $\frac{dy}{dx}=\frac{y}{x}$

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3. If $\sin y=\sin(a+y)$, prove that $\frac{dy}{dx}=\frac{s\in^2\ (a+y)}{\sin a}$

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4. If $x^2+y^2=t-\frac{1}{t}$ and $x^4+y^4=t^2+\frac{1}{t^2}$, then prove that $\frac{dy}{dx}=\frac{1}{x^3y}$.

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5. If $\sin(xy) + \cos(xy) = 0$, then $\frac{dy}{dx}$ is

A. $\frac{y}{x}$

B. $-\frac{y}{x}$

C. $-\frac{x}{y}$

D. $\frac{x}{y}$

Answer: B

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6. If $ax^2 + 2hxy + by^2 = 0$ then $\frac{dy}{dx}$ is

A. $\frac{y}{x}$

B. $\frac{x}{y}$

C. $-\frac{x}{x}$

D. None of these

Answer: A

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7. If $x^2e^y + 2xye^x + 13 = 0$ then $\frac{dy}{dx} =$

A. $\frac{-2xe^{y-x} - 2y(x-1)}{x(xe^{y-x} + 2)}$

B. $\frac{2xe^{x-y} - 2y(x-1)}{x(xe^{y-x} + 2)}$

C. $\frac{2xe^{x-y} + 2y(x-1)}{x(xe^{y-x} + 2)}$

D. None of these

Answer: A

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8. If $\log(x+y) = 2xy,$ then $y'(0)$ is

A. 1

B. -1

C. 2

D. 0

Answer: A

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9. If $x \log_e y + y \log_e x = 5$, then $\frac{dy}{dx}$ is

A. $-\frac{y}{x}\left(\frac{x \log y + y}{x + y \log x}\right)$

B. $-\frac{x}{y}\left(\frac{x \log y + y}{x + y \log x}\right)$

C. $-\frac{y}{x}\left(\frac{x \log y - y}{x + y \log x}\right)$

D. None of these

Answer: A

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Exercise For Session 4

1. If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is

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2. If $y = \cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right)$, then $\frac{dy}{dx}$ is

A. $\begin{cases} \frac{2}{1+x^2}, & x > 0 \\ -\frac{2}{1+x^2}, & x < 0 \end{cases}$

B. $\begin{cases} \frac{2}{1+x^2}, & x > 0 \\ \text{does not exist}, & x = 0 \\ \frac{-2}{1+x^2}, & x < 0 \end{cases}$

C. $\begin{cases} \frac{2}{1+x^2}, & x < 0 \\ \text{does not exist}, & x = 0 \\ \frac{-2}{1+x^2}, & x > 0 \end{cases}$

D. None of these

Answer: C

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3. If $y = \tan^{-1}\dfrac{\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}{\left(\sqrt{1+\sin x}-\sqrt{1-\sin x}\right)}$, find $\dfrac{dy}{dx}$.

A.

$$\left\{\left(\frac{1}{2},\quad \cos\frac{x}{2} > \sin\frac{x}{2}\right), \left(-\frac{1}{2},\quad \cos\frac{x}{2} < \sin\frac{x}{2}\right), (\text{doesnotexist}, x =$$

B.

$$\left\{\left(-\frac{1}{2},\quad \cos\frac{x}{2} > \sin\frac{x}{2}\right), \left(\frac{1}{2},\quad \cos\frac{x}{2} < \sin\frac{x}{2}\right), (\text{doesnotexist}, x$$

C.
$$\begin{cases} -\frac{1}{2}, & \cos\frac{x}{2} \geq \sin\frac{x}{2} \\ \frac{1}{2}, & \cos\frac{x}{2} < \sin\frac{x}{2} \end{cases}$$

D. None of these

Answer: B

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4. If $y = \cot^{-1}(\cot x)$, then $\dfrac{dy}{dx}$ is

A. $1, x \in R$

B. $1, x \in R - \{n\pi\}$

C. $\{(1,, x \in R - \{n\pi\}), (\text{does not exist}, x \in \{n\pi\}, n \in \text{integer})$

D. None of these

Answer: C

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5. Sketch for the curve $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

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6. Sketch the graph for $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

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7. Sketch the graph for $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

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8. Sketch the graph for $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$

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9. Sketch for the curve $y = \sin^{-1}(3x - 4x^3)$

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10. Sketch for the curve $y = \sin^{-1}(3x - 4x^3)$

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Exercise For Session 5

1. If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$, prove that $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$

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2. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$

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3. If $x = \cos t$ and $y = \sin t$, then prove that $\frac{dy}{dx} = \frac{1}{\sqrt{3}}$, at $t = \frac{2\pi}{3}$.

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4. If $x = a\left(t + \frac{1}{t}\right)$ and $y = a\left(t - \frac{1}{t}\right)$, prove that $\frac{dy}{dx} = \frac{x}{y}$

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5. If $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$ and $y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$, then prove that $\frac{dy}{dx} = 1$.

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6. If $x = a\sec^3\theta and y = a\tan^3\theta$, $f \in d\frac{dy}{dx} ah\eta = \frac{\pi}{3}$.

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7. Let $y = x^3 - 8x + 7 and x = f(t) \dot{I} \frac{f(dy)}{dx} = 2 and x = 3a = 0,$ then find the value of $\frac{dx}{dt} a = 0.$

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Exercise For Session 6

1. Differentiate the following w.r.t.x.

x^x

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2. Differentiate the following w.r.t.x.

$x^{\sqrt{x}}$

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3. Differentiate the following w.r.t.x.

x^{x^x}

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4. Differentiate the following w.r.t.x. x^{x^2}

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5. Differentiate the following w.r.t.x.

$x^x\sqrt{x}$

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6. Differentiate the following w.r.t.x.

$(\cos x)^x$

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7. Differentiate the following w.r.t.x.

$(\sin x)^{\cos x}$

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8. Differentiate the following w.r.t.x.

$x^{\cos^{-1}x}$

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9. Differentiate the following w.r.t.x.

$\cos(x^x)$

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10. Differentiate the following w.r.t.x.

$\log(x^x + \cos ec^2 x)$

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11. If $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$, find $\frac{dy}{dx}$.

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12. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

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13. If $x^y + y^x = 2$, find $\frac{dy}{dx}$.

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14. If $(\cos x)^y = (\sin y)^x$, then find $\frac{dy}{dx}$.

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15. If $\sin(a+y) + \sin a.\cos(a+y) = 0$. Prove that : $\frac{dy}{dx} = \left(\frac{\sin^2(a+y)}{\sin a}\right)$

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16. `"If "y=sqrt(cosx+sqrt(cosx+sqrt(cosx+...oo)))", prove that "(dy)/(dx)= (sinx)/((2y-1)).

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17. If $y = (\tan x)^{(\tan x)^{(\tan x)^{\cdots \infty}}}$, then prove that $\frac{dy}{dx} = 2$ at $x = \frac{\pi}{4}$.

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18. if $y = e^{(x)^{e^x}} + x^{e^{e^x}} + e^{x^{x^e}}$, then dy/dx $= e^{(x)^{e^x}} x^{e^x}\left[e^x \log x + \frac{e^x}{x}\right] + x^{e^{e^x}} e^{e^x}\left[\frac{1}{x} + e^x \log x\right] + e^{x^{x^e}} x^{x^e} x^{e-1}[1 + e$

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19. If `0

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Exercise For Session 7

1. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$, if $x \in (-1, 1)$

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2. Find the derivative of $\frac{\tan^{-1}(2x)}{1-x^2}$ $w\dot{r}t\frac{\sin^{-1}(2x)}{1+x^2}$

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3. Differentiate $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ w.r.t. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

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4. Find the differential coefficient of thefollowing functions $x^{\sin^{-1}x}$ w.r.t. $\sin^{-1}x$

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5. Differentiate $\sin^{-1}\left(2ax\sqrt{1-a^2x^2}\right) w.\ r.\ t.\ \sqrt{1-a^2x^2}$.

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6. Differentiate $\log \sin x$ w.r.t. $\sqrt{\cos x}$

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7. Differentiate $\tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$ with respect to $\cos^{-1} x^2$

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8. Differnetiate $x^x w.\ r.\ t.\ x \log x.$

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9. Differentiate $\sin^{-1}\left(4x\sqrt{1-4x^2}\right) w.\ r.\ t.\ \sqrt{1-4x^2}$, if $x \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$

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10. Differentiate $\sin^{-1}\left(4x\sqrt{1-4x^2}\right) w.r.t. \sqrt{1-4x^2}$, if $x \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$

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11. Differentiate $\sin^{-1}\left(4x\sqrt{1-4x^2}\right) w.r.t. \sqrt{1-4x^2}$, if $x \in \left(-\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$

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Exercise For Session 8

1. If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$

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2. If $y = A\cos(\log x) + B\sin(\log x)$ then prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0.$

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3. If $y = x\log\left(\frac{x}{a+bx}\right)$, then prove that $x^3\frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

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4. If $y = \log\left(x + \sqrt{x^2+a^2}\right)$, then prove that $y = \log\left(x + \sqrt{x^2+a^2}\right)$,

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5. $y = \left[\log\left(x + \sqrt{x^2+1}\right)\right]^2$ then prove that $\left(x^2+1\right)y_2 + xy_1 = 2$

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6. If $x = at^2,\ y = 2\,at$, then $\frac{d^2y}{dx^2} = -\frac{1}{t^2}$ (b) $\frac{1}{2\,at^3}$ (c) $-\frac{1}{t^3}$ (d) $-\frac{1}{2\,at^3}$

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7. $x = a\cos^3\theta, y = a\sin^3\theta$ then find $\frac{d^2y}{dx^2}$

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8. If $x = \tan\left(\frac{1}{a}\log y\right)$, show that $\left(1 - x^2\right)\frac{d^2\,y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$

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9. If $x = a\cos\theta + b\sin\theta$ and $y = a\sin\theta - b\cos\theta$, then prove that $y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

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10. If $y = \dfrac{ax + b}{cx + d}$, then prove that $2y_1y_3 = 3(y_2)^2$

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11. If $x = f(t)$ and $y = \phi(t)$, prove that $\dfrac{d^2y}{dx^2} = \dfrac{f_1\phi_2 - f_2\phi_1}{f_1^3}$ where suffixes denote differentiation $w.\ r.\ t.\ t.$

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12. If $x = \sin t, y = \sin Kt$ then show that $\left(1 - x^2\right)y_2 - xy_1 + K^2y = 0$

.

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13. If $x^2 + y^2 = 1, then \quad yy^{-2}\left(y'\right)^2 + 1 = 0 \quad y^+\left(y'\right)^2 + 1 = 0$

$yy^+\left(y'\right)^{-2} - 1 = 0\ yy^+2\left(y'\right)^2 + 1 = 0$

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14. Let f(x) be polynomial function of defree 2 such that $f(x) > 0$ for all $x \in R$. If $g(x) = f(x) + f'(x) + f''(x)$ for all x, then

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Exercise For Session 9

1. If f(x) g(x) and h(x) are three polynomials of degree 2 and $\Delta = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$ then $\Delta(x)$ is a polynomial of degree (dashes denote the differentiation).

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2. Let f, g, h be differentiable functions of x. if $\Delta = \begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ (x^2f)'' & (x^2g)'' & (x^2h)'' \end{vmatrix}$ and , $\Delta' = \begin{vmatrix} f & g & \\ f' & g' & \\ (x^nf'')' & (x^ng'')' & (x^n \end{vmatrix}$

, then n =

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3. Find $\frac{dy}{dx}$ at $x = -1$,when $(\sin y)^{\sin\left(\left(\frac{\pi}{2}\right)x\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x \tan(\ln(x+2)) = 0$

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Exercise For Session 10

1. The function $f(x) = e^x + x,$ being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)\cdot$ The value of $\frac{d}{dx}\left(f^{-1}\right)$ at the point $f(\log 2)$ is $\frac{1}{1n2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) none of these

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2. Let g(x) be the inverse of an invertible function f(x), which is differentiable for all x, then $g''f(x)$ is equal to

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3. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$. Then $g'(f(x))$ equal. $f'(c)$ (b) $\frac{1}{f'(c)}$ (c) $f(c)$ (d) none of these

A. $f'(c)$

B. $\frac{1}{f'(c)}$

C. $f(c)$

D. None of these

Answer: B

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4. If $f(x) = x + \tan x$ and $g(x)$ is inverse of $f(x)$ then $g'(x)$ is equal to

A. $\frac{1}{[g(x) - x]^2}$

B. $\frac{1}{2 - [g(x) - x]^2}$

C. $\frac{1}{2 + [g(x) - x]^2}$

D. None of these

Answer: C

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