MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

ELLIPSE

Examples

1. If PSQ is a focal chord of the ellipse $16x^2 + 25y^2 = 400$, such that SP=8, then find the length of SQ.

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2. If the latursrectum of an ellipse is equal to half of its minor-axis, then find its eccentricity.

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3. If the distance between the directrices is thrice the distance between the foci, then find eccentricity of the ellipse.

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4. If $P(x, y)$ is any point on the ellipse $16x^2 + 25y^2 = 400$ and $f_1 = (3, 0) F_2 = (-3, 0)$, then find the value of $PF_1 + PF_2$.

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5. Find the eccentric angle of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ whose distance from the center of the ellipse is $\sqrt{5}$

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6. An ellipse having foci at (3, 3) and $(-4, 4)$ and passing through the origin has eccentricity equal to $\frac{3}{7}$ (b) $\frac{2}{7}$ (c) $\frac{5}{7}$ (d) $\frac{3}{5}$

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7. Find the lengths of major and minor axes,the coordinate of foci, vertices and the eccentricity of the ellipse $3x^2 + 2y^2 = 6$. Also the equation of the directries.

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8. Find the equation to the ellipse, whose focus is the point $(-1, 1)$, whose directrix is the straight line $x - y + 3 = 0$, and whose eccentricity is $\frac{1}{2}$.

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9. If the line $lx + my + n = 0$ cuts the ellipse $\left(\frac{x^2}{a^2}\right) + \left(\frac{y^2}{b^2}\right) = 1$ at points whose eccentric angles differ by $\frac{\pi}{2}$, then find the value of $\frac{a^2l^2 + b^2m^2}{n^2}$.

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10. If the chord, joining two points whose eccentric angles are α and β, cuts the major axis ofthe ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at a distance c fromn the centre, then $\tan\alpha/2.\tan\beta/2$ is equal to

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11. If the angle between the straight lines joining foci and the ends of minor axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{\pi}{2}$ then the eccentricity is

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12. Find the equation of the ellipse whose minor axis is equal to distance between the foci and latus rectum is 10.

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13. The ratio of any triangle PQR inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and that of triangle formed by the corresponding points on the auxilliary circle is $\frac{b}{a}$.

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14. If the extremities of a line segment of length l moves in two fixed perpendicular straight lines, then the locus of the point which divides this line segment in the ratio 1 : 2 is-

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15. Find the lengths of and the equations to the focal radii drawn to the point $\left(4\sqrt{3}, 5\right)$ of the ellipse $25x^2 + 16y^2 = 1600$

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16. Find the position of the point (4,-3) relative to the ellipse $5x^2 + 7y^2 = 140$.

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17. Find the set of those value(s) of α for which $\left(7 - \frac{5\alpha}{4}, \alpha\right)$ lies inside the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$

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18. If straight line $lx + my + n = 0$ is a tangent of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then prove that $a^2l^2 + b^2m^2 = n^2$.

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19. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then prove that $a^2\cos^2\alpha - b^2\sin^2\alpha = p^2$.

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20. The values of λ for which the line y=x+λ touches the ellipse $9x^2 + 16y^2 = 144$, are

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21. If the line $3x + 4y = \sqrt{7}$ touches the ellipse $3x^2 + 4y^2 = 1,$ then the point of contact is

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22. Find the equations of the tangents to the ellipse $3x^2 + 4y^2 = 12$ which are perpendicular to

the line $y + 2x = 4$.

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23. Find the locus of the foot of the perpendicular drawn from the center upon any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

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24. Find the point on the ellipse $16x^2 + 11y^2 = 256$ where the common tangent to ti and the circle $x^2 + y^2 - 2x = 15$ toch.

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25. Find the maximum area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which touches the line $y=3x+2.$

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26. Find the slope of a common tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and a concentric circle of radius $r.$

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27. Prove that the product of the perpendicular from the foci on any tangent to an ellipse is equal to the square of the semi-minor axis.

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28. The locus of the middle point of the portion of a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ included between axes is the curve

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29. Show that the tangents at the extremities of the latus rectum of an ellipse intersect on the corresponding directrix.

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30. The normal at an end of a latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through an end of the minor axis if (A) $e^4+e^2=1$ (B) $e^3+e^2=1$ (C) $e^2+e=1$ (D) $e^3+e=1$

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31. The line $lx + my + n = 0$ is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. then prove that $\frac{a^2}{l^2} + \frac{b^2}{m^2} = \frac{\left(a^2 - b^2\right)^2}{n^2}$

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32. A normal inclined at 45° to the axis of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is drawn. It meets the x-axis & the y-axis in P & Q respectively. If C is the centre of the ellipse, show that the area of triangle CPQ is $\frac{\left(a^2 - b^2\right)^2}{2(a^2 + b^2)}$ sq units

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33. Any ordinate MP of an ellipse meets the auxillary circle in Q. Ptove that the locus of the point of intersection of the normals at P and Q is the circle $x^2 + y^2 = (a + b)^2$.

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34. Prove that the chord of contact of tangents drawn from the point (h,k) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ will subtend a right angle at the centre, if $\frac{h^2}{a^4} + \frac{k^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$

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35. Show that the locus of the middle points of chord of an ellipse which paas through a fixed point, is another ellipse

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36. Show that the tangents at the ends of conjugate diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersect on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$.

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37. Tangents at right angle are drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Show that the focus of the middle points of the chord of contact is the curve $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2=\frac{x^2+y^2}{a^2+b^2}$.

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38. A ray emanating from the point (-3,0) is incindent on the ellipse $16x^2+25y^2=400$ at the point p with ordinate 4. Find the equation of the reflected ray after first reflection.

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39. For the ellipse $4(x-2y+1)^2+9(2x+y+2)^2=180$, lengths of major and minor axes are respectively

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40. Point 'O' is the centre of the ellipse with major axis AB & minor axis CD. Point F is one focus of the ellipse. If OF = 6 & the diameter of the inscribed circle of triangle OCF is 2, then find the product $(AB).(CD)$

A. 52

B. 56

C. 78

D. None of these

Answer: A::B::C

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41. Let P_i and Π' be the feet of the perpendiculars drawn from the foci $SandS'$ on a tangent T_i to an ellipse whose length of semi-major axis is 20. If

$\sum_{i=0}^{10}(SP_i)\left(S'\Pi'\right)=2560,$ then the value of eccentricity is $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{4}{5}$

Answer: B::C

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42. The coordinates of the vertices $BandC$ of a triangle ABC are (2, 0) and (8, 0), respectively. Vertex A is moving in such a way that $4\frac{\tan B}{2}\frac{\tan C}{2} = 1$. Then find the locus of A

A. $\frac{x-5^2}{25} + \frac{y^2}{16} = 1$

B. $\frac{(x-5)^2}{16} + \frac{y^2}{9} = 1$

C. $\frac{(x-5)^2}{25} + \frac{y^2}{9} = 1$

D. $\frac{(x-5)^2}{16} + \frac{y^2}{25} = 1$

Answer: A::B

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43. A ray emanating from the point (0,6) is incident on the ellipse $25x^2 + 16y^2 = 1600$ at the point P with ordinate S. After reflection, ray cuts the Y-axis at B. The length of PB is

A. 5

B. 7

C. 12

D. 13

Answer: A::B::C::D

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44. If the ellipse $\frac{x^2}{4} + y^2 = 1$ meets the ellipse $x^2 + \frac{y^2}{a^2} = 1$ in four distinct points and $a = b^2 - 5b + 7$, then b does not lie between

A. (1,4)

B. $(-\infty, 2) \cup (3, \infty)$

C. $(2, 3)$

D. None of these

Answer: B

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45. The normal at a variable point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e meets the axes of the ellipse at $QandR\cdot$ Then the locus of the midpoint of QR is a conic with eccentricity e' such that e' is independent of e (b) $e'=1$ $e'=e$ (d) $e'=\frac{1}{e}$

A. e' is indipendant of e

B. e'=1

C. e'=e

D. e'=1/e

Answer:

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46. If the curves $\frac{x^2}{4}+y^2=1$ and $\frac{x^2}{a^2}+y^2=1$ for a suitable value of a cut on four concyclic points, the equation of the circle passing through these four points is $x^2+y^2=2$ (b) $x^2+y^2=1$ $x^2+y^2=4$ (d) none of these

A. $x^2+y^2=8$

B. $x^2+y^2=4$

C. $x^2=y^2=2$

D. $x^2+y^2=1$

Answer: A::B

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47. If P is the length of perpendicluar drawn from the origin to any normal to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, then the maximum value of p is

A. 5

B. 4

C. 2

D. 1

Answer: D

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48. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)}+\frac{y^2}{f(k+11)}=1$. If $f(x)$ is a positive decr4easing function, then the set of values of k for which the major axis is the x-axis is $(-3,2)\cdot$ the set of values of k for which the major axis is the y-axis is $(-\infty,2)\cdot$ the set of values of k for which the major axis is the y-axis is $(-\infty,-3)\cup(2,\infty)$ the set of values of k for which the major axis is the y-axis is $(-3,-\infty,)$

A. $k \in (-2, 3)$

B. $k \in (-3, 2)$

C. $k \in (-\infty, -3) \cup (2, \infty)$

D. $k \in (-\infty, -2) \cup (3, \infty)$

Answer: B::C

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49. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle

$x^2 + y^2 + 4x + 1 = 0$, then the maximum value of ab is

A. 1

B. 2

C. 4

D. 8

Answer: D

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50. Extremities of the latera recta of the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ having a given major axis 2a lies on

A. $x^2=a(a-y)$

B. `x=a(a+y)

C. $y^2=a(a+x)$

D. y^(2)=a(a-x)`

Answer: A::B

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51. The locus pf the image of the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, with the respect to any of the tangent to the ellipse is

A. $(x+4)^2+y^2=100$

B. (x+2)^(2)+y^(2)=50`

C. $(x-4)^2+y^2=100$

D. $\left(x-20^2+y^2=50\right.$

Answer: A::B::D

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52. A tangent to the ellipse $4x^2 + 9y^2 = 36$ is cut by the tangent at the extremities of the major axis at T and T^1, the circle on TT^1 as diameter passes through the point

A. $(-\sqrt{5}, 0)$

B. $(\sqrt{5}, 0)$

C. $(\sqrt{3}, 0)$

D. $(-\sqrt{3}, 0)$

Answer: A::B::D

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53. Consider the ellipse $\frac{x^2}{\tan^2\alpha}+\frac{y^2}{\sec^2\alpha}=1$ where$\alpha \in \left(0, \frac{\pi}{2}\right)$. Which of the following quantities would vary as α varies?

A. degree of flatness

B. ordinate of the vertex

C. coordinate of the foci

D. length of latusrectum

Answer: A::C::D

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54. Let $A(\theta)$ and $B(\phi)$be the extrenities of a chord of an emplise. If the slope of AB is equal to the slope of the tangent at a point C(alpha) on the ellipse, then value of alpha is

A. $\frac{\theta+\phi}{2}$

B. $\frac{\theta-\phi}{2}$

C. $\frac{\theta+\phi}{2}+\pi$

D. $\frac{\theta+\phi}{2}-\pi$

Answer: A::B::C

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55. A series of concentric ellipses E_1, E_2, E_3..., E_n are drawn such that E touches the extremities of the major axis of E_{n-1}, and the foci of E_n coincide with the extremities of minor axis of E_{n-1} If the eccentricity of the ellipses is independent of n, then the value of the eccentricity, is (A) $\frac{\sqrt{5}}{3}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{5}+1}{2}$ (D) $\frac{1}{\sqrt{5}}$

A. $\frac{3-\sqrt{5}}{2}$

B. $\frac{\sqrt{5}-1}{2}$

C. $\frac{2-\sqrt{3}}{2}$

D. $\frac{\sqrt{3}-1}{2}$

Answer: A::B

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56. A series of concentric ellipse $E_1, E_2, E_3, \ldots, E_n$ is constructed as follows: Ellipse E_n touches the extremities of the major axis of E_{n-1} and have its focii at the extremities of the minor axis of E_{n-1}. If eccentricity of ellipse E_n is e_n, then the locus of $\left(e_n^2, e_{n-1}^2\right)$ is

A. a parabola

B. an ellipse

C. a hyperbola

D. a rectangular hyperbola

Answer: A::B::C

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57. A series of concentric ellipse $E_1, E_2, E_3, \ldots, E_n$ is constructed as follows: Ellipse E_n touches the extremities of the major axis of E_{n-1} and have its focii at the extremities of the minor axis of E_{n-1} If equation of ellipse E_1 is $\frac{x^2}{9} + \frac{y^2}{16} = 1$, then equation pf ellipse E_3 is

A. $\frac{x^2}{9}+\frac{y^2}{16}=1$

B. $x^2+y^{49}=1$

C. $\frac{x^2}{25}+\frac{y^2}{41}=1$

D. $\frac{x^2}{16}+\frac{y^2}{25}=1$

Answer: A::B::D

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58. Consider an ellipse E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, centered at point 'O' and having AB and CD as its major and minor axes respectively if S_1 be one of the focus of

the ellipse, radius of the incircle of ΔOCS_1 be 1 unit and $OS_1 = 6$ units.

Q. if the ellipse (E) is Δsq unit, then the value of 4 Δ is

A. 63π

B. 64π

C. 65π

D. 66π

Answer: C

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59. An ellipse E, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centred at point O has AB and CD as its major and minor axes, respectively. Let S_1 be one of the foci of the ellipse, the radius of the incircle of traingle OCS_1 be 1 unit, adn $OS_1 = 6$ units

The perimeter of ΔOCS_1 is

A. 10

B. 15

C. 20

D. 25

Answer: A::B::C

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60. Consider an ellipse E: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, centered at point 'O' and having AB and CD as its major and minor axes respectively if S_1 be one of the focus of the ellipse, radius of the incircle of ΔOCS_1 be 1 unit and $OS_1 = 6$ units. Q. The equation of the director circle of (E) is

A. $x^2 + y^2 = 48.5$

B. $x^2 + y^2 = 97$

C. $x^2 + y^2 = \sqrt{48.5}$

D. $x^2 + y^2 = \sqrt{97}$

Answer: A::B::D

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61. If the normals at the four points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are concurrent, then the value of $\left(\sum_{i+1}^{4} x_1\right)\left(\sum_{i=1}^{4} \frac{1}{2}\right)$

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62. If $x, y \in R,$ satisfies the equation $\frac{(x-4)^2}{4} + \frac{y^2}{9} = 1$, then the difference between the largest and the smallest valus of the expression $\frac{x^2}{4} + \frac{y^2}{9}$ is_____

A. 8

B. 7

C. 5

D. 9

Answer: A

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63. Statement 1 Feet of prependiculars drawn from foci of an ellipse $4x^2 + y^2 = 16$ on the line $2\sqrt{3}x + y = 8$ lie on the circle $x^2 + y^2 = 16$

Statement 2 If prependiculars are from foci of an ellipse to its any tangent, the feet of these perpendicular lie on director circle of the ellipse.

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64. Statement 1 the condition on a and b for which two distinct chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ passing through (a,-b) are bisected by the line

x+y=b is $a^2 + 6ab - 7b^2 > 0$.

Statement 2 Equation of chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose mid-point (x_1, y_1) is $T = S_1$

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65. Find the locus of the centriod of an equilateral triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

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66. If the normals to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the ends of the choeds $lx+my=1$ and $l'x+m'y=1$.

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67. An ellipse slides between two perpendicular straight lines. Then identify the locus of its center.

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68. Triangles are formed by pairs of tangent dreawn from any point on the ellipse

$a^2x^2 + b^2y6(2) = \left(a^2 + b^2 \hat{} (2)\right.$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the chord of contact. Show that the orthocentre of each such triangles lies

triangle lies on the ellipse.

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69. Show that the tangents drawn at those points of the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = (a + b)$, where it is cut by any tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, intersect at right angles.

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70. A variable point P on the ellipse of eccentricity e is joined to the foci S and S'. The eccentricity of the locus of incentre of the triangle PSS' is (A) $\sqrt{\frac{2e}{1+e}}$ (B) $\sqrt{\frac{e}{1+e}}$ (C) $\sqrt{\frac{1-e}{1+e}}$ (D) $\frac{e}{2(1+e)}$

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71. If ω is one of the angles between the normals to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point whose eccentric angles are θ and $\frac{\pi}{2}+\theta$, then prove that $\frac{2\cot\omega}{\sin 2\theta}=\frac{e^2}{\sqrt{1-e^2}}$

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72. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose pendiculars from A, B, C to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ meets the ellipse respectivelily at P, Q, R so that P, Q , R lies on same side of major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P Q nad R are concurrent.

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73. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose pendiculars from A, B, C to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ meets the ellipse respectivelily at P, Q, R so that P, Q , R lies on same side of major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P Q nad R are concurrent.

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74. If two concentric elipse be such that the foci of one be on the and if e and e' be their

eccentricities. Prove that the angle between their axes is $\cos^{-1}\left\{\frac{\sqrt{e^2+e'^2-1}}{ee'}\right\}$

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75. If the normals at the four points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are concurrent. Prove that $(x_1+x_2+x_3+x_4)\left(\left(\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}\right)=4\right.$

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Exercise For Session 1

1. If the length of the major axis of the ellipse $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)=1$ is three times the length of minor axis, its accentricity is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{2\sqrt{2}}{3}$

D. $\frac{2\sqrt{2}}{5}$

Answer: C

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2. The equation $\frac{x^2}{10-a}+\frac{y^2}{4-a}=1$ represents an ellipse , if

A. $a<4$

B. $a>4$

C. $4<a<10$

D. $a>10$

Answer: A

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3. Find the eccentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latus rectum is half of its major axis.

A. 1/sqrt3

B. 1/sqrt2

C. sqrt3/2

D. $\sqrt{\left(\frac{2}{3}\right)}$

Answer: B

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4. If the eccentricity of an ellipse is $\frac{1}{\sqrt{2}}$, then its latusrectum is equal to its

A. minor axis

B. semi minor axis

C. major axis

D. semi major axis

Answer: D

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5. If the distance between the foci of an ellipse is equal to length of minor axis, then its eccentricity is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{3}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

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6. The eccentric angle of a point on the ellipse $\frac{x^2}{6}+\frac{y^2}{2}=1$ whose distance from the centre of the ellipse is 2, is

A. $-\frac{\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{2}$

D. $\frac{5\pi}{3}$

Answer: A::B

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7. If $\tan\theta_1 . \tan\theta_2 = \frac{a^2}{b^2}$ then the chord Joining two points θ_1 and θ_2 on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ will subtend a right angle at (A) focus (B) centre (C) end of the major axis (D) end of the major axis

A. focus

B. center

C. end of major axis

D. end of minor axis

Answer: B

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8. If the eccentricities of the two ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ and $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and equal , then the value $\frac{a}{b}$, is

A. $\frac{5}{13}$

B. $\frac{6}{13}$

C. $\frac{13}{5}$

D. $\frac{13}{6}$

Answer: C

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9. The ratio of the area of triangle inscribed in ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to that of triangle formed by the corresponding points on the auxiliary circle is 0.5. Then, find the eccentricity of the ellipse.

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

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10. If PSQ is a focal chord of the ellipse $16x^2 + 25y^2 = 400$ such that SP=16, then the length SQ is

A. $\frac{2}{9}$

B. $\frac{4}{9}$

C. $\frac{8}{9}$

D. $\frac{16}{9}$

Answer: D

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11. Let P be a variable point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ with foci at S and S'. If A be the area of triangle PSS' then the maximum value of A, is

A. 12 sq units

B. 24 sq units

C. 36 sq units

D. 48 sq units

Answer: A

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12. $S_1 and S_2$ are the foci of an ellipse of major axis of length 10 units, and P is any point on the ellipse such that the perimeter of triangle PS_1 is 15. Then the eccentricity of the ellipse is 0.5 (b) 0.25 (c) 0.28 (d) 0.75

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{7}{25}$

D. $\frac{3}{4}$

Answer: A

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13. Find the latus rectum, eccentricity, coordinates of the foci and the length of axes of the ellipse $4x^2 + 9y^2 - 8x - 36y + 4 = 0$.

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14. The distance between the foci of an ellipse is 10 and its latus rectum is 15, find its equation referred to its axes as axes of coordinates.

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15. Find the equation of the ellipse whose axes are parallel to the coordinate axes having its centre at the point $(2, -3)$ one focus at $(3, -3)$ and vertex at $(4, -3)$.

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16. Find the equation of the ellipse whose foci are $(2, 3), (-2, 3)$ and whose semi-minor axes is $\sqrt{5}$.

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17. Show that the equation $(10x-5)^2+(10y-5)^2=(3x+4y-1)^2$ represents an ellipse, find the eccentricity of the ellipse.

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{\sqrt{2}}{3}$

Answer: B

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18. The locus of extremities of the latus rectum of the family of ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is

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Exercise For Session 2

1. The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + \frac{y^2}{1} = 1$ is 0 (b) 1 (c) 2 (d) infinite

A. 0

B. 1

C. 2

D. infinite

Answer: C

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2. If any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts off intercepts of length h and k on the axes, then $\frac{a^2}{h^2}+\frac{b^2}{k^2}=$ (A) 0 (B) 1 (C) -1 (D) Non of these

A. -1

B. 0

C. 1

D. None of these

Answer: C

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3. The equations of the tangents to the ellipse $3x^2 + y^2 = 3$ making equal intercepts on the axes are

A. $y = \pm x \pm 2$

B. $y = \pm x \pm 4$

C. $y = \pm x \pm \sqrt{30}$

D. $y = \pm x \pm \sqrt{35}$

Answer: A

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4. If $\frac{x}{a} + \frac{y}{b} = \sqrt{2}$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then find the eccentric angle θ of point of contact.

A. 0

B. 45°

C. 60°

D. 90°

Answer: B

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5. The number of values of $\phi \in [0, 2\pi]$ for which the line $2x\cos\phi + 3y\sin\phi = 6$ touches the ellipse $4x^2 + 9y^2 = 36$ is a) four b) two c)one d) infinite

A. 1

B. 2

C. 4

D. infinite

Answer: D

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6. The common tangent of $x^2 + y^2 = 4$ and $2x^2 + y^2 = 2$ is

A. x+y+4=0

B. x-y+7=0

C. 2x+3y+8=0

D. None of these

Answer: D

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7. If the normal at any point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the axes at $Gandg$, respectively, then find the raio $PG:Pg$.

A. a:b

B. $a^2 : b^2$

C. b:a

D. $b^2 : a^2$

Answer: D

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8. The number of distinct normal lines that can be drawn to the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ from the point $P(0, 6)$ is one (b) two (c) three (d) four

A. one

B. two

C. three

D. four

Answer: C

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9. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$, then the maximum value of ab is 4 (b) 2 (c) 1 (d) none of these

A. 4

B. 2

C. 1

D. none of these

Answer: A

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10. If the normal at the point $P(\theta)$ to the ellipse $\frac{x^2}{14} + \frac{y^2}{5} = 1$ intersects it again at the point

$Q(2\theta)$, then $\cos\theta$ is equal to (A) $\frac{2}{3}$ (B) $\frac{-2}{3}$ (C) $\frac{3}{4}$

(D) non of these

A. $\frac{2}{3}$

B. $-\frac{2}{3}$

C. $\frac{3}{2}$

D. $-\frac{3}{2}$

Answer: B

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11. The line $5x - 3y = 8\sqrt{2}$ is a normal to the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, If 'theta' be eccentric angle of the foot of this normal then 'theta' is equal to

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: B

Watch Video Solution

12. If the tangent drawn at point $(t^2, 2t)$ on the parabola $y^2 = 4x$ is the same as the normal drawn at point $(\sqrt{5}\cos\theta, 2\sin\theta)$ on the ellipse $4x^2 + 5y^2 = 20$, then $\theta = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$ (b) $\theta = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ $t = -\frac{2}{\sqrt{5}}$ (d) $t = -\frac{1}{\sqrt{5}}$

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13. If the normal at any point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cuts the major and minor axes in L and M respectively and if C is the centre, then

$a^2CL^2 + b^2CM^2 =$ (A) $(a-b)$ (B) $\left(a^2-b^2\right)$ (C) $(a+b)$ (D) $\left(a^2+b^2\right)$

Watch Video Solution

14. If the normal at the point $P(\theta)$ to the ellipse $\frac{x^2}{14}+\frac{y^2}{5}=1$ intersects it again at the point $Q(2\theta)$, then $\cos\theta$ is equal to (A) $\frac{2}{3}$ (B) $\frac{-2}{3}$ (C) $\frac{3}{4}$ (D) non of these

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15. The tangent and normal at any point P of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cut its major axis in point Q and R respectively. If QR=a prove that the eccentric angle of the point P is given by $e^2\cos^2\phi+\cos\phi-1=0$

Watch Video Solution

Exercise For Session 3

1. Find the angle between the pair of tangents from the point (1,2) to the ellipse $3x^2+2y^2=5$.

A. $\tan^{-1}\left(\frac{12}{5}\right)$

B. $\tan^{-1}\left(\frac{6}{\sqrt{5}}\right)$

C. $\tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$

D. $\tan^{-1}(\sqrt{5})$

Answer: C

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2. If the chords of contact of tangents from two poinst (x_1, y_1) and (x_2, y_2) to the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are at right angles, then find the value of $\frac{x_1 x_2}{y_1 y_2}$.

A. $\frac{a^2}{b^2}$

B. $-\frac{b^2}{a^2}$

C. $-\frac{a^4}{b^4}$

D. $\frac{b^4}{a^4}$

Answer: C

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3. If the tangents from the point $(\lambda, 3)$ to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ are at right angles then λ is

A. ± 1

B. ± 2

C. ± 3

D. ± 4

Answer: B

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4. The eccentric angle of one end of a diameter of $x^2 + 3y^2 = 3$ is $\frac{\pi}{6}$, then the eccentric angle of the other end will be

A. $\frac{5\pi}{6}$

B. $-5\frac{\pi}{6}$

C. $-2\frac{\pi}{3}$

D. $2\frac{\pi}{3}$

Answer: B

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5. The locus of mid-points of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

A. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a}$

B. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a}$

C. $x^2+y^2=a^2+b^2$

D. $x^2-y^2=a^2+b^2$

Answer: A

Watch Video Solution

6. the centre of the ellipse $\frac{(x+y-2)^2}{9}+\frac{(x-y)^2}{16}=1$, is

A. (0,0)

B. (1,0)

C. (0,1)

D. (1,1)

Answer: D

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7. The locus of the point of intersection of two prependicular tangents of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is

A. $x^2 + y^2 = 4$

B. $x^2 + y^2 = 9$

C. $x^2 + y^2 = 13$

D. $x^2 + y^2 = 5$

Answer: C

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8. The area of the parallelogram inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose diaonals are the conjugate diameters of the ellipse is given by

A. 2ab

B. 3ab

C. 4ab

D. 5ab

Answer: A

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9. Find the locus of the vertices of equilateral triangle circumscribing the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

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10. A tangent to the ellipse $x^2+4y^2=4$ meets the ellipse $x^2+2y^2=6$ at P&Q.

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11. The locus of the mid-points of the chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which pass through the

positive end of major axis, is.

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12. Ifchord ofcontact ofthe tangents drawn from the point $(\alpha, \beta)$to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,touches the circle $x^2 + y^2 = c^2$, then the locus of the point

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13. Find the centre and eccentricity of the ellipse $4(x - 2y + 1)^2 + 9(2x + y + 2)^2 = 5.$

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14. A ray emanating from the point $(0,\sqrt{5})$ is incident on the ellipse $9x^2+4y^2=36$ at the point P with abscissa 2. find the equation of the reflected ray after first reflection.

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Exercise Single Option Correct Type Questions

1. Given f is increasing, the equation $\frac{x^2}{f(2a)} + \frac{y^2}{f(a^2-3)} = 1$ represents an ellipse with X-axis as major axis if

A. [-1,3]

B. [1,3]

C. (-1,3)

D. (0,5)

Answer: C

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2. If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2-5)}$ represents an ellipse with major axis as Y-axis and f is a decreasing function,then

A. $\alpha \in (1\infty, 1)$

B. $\alpha \in (5, \infty)$

C. $\alpha \in (1, 4)$

D. $\alpha \in (-1, 5)$

Answer: D

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3. The curve represents by the equation $\frac{x^2}{\sin\sqrt{2}-\cos\sqrt{3}}+\frac{y^2}{\sin\sqrt{3}-\cos\sqrt{2}}=1$ is

A. an ellipse with foci on X-axis

B. an ellipse on focii Y-axis

C. a heyperbola with foci on X-axis

D. an hyperbola with foci on Y-axis

Answer: A

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4. The maximum distance of the centre of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ from the chord of contact of mutually perpendicular tangents of the ellipse is

A. 144/5

B. 16/5

C. $\frac{9}{5}$

D. None of these

Answer: B

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5. S and T are the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and B is an end of the minor axis. If STB is an equilateral triangle, the eccentricity of the ellipse is e then find value of $4e$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$

Answer: C

Watch Video Solution

6. A circle of radius $\frac{5}{\sqrt{2}}$ is concentric with the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, then the acute angle made by the common tangent with the line $\sqrt{3}x-y+6=0$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{12}$

Answer: D

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7. Consider the particle travelling clockwise on the elliptical path $\frac{x^2}{100} + \frac{y^2}{25} = 1$ The particle leaves the orbit at the point $(-8, 3)$ and travels in a straight line tangent to the ellpse. At what point will the particle cross the y-axis?

A. $\left(0, -\frac{25}{3}\right)$

B. $\left(0, \frac{7}{3}\right)$

C. $\left(0, \frac{25}{3}\right)$

D. $(0, 9)$

Answer: A

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8. C is the centre of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and A and B are two points on the ellipse such that angle ACB=90◦, then $\frac{1}{(CA)^2}+\frac{1}{(CB)^2}=$

A. $\frac{7}{12}$

B. $\frac{12}{7}$

C. $\frac{25}{144}$

D. $\frac{144}{25}$

Answer: C

Watch Video Solution

9. Let (alpha,beta) be a point from which two perpendicular tangents can be drawn to the ellipse $4x^2 + 5y^2 = 20$. If $F = 4\alpha + 3\beta$, then

A. $-15 \leq F \leq 15$

B. $F \geq 0$

C. $-5 \leq F \leq 20$

D. $F \leq -5\sqrt{5}$ or $F \geq 5\sqrt{5}$

Answer: A

Watch Video Solution

10. If $a = \left[t^2 - 3t + 4\right]$ and $b = [3 + 5t]$, where [.] donates the greatest integer function, then the latusrectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at $t = \frac{3}{2}$ is

A. 20

B. 10

C. $\frac{1}{5}$

D. $\frac{1}{10}$

Answer: C

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11. If the line $x + 2y + 4 = 0$ cutting the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in points whose eccentric angies are 30° and 60° subtends right angle at the origin then its equation is

A. $\frac{x^2}{8} + \frac{y^2}{4} = 1$

B. $\frac{x^2}{16} + \frac{y^2}{4} = 1$

C. $\frac{x^2}{4} + \frac{y^2}{16} = 1$

D. None of the above

Answer: B

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12. An arc of a bridge is semi-elliptical with the major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from the horizontal, then prove that the best approximation of the height of the acr 2 m from the center of the base is $\frac{8}{3}m$.

A. $\frac{11}{4}m$

B. $\frac{8}{3}m$

C. $\frac{7}{2}m$

D. $2m$

Answer: B

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13. A tangent to the ellipes $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at any points meet the line $x = 0$ at a point Q Let R be the image of Q in the line $y = x$, then circle whose extremities of a dameter are Q and R passes through a fixed point, the fixed point is

A. (3,0)

B. (4,0)

C. (5,0)

D. (0,0)

Answer: D

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14. If tangents are drawn from any point on the circle $x^2 + y^2 = 25$ the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ then the angle between the tangents is

A. $\frac{\sqrt{5}}{4}$

B. $\frac{\sqrt{5}}{3}$

C. $\frac{\sqrt{7}}{4}$

D. $\frac{\sqrt{7}}{3}$

Answer: C

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15. the equation of the chord of contact of the pair of tangents drawn to the ellipse $4x^2 + 9y^2 = 36$ from the point (m, n) where $m\dot{n} = m + n, m, n$ being nonzero positive integers, is $2x + 9y = 18$ (b) $2x + 2y = 1$ $4x + 9y = 18$ (d) none of these

A. $2x + 9y = 18$

B. $2x + 2y = 1$

C. $4x + 9y = 18$

D. $4x + 2y = 1$

Answer: C

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16. $x - 2y + 4 = 0$ is a common tangent to $y^2 = 4x$ and $\frac{x^4}{4} + \frac{y^2}{b^2} = 1$. Then the value of b and the other common tangent are given by : (A) $b = \sqrt{3}$ (B) $x + 2y + 4 = 0$ (C) $b = 3$ (D) $x - 2y - 4 = 0$

A. $b = \sqrt{3}, x + 2y + 4 = 0$

B. $b = 3, x + 2y + 4 = 0$

C. $b = \sqrt{3}, x + 2y - 4 = 0$

D. $b = \sqrt{3}, x - 2y - 4 = 0$

Answer: A

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17. The point on the ellipse $x^2 + 2y^2 = 6$ closest to the line $x + y = 7$ is

A. $(\sqrt{2}, \sqrt{2})$

B. $(0, \sqrt{3})$

C. $(2, 1)$

D. $\left(\sqrt{5}, \frac{1}{\sqrt{2}}\right)$

Answer: C

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18. From a point on the axis of x common tangents are drawn to the parabola y^(2)=4x and the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1(a > b > 0)$. If these tangents from an equilateral trianlge with their chord of contact w.r.t parabola, then set of exhaustive values of a is

A. $(0, 3)$

B. $\left(0, \frac{3}{2}\right)$

C. $\left(\frac{3}{2}, 3\right)$

D. $\left(1, \frac{3}{2}\right)$

Answer: C

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19. If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2} + y^2b^2 = 1,$ with vertices having

eccentric angles $alpna, \beta, \gamma,$ respectively is (x_1, y_1) then $\sum \cos\alpha \cos\beta + \sum \sin\alpha \sin\beta =$

A. $\frac{9h^2}{a^2} + \frac{9k^2}{b^2} + \frac{3}{2}$

B. $9h^2 - 9k^2 + a^2b^2$

C. $\frac{9h^2}{a^2} + \frac{9k^2}{b^2} + 3$

D. $\frac{9h^2}{2a^2} + \frac{9k^2}{2b^2} - \frac{3}{2}$

Answer: D

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20. A parabola is drawn whose focus is one of the foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (where a>b) and whose directrix passes through the other focus and perpendicular to the major axes of the ellipse. Then the eccentricity of the ellipse for which the length of latus-rectum of the ellipse and the parabola are same is

A. $\sqrt{2}-1$

B. $\sqrt{3}-1$

C. $2\sqrt{2}-2$

D. $3\sqrt{3}-5$

Answer: A

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21. If the maximum distance of any point on the ellipse $x^2 + 2y^2 + 2xy = 1$ from its center is r, then r is equal to $3 + \sqrt{3}$ (b) $2 + \sqrt{2}$ $\frac{\sqrt{2}}{\sqrt{3 - \sqrt{5}}}$ (d) $\sqrt{2 - \sqrt{2}}$

A. $\frac{\sqrt{6} + 1}{2}$

B. $\frac{\sqrt{5} + 1}{2}$

C. $\frac{\sqrt{3} + 1}{2}$

D. $\frac{\sqrt{2}+2}{2}$

Answer: B

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22. The length of the common chord of the ellipse $\frac{(x-1)^2}{9}+\frac{(y-2)^2}{4}=1$ and the circle $(x-1)^2+(y-2)^2=1$ is (A) 2 (B) $\sqrt{3}$ (C) 4 (D) none of these

A. zero

B. one

C. three

D. eight

Answer: A

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23. The eccentricity of ellipse $ax^2 + by^2 + 2gx + 2fy + c = 0$ if its axis is parallel to x-axis is (A) $\sqrt{\frac{a+b}{4}}$ (B) $\sqrt{\frac{a-b}{2}}$ (C) $\sqrt{\frac{b-a}{a}}$ (D) $\sqrt{\frac{b-a}{b}}$

A. $\sqrt{\left(\frac{b-a}{b}\right)}$

B. $\sqrt{\left(\frac{a+b}{b}\right)}$

C. $\sqrt{\left(\frac{a+b}{a}\right)}$

D. None of these

Answer: A

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24. A circle has the same center as an ellipse and passes through the foci $F_1 and F_2$ of the ellipse, such that the two cuves intersect at four points. Let P be any one of their point of intersection. If the major axis of the ellipse is 17 and the area of

triangle PF_1F_2 is 30, then the distance between the foci is 13 (b) 10 (c) 11 (d) none of these

A. 13

B. 11

C. 9

D. 7

Answer: A

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25. The area of the rectangle formed by the perpendiculars from the centre of the standard ellipse to the tangent and normal at its point whose eccentric angles $\frac{\pi}{4}$ is

A. $\frac{\left(a^2-b^2\right)ab}{a^2+b^2}$

B. $\left(\frac{a^2-b^2}{a^2+b^2}\right)$

C. $\frac{\left(a^2+b^2\right)ab}{\left(a^2-b^2\right)}$

D. $\frac{\left(a^2+b^2\right)}{\left(a^2-b^2\right)}$

Answer: A

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26. An ellipse is inscribed in a circle and a point within the circle is chosen at random. If the probability that this point lies outside the ellipse is $\frac{2}{3}$ then the eccentricity of the ellipse is: (A) $\frac{2\sqrt{2}}{3}$ (B) $\frac{\sqrt{5}}{3}$ (C) $\frac{8}{9}$ (D) $\frac{2}{3}$

A. $2\frac{\sqrt{2}}{3}$

B. $\frac{\sqrt{5}}{3}$

C. $\frac{8}{9}$

D. $\frac{2}{3}$

Answer: A

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27. An ellipse slides between two perpendicular straight lines. Then identify the locus of its center.

A. parabola

B. ellipse

C. hyperbola

D. circle

Answer: D

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28. The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ where mid point is $\left(\frac{1}{2},\frac{2}{3}\right)$ (A) $\frac{1}{10}$ (B) $\frac{\sqrt{8161}}{10}$ (C) $\frac{\sqrt{8061}}{10}$ (D) 1

A. $\frac{1}{10}$

B. $\frac{\sqrt{8161}}{10}$

C. $\frac{\sqrt{8061}}{10}$

D. None of the above

Answer: D

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29. The equation of the locus of the middle point of the portion of the tangent to the ellipse $x/16 + \frac{y^2}{9} = 1$ included between the co-ordinate axes is the curve

A. $9x^2 + 16y^2 = 4x^2y^2$

B. $16x^2 + 9y^2 = 4x^2y^2$

C. $3x^2 + 4y^2 = 4x^2y^2$

D. $9x^2 + 16y^2 = x^2y^2$

Answer: A

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30. The tangent at a point $P(a\cos\varphi, b\sin\varphi)$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets its auxiliary circle at two points, the chord joining which subtends a right angle at the center. Find the eccentricity of the ellipse.

A. $e^2(1+\cos^2\theta)=1$

B. $e^2(\cos ec^2\theta+1)=1$

C. $e^2(1+\sin^2\theta)=1$

D. $e^2(1+\tan^2\theta)=1$

Answer: C

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Exercise More Than One Correct Option Type Questions

1. The locus of extremities of the latus rectum of the family of ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is

A. $x^2 - ay = a^2$

B. $x^2 - ay = b^2$

C. $x^2 + ay = a^2$

D. $x^2 + ay = b^2$

Answer: A::C

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2. The distance of a point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from the centre is 2 Then the eccentric angle of the point is

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{5\pi}{4}$

D. $\frac{7\pi}{4}$

Answer: A::B::C::D

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3. If the equation of family of ellipse is $x^2 \sec^2\theta + y^2 \operatorname{cosec}^2\theta = 1, where \frac{\pi}{4} < \theta < \frac{\pi}{2},$ then the locus of extremities of the latusrectum is

A. $2y^2(1+x^2) = (1-x^2)^2$

B. $2x^2\left(1+y^2 = (1-y^2\right)^2$

C. $2y(1-x^2) = 1+x^2$

D. $2y^2(1+x^2) = 1+x^4-2x^2$

Answer: B::D

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4. Let $F1, F_2$ be two focii of the ellipse and PT and PN be the tangent and the normal respectively to the ellipse at ponit P.then

A. $PNbi\sec ts\angle F_1PF_2$

B. $PTbi\sec ts\angle F_1PF_2$

C. $PTbi\sec ts\angle(180\circ - \angle F_1PF_2)$

D. None of above

Answer: A::C

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5. $\frac{x^2}{r^2 - r - 6} + \frac{y^2}{r^2 - 6r + 5} = 1$ will represent ellipse if r lies in the interval

A. $(-\infty, -2)$

B. $(1, \infty)$

C. $(3, \infty)$

D. $(5, \infty)$

Answer: A::D

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6. A laturectum of an ellipse is a line

A. passing through a focus

B. passing through the najor axis

C. perpendicular to the major axis

D. parallel to the major axis

Answer: A::B::C

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7. An ellipse passes through the point $(4, -1)$ and touches the line $x + 4y - 10 = 0$. Find its equation if its axes coincide with the coordinate axes.

A. $x^2 + 64y^2 = 80$

B. $x^2 + 4y^2 = 20$

C. $x^2 + 20y^2 = 100$

D. $x^2 + 8y^2 = 40$

Answer: A::B

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8. If P is any point lying on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci are S and S'. Let $\angle PSS' = \alpha$ and $\angle PS'S = \beta$,then

A. $SP + S'P = 2a, \quad \text{if} \quad a > b$

B. $SP + S'P = 2b, \quad \text{if} \quad b > a$

C. $\tan\left(\frac{\theta}{2}\right)\tan\left(\frac{\phi}{2}\right) = \frac{1-e}{e+1}$

D.

Answer: A::B::C

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9. If (5, 12) and (24, 7) are the foci of an ellipse passing through the origin, then find the eccentricity of the ellipse.

A. $\frac{\sqrt{386}}{38}$

B. $\frac{\sqrt{386}}{12}$

C. $\frac{\sqrt{386}}{13}$

D. $\frac{\sqrt{386}}{25}$

Answer: A::B

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10. If a pair of variable straight lines $x^2 + 4y^2 + \alpha xy = 0$ (where α is a real parameter) cut the ellipse $x^2 + 4y^2 = 4$ at two points A and B, then the locus of the point of intersection of tangents at A and B is

A. $x - 2y = 0$

B. $2x - y = 0$

C. $x + 2y = 0$

D. $2x + y = 0$

Answer: A::C

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11. In the ellipse

$$25x^2 + 9y^2 - 150x - 90y + 225 = 0$$

A. foci are at $(3, 1), (3, 9)$

B. $e = \frac{4}{5}$

C. center is $(5, 3)$

D. major axis axis is 6

Answer: A::B

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12. If the tangent to the ellipse $x^2 + 4y^2 = 16$ at the point 0 sanormal to the circle $x^2 + y^2 - 8x - 4y = 0$ then θ is equal to

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. 0

D. $-\frac{\pi}{4}$

Answer: A::C

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13. The product of eccentricities of two conics is unity, one of them can be a/an

A. parabola

B. ellipse

C. hyperbola

D. circle

Answer: A::B::C

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14. The parametric $\angle\alpha$ where $-\pi < \alpha \leq \pi$ of the point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at which the tangent drawn cuts the intercept of minimum length on the coordinates axes, is/are

A. $\tan - \sqrt{\frac{b}{a}}$

B. $-\tan(-1)\sqrt{\frac{b}{a}}$

C. $\pi - \tan^{-1}\sqrt{\frac{b}{a}}$

D. $\pi + \tan^{-1}\sqrt{\frac{b}{a}}$

Answer: A::B::C

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15. If latus recturn of the ellipse $x^2 \tan^2 \alpha + y^2 \sec^2 \alpha = 1$ is $\frac{1}{2}$ then $\alpha(0 < \alpha < \pi)$ is equal to

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $5\frac{\pi}{12}$

D. $\frac{\pi}{2}$

Answer: A::C

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Exercise Passage Based Questions

1. A conic is represented by $C \equiv 9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$ Q. The center of conic C is

A. (0,0)

B. (1,0)

C. (0,1)

D. (1,1)

Answer: D

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2. A conic is represented by $C \equiv 9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$ Q.

The center of conic C is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{2}{3}$

D. $\frac{2}{\sqrt{5}}$

Answer: B

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3. A conic is represented by $C \equiv 9x^2 + 4xy + 6y^2 - 22x - 16y + 9 = 0$ Q. The center of conic C is

A. $2, 2\sqrt{2}$

B. $4, 2\sqrt{3}$

C. $6, 2\sqrt{5}$

D. $5, 2\sqrt{6}$

Answer: A

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4. An ellipse E has its center C(3,1), focus at (3,6) and passing through the point P(7,4) Q. The product of the lengths of the prependicular segeent from the focii on tangent at point P is

A. 20

B. 45

C. 40

D. 90

Answer: A

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5. An ellipse E has its center C(3,1), focus at (3,6) and passing through the point P(7,4) Q. The product of the lengths of the prependicular segeent from the focii on tangent at point P is

A. $\left(5, \frac{5}{3}\right)$

B. $3, \left(\frac{4}{3}\right)$

C. $\left(5, \left(\frac{10}{3}\right)\right.$

D. $3, \left(\frac{8}{3}\right)$

Answer: C

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6. An ellipse E has its center C(3,1), focus at (3,6) and passing through the point P(7,4) Q. The product of the lengths of the prependicular segeent from the focii on tangent at point P is

A. $e_1 = \frac{3}{\sqrt{5}}$

B. $e_1 = \frac{\sqrt{5}}{3}$

C. $e_1 = \frac{3}{\sqrt{10}}$

D. $e_1 = \frac{\sqrt{10}}{3}$

Answer: B

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7. $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ interset at four distinct points A,B,C, and D. Their common tangents form a peaallelogram A'B'C'D'. if A'B'C'D' is a square, then r is equal to

A. 36

B. 72

C. 144

D. 288

Answer: D

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8. $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ interset at four distinct points A,B,C, and D. Their common tangents form a peaallelogram A'B'C'D'. if A'B'C'D' is a square, then r is equal to

A. 12

B. 15

C. 20

D. 25

Answer: D

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9. $C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ interset at four distinct points A,B,C, and D. Their common tangents form a peaallelogram A'B'C'D'. if A'B'C'D' is a square, then r is equal to

A. $1:4$

B. $1:2$

C. $3:4$

D. $9:16$

Answer: B

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10. An ellipse whose distance between foci S and S' is 4 units is inscribed in the ΔABC touching thesides AB, AC and $BCatP, Q$ and R. If centre of ellipse is at origin and major axis along x-axis $SP + S'P = 6$, then

A. $9x^2 + 5y^2 = 45$

B. $4x^2 + 9y^2 = 46$

C. $5x^2 + 9y^2 = 45$

D. $9x^2 + 4y^2 = 36$

Answer: C

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11. An ellipse whose distance between foci S and S' is 4 units is inscribed in the ΔABC touching thesides AB, AC and $BCatP, Q$ and R. If centre of ellipse is at origin and major axis along x-axis $SP + S'P = 6$, then

A. $\left(x^2 + y^2 - 14\right)^2 = 4\left(5x^2 + 9y^2 - 45\right)$

B. $\left(x^2 + y^2 - 14\right)^2 = 4\left(5x^2 + 9y^2 - 54\right)$

C. $\left(x^2 + y^2 - 14\right)^2 = 4\left(9x^2 + 5y^2 - 45\right)$

D. $\left(x^2+y^2-14\right)^2=4\left(9x^2+5y^2-54\right)$

Answer: A

Watch Video Solution

12. An ellipse whose distance between foci S and S' is 4 units is inscribed in the ΔABC touching thesides AB, AC and $BCatP, Q$ and R. If centre of ellipse is at origin and major axis along x-axis $SP+S'P=6$, then

A. $5x^2+9y^2=15$

B. $5x^2 + 9y^2 = 60$

C. $9x^2 + 5y^2 = 14$

D. $9x^2 + 5y^2 = 144$

Answer: B

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13. The line$2px + y\sqrt{1 - p^2} = 1(|p| < 1)$ for different values of p, touches a fixed ellipse whose exes are the coordinate axes. Q. The eccentricity of the ellipse is

A. $\frac{1}{\sqrt{5}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{5}}$

Answer: C

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14. For all real p, the line $2px + y\sqrt{1-p^2} = 1$ touches a fixed ellipse whose axex are the coordinate axes

The foci of the ellipse are

A. $\left(\pm \frac{\sqrt{3}}{2}, 0 \right)$

B. $\left(0, \pm \frac{\sqrt{3}}{2} \right)$

C. $(\pm \sqrt{3}, 0$

D. $(0, \pm \sqrt{3})$

Answer: B

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15. The line $2px + y\sqrt{1 - p^2} = 1 (|p| < 1)$ for different values of p, touches a fixed ellipse whose exes are the coordinate axes. Q. The locus of the

point of intersection of prependicular tangents of the ellipse is

A. $x^2 + y^2 = 2$

B. $x^2 + y^2 = \frac{3}{2}$

C. $x^2 + y^2 = \frac{5}{4}$

D. $x^2 + y^2 = \frac{1}{2}$

Answer: C

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Exercise Single Integer Answer Type Questions

1. Two concentric ellipse be such that the foci of one be on the other and if 3/5 and 4/5 be their eccentricities. If theta be the angle between their axes, then the values of $2\left(1+\sin^2\theta+\sin^4\theta\right)$ must be

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2. Rectangle ABCD has area 200.An ellipse with area 200π passes through A and C and has foci at B and D.Find the perimeter of the rectangle.

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3. Number of points on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ from which pair of perpendicular tangents are drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is

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4. The length of the sides of the square which can be made by four perpendicular tangents to the ellipse $\frac{x^2}{7}+\frac{2y^2}{11}=1$, is

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5. Length of the focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which is inclined to the major axis at angle θ is

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6. The number of distinct normal lines that can be drawn to the ellipse $\frac{x^2}{169}+\frac{y^2}{25}=1$ from the point $P(0,6)$ is one (b) two (c) three (d) four

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7. If p is the length of the perpendicular from a focus upon the tangent at any point P of the the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and r is the distance of P from the foicus , then $\frac{2a}{r}-\frac{b^2}{p^2}$ is equal to

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8. An ellipse passing through the origin has its foci (3,4) and (6,8). Then length of its semi-minor axis is b, then the value of $\frac{b}{\sqrt{2}}$ is

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9. The maximum value of 5λ for which four normals can be drawn to ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ through a point (λ,0) is

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10. An ellipse with major and minor axis $6\sqrt{3}$ and 6 respectively, slides along the coordinates axes and always remains confined in the first quardrant. If the length if arc decribed by center of ellipse is $\frac{\pi\lambda}{6}$ then the value of lambda is

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Exercise Statement I And Ii Type Questions

1. Let the equation of ellipse be $\frac{x^2}{a^2+1}=\frac{y^2}{a^2+2}=1$ Statement 1 If eccentricity of the ellipse be $\frac{1}{\sqrt{6}}$, then length of latusrectum is $\frac{10}{\sqrt{6}}$. Statement 2 Length of latusrectum= $\frac{2\left(a^2+1\right)}{\sqrt{a^2+2}}$

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2. Statement 1 : The area of the ellipse $2x^2 + 3y^2 = 6$ is more than the area of the circle $x^2 + y^2 - 2x + 4y + 4 = 0$. Statement 2 : The length f the semi-major axis of an ellipse is more that the radius of the circle.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: B

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3. Statement 1 The equation of the director circle to the ellipse $4x^2 + 9y^2 = 36 is x^2 + y^2 = 13$

Statement 2 The locus of the point of intersection of perpendicular tangents to an ellipse is called the director circle.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: A

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4. Statement 1 In an ellipse the distance between foci is always less than the sum of focal distances of any point on it.

Statement 2 If e be the eccentricity of the ellipse, then 0lteIt1.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: A

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5. Statement 1 The sum of the focal distances of a point on the ellipse $4x^2 + 5y^2 - 16x - 30y41 = 0is2\sqrt{5}.$

Statement 2 The equation $4x^2 + 5y^2 - 16x - 30y + 41 = 0$ can be expressed as $4(x-2)^2 + 5(y-3)^2 = 20.$

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: B

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6. Statement 1 : The locus of the center of a variable circle touching two circle $(x-1)^2+(y-2)^2=25$ and $(x-2)^2+(y-1)^2=16$ is an ellipse. Statement 2 : If a circle $S_2=0$ lies completely inside the circle $S_1=0$, then the locus of the center of a variable circle $S=0$ that touches both the circles is an ellipse.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: C

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7. Statement 1 The tangent and normal at any point P on a ellipse bisect the external and

internal angles between the focal distance of P.

Statement 2 The straight line joining the foci of the ellipse subtends of a right angle at P.

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: C

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8. Statement 1 In a $\triangle ABC$, if based BC is fixed and perimeter of the triangle is also fixed, then certex moves on an ellipse.

Statement 2 If sum of distance of a point P from two fixed points is constant , then locus of P is an ellipse.

A. Statement I is true, statement II is true: statement II is a correct explanation for

statement I

B. Statement I is true, statement II is true, statement II is not a correct explanation for statement I

C. statement I is true, statement II is false

D. statement I is false, statement II is true

Answer: A

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Exercise Subjective Type Questions

1. Consider the family ol circles $x^2 + y^2 = r^2, 2 < r < 5$. If in the first quadrant, the common tangnet to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A and B, then find the equation of the locus of the mid-point of AB.

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2. A straight line PQ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = r^2(b < r < a)$. RS is a focal chord of

the ellipse. If RS is parallel to PQ and meets the circle at points R and S. Find the length of RS.

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3. Let d be the perpendicular distance from the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the tangent drawn at a point P on the ellipse. If $F_1 \& F_2$ are the two foci of the ellipse, then show the $(PF_1-PF_2)^2=4a^2\left[1-\frac{b^2}{d^2}\right]$.

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Exercise Questions Asked In Previous 13 Years Exam

1. The muinimum area of the triangle formed by the tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the coordinate axes is

A. ad sq units

B. $\frac{a^2+b^2}{2}$ sq units

C. $\frac{(a+b)^2}{2}$ sq units

D. $\frac{a^2+ab+b^2}{3}$ sq units

Answer: A

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2. Find the equation of the common tangent in the 1st quadrant to the circle $x^2 + y^2 = 16$ and the elllise $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Also find the length of the intercept of the tangent between the coordinates axes.

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3. An ellipse has OB as the semi-minor axis, $FandF'$ as its foci, and $\angle FBF'$ a right angle. Then, find the eccentricity of the ellipse.

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{1}{\sqrt{3}}$

Answer: A

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4. In an ellipse, the distances between its foci is 6 and minor axis is 8. Then its eccentricity is

A. $\frac{3}{5}$

B. $\frac{1}{2}$

C. $\frac{4}{5}$

D. $\frac{1}{\sqrt{5}}$

Answer: A

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5. Let $P(x_1, y_1)$ and $Q(x_2, y_2), y_1 < 0, y_2 < 0,$ be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

A. $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$

B. $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

C. $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$

D. $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

Answer: B::C

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6. A focus of an ellipse is at the origin. The directrix is the line $x = 4$ and the eccentricity is $\frac{1}{2}$ Then the length of the semi-major axis is

A. $\frac{8}{3}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{5}{3}$

Answer: A

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7. The line passing through the extremity A of the major exis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$ meets is auxiliary circle at the point M. Then the area of the triangle with

vertices at $A, M,$ and O (the origin) is 31/10 (b) 29/10 (c) 21/10 (d) 27/10

A. $\frac{31}{10}$

B. $\frac{29}{10}$

C. $\frac{21}{10}$

D. $\frac{27}{10}$

Answer: D

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8. The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x-axis at Q. If M is the midpoint of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at points. $\left(\pm \frac{\left(3\sqrt{5}\right)}{2} \pm \frac{2}{7} \right)$ (b) $\left(\pm \frac{\left(3\sqrt{5}\right)}{2} \pm \frac{\sqrt{19}}{7} \right)$ $\left(\pm 2\sqrt{3}, \pm \frac{1}{7} \right)$ (d) $\left(\pm 2\sqrt{3} \pm \frac{4\sqrt{3}}{7} \right)$

A. $\left(\pm \left(3\frac{\sqrt{5}}{2}, \pm \frac{2}{7} \right) \right)$

B. $\left(\pm \left(3\frac{\sqrt{5}}{2}, \pm \frac{\sqrt{19}}{4} \right) \right)$

C. $\left(\pm 2\sqrt{3}, \pm \frac{1}{7} \right)$

D. $\left(\pm 2\sqrt{3}, \pm \left(4\frac{\sqrt{3}}{7} \right) \right.$

Answer: C

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9. a triangle ABC with fixed base BC , the vertex A moves such that $\cos B + \cos C = 4\frac{\sin^2 A}{2}\cdot$ If $a, bandc,$ denote the length of the sides of the triangle opposite to the angles $A, B, andC$, respectively, then $b + c = 4a$ (b) $b + c = 2a$ the locus of point A is an ellipse the locus of point A is a pair of straight lines

A. b+c=4a

B. b+c=2a

C. locus of point A is an ellipse

D. locus of point A is a pair od straight lines

Answer: B::C

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10. The conic having parametric representation $x = \sqrt{3}\left(\left(1 - \frac{t^2}{1+t^2}\right), y\frac{= 2t}{1+t^2}\right.$ is

A. an circle

B. a parabola

C. an ellipse

D. a hyperbola

Answer: C

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11. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which in turn is inscribed in another ellipse that passes through the point (4, 0). Then the equation of the

ellipse is (1) $x^2 + 16y^2 = 16$ (2) $x^2 + 12y^2 = 16$

(3) $4x^2 + 48y^2 = 48$ (4) $4x^2 + 64y^2 = 48$

A. $x^2 + 12y^2 = 16$

B. $4x^2 + 48y^2 = 48$

C. $4x^2 + 64y^2 = 48$

D. $x^2 + 16y^2 = 16$

Answer: A

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12. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at point A and B. Q. The coordinates of A and B are

A. (3,0) amd (0,2)

B. $\left(-\frac{8}{5}, \left(2\frac{\sqrt{161}}{15}\right)\right.$ and $\left(-\frac{9}{8}, \frac{8}{5}\right)$

C. $\left(-\frac{8}{5}, \left(2\frac{\sqrt{161}}{15}\right)\right.$ and $(0, 2)$

D. $(3, 0)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

Answer: D

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13. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at point A and B. Q. The orthocenter of the trianlge PAB is

A. $\left(5, \frac{8}{7}\right)$

B. $\left(\frac{7}{5}, \frac{25}{8}\right)$

C. $\left(\frac{11}{5}, \frac{8}{5}\right)$

D. $\left(\frac{8}{25}, \frac{7}{5}\right)$

Answer: C

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14. Tangents are drawn from the point P(3,4) to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B.

The equation of the locus of the point whose distance from the point P and the line AB are equal, is:

A. $\left(9x^2+y^2-6xy-54x-62y+241=0\right.$

B. $x^2+9y^2+6xy-54x+62y-241=0$

C. $9x^2+9y^2-6xy-54x-62y-241=0$

D. $x^2+y^2-2xy+27x+31y-120=0$

Answer: A

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15. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{\frac{2}{5}}$ is: (1) $3x^2 + 5y^2 - 32 = 0$ (2) $5x^2 + 3y^2 - 48 = 0$ (3) $3x^2 + 5y^2 - 15 = 0$ (4) $5x^2 + 3y^2 - 32 = 0$

A. $5x^2 + 3y^2 - 48 = 0$

B. $3x^2 + 5y^2 - 15 = 0$

C. $5x^2 + 3y^2 - 32 = 0$

D. $3x^2 + 5y^2 - 32 = 0$

Answer: D

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16. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

A. $\frac{\sqrt{2}}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

Answer: C

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17. Statement 1: An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4isy = 2x + 2\sqrt{3}$. Statement 2: If the line $y = mx + \frac{4\sqrt{3}}{m}, (m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies

$m^4 + 2m^2 = 24$. (1) Statement 1 is false, statement 2 is true (2) Statement 1 is true, statement 2 is true; statement 2 is a correct explanation for statement 1 (3) Statement 1 is true, statement 2 is true; statement 2 is not a correct explanation for statement 1 (4) Statement 1 is true, statement 2 is false

A. Statement I is true, statement II is true: statement II is a correct explanation for statement I

B. Statement 1 is true,

Statement 2 is true,

Statement 2 is a correct explanation for statemennt 1

C. Statement 1 is true,statement 2 is true,

Statement 2 is not a correct explanation for statement 1.

D. Statement 1 is true,

Statement 2 is false.

Answer: B

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18. An ellipse is drawn by taking a diameter of the circle $(x1)^2 + y^2 = 1$ as its semiminor axis and a diameter of the circle $x^2 + (y2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is the origin and its axes are the coordinate axes, then the equation of the ellipse is (1) $4x^2 + y^2 = 4$ (2) $x^2 + 4y^2 = 8$ (3) $4x^2 + y^2 = 8$ (4) $x^2 + 4y^2 = 16$

A. $4x^2 + y^2 = 4$

B. $x^2 + 4y^2 = 8$

C. $4x^2 + y^2 = 8$

D. $x^2 + 4y^2 = 16$

Answer: D

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19. the equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0,3) is

A. $x^2 + y^2 - 6y - 7 = 0$

B. $x^2 + y^2 - 6y + 7 = 0$

C. $x^2 + y^2 - 6y - 5 = 0$

D. $x^2 + y^2 - 6y + 5 = 0$

Answer: A

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20. A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q.Let the tangents to the ellipse at P and Q meet at R. If $\delta(h)$ Area of triangle δPQR, and $\delta_1 \max_{\frac{1}{2} \leq h \leq 1} \delta(h)$ A further $\delta_2 \min_{\frac{1}{2} \leq h \leq 1} \delta(h)$ Then $\frac{8}{\sqrt{5}}\delta_1 - 8\delta_2$

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21. The locus of the foot of prependicular drawn from the center of the ellipse $x^2 + 3y^2 = 6$ on any tangent to it is

A. $\left(x^2 + y^2\right)^2 = 6x^2 + 2y^2$

B. $\left(x^2 + y^2\right)^2 = 6x^2 - 2y^2$

C. $\left(x^2 - y^2\right)^2 = 6x^2 + 2y^2$

D. $\left(x^2 - y^2\right)^2 = 6x^2 - 2y^2$

Answer: A

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22. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at the end of latus rectum. Find the area of quadrilateral so formed

A. $\frac{27}{2}$

B. 27

C. $\frac{27}{4}$

D. 18

Answer: B

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23. Let E1 and E2, be two ellipses whose centers are at the origin.The major axes of E1 and E2, lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y-1)^2 = 2$. The straight line x+ y =3 touches the curves S, E1 and E2 at P,Q and R, respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$.If e1 and e2 are the eccentricities of E1 and E2, respectively, then the correct expression(s) is(are):

A. $e_1^2 + e_2^2 = \frac{44}{40}$

B. $e_1 e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$

C. $|e|_1^2 - e_2^2 \mid = \frac{5}{8}$

D. $e_1 e^2 = \frac{\sqrt{3}}{4}$

Answer: A::B

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24. Suppose that the foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ are $(f_1,0)and(f_2,0)$ where $f_1>0andf_2<0$. Let P_1andP_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1.0)and$ (2f_2 , 0), respectively. LetT_1 be a tangent to P_1 which passes through $(2f_2,0)$ and T_2 be a tangents to P_2 which passes through $(f_1,0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m12}+m22\right)$ is

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25. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF1NF2 is

A. $\left(-\frac{9}{10}, 0\right)$

B. $\left(\frac{2}{3}, 0\right)$

C. $\left(\frac{9}{10}, 0\right)$

D. $\left(\frac{2}{3}, \sqrt{6}\right)$

Answer: A

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26. If the tangents to the ellipse at M and N meet at R and the normal to the parabola at M meets the x-axis at Q, then the ratio of area of the triangle MQR to area of the quadrilateral MF1NF2 is

A. $3:4$

B. $4:5$

C. $5:8$

D. $2:3$

Answer: C

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27. Let $F_1(x_1, 0)$ and $F^2(x_2, 0)$ for $x_1 < 0$ and $x_2 > 0$, be the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{8} = 1$. Suppose a parabola having vertex at the origin and the focus at F_2 intersects the ellipse at point M in the first quadrant and at point N in the fourth quadrtant. Q. The eccentricity of an ellipse whose center is at the

origin is $\frac{1}{2}$. If one of its directices is $x = -4$, then the equation of the normal to its $\left(1, \frac{3}{2}\right)$ is

A. $x + 2y = 4$

B. $2y - x = 2$

C. $4x - 2y = 1$

D. $4x + 2y = 7$

Answer: C

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