MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

DY / DX AS A RATE MEASURER AND TANGENTS, NORMALS

Examples

1. If the radius of a circle is increasing at a uniform rate of 2 cm/s, then find the rate of increase of area of circt the instant when the radius is 20 cm.

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2. On the curve $x^3 = 12y$, find the interval at which the abscissa changes at a faster rate than the ordinate.

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3. If the displacement of a particle is givne by $s = \left(\frac{1}{2}t^2 + 4\sqrt{t}\right)m$. Find the velocity and acceleration at t = 4 seconds.

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4. If $s = \frac{1}{2}t^3 - 6t$, then the find the acceleration at time when the velocity tends to zero.

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5. If r is the radius, S the surface area and V the volume of a spherical bubble, prove that

(i) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

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6. If r is the radius, S the surface area and V the volume of a spherical bubble, prove that

(ii) $\frac{dV}{dS} \infty r$

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7. A man who is 1.6 m tall walks away from a lamp which is 4 m above ground at the rate of 30 m/min. How fast is the man's shadow lengthening?

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8. If x and y are the sides of two squares such that $y = x - x^2$, find the rate of the change of the area of the second square with respect to the first square.

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9. Use differential to approximate $\sqrt{10}$.

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10. Use differential to approximate $(66)^{1/3}$.

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11. If the radius of a circle increases from 5 cm to 5.1 cm, find the increase in area.

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12. Find the approximate value of $\tan^{-1}(0.999)$ using differential.

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13. The time T of oscillation of a simple pendulum of length l is given by

$$T = 2\pi.\sqrt{\frac{l}{g}}.$$

Find the percentage error in T corresponding to

(i) on increase of 2% in the value of l.

(ii) decrease of 2% in the value of l.`

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14. In an acute triangle ABC if sides a, b are constants and the base angles $AandB$ vary, then show that

$$\frac{dA}{\sqrt{a^2 - b^2 \sin^2 A}} = \frac{dB}{\sqrt{b^2 - a^2 \sin^2 B}}$$

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15. Find the slopes of the tangent and normal to the curve $x^3 + 3xy + y^3 = 2$ at (1, 1).

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16. Find the point on the curve $y = x^3 - 3x$ at which tangent is parallel to X-axis.

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17. Find the points on the curve $y = x^3 - 2x^2 - x$ at which the tangent lines are parallel to the line $y = 3x - 2$

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18. In which of the following cases, the function f(x) has vertical tangent at x = 0 ?

(ii) $f(x) = sgnx$

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19. In which of the following cases, the function f(x) has vertical tangent at x = 0 ?

$f(x) = x^{2/3}$

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20. In which of the following cases, the function f(x) has vertical tangent at x = 0 ?

(v) $f(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 0 \end{cases}$

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21. Find value of c such that line joining points (0, 3) and (5, -2) becomes tangent to $y = \dfrac{c}{x+1}$.

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22. Find the equation of tangent and normal to the curve $2y = 3 - x^2$ at (1, 1).

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23. Find the equation of tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

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24. The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point :

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25. The sum of the intercepts made on the axes of coordinates by any tangent to the curve $\sqrt{x} + \sqrt{y} = 2$ is equal to

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26. The tangent, represented by the graph of the function $y = f(x)$, at the point with abscissa x = 1 form an angle of $\pi/6$, at the point x = 2 form an angle of $\pi/3$ and at the point x = 3 form and angle of $\pi/4$. Then, find the value of,

$$\int_1^3 f'(x)f''(x)dx + \int_2^3 f''(x)dx.$$

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27. Find the curves for which the length of normal is equal to the radius vector.

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28. If tangent and normal to the curve $y = 2\sin x + \sin x 2x$ are drawn at $p\left(x = \frac{\pi}{3}\right)$, then area of the quadrilaterial formed by the tangent, the normal at p and the cordinate axes is

A. $\frac{\pi}{3}$

B. 3π

C. $\frac{\pi\sqrt{3}}{2}$

D. None of these

Answer: C

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29. The maximum value of the sum of the intercepts made by any tangent to the curve $(a\sin^2\theta, 2a\sin\theta)$ with the axes is $\frac{a}{k}$ find the value of k

A. 2

B. 4

C. 3

D. 5

Answer: B

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30. If $g(x)$ is a curve which is obtained by the reflection of $f(x) = \frac{e^x - e^{-x}}{2}$ then by the line $y = x$ then

A. g(x) has more than one tangent parallel to X-axis

B. g(x) has more than one tangent parallel to Y-axis

C. $y = -x$ is a tangent of g(x) at (0, 0)

D. g(x) has no extermum

Answer: D

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31. Find the angle of intersection of the curves $y = x^2$ and $y = 4 - x^2$.

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32. Find the acute angle between the curves $y = \left|x\hat{2} - 1\right| and$ $y = \left|x^2 - 3\right|$ at their points of intersection.

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33. Find the angle of intersection of curves $y = \left[|\sin x| + |\cos x| and x^2 + y^2 = 5,\right.$ where [.] denotes the greatest integral function.

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34. Prove that for the curve $y = be^{x/a}$, the subtangent is of constant length and the sub-normal varies as the square of the ordinate .

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35. Find the equation of tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

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36. If ST and SN are the lengths of subtangents and subnormals respectively to the curve $by^2 = (x + 2a)^3$. then $\frac{ST^2}{SN}$ equals (A)1 (B) $\frac{8b}{27}$ (C) $\frac{27b}{8}$ (D) $\left(\frac{4b}{9}\right)$

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37. If the length of sub-normal is equal to the length of sub-tangent at any point (3,4) on the curve $y = f(x)$ and the tangent at (3,4) to $y = f(x)$ meets the coordinate axes at $AandB$, then the maximum area of the triangle $OAB,$ where O is origin, is 45/2 (b) 49/2 (c) 25/2 (d) 81/2

A. $\frac{45}{2}$

B. $\frac{49}{2}$

C. $\frac{25}{2}$

D. $\frac{81}{2}$

Answer: B

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38. Verify Rolle's theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in the interval $[0, 2]$.

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39. If $ax^2 + bx + c = 0, a, b, c \in R$, then find the condition that this equation would have atleast one root in (0, 1).

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40. If $f(x) and g(x)$ are continuous functions in $[a, b]$ and are differentiable in (a, b) then prove that there exists at least one $c \in (a, b)$ for which. `|f(a)f(b)g(a)g(b)|=(b-a)|f(a)f^(prime)(c)g(a)g^(prime)(c)|,w h e r ea

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41. Use Rollle's theorem to find the condition for the polynomial equation f(x)=0 to have a repeated real roots, Hence, or otherwise prove that the equation.

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42. Let $f(x) and g(x)$ be differentiable functions such that $f'(x)g(x) \neq f(x)g'(x)$ for any real x. Show that between any two real solution of $f(x) = 0$, there is at least one real solution of $g(x) = 0$.

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43. Consider the function

$f(x) = \begin{cases} x \sin. \frac{\pi}{x}, & \text{for} \quad x > 0 \\ 0, & \text{for} \quad x = 0 \end{cases}$, then the number of points in (0, 1), where the derivative f'(x) tends to zero is

A. 0

B. 1

C. 2

D. infinite

Answer:

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44. Find c of the Lagrange's mean value theorem for which $f(x) = \sqrt{25 - x^2}$ in $[1, 5]$.

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45. Let $f(x) and g(x)$ be differentiable for $0 \leq x \leq 2$ such that $f(0) = 2, g(0) = 1, and f(2) = 8.$ Let there exist a real number c in $[0, 2]$ such that $f'(c) = 3g'(c)\cdot$ Then find the value of $g(2)\cdot$

A. 2

B. 3

C. 4

D. 5

Answer:

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46. If $0 < a < b < \frac{\pi}{2}$ and $f(a, b) = \frac{\tan b - \tan a}{b - a}$ then,

A. $f(a, b) \leq 2$

B. $f(a, b) > 1$

C. $f(a, b) \leq 1$

D. None of these

Answer:

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47. In $[0, 1]$ Lagrange's mean value theorem is not applicable to

A. $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$

B. $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

C. $f(x) = x|x|$

D. $f(x) = |x|$

Answer:

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48. If f(x) satisfies the requirements of Lagrange's mean value theorem on [0, 2] and if f(0)= 0 and $f'(x) \leq \frac{1}{2}$

A. $f(x) \leq 2$

B. $|f(x)| \leq 2x$

C. $|f(x)| \leq 1$

D. $f(x) = 3$

Answer:

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49. Let $f[2, 7] \to [0, \infty)$ be a continuous and differentiable function. Then, the value of

$(f(7) - f(2))\dfrac{(f(7))^2 + (f(2))^2 + f(2).\ f(7)}{3}$ is

(where $c \in (2, 7)$)

A. $3f^2(c)f'(c)$

B. $5f^2(c).\ f(c)$

C. $5f^2(c).\ f'(c)$

D. None of these

Answer:

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50. The equation $\sin x + x\cos x = 0$ has atleast one root in the interval.

A. $\left(-\frac{\pi}{2}, 0\right)$

B. $(0, \pi)$

C. $\left(-\frac{\pi}{2}, \frac{3\pi}{2}\right)$

D. None of these

Answer:

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51. Between any two real roots of the equation $e^x \sin x - 1 = 0$ the equation $e^x \cos x + 1 = 0$ has

A. atleast one root

B. atmost one root

C. exactly one root

D. no root

Answer:

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52. If the equation $x^3 + px + q = 0$ has three real roots then show that $4p^3 + 27q^2 < 0$.

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53. If f(x) is a polynomial of degree 5 with real coefficients such that $f(|x|) = 0$ has 8 real roots, then $f(x) = 0$ has

A. 4 real roots

B. 5 real roots

C. 3 real roots

D. nothing can be said

Answer:

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54. Number of integral value (s) of k for which the equation $4x^2 - 16x + k = 0$ has one root lie between 1 and 2 and other root lies between 2 and 3, is

A. 1

B. 2

C. 3

D. 4

Answer:

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55. The tangent to the hyperbola $y = \frac{x+9}{x+5}$ passing through the origin is

A. $x + 25y = 0$

B. $5x + y = 0$

C. $5x - y = 0$

D. $x - 25y = 0$

Answer:

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56. The corrdinate of the points(s) on the graph of the function, $f(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 7x - 4$ where the tangent drawn cuts offintercepts from the coordinate axes which are equal in magnitude but opposite is sign, is

A. $\left(2, \frac{8}{3}\right)$

B. $\left(3, \frac{7}{2}\right)$

C. $\left(1, \frac{5}{6}\right)$

D. None of these

Answer:

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57. If $f:[0,1] \to [0, \propto)$ is differentiable function with decreasing first derivative suc that f(0)=0 and $f'(x) > 0$, then

A. $f(1) \geq f'(1)$

B. $f'(c) \neq 0 \text{ for any } c \in (0,1)$

C. $f(1/2) > f(1)$

D. None of these

Answer:

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58. If $f(x)$ is continuous and derivable, $\forall x \in R$ and $f'(c) = 0$ for exactly 2 real value of 'c'. Then the number of real and distinct value of 'd' for which $f(d) = 0$ can be

A. 1

B. 2

C. 3

D. 4

Answer:

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59. Let f (x) be twice differentialbe function such that $f''(x) > 0$ in $[0, 2]$. Then :

A. $f(0) + f(2) = 2f(x), 0 < c < 2$

B. $f(0) + f(2) = 2f(1)$

C. $f(0)+f(2)=f(2)>2f(1)$

D. $f(0)+f(2)<2f(1)$

Answer:

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60. Statement I The tangent at $x=1$ to the curve $y=x^3-x^2-x+2$ again meets the curve at $x=-2$

Statement II When an equation of a tangent solved with the curve, repeated roots are obtained at the point of tengency.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer:

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61. Statement I The ratio of length of tangent to length of normal is inversely proportional to the ordinate of the point of tengency at the curve $y^2 = 4ax$.

Statement II Length of normal and tangent to a curve

$y = f(x)$ is $\left|y\sqrt{1+m^2}\right|$ and $\left|\dfrac{y\sqrt{1+m^2}}{m}\right|$,

where $m = \dfrac{dy}{dx}$.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer:

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62. Statement I Tangent drawn at the point (0, 1) to the curve $y = x^3 - 3x + 1$ meets the curve thrice at one point only.

statement II Tangent drawn at the point $(1, -1)$ to the curve $y = x^3 - 3x + 1$ meets the curve at one point only.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer:

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63. Let $f(x) = x^3 + ax^2 + bx + c$ be the given cubic polynomial and $f(x) = 0$ be the corresponding cubic equation, where $a, b, c \in R$. Now, $f'(x) = 3x^2 + 2ax + b$

Let $D = 4a^2 - 12b = 4(a^2 - 3b)$ be the discriminant of the equation $f'(x) = 0$.

IF $D = 4(a^2 - 3b) < 0$. Then,

A. f(x) has all real roots

B. f(x) has one real and two imaginary roots

C. f(x) has repeated roots

D. None of the above

Answer:

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64. Let $f(x) = x^3 + ax^2 + bx + c$ be the given cubic polynomial and $f(x) = 0$ be the corresponding cubic equation, where $a, b, c \in R$. Now, $f'(x) = 3x^2 + 2ax + b$ Let $D = 4a^2 - 12b = 4(a^2 - 3b)$ be the discriminant of the equation $f'(x) = 0$. If $D = 4(a^2 - 3b) > 0$ and $f(x_1).\, f(x_2) > 0$ where x_1, x_2 are the roots of f'(x), then

A. f(x) has all real and distinct roots

B. f(x) has three real roots but one of the roots would be repeated

C. f(x) would have just one real root

D. None of the above

Answer: C

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65. Let $f(x) = x^3 + ax^2 + bx + c$ be the given cubic polynomial and $f(x) = 0$ be the corresponding cubic equation, where $a, b, c \in R$. Now,

$f'(x) = 3x^2 + 2ax + b$

Let $D = 4a^2 - 12b = 4(a^2 - 3b)$ be the discriminant of the equation $f'(x) = 0$.

If $D = 4(a^2 - 3b) > 0$ and $f(x_1).\ f(x_2) > 0$ where x_1, x_2 are the roots of f(x), then

A. f(x) has all real and distinct roots

B. f(x) has three real roots but one of the roots would be repeated

C. f(x) would have just one real root

D. None of the above

Answer:

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66. Let $f(x) = x^3 + ax^2 + bx + c$ be the given cubic polynomial and $f(x) = 0$ be the corresponding cubic equation, where $a, b, c \in R$. Now, $f'(x) = 3x^2 + 2ax + b$

Let $D = 4a^2 - 12b = 4(a^2 - 3b)$ be the discriminant of the equation

$f'(x) = 0.$

If $D = 4(a^2 - 3b) > 0$ and $f(x_1).\ f(x_2) > 0$ where x_1, x_2 are the roots of f(x), then

A. f(x) has all real and distinct roots

B. f(x) has three real roots but one of the roots would be repeated

C. f(x) would have just one real root

D. None of the above

Answer:

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67. Let $f(x) = x^3 + ax^2 + bx + c$ be the given cubic polynomial and $f(x) = 0$ be the corresponding cubic equation, where $a, b, c \in R$. Now, $f'(x) = 3x^2 + 2ax + b$

Let $D = 4a^2 - 12b = 4(a^2 - 3b)$ be the discriminant of the equation $f'(x) = 0.$

If $D = 4(a^2 - 3v) = 0$, then

A. f(x) has all real and distinct roots

B. f(x) has three real roots but one of the roots would be repeated

C. f(x) would have just one real root

D. None of the above

Answer:

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68. If $y = f(x)$ is a curve and if there exists two points $A(x_1, f(x_1))$ and $B(x_2, f(x_2))$ on it such that $f'(x_1) = -\frac{1}{f(x_2)}$, then the tangent at x_1 is normal at x_2 for that curve. Now, answer the following questions.

A. 1

B. 0

C. 2

D. Infinite

Answer:

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69. If $y = f(x)$ is a curve and if there exists two points $A(x_1, f(x_1)$ and $B(x_2, f(x_2)$ on it such that $f'(x_1) = -\frac{1}{f'(x_2)} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$, then the tangent at x_1 is normal at x_2 for that curve. Now, anwer the following questions.

Number of such lines on the curve $y = |\ln x|$, is

A. 1

B. 2

C. 0

D. Infinite

Answer: D

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70. If $y = f(x)$ is a curve and if there exists two points $A(x_1, f(x_1))$ and $B(x_2, f(x_2))$ on it such that $f'(x_1) = -\frac{1}{f'(x_2)} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$, then the tangent at x_1 is normal at x_2 for that curve. Now, anwer the following questions.

Number of such lines on the curve $y^2 = x^3$, is

A. 1

B. 2

C. 3

D. infinite

Answer:

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71. Let $f(x) = \int_0^x (|t-1| - |t+2| + t - 2)dt$, such that $f''(a) \neq 1$. If vectors $a\hat{i} - b^2\hat{j}$ and $\hat{i} + 3b\hat{j}$ are parallel for atleast one a, then

Number of integral value of 'b' can be

A. 5

B. 10

C. 11

D. 13

Answer: D

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72. Let $f(x)=\int_0^x (|t-1|-|t+2|+t-2)dt$, such that $f''(a)\neq 1$. If vectors $a\hat{i}-b^2\hat{j}$ and $\hat{i}+3b\hat{j}$ are parallel for atleast one a, then Maximum value of $\left(1-8b-b^2\right)$ is

A. 4

B. 8

C. 12

D. 16

Answer: D

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73. Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all real values of x. Then the largest value of $f(2)$ can attain is

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74. Let C be the curve $y = x^3$ (where x takes all real values). The tangent at A meets the curve again at B. If the gradient at B is K times the gradient at A, then K is equal to 4 (b) 2 (c) -2 (d) $\frac{1}{4}$

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75. Consider the two graphs $y = 2x$ and $x^2 - xy + 2y^2 = 28$. The absolute value of the tangent of the angle between the two curves at the points where they meet, is ....

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76. At the point $P(a, a^n)$ on the graph of $y = x^n (n \in N)$ in the first quadrant at normal is drawn. The normal intersects the Y-axis at the point (0, b). If $\lim_{a \to 0} b = \frac{1}{2}$, then n equals ....

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77. If the line $x \cos\theta + y \sin\theta = P$ is the normal to the curve $(x + a)y = 1$, then show $\theta \in \left(2n\pi + \frac{\pi}{2}, (2n+1)\right) \cup \left(2n\pi + \frac{3\pi}{2}, (2n+2)\pi\right), n \in Z$

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78. Find the total number of parallel tangents of $f_1(x) = x^2 - x + 1$ and $f_2(x) = x^3 - x^2 - 2x + 1$.

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79. Find the points on the curve $3x^2 - 4y^2 = 72$ which is nearest t the line $3x + 2y + 1 = 0.$

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80. Tangent at a point P_1 [other than (0,0)] on the curve $y = x^3$ meets the curve again at P_2. The tangent at P_2 meets the curve at P_3 & so on. Show that the abscissae of $P_1, P_2, P_3, ...P_n,$ form a GP. Also find the ratio area of $A(P_1P_2P_3.\ )$ area of $\Delta(P_2P_3P_4)$

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81. If at each point of the curve $y = x^3 - ax^2 + x + 1,$ the tangent is inclined at an acute angle with the positive direction of the x-axis, then $a > 0$ (b) $a < \ -\sqrt{3} - \sqrt{3} \leq a \leq \ \sqrt{3}$ (d) $noneofthese$

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82. Show that there is no cubic curve for which the tangent lines at two distinct points coincide.

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83. The tangent at a point p to the reactangular hyperbola $xy = c^2$ meets the lines $x - y = 0, x + y = 0$ at Q and R, Δ_1 is the area of the ΔOQR, where O is the origin. The normal at p meets the X-axis at M and Y -axis at N. If Δ_2 is the area of the ΔOMN, show that Δ_2 varies inversely as the square of Δ_1.

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84. $f:[0.4] \rightarrow R$ is a differentiable function. Then prove that for some $a, b \in (0, 4), f^2(4) - f^2(0) = 8f'(a) \cdot f(b)$.

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85. Given a function $f:[0,4] \to R$ is differentiable ,then prove that for some $\alpha, \beta \varepsilon (0,2), \int_0^4 f(t)dt = 2\alpha f(\alpha^2) + 2\beta f(\beta^2)$.

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Exercise For Session 1

1. The surface area of a spherical bubble is increasing at the rate of 2 cm^2/s . When the radius of the bubble is 6cm, at what rate is the volume of the bubble increasing?

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2. A particle moves along the curve $y = \frac{2}{3}x^3 + 1$. Find the point on the curve at which y-coordinate is changing twice as fast as x-coordinate.

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3. The area of an expanding rectangle at the rate of $48cm^2/s.$ the length of rectangle is always equal to square of the breadth. At which rate the length is increasing at the instant when th ebreadth is 4 cm?

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4. An edge of a variable cube is increasing at the rate of 10 cm/s. How fast the volume of the cube is increasing when the edge is 5 cm long?

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5. A air-foce plane is ascending vertically at the rate of 100 km/h. If the radius of the earth is r km, how fast is the area of the earth, visible from the plane, increasing at 3 min after it started ascending?

Note Visible area $A = \frac{2\pi r^2 h}{r+h}$, where h is the height of the plane above the earth.

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6. Water is dripping out from a conical funnel at a uniform rate of $4cm^3 / cm$ through a tiny hole at the vertex in the bottom. When the slant height of the water is 3cm, find the rate of decrease of the slant height of the water-cone. Given that the vertical angle of the funnel is 120^0.

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7. From a cylindrical drum containing oil and kept vertical, the oil leaking at the rate of $10cm^3 / s$. If the radius of the durm is 10 cm and height is 50 cm, then find the rate at which level of oil is changing when oil level is 20cm.

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8. A kit is 120m high and 130 m of string is out. If the kite is moving away horizontally at the rate of 52 m/sec, find the rate at which the string is being paid out.

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9. A kit is 120m high and 130 m of string is out. If the kite is moving away horizontally at the rate of 52 m/sec, find the rate at which the string is being paid out.

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10. A ladder 13m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall, at the rate of 1.5m/sec. How fast is the angle θ between the ladder and the ground is changing when the foot of the ladder is 12m away from the wall.

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11. Water is running into a conical vessel, 15cm deep and 5cm in radius, at the rate of 0.1 cm^3 / sec. When the water is 6cm deep, find at what rate it.

the water level rising? the water-surface area increasing? the wetted surface of the vessel increasing?

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12. Height of a tank in the form of an inverted cone is 10 m and radius of its circular base is 2 m. The tank contains water and it is leaking through a hole at its vertex at the rate of $0.02m^3/s$. Find the rate at which the water level changes and the rate at which the radius of water surface changes when height of water level is 5 m.

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13. At what points of the ellipse $16x^2 + 9y^2 = 400$ does the ordiante decreases at the same rate at which the abscissa increases?

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14. A satellite travels in a circular orbit of radius R. If its x- coordinate decreases at which the abscissa increases?

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15. The ends of a rod AB which is 5 m long moves along two grooves OX, OY which are at right angles. If A moves 2 at a constant speed of $\frac{1}{2}\frac{m}{s}$, what is the speed of B, when it is 4 m from O?

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Exercise For Session 2

1. Use differential to approximate $\sqrt{51}$.

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2. Use differential to approximate $\log(9.01)$. (Given, $\log 3 = 1.0986$)

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3. If the error committed in measuring the radius of a circle is $0.01\,\%$, find the corresponding error in calculating the area.

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4. The pressure p and the volume V of a gas are connected by the relation $pV^{1/4} = a$ (constant). Find percentage increaase in pressure corresponding to a decrease of $\left(\frac{1}{2}\right)\%$ in volume.

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5. If in a triangle ABC, the side c and the angle C remain constant, while the remaining elements are changed slightly, using differentials show that $\frac{da}{csA}+\frac{db}{\cos B}=0$

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6. If a triangle ABC, inscribed in a fixed circle, be slightly varied in such away as to have its vertices always on the circle, then show that $\frac{da}{casA}+\frac{db}{\cos B}+\frac{dc}{\cos C}=0.$

Watch Video Solution

Exercise For Session 3

1. If the line $ax+by+c=0$ is normal to the $xy+5=0$, then a and b have

A. same sign

B. opposite sign

C. cannot be discussed

D. None of these

Answer: A

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2. The equation of tangent drawn to the curve $y^2 - 2x^3 - 4y + 8 = 0$ from the point (1, 2) is given by

A. $y - 2\left(1 \pm \sqrt{2}\right) = \pm 2\sqrt{3}(x - 2)$

B. $y - 2\left(1 \pm \sqrt{3}\right) = \pm 2\sqrt{2}(x - 2)$

C. $y - 2\left(1 \pm \sqrt{3}\right) = \pm 2\sqrt{3}(x - 2)$

D. None of these

Answer: C

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3. The equation of the tangents to the curve $(1+x^2)y=1$ at the points of its intersection with the curve $(x+1)y=1$, is given by

A. $x+y=1, y=1$

B. $x+2y=2, y=1$

C. $x-y=1, y=1$

D. None of these

Answer: B

Watch Video Solution

4. For $y=f(x)=\int_0^x 2|t|dt$, the tangent lines parallel to the bi-sector of the first quadrant angle are

A. $y=x+\frac{3}{4}, y=x-\frac{1}{4}$

B. $y=-x+\frac{1}{4}, y=-x+\frac{3}{4}$

C. $x - y = 2, x - y = 1$

D. None of these

Answer: A

Watch Video Solution

5. The equation of normal to $x + y = x^y$, where it intersects X-axis, is given by

A. $x + y = 1$

B. $x - y - 1 = 0$

C. $x - y + 1 = 0$

D. None of these

Answer: B

Watch Video Solution

6. The equation of normal at any point o to the curve $x = a\cos\theta + a\sin\theta, y = a\sin\theta - a\cos\theta$ is always at a distance of

A. 2a unit from origin

B. a unit from origin

C. $\frac{1}{2}a$ unit from origin

D. None of these

Answer: B

Watch Video Solution

7. If the tangent at (x_0, y_0) to the curve $x^3 + y^3 = a^3$ meets the curve again at (x_1, y_1), then $\frac{x_1}{x_0} + \frac{y_1}{y_0}$ is equal to

A. Ans

B. 2a

C. 1

D. None of these

Answer: D

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8. The area bounded by the axes of reference and the normal to $y = \log_e x$ at (1,0), is

A. 1 sq units

B. 2 sq units

C. $\frac{1}{2}$ sq units

D. None of these

Answer: C

Watch Video Solution

9. If $\frac{x}{a}+\frac{y}{b}=2$ touches the curve $\frac{x^n}{a^n}+\frac{y^n}{b^n}=2$ at the point (α, β), then

A. $\alpha = a^2 m\beta = b^2$

B. $\alpha = a, \beta = b$

C. $\alpha = -2a, \beta = 2b$

D. $\alpha = 3a = \beta = -2b$

Answer: B

Watch Video Solution

10. The equation of tangents to the curve $y=\cos(x+y), -2\pi \leq x \leq 2\pi$ that are parallel to the line $x+2y=0$, is

A. $x+2y=\frac{\pi}{2}$ $x+2y=-\frac{3\pi}{2}$

B. $x+2y=\frac{\pi}{2}$ $x+2y=\frac{3\pi}{2}$

C. $x + 2y = 0 \qquad x + 2y = \pi$

D. None of these

Answer: A

Watch Video Solution

Exercise For Session 4

1. The angle of intersection of $y = a^x$ and $y = b^x$, is given by

A. $\tan\theta = \left|\dfrac{\log(a/b)}{1 - \log(ab)}\right|$

B. $\tan\theta = \left|\dfrac{\log(a/b)}{1 + \log a \log b}\right|$

C. $\tan\theta = \left|\dfrac{\log(a/b)}{1 - \log(a/b)}\right|$

D. None of these

Answer: B

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2. The angle between the curves $x^2 + 4y^2 = 32 \text{ and } x^2 - y^2 = 12,$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: D

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3. If $ax^2 + by^2 = 1$ cute $a'x^2 + b'y^2 = 1$ orthogonally, then

A. $\frac{1}{a} - \frac{1}{a'} = \frac{1}{b} - \frac{1}{b'}$

B. $\frac{1}{a} + \frac{1}{a'} = \frac{1}{b} + \frac{1}{b'}$

C. $\frac{1}{a} + \frac{1}{b} = \frac{1}{a'} + \frac{1}{b'}$

D. None of these

Answer: A

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4. The length of subtangent to the curve, $y = e^{x/a}$ is

A. 2a

B. a unit from origin

C. 43832

D. a/4

Answer: B

Watch Video Solution

5. The length of normal to the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$, at $\theta = \dfrac{\pi}{2}$ is

A. 2a

B. a unit from origin

C. $\sqrt{2}a$

D. $2\sqrt{2}a$

Answer: C

Watch Video Solution

6. Find the length of the tangent for the curve $y = x^3 + 3x^2 + 4x - 1$ at point $x = 0$.

A. $\frac{1}{4}$

B. $\frac{\sqrt{17}}{4}$

C. $\frac{3}{4}$

D. $\frac{\sqrt{15}}{4}$

Answer: B

Watch Video Solution

7. Find the possible values of p such that the equation $px^2 = \ln x$ has exactly one solution.

A. R

B. R^+

C. $(-\infty, 0) \cup \left\{\frac{1}{2e}\right\}$

D. $[0, \infty)$

Answer: C

Watch Video Solution

8. If curve $y = 1 - ax^2$ and $y = x^2$ intersect orthogonally then the value of a is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. 2

D. 3

Answer: B

Watch Video Solution

9. The length of tangent to the curve $y^3 = a\left(\cos t + \log \tan. \frac{t}{2}\right), y = a(\sin t),$ is

A. ax

B. ay

C. a

D. xy

Answer: C

View Text Solution

10. The line tangent to the curves $y^3 - x^2y + 5y - 2x = 0$ and $x^2 - x^3y^2 + 5x + 2y = 0$ at the origin intersect at an angle θ equal to $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

Watch Video Solution

Exercise For Session 5

1. If $f(x) = \begin{cases} x^{\alpha} \log x & x > 0 \\ 0 & x = 0 \end{cases}$ and Rolle's theorem is applicable to $f(x)$ for $x \in [0, 1]$ then α may equal to (A) -2 (B) -1 (C) 0 (D) $\frac{1}{2}$

A. -2

B. -1

C. 0

D. $\frac{1}{2}$

Answer: D

Watch Video Solution

2. Let a, b, c be nonzero real numbers such that $\int_0^1 \left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx$
$= \int_0^2 \left(1+\cos^8 x\right)\left(ax^2+bx+c\right)dx = 0$ Then show that the equation $ax^2+bx+c=0$ will have one root between 0 and 1 and other root between 1 and 2.

A. one root between 0 and 1 and another between 1 and 2

B. both the roots between 0 and 1

C. both the roots between 1 and 2

D. None of these

Answer: A

Watch Video Solution

3. Let $n \in N$, if the value of c prescribed in Roole's theorem for the function $f(x) = 2x(x-3)^n$ on $[0, 3]$ is $\frac{3}{4}$, then n is equat to

A. 1

B. 3

C. 5

D. 7

Answer: B

Watch Video Solution

4. If $f(x) > x; \forall x \in R.$ Then the equation $f(f(x)) - x = 0,$ has

A. atleast one real root.

B. more than one real root.

C. no real root if f(x) is a polynomial and one real root if f(x) is not a polynomial.

D. no real root.

Answer: D

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5. If f(x) is a twice differentiable function such that f(a)=0, f(b)=2, f(c)=-1,f(d)=2, f(e)=0 where a < b < c < d e, then the minimum number of zeroes of $g(x) = f'(x)^2 + f''(x)f(x)$ in the interval [a, e] is

A. 4

B. 5

C. 6

D. 7

Answer: C

Watch Video Solution

Exercise For Session 6

1. Find the value of a if $x^3 - 3x + a = 0$ has three distinct real roots.

Watch Video Solution

2. f(x) is a polynomial of degree 4 with real coefficients such that $f(x) = 0$ is satisfied by $x = 1, 2, 3$ only, then $f'(1).\ f'(2).\ f'(3)$ is equal to

A. 0

B. 2

C. -1

D. None of these

Answer: A

Watch Video Solution

3. If the function $f(x) = \left|x^2 + a|x| + b\right|$ has exactly three points of non-derivability, then

A. $b = 0, a < 0$

B. $a < 0, a \in R$

C. $b > 0, a \in R$

D. All of these

Answer: A

Watch Video Solution

4. If the equation $e^{|[|x|]-2|+b}=2$ has four solutions, then b lies in

A. $(\log 2, -\log 2)$

B. $(\log 2-2, \log 2)$

C. $(-2, \log 2)$

D. $(0, \log 2)$

Answer: B

Watch Video Solution

Exercise Single Option Correct Type Questions

1. Consider the cubic equation $f(x)=x^3-nx+1=0$ where $n\geq 3, n\in N$ then $f(x)=0$ has

A. atleast one root in (0, 1)

B. atleast one root in (1, 2)

C. atleast one root in $(-1, 0)$

D. data insufficient

Answer: A

Watch Video Solution

2. If the normal to $y=f(x)$ at (0, 0) is given by $y-x=0$, then $\lim_{x\to 0}\frac{x^2}{f(x^2)-20f(9x^2)+2f(99x^2)}$, is

A. $1/19$

B. $-1/19$

C. $1/2$

D. does not exist

Answer: B

Watch Video Solution

3. Tangent to a curve intercepts the y-axis at a point P. A line perpendicular to this tangent through P passes through another point (1,0). The differential equation of the curve is (a) $(b)(c)y(d)\frac{(e)dy}{f}((g)dx)(h)(i) - x(j)(k)\Big((l)(m)(n)\frac{(o)dy}{p}((q)dx)(r)(s)(t$

(y) (b) *[Math Processing Error]* (eee) (c) $(d)(e)y(f)\frac{(g)dx}{h}((i)dy)(j)(k) + x = 1(l)$ (m) (d) None of these

A. $y = dy/dx - x(dy/dx)^2 = 1$

B. $x\frac{d^2y}{dx^2} + (dy/dx)^2 = 1$

C. $ydx/dy + x = 1$

D. None of these

Answer: A

Watch Video Solution

4. The number of points in the rectangle $\{(x, y) : |x| \leq 9, |y| \leq 3\}$ which lie on the curve $y^2 = x + \sin x$ and at which the tangent to the curve is parallel to X-axis is

A. 3

B. 2

C. 4

D. None of these

Answer: B

Watch Video Solution

5. If $3(a + 2c) = 4(b + 3d),$ then the equation $ax^3 + bx^2 + cx + d = 0$ will have no real solution at least one real root in $(-1, 0)$ at least one real root in $(0, 1)$ none of these

A. atleast one root in $(-1, 0)$

B. atleast one root in (0, 1)

C. no root in $(-1, 1)$

D. no root in (0, 2)

Answer: B

Watch Video Solution

6. If $3(a+2c)=4(b+3d)$, then the equation $ax^3+bx^2+cx+d=0$ will have no real solution at least one real root in $(-1, 0)$ at least one real root in $(0, 1)$ none of these

A. no real solution

B. atleast one real root in $(-1, 0)$

C. atleast one real root in $(0, 1)$

D. None of the above

Answer: B

Watch Video Solution

7. Let f(x) be a differentiable function in the interval (0, 2) then the value of $\int_0^2 f(x)dx$

A. f(c) where $c \in (0, 2)$

B. $2f(c)$, where $c \in (0, 2)$

C. $f'(c)$ where $c \in (0, 2)$

D. None of these

Answer: B

Watch Video Solution

8. Let $f(x)$ be a fourth differentiable function such $f(2x^2 - 1) = 2xf(x)\,\forall x \in R,$ then $f^{iv}(0)$ is equal

A. 0

B. 1

C. -1

D. data insufficient

Answer: A

Watch Video Solution

9. $x+y-\ln(x+y)=2x+5$ has a vertical tangent at the point (α, β) then $\alpha+\beta$ is equal to

A. -1

B. 1

C. 2

D. -2

Answer: B

Watch Video Solution

10. Let y = f(x), $f: R \to R$ be an odd differentiable function such that $f'''(x) > 0$ and $g(\alpha, \beta) = \sin^8 \alpha + \cos^8 \beta + 2 - 4 \sin^2 \alpha \cos^2 \beta$ If $f''(g(\alpha, \beta)) = 0$ then $\sin^2 \alpha + \sin^2 \beta$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: B

Watch Video Solution

11. A polynomial of 6th degree f(x) satisfies $f(x) = f(2 - x), \forall x \in R$, if $f(x) = 0$ has 4 distinct and 2 equal roots, then sum of the roots of $f(x) = 0$ is

A. 4

B. 5

C. 6

D. 7

Answer: C

Watch Video Solution

12. Let a curve $y = f(x), f(x) \geq 0, \forall x \in R$ has property that for every point P on the curve length of subnormal is equal to abscissa of p. If $f(1) = 3$, then f(4) is equal to

A. $-2\sqrt{6}$

B. $2\sqrt{6}$

C. $3\sqrt{5}$

D. None of these

Answer: B

Watch Video Solution

13. If a variable tangent to the curve $x^2y = c^3$ makes intercepts $a, bonx - andy - axes,$ respectively, then the value of a^2b is $27c^3$ (b) $\frac{4}{27}c^3$ (c) $\frac{27}{4}c^3$ (d) $\frac{4}{9}c^3$

A. $27c^3$

B. $\frac{4}{27}c^3$

C. $\frac{27}{4}c^3$

D. $\frac{4}{9}c^3$

Answer: C

Watch Video Solution

14. Let $f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 3-x & 5-3x^2 & 3x^3-1 \\ 2x^2-1 & 3x^5-1 & 7x^8-1 \end{vmatrix}$ then the equation of $f(x) = 0$ has

A. no real solution

B. atmost one real root

C. atleast two real root

D. exactly one real root in (0, 1) and no other real root

Answer: C

Watch Video Solution

15. The graphs $y = 2x^3 - 4x + 2$ and $y = x^3 + 2x - 1$ intersect at exacty 3 distinct points. The slope of the line passing through two of these point is

A. equal to 4

B. equal to 6

C. equal to 8

D. not unique

Answer: C

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16. In which of the following function Rolle's theorem is applicable ?

A. $f(x) = \begin{cases} x, & 0 \leq \ < 1 \\ 0, & x = 1 \end{cases}$ on $[0, 1]$

B. $f(x) = \begin{cases} \frac{\sin x}{x}, & -\pi \leq x < 0 \\ 0, & x = 0 \end{cases}$ on $[-\pi, 0]$

C. $f(x) = \dfrac{x^2 - x - 6}{x - 1}$ on $[-2, 3]$

D. $f(x) = \begin{cases} \frac{x^3 - 2x^2 - 5x + 6}{x - 1}, & \text{if } x \neq 1 \\ -6, & \text{if } x = 1 \end{cases}$ on $[-2, 3]$

Answer: D

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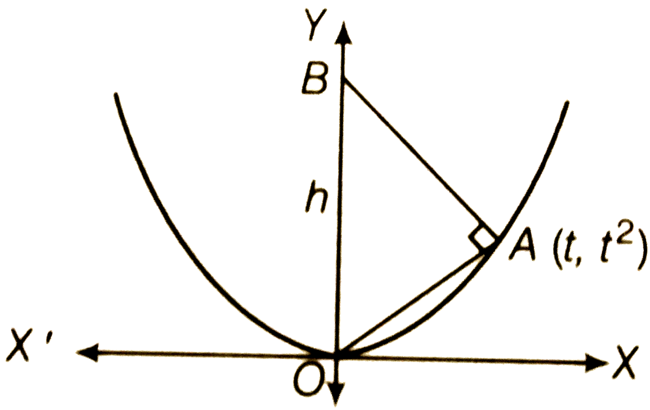
17. The figure shows a right triangle with its hypotenuse OB along the Y-axis and its vertex A on the parabola $y = x^2$.



Let h represents the length of the hypotenuse which depends on the x-coordinate of the point A. The value of $\lim_{t \to 00} (h)$ is equal to

A. 0

B. $1/2$

C. 1

D. 2

Answer: C

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18. Number of positive integral value(s) of 'a' for which the curve $y = a^x$ intersects the line $y = x$ is

A. 0

B. 1

C. 2

D. more than 2

Answer: B

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19. Given $f(x) = 4 - \left(\frac{1}{2} - x\right)^{\frac{2}{3}}, g(x) = \left\{\frac{\tan[x]}{x}, x \neq 01, x = 0\right.$ $h(x) = \{x\}, k(x) = 5^{(\log)_2(x+3)}$ Then in [0,1], lagranges mean value theorem is not applicable to (where [.] and {.} represents the greatest integer functions and fractional part functions, respectively). f (b) g (c) k (d) h

A. f, g, h

B. h, k

C. f, g

D. g, h, k

Answer: A

Watch Video Solution

20. If the function $f(x) = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of x, then the value of b is equal to

A. -1

B. 1

C. 6

D. -6

Answer: D

Watch Video Solution

21. Coffee is coming out from a conical filter, with height and diameter both are 15 cm into a cylindrical coffee pot with a diameter 15 cm. The rate at which coffee comes out from the filter into the pot is 100 cu cm/min.

The rate (in cm/min) at which the level in the pot is rising at the instance when the coffe in the pot is 10 cm, is

A. $\frac{9}{16\pi}$

B. $\frac{25}{9\pi}$

C. $\frac{5}{3\pi}$

D. $\frac{16}{9\pi}$

Answer: D

Watch Video Solution

22. A horse runs along a circle with a speed of $20 km / h$. A lantern is at the centre of the circle. A fence is along the tangent to the circle at the point at which the horse starts. Find the speed with which the shadow of the horse moves along the fence at the moment when it covers 1/8 of the circle in km/h.

A. 20

B. 40

C. 30

D. 60

Answer: B

Watch Video Solution

23. Water runs into an inverted conical tent at the height of the water is three times the radius of the water's surface. The radius of the water surface is increasing when the radius is 5 ft, is

A. $\frac{1}{5\pi}$ ft/mi

B. $\frac{1}{10\pi}$ ft/min

C. $\frac{1}{15\pi}$ ft/min

D. None of these

Answer: A

Watch Video Solution

24. Let $f(x) = x^3 - 3x^2 + 2x$. If the equation $f(x) = k$ has exactly one positive and one negative solution then the value of k equals. $-\frac{2\sqrt{3}}{9}$ (b) $-\frac{2}{9}$ $\frac{2}{3\sqrt{3}}$ (d) $\frac{1}{3\sqrt{3}}$

A. $-\frac{2\sqrt{3}}{9}$

B. $-\frac{2}{9}$

C. $\frac{2}{3\sqrt{3}}$

D. $\frac{1}{3\sqrt{3}}$

Answer: A

Watch Video Solution

25. The x-intercept of the tangent at any arbitarary point of the curve $\frac{a}{x^2} + \frac{b}{y^2} = 1$ is proportioanl to

A. square of the abscissa of the point of tangency

B. square root of the abscissa of the point of tangency

C. cube of the abscissa of the point of tangency

D. cube root of the abscissa of the point of tangency

Answer: C

Watch Video Solution

26. If f(x) is continuous and differentible over $[-2, 5]$ and $-4 \leq f'(x) \leq 3$ for all x in $(-2, 5)$, then the greatest possible value of $f(5) - f(-2)$ is

A. 7

B. 9

C. 15

D. 21

Answer: D

Watch Video Solution

27. A curve is represented parametrically by the equations $x = t + e^{at}$ and $y = -t + e^{at}$ when $t \in R$ and $a > 0$. If the curve touches the axis of x at the point A, then the coordinates of the point A are

A. $(1, 0)$

B. $(1/e, 0)$

C. $(e, 0)$

D. $(2e, 0)$

Answer: D

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28. At any two points of the curve represented parametrically by $x = a(2\cos t - \cos 2t); y = a(2\sin t - \sin 2t)$ the tangents are parallel to the axis of x corresponding to the values of the parameter t differing from each other by :

A. $2\pi/3$

B. $3\pi/4$

C. $\pi/2$

D. $\pi/3$

Answer: A

Watch Video Solution

29. Let $F(x)=\int_{\sin x}^{\cos x} e^{(1+\sin^{-1}(t))\,dt}$ on $\left[0,\frac{\pi}{2}\right]$, then

A. $F''(c)=0 \quad \text{for all} c \in \left(0,\frac{\pi}{2}\right)$

B. $F''(c)=0 \quad \text{for some} c \in \left(0,\frac{\pi}{2}\right)$

C. $F''(c)=0 \quad \text{for no value of} \quad c \in \left(0,\frac{\pi}{2}\right)$

D. $F(c)\neq 0 \quad \text{for all} \quad c \in \left(0,\frac{\pi}{2}\right)$

Answer: B

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30. Given $f'(1)=1$ and $\left.\frac{d}{dx}f(2x)\right)=f'(x)\,\forall x>0.$ If $f'(x)$ is differentiable then there exists a numberd $x \in (2,4)$ such that $f''(c)$ equals

A. $-1/4$

B. $-1/8$

C. $1/4$

D. $1/8$

Answer: B

Watch Video Solution

31. Let $f(x) and g(x)$ be two functions which are defined and differentiable for all $x \geq x_0$. If $f(x_0) = g(x_0) and f'(x) > g'(x)$ for all $x > x_0$, then prove that $f(x) > g(x)$ for all $x > x_0$.

A. $f(x) < g(x)$ for some $x > x_0$

B. $f(x) = g(x)$ for some $x > x_0$

C. $f(x) > g(x)$ only for some $x > x_0$

D. $f(x) > g(x)$ for all $x > x_0$

Answer: B

Watch Video Solution

32. The range of values of m for which the line y = mx and the curve $y=\frac{x}{x^2+1}$ enclose a region, is

A. $(-1, 1)$

B. $(0, 1)$

C. $[0, 1]$

D. $(1, \infty)$

Answer: B

View Text Solution

33. Let S be a square with sides of length x. If we approximate the change in size of the area of S by $h\frac{dA}{dx}\Big|_{x=x_0}$, when the sides are changed from

x_0 to $x_0 + h$, then the absolute value of the error in our approximation, is

A. h^2

B. $2hx_0$

C. x_0^2

D. h

Answer: A

Watch Video Solution

34. Consider $f(x) = \int_1^x \left(t + \frac{1}{t}\right)dt$ and $g(x) = f'(x)$ If P is a point on the curve $y = g(x)$ such that the tangent to this curve at P is parallel to the chord joining the point $\left(\frac{1}{2}, g\left(\frac{1}{2}\right)\right)$ and $(3, g(3))$ of the curve then the coordinates of the point P

A. can't be found out

B. $\left(\frac{7}{4}, \frac{65}{28}\right)$

C. $(1, 2)$

D. $\left(\sqrt{\frac{3}{2}}, \frac{5}{\sqrt{6}}\right)$

Answer: D

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Exercise More Than One Correct Option Type Questions

1. For the curve represented parametrically by the equation, `x=2log(cott)+1 and y=tant+cott.

A. tangent at $t = \frac{\pi}{4}$ is parallel to X - axis

B. normal at $t = \frac{\pi}{4}$ is parallel to Y - axis

C. tangent at $t = \frac{\pi}{4}$ is parallel to $y = x$

D. normal at $t = \frac{\pi}{4}$ is parallel to y = x

Answer: A::B

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2. Consider the curve $f(x) = x^{\frac{1}{3}}$, then

A. the equation of tangent at (0, 0) is x = 0

B. the equation of normal at (0, 0) is y = 0

C. normal to the curve does not exist at (0, 0)

D. f(x) and its inverse meet at exactly 3 points

Answer: A::B::D

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3. The angle at which the curve $y = ke^{kx}$ intersects Y-axis is

A. $\tan^{-1}(k^2)$

B. $\cot^{-1}(k^2)$

C. $\sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$

D. $\sec^{-1}\left(\sqrt{1+k^4}\right)$

Answer: B::C

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4. Let $f(x) = 8x^3 - 6x^2 - 2x + 1,$ then

A. $f(x) = 0$ has no root in (0, 1)

B. $f(x) = 0$ has atleast one root in (0, 1)

C. f'(c) vanishes for some $c \in (0, 1)$

D. None of tha above

Answer: B::C

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5. If $f(0) = f(1) = f(2) = 0$ and function f(x) is twice differentiable in (0, 2) and continuous in [0, 2], then which of the following is/are definitely true ?

A. $f''(c) = 0, \ \forall c \in (0, 2)$

B. $f'(c) = 0, \text{ for atleast two } \ c \in (0, 2)$

C. $f'(c) = 0, \text{ for exactly one } \ c \in (-0, 2)$

D. $f''(c) = 0,$ for atleast one $c \in (0, 2)$

Answer: B::D

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6. Equation $\dfrac{1}{(x+1)^3} - 3x + \sin x = 0$ has

A. no real solution

B. two real and distinct roots

C. exactly one negative root

D. exactly one root between -1 and 1

Answer: B::C::D

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7. If f is an odd continuous function in $[-1, 1]$ and differentiable in $(-1, 1)$ then

A. $f'(A) = f(1) \text{ for some } A \in (-1, 0)$

B. $f'(B) = f(1) \text{ for some } B \in (0, 1)$

C. $n(f(A))^{n-1} f'(A) = (f(1))^n \text{ for some } A \in (-1, 0), n \in N$

D. $n(f(B))^{n-1} f'(B) = (f(1))^n \text{ for some} B \in (0, 1), n \in N$

Answer: A::B::D

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8. The parabola $y = x^2 + px + q$ intersects the straight line $y = 2x - 3$ at a point with abscissa 1. If the distance between the vertex of the parabola nad the X-axis is least, then

A. p 0 and $q = -2$

B. $p = -2$ and q = 0

C. least distance between the vertex of the parabola and X-axis is 1

D.

Answer: B::D

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9. The abscissa of the point on the curve $\sqrt{xy} = a + x$ the tangent at which cuts off equal intercepts from the coordinate axes is $-\frac{a}{\sqrt{2}}$ (b) $a / \sqrt{2}$ (c) $-a\sqrt{2}$ (d) $a\sqrt{2}$

A. $\frac{a}{\sqrt{2}}$

B. $-\dfrac{a}{\sqrt{2}}$

C. $a\sqrt{2}$

D. $-a\sqrt{2}$

Answer: A::B

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10. If the sides and angles of a plane triangle vary in such a way that its circumradius remains constant then that $\dfrac{da}{\cos A} + \dfrac{db}{\cos B} + \dfrac{dc}{\cos C} =$

A. 6 R

B. 2 R

C. 0

D. $2R(dA + dB + dC)$

Answer: C::D

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11. Let $f(x)$ satisfy the requirements of Lagrange's mean value theorem in [0,1], f(0) = 0 and $f'(x) \le 1 - x, \forall x \in (0, 1)$ then

A. $f(x) \ge x$

B. $|f(x)| \ge 1$

C. $f(x) \le x(1 - x)$

D. $f(x) \le 1/4$

Answer: C::D

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12. For function $f(x) = \frac{\ln x}{x}$, which of the following statements are true?

A. f(x) has horizontal tangent at x = e

B. f(x) cuts the X-axis only at one point

C. f(x) is many-one function

D. f(x) has one vertical tangent

Answer: A::B::C

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13. Equation of a line which is tangent to both the curve $y = x^2 + 1\ and\ y = x^2$ is $y = \sqrt{2}x + \frac{1}{2}$ (b) $y = \sqrt{2}x - \frac{1}{2}$ $y = -\sqrt{2}x + \frac{1}{2}$ (d) $y = -\sqrt{2}x - \frac{1}{2}$

A. $y = \sqrt{2}x + \frac{1}{2}$

B. $y = \sqrt{2}x - \frac{1}{2}$

C. $y = -\sqrt{2}x + \frac{1}{2}$

D. $y - \sqrt{2}x - \frac{1}{2}$

Answer: A::C

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14. Let $f(x) = (f(x))^2 + (f'(x))^2, F(0) = 0$ where f(x) is thrice differentiable function such that $|f(x)| \leq 1$ for all $x \in [-1, 1]$, then choose the correct statement(s)

A. There is atleast and point in each of the intervals $(-1, 0)$ and (0, 1) where $|f'(x)| \leq 2$

B. There is atleast one point in each of the intervals $(-1, 0)$ and 0, 1) where $F(x) \leq 5$

C. There is no point of local maxima of F(x) in $(-1, 1)$

D. For some $c \in (-1, 1), F(c) \geq 6, F'(c) = 0$ and $F''(c) \leq 0$

Answer: A::B::D

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15. If the Rolle's theorem is applicable to the function f defined by

$$f(x) = \begin{cases} ax^2 + b, & |x| \leq 1 \\ 1, & |x| = 1 \\ \frac{c}{|c|}, & |x| > 1 \end{cases}$$

in the interval $[-3, 3]$, then which of the following alternative(s) is/are correct?

A. $a + b + c = 2$

B. $|a| + |b| + |c| = 3$

C. $2a + 4b + 3c = 8$

D. $4a^2 + 4b^2 + 5c^2 = 15$

Answer: A::B::C::D

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Exercise Statement I And Ii Type Questions

1. Statement I If g(x) is a differentiable function $g(1) \neq 0, g(-1) \neq 0$ and Rolle's theorem is not applicable to $f(x) = \dfrac{x^2 - 1}{g(x)}$ in $[-1, 1]$, then g(x) has atleast one root in $(-1, 1)$.

Statement II if $f(a) = f(b)$, then Rolle's theorem is applicable for $x \in (a, b)$.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: C

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2. Shortest distance between $|x| + |y| = 2$ and $x^2 + y^2 = 16$ is

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: D

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3. Statement I If $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$ are the n real roots of a polynomial equation of nth degree with real coefficients such that sum of the roots taken $r(1 \leq r \leq n)$ at a time is positive, then all the roots are positive.

Statement II The number of times sign of coefficients change while going left to right of a polynomial equation is the number of maximum positive roots.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: A

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4. Statements I Tangents at two distinct points of a cubic polynomial cannot coincide.

Statement II If p(x) is a polynomial of degree $n(n \geq 2)$, then $p'(x) + k$ cannot hold for n or more distinct values of x.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: D

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5. Statement I for $f(x) = \begin{cases} \frac{1}{2} - x, & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2, & x \geq \frac{1}{2} \end{cases}$ mean value theorem is applicable in the interval $[0, 1]$

Statement II For application of mean value theorem, f(x) must be continuous in [0,1] and differentiable in (0, 1).

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: D

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6. Let $f(x) = \ln(2 + x) - \dfrac{2x + 2}{x + 3}$.

Statement I The function f(x) = has a unique solution in the domain of f(x).

Statement II f(x) is continuous in [a, b] and is strictly monotonic in (a, b), then f has a unique root in (a, b).

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7. Consider the polynomial function $f(x) = \frac{x^7}{7} - \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x$ Statement-1: The equation $f(x) = 0$ can not have two or more roots.Statement-2: Rolles theorem is not applicable for $y = f(x)$ on any interval $[a, b]$ where $a, b \in R$

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Exercise Passage Based Questions

1. We say an equation $f(x) = g(x)$ is consistent, if the curves $y = f(x)$ and $y = g(x)$ touch or intersect at atleast one point. If the curves $y = f(x)$ and $y = g(x)$ do not intersect or touch, then the equation $f(x) = g(x)$ is said to be inconsistent i.e. has no solution.

The equation $\cos x + \cos^{-1} x = \sin x + \sin^{-1} x$ is

A. consistent and has infinite number of solutions

B. consistent and has finite number of solutions

C. inconsistent

D. None of the above

Answer: B

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2. We say an equation $f(x) = g(x)$ is consistent, if the curves $y = f(x)$ and $y = g(x)$ touch or intersect at atleast one point. If the curves $y = f(x)$ and $y = g(x)$ do not intersect or touch, then the equation $f(x) = g(x)$ is said to be inconsistent i.e. has no solution.

The equation $\sin x = x^2 + x + 1$ is

A. consistent and has infinite number of solutions

B. consistent and has finite number of solutions

C. inconsistent

D. None of the above

Answer: C

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3. We say an equation $f(x) = g(x)$ is consistent, if the curves $y = f(x)$ and $y = g(x)$ touch or intersect at atleast one point. If the curves $y = f(x)$ and $y = g(x)$ do not intersect or touch, then the equation $f(x) = g(x)$ is said to be inconsistent i.e. has no solution.

Among the following equations, which is consistent in $(0, \pi/2)$?

A. $\sin x + x^2 = 0$

B. $\cos x = x$

C. $\tan x = x$

D. All of these

Answer: B

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4. To find the point of contact $P(x_1, y_1)$ of a tangent to the graph of $y = f(x)$ passing through origin O, we equate the slope of tangent to $y = f(x)$ at P to the slope of OP. Hence we solve the equation $f'(x) = \frac{f(x_1)}{x_1}$ to get x_1 and y_1.Now answer the following questions (7 -9): The equation $|\ln mx| = px$ where m is a positive constant has a single root for

A. $0 < p < \frac{m}{e}$

B. $p < \frac{e}{m}$

C. $0 < p < \frac{e}{m}$

D. $p > \frac{m}{e}$

Answer: D

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5. To find the point of contact $P(x_1, y_1)$ of a tangent to the graph of $y = f(x)$ passing through origin O, we equate the slope of tangent to

$y = f(x)$ at P to the slope of OP. Hence we solve the equation $f'(x) = \frac{f(x_1)}{x_1}$ to get x_1 and y_1.Now answer the following questions (7 -9): The equation $|\ln mx| = px$ where m is a positive constant has a single root for

A. $p\frac{m}{e}$

B. $p = \frac{e}{m}$

C. $0 < p \le \frac{e}{m}$

D. $0 < p \le \frac{m}{e}$

Answer: A

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6. To find the point of contact $P(x_1, y_1)$ of a tangent to the graph of $y = f(x)$ passing through origin O, we equate the slope of tangent to $y = f(x)$ at P to the slope of OP. Hence we solve the equation $f'(x) = \frac{f(x_1)}{x_1}$ to get x_1 and y_1.Now answer the following questions (7

-9): The equation $|\ln mx| = px$ where m is a positive constant has a single root for

A. $p < \frac{m}{e}$

B. $0 < p < \frac{m}{e}$

C. $0 < p < \frac{e}{m}$

D. $p < \frac{e}{m}$

Answer: B

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7. Consider the family of circles : $x^2 + y^2 - 3x - 4y - c_1 = 0, c_1 \in N$ $(i = 1, 2, 3, \ldots, n)$

Also, let all circles intersects X-axis at integral points only and $c_1 < c_2 < c_3 < c_4 \ldots < c_n A$ point (x, y) is said to be integral point, if both coordinates x and y are integers.

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8. Consider the family of circles : $x^2 + y^2 - 3x - 4y - c_1 = 0,\ c_1 \in N$

$(i = 1, 2, 3, \ldots, n)$

Also, let all circles intersects X-axis at integral points only and $c_1 < c_2 < c_3 < c_4 \ldots < c_n A$ point (x, y) is said to be integral point, if both coordinates x and y are integers.

If circle $x^2 + y^2 - 3x - 4y - (c_2 - c_1) = 0$ and circle $x^2 + y^2 = r^2$ have only one common tangent, then

A. $r = 1/2$

B. tangent passes through $(10, 0)$

C. (3, 4) lies outside the circle $x^2 + y^2 = r^2$

D. $c_2 = 2r + c_1$

Answer: D

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Exercise Matching Type Questions

1. Match the statements of Column I with values of Column II.

	Column I		Column II
(A)	A circular plate is expanded by heat from radius 5 cm to 5.06 cm. Approximate increase in area is	(p)	4
(B)	If an edge of a cube increases by 1%, then percentage increase in volume is	(q)	0.6π
(C)	If the rate of decrease of $\frac{x^2}{2} - 2x + 5$ is twice the rate of decrease of x, then x is equal to (rate of decrease is non-zero)	(r)	3
(D)	Rate of increase in area of equilateral triangle of side 15 cm, when each side is increasing at the rate of 0.1 cm/s, is	(s)	$\frac{3\sqrt{3}}{4}$

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Exercise Single Integer Answer Type Questions

1. A point is moving along the curve $y^3 = 27x$. The interval in which the abscissa chnages at alower rate than ordinate, is (a, b). Then $(a + b)$ is ....

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2. The slope of the curve $2y^2 = ax^2 + bat(1, \, -1)$ is -1 Find a, b

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3. Let $f(1) = -2, f'(x) \geq 4.2$ for $1 \leq x \leq 6$. The smallest possible value of $f(6) - 16$ is ....

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4. Let $f(x) = \begin{cases} -x^2, & \text{for} \quad x < 0 \\ x^2 + 8, & x \geq 0 \end{cases}$. Then the absolute value of x-intercept of the line that is tangent to the graph of f(x) is ....

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5. The tangent to the graph of the function $y = f(x)$ at the point with abscissa $x = a$ forms with the x-axis an angle of $\frac{\pi}{3}$ and at the point with abscissa $x = b$ at an angle of $\frac{\pi}{4}$, then the value of the integral, $\int_1^b f'(x).\ f''(x)dx$ is equal to

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6. Two curves $C_1 : y = x^2 - 3\ and\ C_2 : y\ kx^2$, $k \in R$ intersect each other at two different points. The tangent drawn to C_2 at one of the points of intersection $A \equiv (a, y_1), (a > 0)$ meets C_1 again at $B(1, y_2)\ (y_1 \neq y_2)$. The value of $'a'$ is 1 (b) 3 (c) 5 (d) 7

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7. Consider the function $f(x) = 8x^2 - 7x + 5$ on the interval $[-6, 6]$. Find the value of c that satisfies the conclusion of Lagranges mean value theorem.

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8. Suppose that f is differentiable for all x and that $f'(x) \leq 2$ for all x, If $f(1) = 2$ and $f(4) = 8$, then f(2)has the value equal to ....

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9. Suppose a, b, c are positive integers with $a < b < c$ such that $1/a + 1/b + 1/c = 1$. The value of $(a + b + c - 5)$ is ....

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Exercise Subjective Type Questions

1. Show that a tangent to an ellipse whose tangent intercepted by the axes is the shortest, is divided at the point of tangency into two parts respectively, is equal to the semi-axes of the ellipse.

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2. Tangents are drawn from the origin to curve $y = \sin x$. Prove that points of contact lie on $y^2 = \frac{x^2}{1 + x^2}$

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3. If" f, is a continuous function with $\int_0^x f(t)dt \to \infty$ as $|x \mapsto \infty$then show that every line $y = mx$ intersects the curve $y^2 + \int_0^x f(t)dt = 2$

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4. Find the equation of the straight line which is a tangent at one point and normal at another point to the curve $y = 8t^3 - 1, x = 4t^2 + 3$.

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5. Let a curve $y = f(x)$ pass through (1,1), at any point p on the curve tangent and normal are drawn to intersect the X-axis at Q and R respectively. If QR = 2 , find the equation of all such possible curves.

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6. Show that the angle between the tangent at any point P and the line joining P to the origin O is same at all points on the curve $\log(x^2 + y^2) = k\tan^{-1}\left(\frac{y}{x}\right)$

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7. If the equation of two curve is $y^2 = 4ax$ and $x^2 = 4ay$

(i) Find the angle of intersection of two curves.

(ii) Find the equation of common tangents to these curves.

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8. A straight line intersects thethree concentric circles at A, B, C. if the distance of line from the centre of the circles is 'P', prove that the area of the triangle formed by tangents to the circle at A, B, C is $\left(\frac{1}{2P} . AB. BC. CA\right)$.

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9. Find the equation of all possible curves such that length of intercept made by any tangent on x-axis is equal to the square of X-coordinate of the point of tangency. Given that the curve passes through (2,1)

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10. The tangent to the curve $y = x - x^3$ at a point P meets the curve again at Q. Prove that one point of trisection of PQ lies on the Y-axis. Find the locus of the other points of trisection.

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11. The curve for which the ratio of the length of the segment intercepted by any tangent on the Y-axis to the length of the radius vector is constant (k), is

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12. If t is a real number satisfying the equation $2t^3 - 9t^2 + 30 - a = 0$, then find the values of the parameter a for which the equation $x + \frac{1}{x} = t$ gives six real and distinct values of x .

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Exercise Questions Asked In Previous 13 Years Exam

1. The slope of the tangent to the curve $\left(y - x^5\right)^2 = x\left(1 + x^2\right)^2$ at the point $(1, 3)$ is.

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2. let $f(x) = 2 + \cos x$ for all real x Statement 1: For each real t, there exists a pointc in $[t, t + \pi]$ such that $f'(c) = 0$ Because statement 2: $f(t) = f(t + 2\pi)$ for each real t

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I

C. Statement I is true, Statement II is false

D. Statement I is false, Statement II is true

Answer: b

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3. if $|f(x_1) - f(x_2)| \leq (x_1 - x_2)^2$ Find the equation of gent to the curve $y = f(x)$ at the point $(1, 2)$.

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4. The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is(are) ?? $\left(\pm\frac{4}{\sqrt{3}},\ -2\right)$ (b) $\left(\pm\sqrt{\frac{11}{3}},\ 1\right)$ $(0,\ 0)$ (d) $\left(\pm\frac{4}{\sqrt{3}},\ 2\right)$

A. $\left(\pi\frac{4}{\sqrt{3}},\ -2\right)$

B. $\left(\pi\sqrt{\frac{11}{3}}, 0\right)$

C. $(0, 0)$

D. $\left(\pi\frac{4}{\sqrt{3}}, 2\right)$

Answer: D

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5. If the normal to the curve $y = f(x)$ at the point $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3) =$ (a) -1 (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 1

A. -1

B. $-3/4$

C. $4/3$

D. 1

Answer: D

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6. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the $y-a\xi s$, passes through the point : $\left(\frac{1}{2}, -\frac{1}{3}\right)$

(2) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (3) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$ (4) $\left(\frac{\frac{1}{2,1}}{2}\right)$

A. $\left(\frac{1}{2}, \frac{1}{3}\right)$

B. $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

C. $\left(\frac{-1}{2}, \frac{1}{2}\right)$

D. $\left(\frac{1}{2}, -\frac{1}{3}\right)$

Answer: C

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7. The normal to the curve $x^2 + 2xy - 3y^2 = 0$ at (1, 1)

A. does not meet the curve again

B. meets in the curve again the second quadrant

C. meets the curve again in the third quadrant.

D. meets the curve again in the fouth quadrant

Answer: D

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