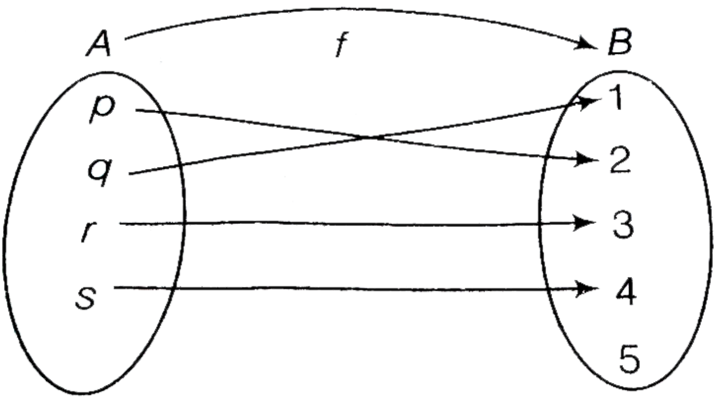
MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

FUNCTIONS

Example

1. In the given figure, find the domain, codomain and range.



2. The number of functions $f:\{1, 2, 3, \dots n\} \rightarrow \{2016, 2017\}$, where ne N, which satisfy thecondition f1)+f(2)+ ...+ f(n) is an odd number are

A. 2^{n}

B. $n \cdot 2^{n-1}$

C. 2^{n-1}

D. n!

Answer: C

Watch Video Solution

3. Find whether $f(x) = x^{3}$ forms a mapping or not.

Watch Video Solution

4. Find whether $\frac{x^{2}}{a^{2}} + \frac{y^{2}}{b^{2}} = 1$ forms a mapping or not.

Watch Video Solution

5. Find the domain of the following functions.

$(i) y = \sqrt{5x - 3}$ $(ii) y = \sqrt[3]{5x - 3}$

$(iii) y = \frac{1}{(x-1)(x-2)}$ $(iv) y = \frac{1}{\sqrt[3]{x-1}}$

Watch Video Solution

6. Find the domain of $f(x) = \sqrt{\left(\frac{1 - 5^x}{7^{-1} - 7}\right)}$

Watch Video Solution

7. Draw the graph of polynominal funtions

$(i) y = x + 1$ $(ii) y = x^2$

$(iii) y = x^3 + 1$ $(iv) y = x(x - 1)(x - 2)$

Watch Video Solution

8. Find domain of the function $10^x + 10^y = 10$

Watch Video Solution

9. Find the domain of the function : $f(x) = \dfrac{1}{\sqrt{(\log)_{\frac{1}{2}}(x^2 - 7x + 13)}}$

Watch Video Solution

10. Find domain of the function $f(x) = \dfrac{1}{\log_{10}(1 - x)} + \sqrt{x + 2}$

Watch Video Solution

11. Find domain of $f(x) = \log_{10}\left(1 + x^3\right)$.

Watch Video Solution

12. Find domain of $f(x) = \log_{10} \log_{10}(1 + x^3)$.

Watch Video Solution

13. Find domain of the function $\log_{10} \log_{10} \log_{10} \log_{10} \log_{10} x$

Watch Video Solution

14. Find the domain of $f(x) = \sqrt{(\log)_{0.4}\left(\frac{x-1}{x+5}\right)}$

Watch Video Solution

15. Find the domain $f(x) = \log_{100x}\left(\frac{2\log_{10} x + 1}{-x}\right)$

Watch Video Solution

16. The domain of definition of f(x) = $\frac{\log_2(x+3)}{x^2+3x+2}$

Watch Video Solution

17. Find the domain for $f(x) = \sin^{-1}\left(\frac{x^2}{2}\right)$.

Watch Video Solution

18. The domain of definition of the function

$f(x) = \sin^{-1}\left\{\log_2\left(\frac{x^2}{2}\right)\right\}$, is

Watch Video Solution

19. Find domain for $f(x) = \sqrt{\cos(\sin x)}$

Watch Video Solution

20. Find the domain for $f(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$

Watch Video Solution

21. Find range and domain of $f(x) = \sqrt{sin^{-1}(\log_2 x)}$

Watch Video Solution

22. Find the domain of the function,

$$f(x) = \log\left\{\log_{|\sin x|}\{x^2 - 8x + 23) - \frac{3}{\log_2|\sin x|}\right\}.$$

Watch Video Solution

23. Solve for x.

$(i)(|x|-1)(|x|-2) \leq 0 \quad (ii)\frac{|x|-1}{|x|-2} \geq 0$

$(iii)|x-3| + |4-x| = 1 \quad (iv)|x| + |x+4| = 4$

Watch Video Solution

24. solve $\left|x^2-1+\sin x\right| = \left|x^2-1\right| + |\sin x|$, where $x \in [-2\pi, 2\pi]$.

View Text Solution

25. Solve $\left|\frac{x}{x-1}\right| + |x| = \frac{x^2}{|x-1|}$.

View Text Solution

26. Find domain for $y = \frac{1}{\sqrt{|x|-x}}$.

Watch Video Solution

27. Find domain for

$$y = \cos^{-1}\left(\frac{1-2|x|}{3}\right) + \log_{|x-1|} x.$$

Watch Video Solution

28. The domain of the function $f(x) = \frac{1}{\sqrt{4x = | x^2 - 10x + 9}})$ is $(7 - \sqrt{40}, 7 + \sqrt{40})$ $(0, 7 + \sqrt{40})$ $(7 - \sqrt{40}, \infty)$ (d) none of these

A. $(7 - \sqrt{40}, 7 + \sqrt{40})$

B. $(0, 7 + \sqrt{40})$

C. $(7 - \sqrt{40}, \infty)$

D. None of these

Answer: D

Watch Video Solution

29. The domain of the function

$f(x) = \sqrt{|\sin^{-1}(\sin x)| - \cos^{-1}(\cos x)}$ in $[0, 2\pi]$ is

A. $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

B. $[\pi, 2\pi]$

C. $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

D. $[0, 2\pi] - \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$

Answer: a

Watch Video Solution

30. Sketch the graph of

$(i)\, f(x) = sgn\left(x^2 + 1\right) \quad (ii)\, f(x) = sgn(\log_e x)$
$(iii)\, f(x) = sgn(\sin x) \quad (iv)\, f(x) = sgn(\cos x)$

Watch Video Solution

31. Find domain for, $f(x) = \cos^{-1}[x]$.

Watch Video Solution

32. Find the value of

$\left[\frac{3}{4}\right] + \left[\frac{3}{4} + \frac{1}{100}\right] + \left[\frac{3}{4} + \frac{2}{100}\right] + \ldots + \left[\frac{3}{4} + \frac{99}{100}\right]$.

Watch Video Solution

33. Given that $y = 2[x] + 3$ and $y = 3[x-2] + 5$ then find the value of $[x+y]$

Watch Video Solution

34. Find domain for $f(x) = [\sin x]\cos\left(\frac{\pi}{[x-1]}\right)$.

Watch Video Solution

35. The domain of the function

$$f(x) = \frac{\log_4\left(5 - [x-1] - [x]^2\right)}{x^2 + x - 2}$$ is

(where [x] denotes greatest integer function)

Watch Video Solution

36. Let [x] represent the greatest integer less than or equal to x If $\left[\sqrt{n^2+\lambda}\right] = \left[n^2+1\right] + 2$, where $\lambda, n \in N$, then λ can assume

$(2n+4)d \Leftrightarrow erentvalus$ $(2n+5)d \Leftrightarrow erentvalus$

$(2n+3)d \Leftrightarrow erentvalus$ $(2n+6)d \Leftrightarrow erentvalus$

Watch Video Solution

37. $f(x) = \dfrac{1}{\sqrt{[x]-x}}$, where $[\cdot]$ denotes the greatest integeral function less than or equals to x. Then, find the domain of f(x).

Watch Video Solution

38. The function $f(x)$ is defined in $[0, 1]$. Find the domain of $f(tanx)$.

Watch Video Solution

39. If the domain of $y = f(x) is[-3, 2]$, then find the domain of $g(x) = f(|[x]|), wher[]$ denotes the greatest integer function.

Watch Video Solution

40. Find the domain of function $f(x) = \frac{1}{[|x-1|] + [|7-x|] - 6}$ where $[\cdot]$ denotes the greatest integral function .

Watch Video Solution

41. If the function $f(x) = [3.5 + b\sin x]$ (where [.] denotes the greatest integer function) is an even function, the complete set of values of b is

A. $(-0.5, 0.5)$

B. $[-0.5, 0.5]$

C. $(0, 1)$

D. $[-1, 1]$

Answer: A

Watch Video Solution

42. The domain of the function

$$f(x) = \log_3 \log_{1/3}(x^2 + 10x + 25) + \frac{1}{[x] + 5}$$

(where [.] denotes the greatest integer function) is

A. (-4,-3)

B. (-6,-5)

C. (-6,-4)

D. None of these

Answer: B

Watch Video Solution

43. The equation $\sin x = [1 + \sin x] + [1 - \cos x]$ has (where [x] is the greatest integer less than or equal to $'x'$)

A. one solution in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

B. one solution in $\left[\frac{\pi}{2}, \pi\right]$

C. One solution in R

D. no solution in R

Answer: d

Watch Video Solution

44. If $\{x\}$ and $[x]$ represent fractional and integral part of x respectively, find the value of $[x] + \sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$

Watch Video Solution

45. Solve the equation $4[x] = x + \{x\}$

Watch Video Solution

46. Prove that $[x] + [y] \leq [x + y]$, where x=[x]+{x} and y=[y]+{y} $[\cdot]$ represents greatest integer function and $\{\cdot\}$ represents fractional part of x.

Watch Video Solution

47. The solution set of x which satisfies the equation $x^2 + (x + 1)^2 = 25$ where (x) is a least integer greater than or equal to x

Watch Video Solution

48. If $[x]$ is the greatest integer less than or equal to x and (x) be the least integer greater than or equal to x and $[x]^2 + (x)^2 > 25$ then x

belongs to

Watch Video Solution

49. The number of solutions of $|[x]-2x|=43$, where $[x]$ denotes the greatest integer $\leq x$ is

Watch Video Solution

50. Find the range for $y=\dfrac{x-[x]}{1-[x]+x}$.

Watch Video Solution

51. Find the range for $f(x)=\dfrac{e^{x}}{1+[x]}$ when $x\geq 0.$

Watch Video Solution

52. Find the domain and range of the function $y = \log_e\left(3x^2 - 4x + 5\right)$.

Watch Video Solution

53. Find the range of $f(x)\sqrt{x-1} + \sqrt{5-1}$

Watch Video Solution

54. Find the range of $\log_3\left\{\log_{\frac{1}{2}}\left(x^2 + 4x + 4\right)\right\}$

Watch Video Solution

55. Range of the function

$f(x) = \left(\cos^{-1}\left|1 - x^2\right|\right)$ is

A. $\left[0, \frac{\pi}{2}\right]$

B. $\left[0, \frac{\pi}{3}\right]$

C. (o, π)

D. $\left(\frac{\pi}{2}, \pi\right)$

Answer: A

Watch Video Solution

56. If x, y and$z$ are three real numbers such that $x + y + z = 4$ and $x^2 + y^2 + z^2 = 6$,then find closed interval in which each of x,y and z lie

A. (-1,1)

B. [0,2]

C. [2,3]

D. $\left[\frac{2}{3}, 2\right]$

Answer: D

Watch Video Solution

57. The range of the function

$f(x) = \frac{1}{|\sin x|} + \frac{1}{|\cos x|}$ is

A. $[2\sqrt{2}, \infty)$

B. $(\sqrt{2}, 2\sqrt{2})$

C. $(0, 2\sqrt{2})$

D. $(2\sqrt{2}, 4)$

Answer: A

Watch Video Solution

58. If z=x+iy and $x^2 + y^2 = 16$, then the range of $||x| - |y||$ is

A. [0,4]

B. [0,2]

C. [2,4]

D. None of these

Answer: A

Watch Video Solution

59. Find the range of f(x)=$\frac{1}{\pi}\sin^{-1}x+\tan^{-1}+\frac{x+1}{x^2+2x+5}$

A. $\left[-\frac{3}{4},\frac{1}{5}\right]$

B. $\left[-\frac{5}{4},\frac{3}{4}\right]$

C. $\left[-\frac{3}{4},\frac{5}{4}\right]$

D. $\left[-\frac{3}{4},1\right]$

Answer: D

Watch Video Solution

60. The range of the function $\sin^2 x-5\sin x-6$ is

A. [-10,0]

B. [-1,1]

C. $[0, \pi]$

D. $\left[-\frac{49}{4}, 0\right]$

Answer: A

Watch Video Solution

61. If $f(x) = [x^2] - [x]^2$, where [ ] denote the greatestinteger function and $x \varepsilon [0, n], \neq N$ then thenumber of elements In the range of f(x) are

A. (2n+1)

B. 4n-3

C. 3n-3

D. 2n-1

Answer: D

Watch Video Solution

62. Range of the function

$f(x) = \sqrt{|\sin^{-1}|\sin x||\cos^{-1}|\cos x|}$ is

A. {0}

B. $\left[0, \sqrt{\frac{\pi}{2}}\right]$

C. $[0, \sqrt{\pi}]$

D. None of these

Answer: A

View Text Solution

63. The number of values of y in $[-2\pi, 2\pi]$ satisfying the equation $|\sin 2x| + |\cos 2x| = |\sin y|$ is

A. 3

B. 4

C. 5

D. 6

Answer: B

Watch Video Solution

64. $f(x) = \cot^{-1}(x^2 - 4x + 5)$ then range of $f(x)$ is equal to :

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(0, \frac{\pi}{4}\right]$

C. $\left[0, \frac{\pi}{4}\right)$

D. None of these

Answer: B

Watch Video Solution

65. Find the range of $f(x) = \frac{x^2 + 14x + 9}{x^2 + 2x + 3}$, where x $\in$ R.

Watch Video Solution

66. For what real values of a does the range of $f(x) = \frac{x+1}{a + x^2}$ contains the interval [0,1]?

Watch Video Solution

67. Find the range of the function

$$f(x) = \frac{\sin^2 x + \sin x - 1}{\sin^2 x - \sin x + 2}.$$

View Text Solution

68. If f is an even function, then find the realvalues of x satisfying the equation $f(x) = f\left(\frac{x+1}{x+2}\right)$

Watch Video Solution

69. Find out whether the given function is even, odd or neither even nor odd

$$\text{where} \quad f(x) = \begin{cases} x|x| & , \quad x \leq -1 \\ [1+x] + [1-x] & , \quad -1 < x < 1 \\ -x|x| & , \quad x \geq 1 \end{cases}$$

where || and [] represent then modulus and greater integer functions.

Watch Video Solution

70. Find whether the given function is even or odd: $f(x) = \left(x \dfrac{\sin x + \tan x}{\left[\frac{x}{\pi}\right] - \frac{1}{2}} \right.$; whether [] denotes the greatest integer function.

Watch Video Solution

71. Prove sin x is periodic and find its period.

Watch Video Solution

72. Prove that f(x)=x-[x] is periodic function. Also, find its period.

Watch Video Solution

73. Let $f(x)$ be periodic and k be a positive real number such that $f(x+k)+f(x)=0$ for all $x \in R$. Then the period of $f(x)$ is

Watch Video Solution

74. Find periods for

$(i)\cos^4 x. \quad (ii)\sin^3 x. \quad (iii)\cos\sqrt{x}. \quad (iv)\cos x.$

Watch Video Solution

75. Find the period $f(x)=\sin x+\{x\},$ where {x} is the fractional part of $x\cdot$

Watch Video Solution

Watch Video Solution

76. Find period of $f(x) = \tan 3x + \sin\left(\frac{x}{3}\right)$.

Watch Video Solution

77. Find the period of

$f(x) = \sin x + \frac{\tan x}{2} + \frac{\sin x}{2^2} + tan\frac{x}{2^3} + + \frac{\sin x}{2^{n-1}} + \frac{\tan x}{2^n}$

Watch Video Solution

78. Find the period of $f(x) = |\sin x| + |\cos x|$.

Watch Video Solution

79. Period of $f(x) = \sin^4 x + \cos^4 x$

Watch Video Solution

80. Find the period of $\cos(\cos x) + \cos(\sin x)$.

Watch Video Solution

81. Find the period of $f(x) = \cos^{-1}(\cos x)$

Watch Video Solution

82. The period of $f(x) = \cos(|\sin x| - |\cos x|)$ in degree is

A. 360

B. 180

C. 90

D. None of these

Answer: C

Watch Video Solution

83. Period of the function $f(x) = \sin(\sin(\pi x)) + e^{\{3x\}}$, where {.} denotes the fractional part of x is

A. 1

B. 2

C. 3

D. None of these

Answer: B

Watch Video Solution

84. $\sin \alpha x + \cos \alpha x$ and $|\cos x| + |\sin x|$ are periodic functions of same fundamental period, if 'a' equals

A. 0

B. 1

C. 2

D. 4

Answer: D

Watch Video Solution

85. Let $f(x) = \sin x + \cos\left(\sqrt{4-a^2}\right)x$. Then, the integral values of 'a' for which f(x) is a periodic function, are given by

A. {2,-2}

B. (-2,2]

C. [-2,2]

D. None of these

Answer: D

View Text Solution

86. Let $f(x) = \begin{cases} -1 + \sin K_1 \pi x, & \text{x is rational.} \\ 1 + \cos K_2 \pi x, & x \end{cases}$

If f(x) is a periodic function, then

A. either $K_1, K_2 \in$ rational or $K_1, K_2 \in$ irrational

B. $K_1, K_2 \in$ rational only

C. $K_1, K_2 \in$ irrational only

D. $K_1, K_2 \in$ irrational such that $\frac{K_1}{K_2}$ is rational

Answer: B

View Text Solution

87. If $f(x) = \tan^2\left(\frac{\pi x}{n^2 - 5n + 8}\right) + \cot(n+m)\pi x; (n \in N, m \in Q)$ is a periodic function with 2 as its fundamental period, then m can't belong to

A. $(-\infty, -2) \cup (-1, \infty)$

B. $(-\infty, -3) \cup (-2, \infty)$

C. $(-2, -1) \cup (-3, -2)$

D. $\left(-3, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, -2\right)$

Answer: C

Watch Video Solution

88. Let $f(x)$ be a periodic function with period $\int_0^x f(t+n)dt3 \text{ and } f\left(-\frac{2}{3}\right) = 7 \text{ and } g(x) =$ .where $n = 3k, k \in N$. Then $g'\left(\frac{7}{3}\right) =$

A. $-\frac{2}{3}$

B. 7

C. -7

D. $\frac{7}{3}$

Answer: B

Watch Video Solution

89. Let $f:[-\pi/2, \pi/2] \rightarrow [-1, 1]$ where f(x)=sinx. Find whether f(x) is one-one or not.

Watch Video Solution

90. If $f(x) = x^3 + 3x^2 + 4x + b \sin x + c \cos x \,\forall x \in R$ is a one-one function then the value of $b^2 + c^2$ is

A. ≥ 1

B. ≥ 2

C. ≤ 1

D. None of these

Answer: C

Watch Video Solution

91. Show $f: R \rightarrow R$ defined by $f(x) = x^2 + x$ for all $x \in R$ is many-one.

Watch Video Solution

92. Show that $f: R\overrightarrow{R}$ defined by $f(x) = (x-1)(x-2)(x-3)$ is surjective but not injective.

Watch Video Solution

93. If $f: R \rightarrow \left[\frac{\pi}{6}, \frac{\pi}{2}\right], f(x) = \sin^{-1}\left(\frac{x^2 - a}{x^2 + 1}\right)$ is an onto function, the set of values a is

A. $\left\{-\frac{1}{2}\right\}$

B. $\left[-\frac{1}{2}, -1\right)$

C. $(-1, \infty)$

D. None of these

Answer: C

Watch Video Solution

94. Show $f: R \rightarrow R$ defined by $f(x) = x^2 + 4x + 5$ is into

Watch Video Solution

95. Let $A = \{x: -1 \leq x \leq 1\} = B$ be a function $f: A \rightarrow B$. Then find the nature of each of the following functions.

(i) $f(x) = |x|$ $\quad$ (ii) $f(x) = x|x|$

(iii) $f(x) = x^3$ $\quad$ (iv) $f(x) = \sin\frac{\pi x}{2}$

Watch Video Solution

96. The function $f: R \rightarrow R$ defined as $f(x) = \frac{1}{2} In\left(\sqrt{\sqrt{x^2+1}+x} + \sqrt{\sqrt{x^2+1}-x}\right)$ is

A. one-one and onto both

B. one-one but not onto

C. onto but not one-one

D. Neither one-one nor onto

Answer: D

Watch Video Solution

97. If X={1,2,3,4,5} and Y={a,b,c,d,e,f} and $f: X \to Y$, find the total number of

(i) functions (ii) one to one functions
(iii) many-one functions (iv) constant functions
(v) onto functions (vi) into functions

Watch Video Solution

98. Find the number of surjections from A to B, where A={1,2,3,4}, B={a,b}.

Watch Video Solution

99. $f(x) = \log_{x^2} 25$ and $g(x) = \log_x 5$. Then f(x)=g(x) holds for x belonging to

A. $x \in (-\infty, \infty)$

B. $x \in (-\infty, 0)$

C. $x \in (0, \infty)$

D. $x \in (0, \infty) - \{1\}$

Answer: D

Watch Video Solution

100. Let $A = \{1, 2\}, \ B = \{3, 6\}$ and $f: A \to B$ given by $f(x) = x^2 + 2 \, nd \, g: A \to B$ given by $g(x) = 3x$. Then we observe that $f \, and \, g$ have the same domain and co-domain. Also we hve, $f(1) = 3 = g(1) and \ f(2) = 6 = g(2)$. Hence $f = g$.

Watch Video Solution

101. Which pair of functions is identical?

A. $\sin^{-1}(\sin x) \text{ and } \sin(\sin^{-1} x)$

B. $\log_e e^x, e^{\log_e x}$

C. $\log_e x^2, 2\log_e x$

D. None of the above

Answer: D

Watch Video Solution

102. Let $f: A \to B$ and $g: B \to C$ be two functions and $gof: A \to C$ is define statement(s) is true?

A. If gof is one-one, then f anf g both are one-one

B. if gof is one-one, then f is one-one

C. If gof is a bijection, then f is one-one and g is onto

D. If f and g are both one-one, then gof is one-one.

Answer: B::C::D

Watch Video Solution

103. Let R be the set of real numbes. If $f: R\overrightarrow{R}; f(x) = x^2$ and $g: R\overrightarrow{R}; g(x) = 2x + 1$. Then, find $fogandgof$. Also, show that $fog \neq gof$.

Watch Video Solution

104. Let g(x)=1+x-[x]

and $$f(x) = \begin{cases} -1, x < 0 \\ 0, x = 0 \\ 1, x > 0 \end{cases}$$

Then, for all x, find f(g(x)).

Watch Video Solution

105. Let $f(x) = \begin{cases} 1 + x, & 0 \le x \le 2 \\ 3 - x, & 2 < x \le 3 \end{cases}$

find (fof) (x).

Watch Video Solution

106. Two functions are defined as under : $f(x) = \begin{cases} x + 1 & x \le 1 \\ 2x + 1 & 1 < x \le 2 \end{cases}$

and $g(x) = \begin{cases} x^2 & -1 \le x \le 2 \\ x + 2 & 2 \le x \le 3 \end{cases}$ Find fog and gof

Watch Video Solution

107. If $f(x) = 2x + |x|$, $g(x) = \frac{1}{3}(2x - |x|)$ and h(x)=f(g(x)), domain of $\underbrace{\sin^{-1}(h(h(h(h...h(x)...))))}_{\text{n times}}$ is

A. [-1,1]

B. $\left[-1, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right]$

C. $\left[-1, -\frac{1}{2}\right]$

D. $\left[\frac{1}{2}, 1\right]$

Answer: A

Watch Video Solution

108. A function $f : R\overrightarrow{R}$ satisfies $x\cos y(f(2x+2y)-f(2x-2y)=\cos x\sin y(f(2x+2y)+f(2x-2y))$.

If $f'(0)=\frac{1}{2}, then\ f'(x)=f(x)=0\ 4f^x+f(x)=0\ f^x+f(x)=0$
$4f^x-f(x)=0$

A. $f(x)''(x)=f(x)=0$

B. $4f''(x)+f(x)=0$

C. $f''(x)+f(x)=0$

D. $4f''(x)-f(x)=0$

Answer: B

Watch Video Solution

109. If f(x)=3x-5, find $f^{-1}(x)$.

Watch Video Solution

110. If $f:[1,\infty)\to[2,\infty]$ is given by $f(x)=x+\frac{1}{x}$, find $f^{-1}(x)$ (assume bijection).

Watch Video Solution

111. Let $f(x)=x^3+3$ be bijective, then find its inverse.

Watch Video Solution

112. The inverse of the function of $f:R\to R$ given by $f(x)=\log_a\left(x+\sqrt{x^2+1}(a>0,\,a\neq 1)\right.$ is

Watch Video Solution

113. Let $f: R\vec{R}$ be defined by $f(x) = \left(e^{x} - e^{-x}\right)/2$. Is $f(x)$ invertible? If so, find is inverse.

Watch Video Solution

114. Let $f: \left[\frac{1}{2}, \infty\right) \to \left[\frac{3}{4}, \infty\right)$, where $f(x) = x^2 - x + 1$. Find the inverse of f(x). Hence or otherwise solve the equation, $x^2 - x + 1 = \frac{1}{2} + \sqrt{x - \frac{3}{4}}$.

Watch Video Solution

115. Let g(x) be the inverse of f(x) and $f'(x) = \frac{1}{1 + x^3}$.Find g'(x) in terms of g(x).

Watch Video Solution

116. If $f: R \rightarrow R$ be defined by $f(x) = x^2 + 1$, then find $f^{-1}(17)$ and $f^{-1}(-3)$.

Watch Video Solution

117. If the function f and g are defined as $f(x) = e^x$ and g(x)=3x-2, where $f: R \rightarrow R$ and $g: R \rightarrow R$, find the function fog and gof. Also, find the domain of $(fog)^{-1}$ and $(gof)^{-1}$.

Watch Video Solution

118. If f(x)=ax+b and the equation $f(x) = f^{-1}(x)$ be satisfied by every real value of x, then

A. a=2, b=-1

B. $a = -1, b \in R$

C. $a = 1, b \in R$

D. a=1, b=-1

Answer: B

Watch Video Solution

119. If g is inverse of function f and $f'(x) = \sin x$, then $g'(x)$=

A. sin(g(x))

B. cosec(g(x))

C. tan(g(x))

D. None of these

Answer: B

Watch Video Solution

120. If A and B are the points of intersection of y=f(x) and $y = f^{-1}(x)$, then

A. A and B necessarity lie on the line y=x

B. A and B must be coincident

C. slope of line AB may be -1

D. None of these above

Answer: C

View Text Solution

121. For $x \in R$, the functions f(x) satisfies $2f(x) + f(1-x) = x^2$. The value of f(4) is equal to

A. $\frac{13}{3}$

B. $\frac{43}{3}$

C. $\frac{23}{3}$

D. None of these

Answer: C

Watch Video Solution

122. if $f(x) = ax^7 + bx^3 + cx - 5, f(-7) = 7$ then $f(7)$ is

Watch Video Solution

123. $f(x) + f\left(1 - \frac{1}{x}\right) = 1 + x$ for $x \in R - \{0, 1\}$. Find the value of $4f(2)$.

Watch Video Solution

124. Let $f(x) = \max\left\{x, x^2\right\}$. Then, equivalent definition of f(x).

Watch Video Solution

125. Let

$f(x) = \max(1 + s \in x, 1, 1 - \cos x), x \in [0, 2\pi], andg(x) = \max\{1, |x$

Then $g(f(0)) = 1$ (b) $g(f(1)) = 1$ $f(f(1)) = 1$ (d) $f(g(0)) + 1\sin 1$

Watch Video Solution

126. Let $f(x) = \frac{a_{2k}x^{2k} + a_{2k-1}x^{2k-1} + \ldots + a_1x + a_0}{b_{2k}x^{2k} + b_{2k-1}x^{2k-1} + \ldots + b_1x + b_0}$, where k is a positive integer, $a_i, b_i \in R$ and $a_{2k} \neq 0, b_{2k} \neq 0$ such that $b_{2k}x^{2k} + b_{2k-1}x^{2k-1} + \ldots + b_1x + b_0 = 0$ has no real roots, then

A. f(x) must be one to one

B. $a_{2k}x^{2k} + a_{2k-1} + \ldots + a_1x + a_0 = 0$

must have real roots

C. f(x) must be many to one

D. Nothing can be said about the above options

Answer: C

View Text Solution

127. If $\log_{10}\left(\sin\left(x+\frac{\pi}{4}\right)\right)=\frac{\log_{10}6-1}{2}$, the value of $\log_{10}(\sin x)+\log_{10}(\cos x)$ is

A. -1

B. -2

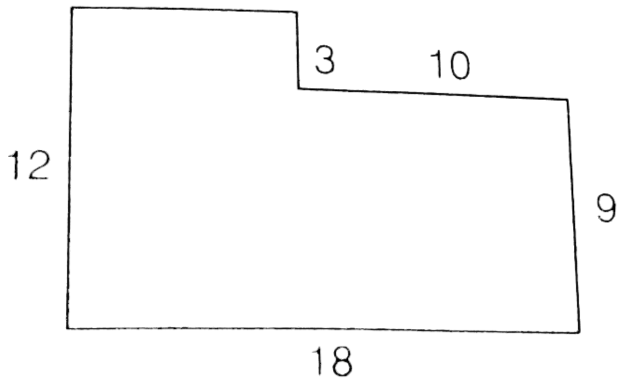
C. 2

D. 1

Answer: A

Watch Video Solution

128. The diagram shows the dimensions of the floor of an L-shaped room. (All the angles are right angles). The area of the largest circle that can be drawn on the floor of this room is



A. 16π

B. 25π

C. $\frac{81\pi}{4}$

D. $\frac{145\pi}{4}$

Answer: B

Watch Video Solution

129. Suppose that the temperature T at every point (x,y) in the plane cartesian is given by the formula $T = 1 - x^2 + 2y^2$. The correct statement about the maximum and minimum temperature along the line x+y=1 is

A. Minimum is -1. There is no maximum

B. Maximum is -1. There is no minimum

C. Maximum is 0. Minimum is -1

D. There is neither a maximum nor a minimum along the line

Answer: A

Watch Video Solution

130. The domain of the function f(x)=max{sin x, cos x} is $(-\infty, \infty)$. The range of f(x) is

A. $\left[-\frac{1}{\sqrt{2}}, 1\right]$

B. $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

C. [0,1]

D. [-1,1]

Answer: A

Watch Video Solution

131. Consider the function $f: A \to A$ where $A: \{1, 2, 3, 4, 5\}$ which satisfy the condition $f(f(x)) = x$, If the number of such functions are λ, then

A. 10

B. 40

C. 41

D. 31

Answer: C

Watch Video Solution

132. Area bounded by the relation [2x]+[y]=5, x,y $>$ 0 is (where $[\cdot]$ represent greatest integer function)

A. 2

B. 3

C. 4

D. 5

Answer: B

Watch Video Solution

133. If the integers a,b,c,d are in arithmetic progression and $a < b < c < d$ and $d = a^2 + b^2 + c^2$, the value of (a+10b+100c+1000d) is

A. 2008

B. 2010

C. 2009

D. 2016

Answer: C

Watch Video Solution

134. Let $f(n)$ denotes the square of the sum of the digits of natural number n, where $f^2(n)$ denotes $f(f(n))$. $f^3(n)$ denote $f(f(f(n)))$ and so on.the value of $\frac{f^{2011}(2011) - f^{2010}(2011)}{f^{2013}(2011) - f^{2012}(2011)}$ is....

A. 1

B. 3

C. 5

D. 7

Answer: A

Watch Video Solution

135. If $\sum_{i=1}^{4} a_i^2 x^2 - 2\sum_{i=1}^{4} a_i a_{i+1} x + \sum_{i=1}^{4} a_i^2 + 1\Big) \leq 0, \quad \text{where} \quad a_i > 0$

and all are distinct. Then,

A. $a_1 + a_5 > 2a_3$

B. $\sqrt{a_1 a_5} = a_3$

C. $\dfrac{2}{\sqrt{a_1 a_4}} > \dfrac{1}{a_1} + \dfrac{1}{a_4}$

D. $\prod_{i=1}^{5} a_i = a_3^5$

Answer: A::B::C

View Text Solution

136. If $f(x-y), f(x) \cdot f(y), f(x+y)$ are in for all $x, y \in R$ and $f(0) \neq 0$, then

A. f'(x) is an even function

B. f'(1)+f'(-1)=0

C. f'(2)-f'(-2)=0

D. f'(3)+f'(-3)=0

Answer: B::D

View Text Solution

137. $x^2 + 4 + 3\cos(ax + b) = 2x$ has atleast on solution then the value of a+b is :

A. 5π

B. 3π

C. 2π

D. π

Answer: B::D

Watch Video Solution

138. Which of following functions have the same graph?

A. $f(x) = \log_e e^x$

B. $g(x) = |x| sgn x$

C. $h(x) = \cot^{-1}(\cot x)$

D. $k(x) = \lim_{n \to \infty} \frac{2|x|}{\pi} \tan^{-1}(nx)$

Answer: A::B::D

View Text Solution

139. Let

$f(\theta) = \frac{\sin^2\theta\cos\theta}{(\sin\theta + \cos\theta)} - \frac{1}{4}\tan\left(\frac{\pi}{4} - \theta\right), \forall\theta \in R - \left\{n\pi - \frac{\pi}{4}\right\}, n \in I.$

Statement I The largest and smallest value of $f(\theta)$ differ by $\frac{1}{\sqrt{2}}$

Statement II $a\sin x + b\cos x + c \in \left[c - \sqrt{a^2 + b^2}, c + \sqrt{a^2 + b^2}\right], \forall x$

View Text Solution

140. Let $a_m(m = 1, 2, , p)$ be the possible integral values of a for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$meets at some poin for which the graphs o t for all real values of b Let $t_r = \prod_{m=1}^{p} (r - a_m)$ and $S_n = \sum_{r=1}^{n} t_r.\ n \in N$ The minimum possible value of a i

A. $\frac{1}{5}$

B. $\frac{5}{26}$

C. $\frac{3}{28}$

D. $\frac{2}{43}$

Answer: A

Watch Video Solution

141. Let $a_m(m = 1, 2, , p)$ be the possible integral values of a for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$meets at some poin for which the graphs o t for all real values of b Let

$t_r = \prod_{m=1}^{p} (r - a_m)$ and $S_n = \sum_{r=1}^{n} t_r.\ n \in N$ The minimum possible value of a i

A. 8

B. 9

C. 10

D. 15

Answer: C

Watch Video Solution

142. Let $a_m (m = 1, 2, , p)$ be the possible integral values of a for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$meets at some poin for which the graphs o t for all real values of b Let $t_r = \prod_{m=1}^{p} (r - a_m)$ and $S_n = \sum_{r=1}^{n} t_r.\ n \in N$ The minimum possible value of a i

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{15}$

D. $\frac{1}{18}$

Answer: D

Watch Video Solution

143. Let w be non-real fifth root of 3 and $x = w^3 + w^4$. If $x^5 = f(x)$, where f(x) is real quadratic polynominal, with roots α and $\beta, (\alpha, \beta \in C)$, then determine f(x) and answer the following questions.

Every term of the sequence $\{f(x)\}, n \in N$ is divisible by

A. 12

B. 18

C. 24

D. 27

Answer: B

View Text Solution

144. Let w be non-real fifth root of 3 and $x = w^3 + w^4$. If $x^5 = f(x)$, where f(x) is real quadratic polynominal, with roots α and $\beta, (\alpha, \beta \in C)$, then determine f(x) and answer the following questions.

Which of the following is not true?

A. $\alpha + \beta = -3$

B. $\alpha\beta = 12/5$

C. $|\alpha - \beta| = \sqrt{3/5}$

D. $|\alpha| + |\beta| = 2\sqrt{3/5}$

Answer: D

View Text Solution

145. Let w be non-real fifth root of 3 and $x = w^3 + w^4$. If $x^5 = f(x)$, where f(x) is real quadratic polynominal, with roots α and $\beta, (\alpha, \beta \in C)$, then determine f(x) and answer the following questions.

If α and β are represented by points A and B in argand plane, then circumradius of $\triangle OAB$, where O is origin, is

A. $4/5$

B. $8/5$

C. $16/5$

D. $32/5$

Answer: A

View Text Solution

146. Let A={1,2,3,4,5} and B={-2,-1,0,1,2,3,4,5}.

Increasing function from A to B is

A. 120

B. 72

C. 60

D. 56

Answer: D

Watch Video Solution

147. Let A={1,2,3,4,5} and B={-2,-1,0,1,2,3,4,5}.

Non-decreasing functions from A to B is

A. 216

B. 540

C. 792

D. 840

Answer: C

Watch Video Solution

148. Let A={1,2,3,4,5} and B={-2,-1,0,1,2,3,4,5}.

Onto functions from A to A such that $f(i) \neq i$ for all i, is

A. 44

B. 120

C. 56

D. 76

Answer: A

Watch Video Solution

149. Let $f(x) = \sin^{23} x - \cos^{22} x and g(x) = 1 + \frac{1}{2}\tan^{-1}|x|$. Then the number of values of x in the interval $[-10\pi, 8\pi]$ satisfying the equation $f(x) = sgn(g(x))$ is ________

Watch Video Solution

150. Consider the function g(x) defined as $g(x)\cdot\left(x^{\left(2^{2008}-1\right)}-1=(x+1)\left(x^{2}+1\right)\left(x^{4}+1\right)\ldots\left(x^{2^{2007}}+1\right)-1\right.$ the value of g(2) equals ....

View Text Solution

151. if $f(x)=\left(\frac{9}{\log_2(3-2x)}-1\right)^{\frac{1}{3}}$ then the value of a which satisfies $f^{-1}(2a-4)=\frac{1}{2}$ is

Watch Video Solution

152. Let f be defined on the natural numbers as follow: f(1)=1 and for $n>1, f(n)=f[f(n-1)]+f[n-f(n-1)]$, the value of $\frac{1}{30}\sum_{r=1}^{20}f(r)$ is

Watch Video Solution

153. If a,b,c are real roots of the cubic equation f(x)=0 such that $(x-1)^2$ is a factor of f(x)+2 and $(x+1)^2$ is a factor of f(x)-2, then $|ab+bc+ca|$ is equal to

Watch Video Solution

154. Minimum integral value of k for which the equation $e^x = kx^2$ has exactly three real distinct solution,

Watch Video Solution

155. $x = \sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + \dots}}}}$

Watch Video Solution

156. Let a sequence $x_1, x_2, x_3, \dots$ of complex numbers be defined by $x_1 = 0, x_{n+1} = x_n^2 - i$ for all $n > 1$, where $i^2 = -1$. Find the distance of x_{2000} from x_{1997} in the complex plane.

View Text Solution

157. If a, b, c, d, e are +ve real numbers such that $a+b+c+d+e=8$ and $a^2+b^2+c^2+d^2+e^2=16$, then the range of 'e' is

Watch Video Solution

158. Find the set of all solutions of the equation $2^{|y|}-|2^{y-1}-1|=2^{y-1}+1$

Watch Video Solution

159. Solve the equation [x]{x}=x, where [] and {} denote the greatest integer function and fractional part, respectively.

Watch Video Solution

160. Sum of all the solution of the equation $\frac{[x]}{[x-2]} - \frac{[x-2]}{[x]} = \frac{8\{x\}+12}{[x-2][x]}$ is (where{*} denotes greatest integer function and {*} represent fractional part function)

Watch Video Solution

161. If $f(x)$ is a polynomial function satisfying $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65, then f \in df(6)$.

Watch Video Solution

162. If f(x) satisfies the relation, f(x+y)=f(x)+f(y) for all x,y $\in$ R and f(1)=5, then find $\sum_{n=1}^{m} f(n)$. Also, prove that f(x) is odd.

Watch Video Solution

163. Let $f(x) = \frac{9^x}{9^x + 3}$. Show $f(x) + f(1-x) = 1$ and, hence, evaluate. $f\left(\frac{1}{1996}\right) + f\left(\frac{2}{1996}\right) + f\left(\frac{3}{1996}\right) + + f\left(\frac{1995}{1996}\right)$

Watch Video Solution

164. ABCD is a square of side a. A line parallel to the diagonal BD at a distance x from the vertex A cuts the two adjacent sides. Express the area of the segment of the square with A at a vertex, as a function of x.

Watch Video Solution

165. If $f : R \to R, f(x) = \frac{\alpha x^2 + 6x - 8}{\alpha + 6x - 8x^2}$ is onto then $\alpha \in$

Watch Video Solution

166. Let f(x, y) be a periodic function satisfying $f(x, y) = f(2x + 2y, 2y - 2x)$ for all x, y; Define $g(x) = f(2^x, 0)$. Show that g(x) is a periodic function with period 12.

Watch Video Solution

167. Solve the equation

$10^{(x+1)(3x+4)} - 2 \cdot 10^{(x+1)(x+2)} = 10^{1-x-x^2}$.

Watch Video Solution

168. The real solution of $[x] + 5x + [10x] + [20x] = 36k + 35, k \in I$, if the fractional part of x lies in $\left[\frac{1}{10}, \frac{1}{5}\right)$

Watch Video Solution

169. Let $f: N \to N$ be a function such $x - f(x) = 19\left[\frac{x}{19}\right] - 90\left[\frac{f(x)}{90}\right], \forall x \in N$, where [.] denotes the greatest integer function and [.] denotes the greatest integers function and $1900 < f(1990) < 2000$, then possible value of $f(1990)$ is

Watch Video Solution

170. Solve the system of equations,

$$\left|x^2 - 2x\right| + y = 1, x^2 + |y| = 1.$$

Watch Video Solution

171. Let f and g be real - valued functions such that $f(x+y) + f(x-y) = 2f(x) \cdot g(y), \forall \text{ x , y } \in R.$ Prove that , if f(x) is not identically zero and $|f(x)| \leq 1, \forall x \in R,$ then $|g(y)| \leq 1, \forall y \in R.$

Watch Video Solution

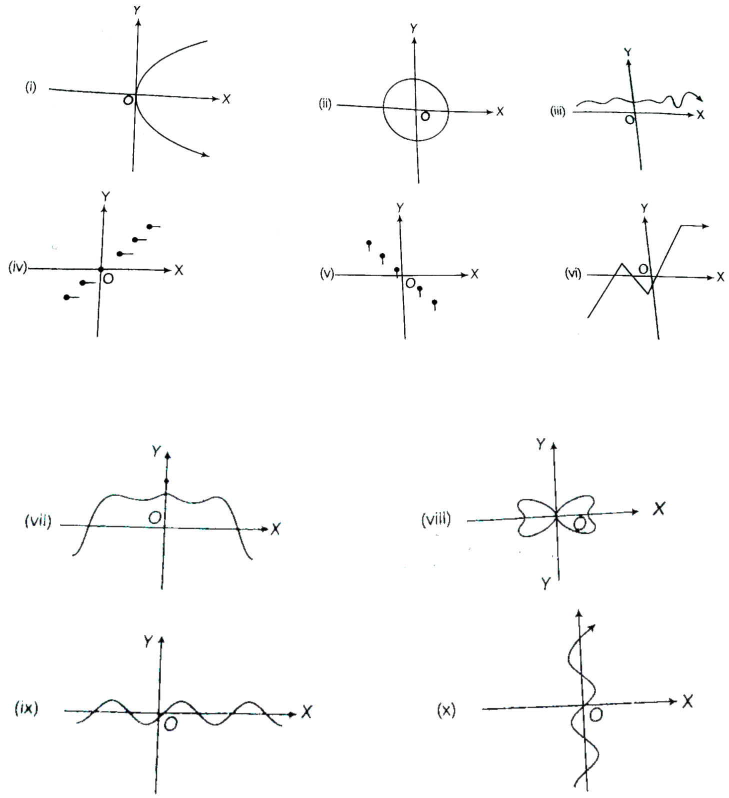
172. If p, q are positive integers, f is a function defined for positive numbers and attains only positive values such that $f(xf(y)) = x^p y^q,$ then prove that $p^2 = q.$

Watch Video Solution

Watch Video Solution

Exercise For Session 1

1. Which of the following graphs are graphs of a function?



Watch Video Solution

2. For which of the following, y can be a function of x, $(x \in R, y \in R)$?

$(i)(x-h)^2+(y-k)^2=r^2$ $(ii)y^2=4ax$

$(iii)x^4=y^2$ $(iv)x^6=y^3$

$(v)3y=(\log x)^2$

Watch Video Solution

3. Let g(x) be a function defined on [-1,1]. If the area of the equilateral triangle with two of its vertices at $(0, 0)$ and $(x, g(x))$ is $\frac{\sqrt{3}}{4}$.then the function g(x) is:

A. $g(x) = \pm\sqrt{1-x^2}$

B. $g(x) = \sqrt{1-x^2}$

C. $g(x) = -\sqrt{1-x^2}$

D. $g(x) = \sqrt{1+x^2}$

Answer: A

Watch Video Solution

4. Represent all possible functions defined from $\{\alpha, \beta\}$ to $\{1, 2\}$.

Watch Video Solution

5. The number of functions from $f: \{a_1, a_2, ..., a_{10}\} \to \{b_1, b_2, ..., b_5\}$ is

A. 10^5

B. 5^{10}

C. $\frac{10!}{5!}$

D. 5!

Answer: B

Watch Video Solution

Exercise For Session 2

1. The domain of the function

$f(x)=\sqrt{x^2-5x+6}+\sqrt{2x+8-x^2}$, is

Watch Video Solution

2. Find domain $f(x)=\sqrt{\frac{2x+1}{x^3-3x^2+2x}}$

Watch Video Solution

3. Find the domain of $f(x)=\sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$

Watch Video Solution

4. The exhaustive domain of $f(x)=\sqrt{x^{12}-x^9+x^4-x+1}$ is

Watch Video Solution

5. The domain of the function $f(x) = {}^{16-x}C_{2x-1} + {}^{20-3x}P_{4x-5}$, where the symbols have their usual meanings, is the set

Watch Video Solution

6. Find the domain? $f(x) = \sqrt{(x^2 + 4x)C_{2x^2+3}}$

Watch Video Solution

Exercise For Session 3

1. The domain of the function

$f(x) = \log_{10}\left(\sqrt{x-4} + \sqrt{6-x}\right)$ is

Watch Video Solution

2. Find domain of $f(x) = \sqrt{\log_{\frac{1}{2}}\left(\frac{5x - x^2}{4}\right)}$

Watch Video Solution

Watch Video Solution

3. $f(x)=\sqrt{\log\left(\frac{3x-x^2}{x-1}\right)}$

Watch Video Solution

4. Find the domain of $\log_{10}\left(1-\log_{10}\left(x^2-5x+16\right)\right)$

Watch Video Solution

5. $f(x)=\sin|x|+\sin^{-1}(\tan x)+\sin\left(\sin^{-1}x\right)$

Watch Video Solution

6. The domain of definition of $f(x)=\sqrt{e^{\cos^{-1}}\left(\log_4 x^2\right)}$ is

Watch Video Solution

7. $f(x) = \sin^{-1}\left(\frac{3-2x}{5}\right) + \sqrt{3-x}$.Find the domain of f(x).

Watch Video Solution

8. Find the domain $f(x) = \frac{\log_{2x} 3}{\cos^{-1}(2x-1)}$

Watch Video Solution

9. Find the domain of $f(x) = (\log)_{10}(\log)_2(\log)_{\frac{2}{\pi}}\left(tan^{-1}x\right)^{-1}$

Watch Video Solution

10. $f(x) = \sqrt{\frac{\log(x-1)}{x^2-2x-8}}$. Find the domain of f(x).

Watch Video Solution

Exercise For Session 4

1. $f(x) = \sqrt{x^2 - |x| - 2}$. Find the domain of f(x).

Watch Video Solution

2. $f(x) = \sqrt{2 - |x|} + \sqrt{1 + |x|}$. Find the domain of f(x).

Watch Video Solution

3. $f(x) = \log_e |\log_e x|$. Find the domain of f(x).

Watch Video Solution

4. $f(x) = \sin^{-1}\left(\frac{2 - 3[x]}{4}\right)$, which $[\cdot]$ denotes the greatest integer function.

Watch Video Solution

5. $f(x) = \log(x - [x])$, where $[\cdot]$ denotes the greatest integer function.

find the domain of f(x).

Watch Video Solution

6. $f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}$, where $[\cdot]$ denotes the greatest integer function.

Watch Video Solution

7. $f(x) = \cos ec^{-1}\left[1 + \sin^2 x\right]$, where $[\cdot]$ denotes the greatest integer function.

Watch Video Solution

8. $f(x) = \cos^{-1}\sqrt{\log_{[x]}\left(\frac{|x|}{x}\right)}$ where [.] denotes the greatest integer function

Watch Video Solution

9. $f(x) = \sqrt{\frac{x-1}{x-2\{x\}}}$, where $\{\cdot\}$ denotes the fractional part.

Watch Video Solution

10. Domain of $f(x) = \sin^{-1}\left(\frac{[x]}{\{x\}}\right)$, where $[\cdot]$ and $\{\cdot\}$ denote greatest integer and fractional parts.

Watch Video Solution

11. $f(x) = \sin^{-1}[2x^2 - 3]$, where $[\cdot]$ denotes the greatest integer function. Find the domain of f(x).

Watch Video Solution

12. $f(x) = \sin^{-1}\left[\log_2\left(\frac{x^2}{2}\right)\right]$ where [.] denotes the greatest integer function.

Watch Video Solution

13. The domain of $f(x) = \sqrt{2\{x\}^2 - 3\{x\} + 1}$ where {.} denotes the fractional part in [-1,1]

Watch Video Solution

14. $f(x) = \frac{1}{[|x-2|] + [|x-10|] - 8}$ where $[\cdot]$ denotes the greatest integer function.

Watch Video Solution

15. If a function is defined as $f(x)=\sqrt{\log_{h(x)} g(x)}$, where $g(x)=|\sin x|+\sin x$, $h(x)=\sin x+\cos x$, $0 \le x \le \pi$. Then find th domain of $f(x)$.

Watch Video Solution

16. The number of solutions of the equation $[y+[y]]=2\cos x$, where $y=\frac{1}{3}[\sin x+[\sin x+[\sin x]]]$ (where [.] denotes the greatest integer function) is

Watch Video Solution

17. Prove that for $n=1,2,3\ldots$ $\left[\frac{n+1}{2}\right]+\left[\frac{n+2}{4}\right]+\left[\frac{n+4}{8}\right]+\left[\frac{n+8}{16}\right]+\ldots=n$ where $[x]$ represents Greatest Integer Function

Watch Video Solution

18. Find the integral solutions to the equation $[x][y] = x + y$. Show that all the non-integral solutions lie on exactly two lines. Determine these lines. Here [.] denotes greatest integer function.

Watch Video Solution

Exercise For Session 5

1. $f(x) = \sqrt{9 - x^2}$. find range of f(x).

Watch Video Solution

2. $f(x) = \frac{x}{1 + x^2}$. Find range of f(x).

Watch Video Solution

3. $f(x) = \sin x + \cos x + 3$. find the range of f(x).

Watch Video Solution

Watch Video Solution

4. $f(x) = |x-1| + |x-2|, \ -1 \leq x \leq 3$. Find the range of f(x).

Watch Video Solution

5. $f(x) = \log_3(5 + 4x - x^2)$. find the range of f(x).

Watch Video Solution

6. $f(x) = \dfrac{x^2 - 2}{x^2 - 3}$. find the range of f(x).

Watch Video Solution

7. $f(x) = \dfrac{x^2 + 2x + 3}{x}$. Find the range of f(x).

Watch Video Solution

8. $f(x) = |x - 1| + |x - 2| + |x - 3|$. Find the range of f(x).

Watch Video Solution

9. Find the domain and range of the following function:

$f(x) = \log_{[x-1]} \sin x,$ where [ ] denotes greatest integer function.

Watch Video Solution

10. $f(x) = \cos^{-1}\sqrt{\log_{[x]}\left(\frac{|x|}{x}\right)}$ where [.] denotes the greatest integer function

Watch Video Solution

11. Let $f(x) = \sqrt{[\sin 2x] - [\cos 2x]}$ (where I I denotes the greatest integer function) then the range of f(x) will be

Watch Video Solution

12. The range of the function $f(x) = \sin^{-1}\left[x^2 + \frac{1}{2}\right] + \cos^{-1}\left[x^2 - \frac{1}{2}\right]$, where [.] denotes the greatest integer function.

Watch Video Solution

13. Range of $f(x) = \sin^{-1}\left(\sqrt{x^2 + x + 1}\right)$ is

Watch Video Solution

14. $f(x) = \cos^{-1}\left(\frac{x^2}{\sqrt{1 + x^2}}\right)$

View Text Solution

15. Find the range of $f(x) = \sqrt{\log(\cos(\sin x))}$

Watch Video Solution

16. $f(x) = \frac{x-1}{x^2-2x+3}$ Find the range of f(x).

Watch Video Solution

17. if: $f(x) = \frac{\sin x}{\sqrt{1+\tan^2 x}} - \frac{\cos x}{\sqrt{1+\cot^2 x}}$, then find the range of $f(x)$

Watch Video Solution

18. Range of $f(x) = \frac{\tan\left(\pi\left[x^2-x\right]\right)}{1+\sin(\cos x)}$

Watch Video Solution

19. $f(x) = \frac{e^x}{[x+1]}, x \geq 0$

Watch Video Solution

20. $f(x) = [|\sin x| + |\cos x|]$, where $[\cdot]$ denotes the greatest integer function.

Watch Video Solution

21. $f(x) = \sqrt{-x^2 + 4x - 3} + \sqrt{\sin\frac{\pi}{2}\left(\sin\frac{\pi}{2}(x-1)\right)}$

Watch Video Solution

22. Find the image of the following sets under the mapping $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 10$ (i) $(-\infty, 1)$

Watch Video Solution

23. Find the domain and range of $f(x) = \log\left[\cos|x| + \frac{1}{2}\right]$,where [.] denotes the greatest integer function.

Watch Video Solution

24. Find the domain and range of $f(x) = \sin^{-1}(\log[x]) + \log\left(\sin^{-1}[x]\right)$,

where [.] denotes the greatest integer function.

Watch Video Solution

25. Find the domain and range of $f(x) = \left[\log\left(\sin^{-1}\sqrt{x^2+3x+2}\right)\right]$.

Watch Video Solution

Exercise For Session 6

1. Determine whether the following functions are even or odd.

$$\left((i) f(x) = \log\left(x+\sqrt{1+x^2}\right), (ii) f(x) = x\left(\frac{a^x+1}{a^x-1}\right)\right), ((iii) f(x) = \text{si}$$

$$\left((v) f(x) = \log\left(\frac{1-x}{1+x}\right), (vi) f(x) = \left\{(sgnx)^{sgnx}\right\}^n, \quad \text{n is an odd integer}\right.$$

$$((vii) f(x) = sgn(x) + x^2,), ((viii) f(x+y) + f(x-y) = 2f(x)\cdot f(y),$$

View Text Solution

2. Determine whether function, $f(x) = (-1)^{[x]}$ is even, odd or neither of two (where $[\cdot]$ denotes the greatest integer function).

Watch Video Solution

3. A function defined for all real numbers is defined for $x \succ 0$ as follows $f(x) = \{x|x|, 0 \leq x \leq 1, 2x, x \geq 1\}$ How if f defined for $x \leq 0$. If (i) f is even ? (ii) f is odd ?

Watch Video Solution

4. Show the function, $f(x) = \frac{2x(\sin x + \tan x)}{2\left[\frac{x+21\pi}{\pi}\right] - 41}$ is symmetric about origin.

Watch Video Solution

5. if $f:[-20, 20] \rightarrow R$ deefined by $f(x) = \left[\frac{x^2}{a}\right] \sin x + \cos x$ is an even fucntion, then set of values of a is

Watch Video Solution

Exercise For Session 7

1. Find the periods of following functions.

$(i) f(x) = [\sin 3x] + |\cos 6x|$ $\quad$ $(ii) f(x) = \frac{1}{2}\left\{\frac{|\sin x|}{\cos x} + \frac{|\cos x|}{\sin x}\right\}$

$(iii) f(x) = e^{\cos^4 \pi x + x - [x] + \cos^2 \pi x}$ $\quad$ $(iv) f(x) = 3 \sin \frac{\pi x}{3} + 4 \cos \frac{\pi x}{4}$

$(v) f(x) = \cos 3x + \sin \sqrt{3} \pi x$ $\quad$ $(vi) f(x) = \sin \frac{\pi x}{n!} - \frac{\cos(\pi x)}{(n+1)!}$

$(vii) f(x) = x - [x - b]$ $\quad$ $(viii) f(x) = e^{In(\sin x)} + \tan^3 x - \cos ec(3x$

Watch Video Solution

2. Find the period of the real-valued function satisfying f(x)+f(x+4)=f(x+2)+f(x+6).

Watch Video Solution

3. Check whether the function defined by $f(x+\lambda)=1+\sqrt{2f(x)-f^2(x)}\ \forall x \in R$ is periodic or not. If yes, then find its period $(\lambda > 0)\cdot$

Watch Video Solution

4. Let f(x) be a real valued periodic function with domain R such that $f(x+p)=1+\left[2-3f(x)+3(f(x))^2-(f(x))^3\right]^{1/3}$ hold good for all $x \in R$ and some positive constant p, then the periodic of f(x) is

Watch Video Solution

5. Let $f(x)$ be a function such that $: f(x-1)+f(x+1)=\sqrt{3}f(x),$ for all $x \varepsilon R.$ If $f(5)=100,$ then prove that the value of $\sum_{r=o}^{99} f(5+12r)$ will be equal to $10000.$

Watch Video Solution

Exercise For Session 8

1. There are exactly two distinct linear functions, which map [-1,1] onto [0,3]. Find the point of intersection of the two functions.

Watch Video Solution

2. Let f be a one-one function with domain $\{x, y, z\}$ and range $\{1, 2, 3\}$. It is given that exactly one of the following statements is true and the remaining two are false $f(X) = 1,\ f(y) \neq 1 f(z) \neq 2$ determine $f^{-1}(1)$

Watch Video Solution

3. Let $A = R - \{3\}, B = R - \{1\}$ and $f: A \rightarrow B$ defined by $f(x) = \dfrac{x-2}{x-3}$. Is 'f' bijective? Give reasons.

Watch Video Solution

4. Let $f: R \rightarrow R$ defined by $f(x) = \frac{x^2}{1+x^2}$. Proved that f is neither injective nor surjective.

Watch Video Solution

5. If the function $f: R\overrightarrow{A}$ given by $f(x) = \frac{x^2}{x^2+1}$ is surjection, then find A.

Watch Video Solution

6. If the function of $f: R \rightarrow A$ is given by $f(x) = \frac{e^x - e^{-|x|}}{e^x + e^{|x|}}$ is surjection, find A

Watch Video Solution

7. Let $f(x) = ax^3 + bx^2 + cx + d\sin x$. Find the condition that $f(x)$ is always one-one function.

Watch Video Solution

8. Let $f: X \rightarrow Y$ be a function defined by f(x) = a sin (x +$\frac{\pi}{4}$) + c. If f is both one-one and onto, then find the set X and Y

Watch Video Solution

Exercise For Session 9

1. $f(x) = In \quad e^x, g(x) = e^{Inx}$. Identical function or not?

Watch Video Solution

2. $f(x) = \sec x, g(x) = \frac{1}{\cos x}$ Identical or not?

Watch Video Solution

3. f(x) and and g(x) are identical or not ?

$f(x) = \sec^{-1}x + \cos ec^{-1}x, \, g(x) = \frac{\pi}{2}$

Watch Video Solution

4. $f(x) = \cot^2 x \cdot \cos^2 x, \, g(x) = \cot^2 x - \cos^2 x$

Watch Video Solution

5. $f(x) = sgn(\cot^{-1} x), \, g(x) = sgn(x^2 - 4x + 5)$

Watch Video Solution

6. $f(x) = \log_e x, \, g(x) = \frac{1}{\log_x e}$. Identical function or not?

Watch Video Solution

7. $f(x) = \sqrt{1-x^2}, g(x) = \sqrt{1-x} \cdot \sqrt{1+x}$. Identical functions or not?

Watch Video Solution

8. $f(x) = \frac{1}{|x|}, g(x) = \sqrt{x^{-2}}$

Watch Video Solution

9. Check for identical`f(x)=[{x}],g(x)={[x]} [Note that f(x) and g(x) are constant functions]

Watch Video Solution

10. $f(x) = e^{\ln \cot}, g(x) = \cot^{-1} x$

Watch Video Solution

Exercise For Session 10

1. Consider the real-valued function satisfying $2f(\sin x) + f(\cos x) = x$. then the domain of $f(x) is R$ domain of $f(x) is [-1, 1]$ range of $f(x)$ is $\left[-\frac{2\pi}{3}, \frac{\pi}{3}\right]$ range of $f(x) is R$

Watch Video Solution

2. If f(x) is defined in [-3,2], find the domain of definition of $f([(|x|])$ and $f([2x+3])$.

Watch Video Solution

3. $f(x) = \begin{cases} x-1, & -1 \leq x0 \\ x^2, & 0 < x \leq 1 \end{cases}$ and g(x)=sinx. Find $h(x) = f(|g(x)|) + |f(g(x))|$.

Watch Video Solution

4. Let f(x) be defined on [-2,2] and is given by

$$f(x) = \begin{cases} -1 & -2 \le x \le 0 \\ x-1 & 0 < x \le 2 \end{cases}$$

and g(x) $= f(|x|) + |f(x)|$. Then g(x) is equal to

Watch Video Solution

5. Two functions are defined as under : $f(x) = \begin{cases} x+1 & x \le 1 \\ 2x+1 & 1 < x \le 2 \end{cases}$

and $g(x) = \begin{cases} x^2 & -1 \le x \le 2 \\ x+2 & 2 \le x \le 3 \end{cases}$ Find fog and gof

Watch Video Solution

Exercise For Session 11

1. Find the inverse of the following function. (i) $f(x) = \sin^{-1}\left(\frac{x}{3}\right), x \in [-3, 3]$ (ii) $f(x) = 5^{\log_e x}, x > 0$ (iii) $f(x) = \log_e\left(x + \sqrt{x^2+1}\right)$

Watch Video Solution

2. If the function $f: [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then find $f^{-1}(x)$.

Watch Video Solution

Exercise For Session 12

1. For $x \in R - \{1\}$, the function f(x) satisfies $f(x) + 2f\left(\frac{1}{1-x}\right) = x$. Find $f(2)$.

Watch Video Solution

2. Let $f(x)$ and $g(x)$be functions which take integers as arguments. Let $f(x+y) = f(x) + g(y) + 8$ for all intege x and y. Let f(x) = x for all negative integers x and let $g(8) = 17$. Find f(0).

Watch Video Solution

3. The function $f: R \to R$ satisfies the condition $mf(x-1) + nf(-x) = 2|x| + 1$. If $f(-2) = 5$ and $f(1) = 1$ find

Watch Video Solution

4. Find the equivalent definition of $f(x) = max x^2, (-x)^{2^{\cdot}}, 2x(1-x) whre 0 \leq x \leq 1$

Watch Video Solution

Exercise (Single Option Correct Type Questions)

1. Let $f_1(x) = \{x, x \leq x \leq 1$ and $1x > 1$ and 0,otherwise $f_2(x) = f_1(-x)$ for all x abd $f_3(x) = -f_2(x)$ for all x and $f_4(x) = -f_3(-x)$ for all x Which of the following is necessarily true?

A. $f_4(x) = f_1(x)$, for all x

B. $f_1(x) = -f_3(-x)$, for all x

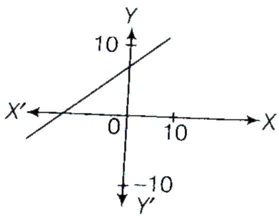
C. $f_2(-x) = f_4(x)$, for all x

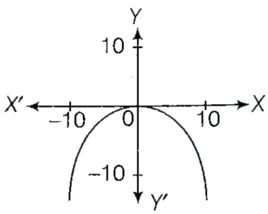
D. $f_1(x) + f_3(x) = 0$, for all x

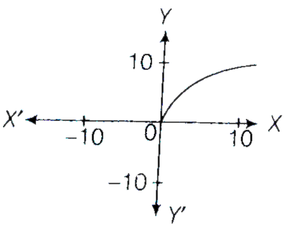
Answer: B

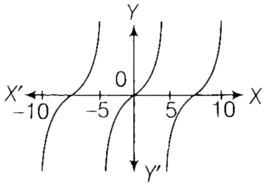
Watch Video Solution

2. Which of the following functions is an odd function?

A. 

B. 

C. 

D. 

Answer: D

Watch Video Solution

3. Given $f(x)=\sqrt{\frac{8}{1-x}+\frac{8}{1+x}}$ and $g(x)=\frac{4}{f(\sin x)}+\frac{4}{f(\cos x)}$ then $g(x)$ is

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. 2π

Answer: A

Watch Video Solution

4. Let f be a function satisfying of x. Then $f(xy)=\frac{f(x)}{y}$ for all positive real numbers $x and y$. If $f(30)=20,$ then find the value of $f(40)$.

A. 15

B. 20

C. 40

D. 60

Answer: A

Watch Video Solution

5. Let $f(x)=e^{e^{(|x|)\sin x}} and g(x)=e^{e^{(|x|)\sin x}}, x \in R,$ where { } and [ ] denote the fractional and integral part functions, respectively. Also, $h(x)=\log(f(x))+\log(g(x))$. Then for real $x, h(x)$ is an odd function an even function neither an odd nor an even function both odd and even function

A. an odd function

B. an even function

C. neither odd nor even function

D. both odd as well as even function

Answer: A

Watch Video Solution

6. Which of the following function is surjective but not injective. (a) $f: R \to R, f(x) = x^4 + 2x^3 - x^2 + 1$ (b) $f: R \to R, f(x) = x^2 + x + 1$ (c) $f: R \to R^+, f(x) = \sqrt{x^2 + 1}$ (d) $f: R \to R, f(x) = x^3 + 2x^2 - x + 1$

A. $f: R \to R, f(x) = x^4 + 2x^3 - x^2 + 1$

B. $f: R \to R, f(x) = x^3 + x + 1$

C. $f: R \to R^+, f(x) = \sqrt{1 + x^2}$

D. $f: R \to R, f(x) = x^3 + 2x^2 - x + 1$

Answer: D

Watch Video Solution

7. If $f(x) = 2x^3 + 7x - 5$ then $f^{-1}(4)$ is :

A. 1

B. 2

C. $1/3$

D. non-existent

Answer: A

Watch Video Solution

8. The range of the function

$f(x) = \dfrac{e^x \cdot \log x \cdot 5^{x^2+2} \cdot \left(x^2 - 7x + 10\right)}{2x^2 - 11x + 12}$ is

A. $(-\infty, \infty)$

B. $[0, \infty)$

C. $\left(\frac{3}{2}, \infty\right)$

D. $\left(\frac{3}{2}, 4\right)$

Answer: A

Watch Video Solution

9. If $x = \cos^{-1}(\cos 4)$ and $y = \sin^{-1}(\sin 3)$, then which of the following holds?

A. x-y=1

B. x+y+1=0

C. x+2y=2

D. $x + y = 3\pi - 7$

Answer: D

Watch Video Solution

10. Let $f(x)=\left(\dfrac{2\sin x+\sin 2x}{2\cos x+\sin 2x}\cdot\dfrac{1-\cos x}{1-\sin x}\right):x\in R.$

Consider the following statements.

I. Domain of f is R.

II. Range of f is R.

III. Domain of f is $R-(4n-1)\dfrac{\pi}{2}, n\in I.$

IV. Domain of f is $R-(4n+1)\dfrac{\pi}{2}, n\in I.$

Which of the following is correct?

A. I and II

B. II and III

C. III and IV

D. II, III and IV

Answer: D

Watch Video Solution

11. If $f(x) = e^{\sin(x - [x]) \cos \pi x}$, where [x] denotes the greatest integer function, then f(x) is

A. non-periodic

B. periodic with no fundamental period

C. periodic with period 2

D. periodic with period π

Answer: C

Watch Video Solution

12. Find the range of the function $f(x) = \cot^{-1}(\log)_{0.5}\left(x^4 - 2x^2 + 3\right)$

A. $(0, \pi)$

B. $\left(0, \frac{3\pi}{4}\right]$

C. $\left[\frac{3\pi}{4}, \pi\right)$

D. $\left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$

Answer: C

Watch Video Solution

13. Range of $f(x) = \left[\frac{1}{\log(x^2+e)}\right] + \frac{1}{\sqrt{1+x^2}}$, where $[\cdot]$ denotes greatest integer function, is

A. $\left(0, \frac{e+1}{e}\right) \cup \{2\}$

B. (0,1)

C. $(0, 1] \cup \{2\}$

D. $(0, 1) \cup \{2\}$

Answer: D

Watch Video Solution

14. The period of the function $f(x) = \sin(x + 3 - [x+3])$where [] denotes the greatest integer function

A. $2\pi + 3$

B. 2π

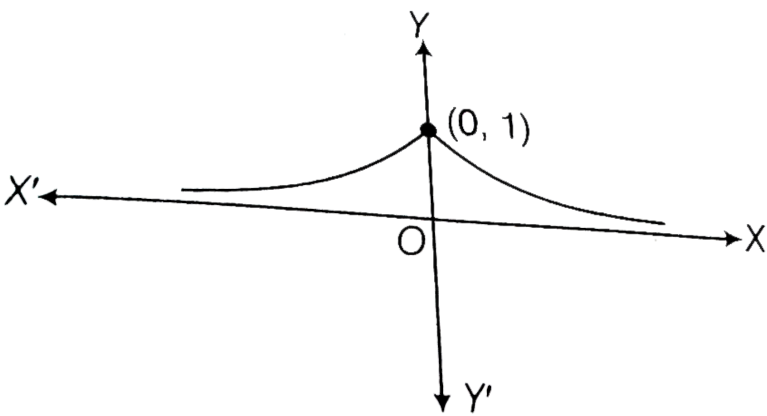
C. 1

D. 4

Answer: C

Watch Video Solution

15. Which one of the following function best represent the graphs as shown below?



A. $f(x) = \frac{1}{1 + x^2}$

B. $f(x) = \frac{1}{\sqrt{1 + |x|}}$

C. $f(x) = e^{-|x|}$

D. $f(x) = a^{|x|}, a > 1$

Answer: C

Watch Video Solution

16. The solution set for [x]{x}=1, where {x} and [x] denote fractional part and greatest integer functions, is

A. $R^{+} - (0, 1)$

B. $R^{+} - \{1\}$

C. $\left\{m + \frac{1}{m} : m \in I - \{0\}\right\}$

D. $\left\{m + \frac{1}{m} : m \in N - \{1\}\right\}$

Answer: D

Watch Video Solution

17. The domain of definition of function

$f(x) = \log\left(\sqrt{x^2 - 5x - 24} - x - 2\right)$, is

A. $(-\infty, -3]$

B. $(-\infty, -3] \cup [8, \infty)$

C. $\left(-\infty, \frac{-28}{9}\right)$

D. None of these

Answer: A

Watch Video Solution

18. If $f(x)$ is a function $f: R \to R$, we say $f(x)$ has property I. If $f(f(x)) = x$ for all real numbers x. II. $f(-f(x)) = -x$ for all real numbers x. How many linear functions, have both property I and II ?

A. 0

B. 2

C. 3

D. Infinite

Answer: B

Watch Video Solution

19. Let $f(x)=\frac{x}{1+x}$ and let $g(x)=\frac{rx}{1-x}$, Let S be the set off all real numbers r such that $f(g(x))=g(f(x))$ for infinitely many real number x. The number of elements in set S is

A. 1

B. 2

C. 3

D. 5

Answer: B

Watch Video Solution

20. Let f(x) be linear functions with the properties that $f(1) \leq f(2), f(3) \geq f(4)$ and $f(5) = 5$. Which one of the following statements is true?

A. $f(0) < 0$

B. f(0)=0

C. $f(1) < f(0) < f(- 1)$

D. f(0)=5

Answer: D

Watch Video Solution

21. Suppose R is relation whose graph is symmetric to both X-axis and Y-axis and that the point (1,2) is on the graph of R. Which one of the following is not necessarily on the graph of R?

A. (-1,2)

B. (1,-2)

C. (-1,-2)

D. (2,1)

Answer: D

Watch Video Solution

22. The area between the curve $2\{y\} = [x] + 1, 0 \leq y < 1,$ where $\{.\}$ and $[.]$ are the fractional part andgreatest integer functions, respectively and the X-axis is

A. $\frac{1}{2}$

B. 1

C. 0

D. $\frac{3}{2}$

Answer: A

Watch Video Solution

23. If f(x)=$\sin^{-1}x$ and g(x)=[sin(cosx)]+[cos(sinx)], then range of f(g(x)) is (where $[\,\cdot\,]$ denotes greatest integer function)

A. $\left\{\frac{-\pi}{2}, \frac{\pi}{2}\right\}$

B. $\left\{\frac{-\pi}{2}, 0\right\}$

C. $\left\{0, \frac{\pi}{2}\right\}$

D. $\left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

Answer: C

Watch Video Solution

24. The number of solutions of the equation $e^{2x}+e^{x}-2=\left[\left\{x^2+10x+11\right)\right]$ is(where, fx) denotes fractional part of x and [x] denotes greatest integer function)

A. 0

B. 1

C. 2

D. 3

Answer: B

Watch Video Solution

25. Total number of values of $x,$ of the form $\frac{1}{n}, n \in N$ in the interval $x \in \left[\frac{1}{25}, \frac{1}{10}\right]$ which satisfy the equation $\{x\} + \{2x) + \ldots\ldots + \{12x\} = 78x$ is $K.$ then K is less than,(where { } represents fractional part function)

A. 12

B. 13

C. 14

D. 15

Answer: B

Watch Video Solution

26. The sum of the maximum and minimum values of the function $f(x) = \frac{1}{1 + (2\cos x - 4\sin x)^2} is$

A. $\frac{22}{21}$

B. $\frac{21}{20}$

C. $\frac{22}{20}$

D. $\frac{21}{11}$

Answer: A

Watch Video Solution

27. Let $f : X \to Y$ be an invertible function. Show that the inverse of f^{-1} is f, i.e., $\left(f^{-1}\right)^{-1} = f$.

A. $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

B. $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

C. $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right] \to \left[\sqrt{2}, -3\sqrt{2}\right]$

D. $\left[-\frac{3\pi}{4}, -\frac{\pi}{4}\right] \to \left[\sqrt{2}, 3\sqrt{2}\right]$

Answer: A

Watch Video Solution

28. The range of values of a so that all the roots of the equations $2x^3 - 3x^2 - 12x + a = 0$ are real and distinct, belongs to

A. (7,20)

B. (-7,20)

C. (-20,7)

D. (-7,7)

Answer: B

Watch Video Solution

29. If f(x) is continuous such that $|f(x)| \leq 1, \forall x \in R$ and $g(x) = \dfrac{e^{f(x)} - e^{-|f(x)|}}{e^{f(x)} + e^{-|f(x)|}}$, then range of g(x) is

A. [0,1]

B. $\left[0, \dfrac{e^2+1}{e^2-1}\right]$

C. $\left[0, \dfrac{e^2-1}{e^2+1}\right]$

D. $\left[\dfrac{1-e^2}{1+e^2}, 0\right]$

Answer: C

Watch Video Solution

30. Let $f(x) = \sqrt{|x| - \{x\}}$, where $\{.\}$ denotes the fractional part of x an X,Y and its domain and range respectively, then

A. $f: X \to Y : y = f(x)$ is one-one function

B. $X \in \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$ and $Y \in \left[\frac{1}{2}, \infty\right)$

C. $X \in \left(-\infty, -\frac{1}{2}\right] \cup [0, \infty)$ and $Y \in [0, \infty)$

D. None of the above

Answer: C

Watch Video Solution

31. If the graphs of the functions$y = \log_e x$ and $y = ax$ intersect at exactly two points,then find the value of a.

A. (0,e)

B. $\left(\frac{1}{e}, 0\right)$

C. $\left(0, \frac{1}{e}\right)$

D. None of these

Answer: C

Watch Video Solution

32. A quadratic polynominal maps from [-2,3] onto [0,3] and touches X-axis at x=3, then the polynominal is

A. $\frac{3}{16}\left(x^2 - 6x + 16\right)$

B. $\frac{3}{25}\left(x^2 - 6x + 9\right)$

C. $\frac{3}{25}\left(x^2 - 6x + 16\right)$

D. $\frac{3}{16}\left(x^2 - 6x + 9\right)$

Answer: B

View Text Solution

33. The range of the function $y = \sqrt{2\{x\} - \{x\}^2 - \frac{3}{4}}$ (where, denotes the fractional part) is

A. $\left[-\frac{1}{4}, \frac{1}{4}\right]$

B. $\left[0, \frac{1}{2}\right)$

C. $\left[0, \frac{1}{4}\right]$

D. $\left[\frac{1}{4}, \frac{1}{2}\right]$

Answer: C

Watch Video Solution

34. Let $f(x)$ be a fourth differentiable function such $f(2x^2 - 1) = 2x f(x) \forall x \in R,$ then $f^{iv}(0)$ is equal

A. 0

B. 1

C. -1

D. Data insufficient]

Answer: A

Watch Video Solution

35. The number of solutions of the equation $[y+[y]] = 2\cos x$, where $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$ (where [.] denotes the greatest integer function) is

A. 1

B. 2

C. 3

D. None of these

Answer: D

Watch Video Solution

36. If a function satisfies $f(x+1)+f(x-1)=\sqrt{2}f(x)$, then period of f(x) can be

A. 2

B. 4

C. 6

D. 8

Answer: D

Watch Video Solution

37. If x and α are real, then the inequation

$\log_2 x+\log_x 2+2\cos\alpha \le 0$

A. has no solution

B. has exactly two solutions

C. is satisfied for any real α and any real x in (0,1)

D. None of these

Answer: D

Watch Video Solution

38. The range of values of 'a' such that $\left(\frac{1}{2}\right)^{|x|} = x^2 - a$ is satisfied for maximum number of values of 'x'

A. $(-\infty, -1)$

B. $(-\infty, \infty)$

C. (-1,1)

D. $(-1, \infty)$

Answer: D

Watch Video Solution

39. Let $f: R \to R$ be a function defined by $f(x) = \{|\cos x|\}$, where {x} represents fractional part of x. Let S be the set containing all real values x lying in the interval $[0, 2\pi]$ for which $f(x) \neq |\cos x|$. The number of elements in the set S is

A. 0

B. 1

C. 3

D. infinite

Answer: C

Watch Video Solution

40. The domain of the function

$f(x) = \sqrt{\log_{\sin x + \cos x}(|\cos x| + \cos x)}, 0 \leq x \leq \pi$ is

A. $(0, \pi)$

B. $\left(0, \frac{\pi}{2}\right)$

C. $\left(0, \frac{\pi}{3}\right)$

D. None of these

Answer: D

View Text Solution

41. If $f(x) = \left(x^2 + 2\alpha x + \alpha^2 - 1\right)^{1/4}$ has its domain and range such that their union is set of real numbers, then α satisfies

A. $-1 < \alpha < 1$

B. $\alpha \leq -1$

C. $\alpha \geq 1$

D. $\alpha \leq 1$

Answer: B

Watch Video Solution

42. If $f:(e,\infty)\to R \& f(x)=\log[\log(\log x)],$ then f is -

A. f is one-one and onto

B. f is one-one but onto

C. f is onto but not one-one

D. the range of f is equal to its domain

Answer: A

Watch Video Solution

43. The expression $x^2-4px+q^2>0$ for all real x and also $r^2+p^2<qr$ the range of $f(x)=\dfrac{x+r}{x^2+qx+p^2}$ is

A. $\left[\dfrac{p}{2r},\dfrac{q}{2r}\right]$

B. $(0,\infty)$

C. $(-\infty,0)$

D. $(-\infty, \infty)$

Answer: D

Watch Video Solution

44. Let $f(x) = \frac{x^4 - \lambda x^3 - 3x^2 + 3\lambda x}{x - \lambda}$. If range of f(x) is the set of entire real numbers, the true set in which λ lies is

A. [-2,2]

B. [0,4]

C. (1,3)

D. None of these

Answer: A

Watch Video Solution

45. Let $a = 3^{1/224} + 1$ and for all $n \geq 3$,

let

$f(n) = {}^nC_0 a^{n-1} - {}^nC_1 a^{n-2} + {}^nC_2 a^{n-3} + ... + (-1)^{n-1} \cdot {}^nC_{n-1} \cdot a^0$.

If the value of f(2016)+f(2017)=3^k, the value of K is

A. 6

B. 8

C. 9

D. 10

Answer: C

View Text Solution

46. The area bounded by $f(x) = \sin^{-1}(\sin x)$ and

$g(x) = \frac{\pi}{2} - \sqrt{\frac{\pi^2}{2} - \left(x - \frac{\pi}{2}\right)^2}$ is

A. $\frac{\pi^3}{8}$ sq units

B. $\frac{\pi^2}{8}$ sq units

C. $\frac{\pi^3}{2}$ sq units

D. $\frac{\pi^2}{2}$ sq units

Answer: A

Watch Video Solution

47. If $f. R \to R, f(x) = \frac{x^2 + bx + 1}{x^2 + 2x + b}, (b > 1)$ and $f(x), \frac{1}{f(x)}$ have the same bounded set as their range, the value of b is

A. $2\sqrt{3} - 2$

B. $2\sqrt{3} + 2$

C. $2\sqrt{2} - 2$

D. $2\sqrt{2} + 2$

Answer: A

View Text Solution

48. The period of $\sin\frac{\pi[x]}{12}+\cos\frac{\pi[x]}{4}+\tan\frac{\pi[x]}{3}$, where [x] represents the greatest integer less than or equal to x is

A. 12

B. 4

C. 3

D. 24

Answer: D

Watch Video Solution

49. If f(2x+3y,2x-7y)=20x, then f(x,y) equals

A. x-y

B. 7x+3y

C. 3x-7y

D. None of these

Answer: B

Watch Video Solution

50. The range of the function $f(x) = \sqrt{x-1} + 2\sqrt{3-x}$ is

A. $[\sqrt{2}, 2\sqrt{2}]$

B. $[\sqrt{2}, \sqrt{10}]$

C. $[2\sqrt{2}, \sqrt{10}]$

D. [1,3]

Answer: B

Watch Video Solution

51. The domain of the function

$f(x) = \cos^{-1}(\sec(\cos^{-1} x)) + \sin^{-1}(\cos ec(\sin^{-1} x))$ is

A. $x \in R$

B. x=1,-1

C. $-1 \leq x \leq 1$

D. $x \in \phi$

Answer: B

Watch Video Solution

52. Let f(x) be a polynominal one-one function such that

$f(x)f(y) + 2 = f(x) + f(y) + f(xy), \forall x, y \in R - \{0\}, f(1) \neq 1, f'(1) =$

Let $g(x) = \frac{x}{4}(f(x) + 3) - \int_0^x f(x)dx$, then

A. g(x)=0 has exactly one root for $x \in (0, 1)$

B. g(x)=0 has exactly two roots for $x \in (0, 1)$

C. $g(x) \neq 0, x \in R - \{0\}$

D. $g(x) = 0, x \in R - \{0\}$

Answer: D

Watch Video Solution

53. Let f(x) be a polynominal with real coefficients such that $f(x) = f'(x) \times f'''(x)$. If f(x)=0 is satisfied x=1,2,3 only, then the value of f'(1)f'(2)f'(3) is

A. positive

B. negative

C. 0

D. Inadequate data

Answer: C

Watch Video Solution

54. Let A={1,2,3,4,5} and $f: A \rightarrow A$ be an into function such that $f(i) \neq$, $\forall i \in A$, then number of such functions f are

A. 1024

B. 904

C. 980

D. None of these

Answer: C

View Text Solution

55. If functions $f: \{1, 2, \ldots, n\} \rightarrow \{1995, 1996\}$ satisfying f(1)+f(2)+... +f(1996)=odd integer are formed, the number of such functions can be

A. 2^n

B. $2^{n/2}$

C. n^2

D. 2^{n-1}

Answer: D

Watch Video Solution

56. The range of $y = \sin^3 x - 6\sin^2 x + 11\sin x - 6$ is

A. [-24,2]

B. [-24,0]

C. [0,24]

D. None of these

Answer: B

Watch Video Solution

57. Let $f(x) = x^2 - 2x$ and g(x)=f(f(x)-1)+f(5-f(x)), then

A. $g(x) < 0, \ \forall x \in R$

B. $g(x) < 0 \ \text{ for some } \ x \in R$

C. $g(x) \geq 0 \ \text{ for some } \ x \in R$

D. $g(x) \geq 0, \ \forall x \in R$

Answer: D

Watch Video Solution

58. If f(x) and g(x) are non-periodic functions, then h(x)=f(g(x)) is

A. non-periodic

B. periodic

C. may be periodic

D. always periodic, if domain of h(x) is a proper subset of real numbers

Answer: C

Watch Video Solution

59. If f(x) is a real-valued function discontinuous at all integral points lying in [0,n] and if $(f(x))^2 = 1, \forall x \in [0, n]$, then number of functions f(x) are

A. 2^{n+1}

B. 6×3^n

C. $2 \times 3^{n-1}$

D. 3^{n+1}

Answer: C

Watch Video Solution

60. A function f from integers to integers is defined as $f(x) = \{n+3, n \in odd\frac{n}{2}, n \in even$ suppose $k \in$ odd and $f(f(f(k))) = 27$. Then the sum of digits of k is________

A. 3

B. 6

C. 9

D. 12

Answer: B

Watch Video Solution

61. If $f: R \to R$ and $f(x) = \frac{\sin(\pi\{x\})}{x^4 + 3x^2 + 7}$, where $\{\}$ is a fractional part of x , then

A. f is injective

B. f is not one-one and non-constant

C. f is a surjective

D. f is a zero function

Answer: B

Watch Video Solution

62. Let $f: R \rightarrow R$ and $g: R \rightarrow R$ be two one-one and onto functions such that they are mirror images of each other about the line $y = a$. If $h(x) = f(x) + g(x)$, then $h(x)$ is (A) one-one onto (B) one-one into (D) many-one into (C) many-one onto

A. one -one and onto

B. only one-one and not onto

C. only onto but not one-one

D. None of the above

Answer: D

Watch Video Solution

Watch Video Solution

63. The domain of the funciton $f(x)$ given by $3^x + 3^f = \min\left(2t^3 - 15t^2 + 36 + \; - 25, 2 + |\sin t|, 2 \le t \le 4\right)$ is

A. $(\,-\infty, 1)$

B. $(\,-\infty, \log_3 e)$

C. $(0, \log_3 2)$

D. $(\,-\infty, \log_3 2)$

Answer: D

Watch Video Solution

64. Let x be the elements of the set A= {1,2,3,4,5,6,8,10,12,15,20,24,30,40,60,120} and x_1, x_2, x_3 be positive integers and d be the number of integral solutions of $x_1, x_2, x_3 = x$, then d is

A. 100

B. 150

C. 320

D. 250

Answer: C

View Text Solution

65. If A $>$ 0, c,d,u.v are non-zero constants and the graph of $f(x) = |Ax + c| + d$ and $g(x) = -|Ax + u| + v$ intersect exactly at two points (1,4) and (3,1), then the value of $\frac{u+c}{A}$ equals

A. 4

B. -4

C. 2

D. -2

Answer: B

View Text Solution

66. If $f(x)=x^3+3x^2+4x+a\sin x+b\cos x, \forall x \in R$ is a one-one fuction, then the greatestn value of (a^2+b^2) is

A. 1

B. 2

C. $\sqrt{2}$

D. None of these

Answer: A

Watch Video Solution

67. If two roots of the equation $(p-1)(x^2+x+1)^2-(p+1)(x^4+x^2+1)=0$ are real and distinct and $f(x)=\frac{1-x}{1+x}$ then $f(f(x))+f\left(f\left(\frac{1}{x}\right)\right)$ is equal to

A. p

B. $-p$

C. 2p

D. $-2p$

Answer: A

Watch Video Solution

68. Let $f(x)=x^{13}+2x^{12}+3x^{11}+\ldots+13x+14 \text{ and } \alpha=\cos\frac{2\pi}{15}+i\sin\frac{2\pi}{15}$. If $N=f(\alpha)f(\alpha^2)\ldots f(\alpha^{14})$, then

A. number of division of N is 144

B. number of divisors of N is 196

C. number of divisors of N which are perfect squares of 49

D. number of divisors of N which are perfect square of 12

Answer: B

View Text Solution

69. Let $f(x) = \sin^{-1} 2x + \cos^{-1} 2x + \sec^{-1} 2x$. Then the sum of the maximum and minimum values of f(x) is

A. π

B. $\frac{\pi}{2}$

C. 2π

D. $\frac{3\pi}{2}$

Answer: C

Watch Video Solution

70. The complete set of values of a for which the function $f(x) = \tan^{-1}(x^2 - 18x + a) > 0 \forall x \in R$ is

A. $(81, \infty)$

B. $[81, \infty)$

C. $(-\infty, 81)$

D. (-infty,81]`

Answer: A

Watch Video Solution

71. The domain of the function

$$f(x) = \sin^{-1}\frac{1}{|x^2 - 1|} + \frac{1}{\sqrt{\sin^2 x + \sin x + 1}}$$ is

A. $(-\infty, \infty)$

B. $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right)$

C. $\left(-\infty, -\sqrt{2}\right] \cup \left[\sqrt{2}, \infty\right) \cup \{0\}$

D. None of the above

Answer: C

Watch Video Solution

72. The domain of $f(x)=\dfrac{\log\left(\sin^{-1}\sqrt{x^2+x+1}\right)}{\log(x^2-x+1)}$ is

A. (-1,1)

B. $(-1, 0)\cup(0, 1)$

C. $(-1, 0)\cup\{1\}$

D. None of these

Answer: D

Watch Video Solution

73. The domain of $f(x)=\sqrt{\sin^{-1}(3x-4x^3)}+\sqrt{\cos^{-1}x}$ is equal to

A. $\left[-1, -\dfrac{\sqrt{3}}{2}\right]\cup\left[0, \dfrac{\sqrt{3}}{2}\right]$

B. $\left[-1, -\dfrac{1}{2}\right]\cup\left[0, \dfrac{1}{2}\right]$

C. $\left[0, \frac{1}{2}\right]$

D. None of these

Answer: A

Watch Video Solution

74. The domain of the function

$f(x) = \sqrt[6]{4^x + 8^{2/3(x-2)} - 52 - 2^{2(x-1)}}$ is

A. (0,1)

B. $[3, \infty]$

C. [1,0)

D. None of these

Answer: B

Watch Video Solution

75. The domain of derivative of the function

$f(x) = \left|\sin^{-1}(2x^2 - 1)\right|$, is

A. (-1,1)

B. $(-1, 1) \sim \left\{0, \pm \frac{1}{\sqrt{2}}\right\}$

C. $(-1, 1) \sim \{0\}$

D. $(-1, 1) \sim \left\{\pm \frac{1}{\sqrt{2}}\right\}$

Answer: B

Watch Video Solution

76. The range of a function

$f(x) = \tan^{-1}\left\{\log_{5/4}(5x^2 - 8x + 4)\right\}$ is

A. $\left(\frac{-\pi}{4}, \frac{\pi}{2}\right)$

B. $\left[\frac{-\pi}{4}, \frac{\pi}{2}\right)$

C. $\left(\frac{-\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[\frac{-\pi}{4}, \frac{\pi}{2}\right]$

Answer: B

Watch Video Solution

Exercise (More Than One Correct Option Type Questions)

1. Which of the following fuunction(s) is/are transcendal?

A. $f(x) = 5\sin(\sqrt{x})$

B. $f(x) = \frac{2\sin 3x}{x^2 + 2x - 1}$

C. $f(x) = \sqrt{x^2 + 2x + 1}$

D. $f(x) = (x^2 + 3) \cdot 2^x$

Answer: A::B

Watch Video Solution

2. Let $f(x) = \frac{\sqrt{x - 2\sqrt{x - 1}}}{\sqrt{x - 1} - 1}. x$ then

A. domain of f(x) is $x \geq 1$

B. domain of f(x) is $[1, \infty) - \{2\}$

C. f'(10)=1

D. $f'\left(\frac{3}{2}\right) = -1$

Answer: B::C::D

Watch Video Solution

3. $f(x) = \cos^2 x + \cos^2\left(\frac{\pi}{3} + x\right) - \cos x \cdot \cos\left(x + \frac{\pi}{3}\right)$ is

A. an odd function

B. an even function

C. a periodic function

D. f(0)=f(1)

Answer: B::C::D

Watch Video Solution

4. If the following functions are defined from $[-1, 1] \rightarrow [-1, 1]$, select those which are not objective. $\sin(s \in^{-1} x)$ (b) $\frac{2}{\pi}\sin^{-1}(\sin x)$ $(sgn(x))1N(e^x)$ (d) $x^3(sgn(x))$

A. $\sin(\sin^{-1} x)$

B. $\frac{2}{\pi} \cdot \sin^{-1}(\sin x)$

C. $sgn(x) \cdot \log(e^x)$

D. $x^3 sgn(x)$

Answer: B::C::D

Watch Video Solution

5. Let $f(x) = \begin{cases} x^2 - 4x + 3, & x < 3 \\ x - 4, & x \geq 3 \end{cases}$

and $g(x) = \begin{cases} x - 3, & x < 4 \\ x^2 + 2x + 2, & x \leq 4 \end{cases}$, which one of the following is/are true?

A. (f+g)(3.5)=0

B. f(gh(3))=3

C. f(g(2))=1

D. (f-g)(4)=0

Answer: A::B

Watch Video Solution

6. If $f(x) = x^2 - 2ax + a(a+1)$, $f: [a, \infty) \rightarrow [a, \infty)$. If one of the solutions of the equation $f(x) = f^{-1}(x)$ is 5049, the other may be

A. 5051

B. 5048

C. 5052

D. 5050

Answer: B::D

Watch Video Solution

7. The function 'g' defined by $g(x) = \sin\left(\sin^{-1}\sqrt{\{x\}}\right) + \cos\left(\sin^{-1}\sqrt{\{x\}}\right) - 1$ (where {x} denotes the functional part function) is (1) an even function (2) a periodic function (3) an odd function (4) neither even nor odd

A. an even function

B. periodic function

C. odd function

D. neither even or odd

Answer: A::B

Watch Video Solution

Watch Video Solution

8. The graph of $f: R \rightarrow R$ defined by y=f(x) is symmetric with respect to x=a and x=b. Which of the following is true ?

A. f(2a-x)=f(x)

B. f(2a+x)=f(-x)

C. f(2b+x)=f(-x)

D. f is periodic

Answer: A,B,C,D

Watch Video Solution

9. Let f be the continuous and differentiable function such that f(x)=f(2-x), $\forall x \in R$ and g(x)=f(1+x), then

A. g(x) is an odd function

B. f(x) is an even function

C. f(x) is symmetric about x=1

D. None of the above

Answer: B::C

View Text Solution

10. Let $f(x) = |x-1| + |x-2| + |x-3| + |x-4|,$ then

A. least value of f(x) is 4

B. least value is not attained at unique point

C. the number of integral solution of f(x)=4 is 2

D. the value of $\dfrac{f(\pi-1)+f(e)}{2f\left(\frac{12}{5}\right)}$ is 1

Answer: A::B::C::D

Watch Video Solution

11. Let A={1,2,3,4,5}, B={1,2,3,4) and $f: A \to B$ is a function, the

A. number of onto functions, if n(f(A))=4 is 240

B. number of onto functions, if n(f(A))=3 is 600

C. number of onto functions, if n(f(A))=2 is 180

D. number of onto functions, if n(f(A))=1 is 4

Answer: A::B::C::D

View Text Solution

12. In a function $2f(x) + xf\left(\frac{1}{x}\right) - 2f\left(\left|\sqrt{2}\sin\left(\pi\left(x + \frac{1}{4}\right)\right)\right|\right) = 4\cos^2\left[\frac{\pi x}{2}\right] + x\cos\left(\frac{\pi}{x}\right.$

. Prove that: 1. f(2)+f(1/2)=1 2. f(2)+f(1)=0

A. $f(2) + f\left(\frac{1}{2}\right) = 1$

B. f(2)+f(1)=0

C. $f(2) + f(1) = f\left(\frac{1}{2}\right)$

D. $f(1)\cdot f\left(\frac{1}{2}\right)\cdot f(2)=1$

Answer: A::B::C

Watch Video Solution

13. If f(x) is a differntiable function satisfying the condition f(100x)=x+f(100x-100), $\forall x\in R$ and f(100)=1, then $f(10^4)$ is

A. 5049

B. $\sum_{r=1}^{100} r$

C. $\sum_{r=2}^{100} r$

D. 5050

Answer: B::D

Watch Video Solution

14. If [x] denotes the greatest integer function then the extreme values of the function

$$f(x) = [1 + \sin x] + [1 + \sin 2x] + \ldots + [1 + \sin nx], n \in I^+, x \in (0, \pi)$$

are

A. (n-1)

B. n

C. (n+1)

D. (n+2)

Answer: B::C

View Text Solution

15. Which of the following is/are periodic?

A. $f(x) = \begin{cases} 1, & \text{if x is rational} \\ 0, & \text{if x is irrational} \end{cases}$

B. $f(x) = \begin{cases} x - [x], & 2n \le x < 2n+1 \\ \frac{1}{2}, & 2n+1 \le x < 2n+2 \end{cases}$ where $[\cdot]$ denotes the greatest integer function

C. $f(x) = (-1)^{\left[\frac{2x}{\pi}\right]}$, where $[\cdot]$ denotes the greatest integer function

D. $f(x) = ax - [ax + a] + \tan\left(\frac{\pi x}{2}\right)$, where $[\cdot]$ denotes the greatest integer function

Answer: B::C::D

Watch Video Solution

16. If $f(x)$ is a polynomial of degree n such that $f(0) = 0, f(x) = \frac{1}{2}, \ldots, f(n) = \frac{n}{n+1}$, ,then the value of $f(n+1)$ is

A. 1, when n is even

B. $\frac{n}{n+2}$, when n is odd

C. 1, when n is odd

D. $\frac{n}{n+2}$, when n is even

Answer: C::D

Watch Video Solution

17. Let $f: R \rightarrow R$ be a function defined by $f(x+1) = \frac{f(x)-5}{f(x)-3}, \forall x \in R.$ Then, which of the following statements is/are true?

A. f(2008)=f(2004)

B. f(2006)=f(2010)

C. f(2006)=f(2002)

D. f(2006)=f(2018)

Answer: A::B::C::D

View Text Solution

18. Let $f(x) = 1 - x - x^3$.Find all real values of x satisfying the inequality, $1 - f(x) - f^3(x) > f(1 - 5x)$

A. (-2,0)

B. (0,2)

C. $(2, \infty)$

D. $(- i\infty, - 2)$

Answer: A::C

Watch Video Solution

19. If a function satisfies $(x - y)f(x + y) - (x + y)f(x - y) = 2(x^2y - y^3)\ \forall x, y \in R \text{ and } f(1) =$, then

A. f(x) must be polynominal function

B. f(3)=12

C. f(0)=0

D. f(x) may not be differentiable

Answer: A::B::C

Watch Video Solution

20. If the fundamental period of function $f(x) = \sin x + \cos\left(\sqrt{4-a^2}\right)x$ is 4π, then the value of a is/are

A. $\frac{\sqrt{15}}{2}$

B. $-\frac{\sqrt{15}}{2}$

C. $\frac{\sqrt{7}}{2}$

D. $-\frac{\sqrt{7}}{2}$

Answer: A::B::C::D

Watch Video Solution

21. Let f(x) be a real valued function such that $f(0)=\frac{1}{2}$ and f(x+y)=f(x)f(a-y)+f(y)f(a-x), $\forall x, y \in R$, then for some real a,

A. f(x) is a periodic function

B. f(x) is a constant function

C. $f(x)=\frac{1}{2}$

D. $f(x)=\frac{\cos x}{2}$

Answer: A::B::C

Watch Video Solution

22. if $f(g(x))$ is one-one function, then (1) g(x) must be one- one (2) f(x) must be one - one (3) f(x) may not be one-one (4) g(x) may not be one-one

A. g(x) must be one-one

B. f(x) must be one-one

C. f(x) may not be one-one

D. g(x) may not be one-one

Answer: A::C

Watch Video Solution

23. Which of the following functions have their range equal to R(the set of real numbers)?

A. xsinx

B. $\frac{x}{\tan 2x} \cdot x \in \left(-\frac{\pi}{4} \cdot \frac{\pi}{4}\right) - \{0\}$, where $[\,\cdot\,]$ denotes the greatest integer function

C. $\frac{x}{\sin x}$

D. $[x] + \sqrt{\{x\}}$, where $\{\,\cdot\,\}$, respectively denote the greatest integer and fractional part functions

Answer: A::D

Watch Video Solution

24. Which of the following pairs of function are identical?

A. $f(x) = e^{In \sec^{-1} x}$ and $g(x) = \sec^{-1} x$

B. $f(x) = \tan(\tan^{-1} x)$ and $g(x) = \cot(\cot^{-1} x)$

C. f(x)=sgn(x) and g(x)=sgn(sgn(x))

D. $f(x) = \cot^2 \cdot \cos^2 x$ and $g(x) = \cot^2 x - \cos^2 x$

Answer: B::C::D

Watch Video Solution

25. Let $f: R \to R$ defined by $f(x) = \cos^{-1}(-\{-x\})$, where {x} denotes fractional part of x. Then, which of the following is/are correct?

A. f is many one but not even function

B. Range of f contains two prime numbers

C. f is non-periodic

D. Graphs of f does not lie below X-axis

Answer: B::D

Watch Video Solution

Exercise (Statement I And Ii Type Questions)

1. $\textbf{Statement I}$ The function f(x) =xsinx and f'(x)=xcosx+sinx are both non-periodic.

$\textbf{Statement II}$ The derivative of differentiable functions (non-periodic) is non-periodic funciton.

Watch Video Solution

2. $\textbf{Statement I}$ The maximum value of $\sin\sqrt{2}x + \sin ax$ cannot be 2 (where a is positive rational number).

$\textbf{Statement II}$ $\frac{\sqrt{2}}{a}$ is irrartional.

Watch Video Solution

3. Let $f: R \to R$ be a function defined by, $f(x)$=$\frac{e^{|x|} - e^{-x}}{e^{x} + e^{-x}}$ then

Watch Video Solution

4. $\textbf{Statement I}$ The range of

$$f(x) = \sin\left(\frac{\pi}{5} + x\right) - \sin\left(\frac{\pi}{5} - x\right) - \sin\left(\frac{2\pi}{5} + x\right) + \sin\left(\frac{2\pi}{5} - x\right)$$

is [-1,1].

$\textbf{Statement II}$ $\cos\frac{\pi}{5} - \cos\frac{2\pi}{5} = \frac{1}{2}$

Watch Video Solution

5. $\textbf{Statement I}$ The period of

$f(x) = 2\cos\frac{1}{3}(x - \pi) + 4\sin\frac{1}{3}(x - \pi)$ is 3π.

$\textbf{Statement II}$ If T is the period of f(x), then the period of f(ax+b) is $\frac{T}{|a|}$.

Watch Video Solution

6. f is a function defined on the interval [-1,1] such that f(sin2x)=sinx+cosx.

Statement I If $x \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, then $f\left(\tan^2 x\right) = \sec x$

Statement II $f(x) = \sqrt{1+x}, \forall x \in [-1, 1]$

Watch Video Solution

7. **Statement I** The equation $f(x) = 4x^5 + 20x - 9 = 0$ has only one real root.

Statement II $f'(x) = 20x^4 + 20 = 0$ has no real root.

Watch Video Solution

8. **Statement I** The range of $\log\left(\frac{1}{1+x^2}\right)$ is $(-\infty, \infty)$.

Statement II when $0 < x \leq 1, \log x \in (-\infty, 0]$.

Watch Video Solution

9. Let $f: X \to Y$ be a function defined by

$f(x) = 2\sin\left(x + \frac{\pi}{4}\right) - \sqrt{2}\cos x + c.$

Statement For set X, $x \in \left[0, \frac{\pi}{2}\right] \cup \left[\pi, \frac{3\pi}{2}\right]$, f(x) is one-one function.

Statement II $f'(x) \geq 0$, $x \in \left[0, \frac{\pi}{2}\right]$

View Text Solution

10. Let f(x)=sin x

Statement I f is not a polynominal function.

Statement II nth derivative of f(x), w.r.t. x, is not a zero function for any positive integer n.

Watch Video Solution

11. The inverse of the function of $f: R \to R$ given by $f(x) = \log_a\left(x + \sqrt{x^2 + 1}(a > 0, a \neq 1)\right.$ is

Watch Video Solution

Exercise (Passage Based Questions)

1. Let $f: R \to R$ be a continuous function such that

$$f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2.$$

f(3) is equal to

A. f(0)

B. 4+f(0)

C. 9+f(0)

D. 16+f(0)

Answer: d

View Text Solution

2. Let $f: R \to R$ be a continuous function such that

$$f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2.$$

The equation f(x)-x-f(0)=0 have exactly

A. no solution

B. one solution

C. two solution

D. infinite solution

Answer: c

View Text Solution

3. Let $f: R \to R$ be a continuous function such that

$$f(x) - 2f\left(\frac{x}{2}\right) + f\left(\frac{x}{4}\right) = x^2.$$

f'(0) is equal to

A. 0

B. 1

C. f(0)

D. $-f(0)$

Answer: a

View Text Solution

4. Consider the equation x+y-[x][y]=0, where $[\cdot]$ is the greatest integer function.

The number of integral solutions to the equation is

A. 0

B. 1

C. 2

D. None of these

Answer: c

Watch Video Solution

5. Consider the equation x+y-[x][y]=0, where $[\cdot]$ is the greatest integer function.

Equation of one of the lines on which the non-integral solution of given equation lies, is

A. x+y=-1

B. x+y=0

C. x+y=1

D. x+y=5

Answer: b

Watch Video Solution

6. Let $f(x)=\frac{1}{2}\left[f(xy)+f\left(\frac{x}{y}\right)\right]$ for $x, y \in R^{+}$ such that f(1)=0,f'(1)=2.`

f(x)-f(y) is equal to

A. $f\left(\frac{y}{x}\right)$

B. $f\left(\frac{x}{y}\right)$

C. f(2x)

D. f(2y)

Answer: b

Watch Video Solution

7. Let $f(x)=\frac{1}{2}\left[f(xy)+f\left(\frac{x}{y}\right)\right]$ for $x, y \in R^{+}$ such that f(1)=0,f'(1)=2.`

f'(3) is equal to

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: b

Watch Video Solution

8. Let $f(x)=\frac{1}{2}\left[f(xy)+f\left(\frac{x}{y}\right)\right]$ for $x, y \in R^{+}$ such that f(1)=0,f'(1)=2.`

f(e) is equal to

A. 2

B. 1

C. 3

D. 4

Answer: a

Watch Video Solution

9. If $f: R \rightarrow R$ and f(x)=g(x)+h(x) where g(x) is a polynominal and h(x) is a continuous and differentiable bounded function on both sides, then f(x) is one-one, we need to differentiate f(x). If f'(x) changes sign in domain of f, then f, if many-one else one-one.

If $f : R \to R$ and $f(x) = a_1 x + a_3 x^3 + a_5 x^5 + \ldots + a_{2n+1} - \cot^{-1} x$ where $0 < a_1 < a_3$

, then the function f(x) is

A. one-one into

B. many-one onto

C. one-one onto

D. many-one into

Answer: c

View Text Solution

10. If $f : R \to R$ and f(x)=g(x)+h(x) where g(x) is a polynominal and h(x) is a continuous and differentiable bounded function on both sides, then f(x) is one-one, we need to differentiate f(x). If f'(x) changes sign in domain of f, then f, if many-one else one-one.

$f : R \to R$ and $f(x) = \dfrac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then f(x) is

A. one-one into

B. many-one onto

C. one-one onto

D. many-one into

Answer: d

View Text Solution

11. If $f: R \to R$ and f(x)=g(x)+h(x) where g(x) is a polynominal and h(x) is a continuous and differentiable bounded function on both sides, then f(x) is one-one, we need to differentiate f(x). If f'(x) changes sign in domain of f, then f, if many-one else one-one.

If $f: R \to R$ and f(x)=2ax +sin2x, then the set of values of a for which f(x) is one-one and onto is

A. $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

B. $a \in (-1, 1)$

C. $a \in R - \left(-\frac{1}{2}, \frac{1}{2}\right)$

D. $a \in R - (-1, 1)$

Answer: d

View Text Solution

12. Let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $f(x) = \sqrt{g(x)}$, $f(x)$ have its non-zero local minimum and maximum values at -3 and 3 respectively. If $a_3 \in$ the domain of the function $h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$

The value of $a_1 + a_2$ is equal to

A. 30

B. -30

C. 27

D. -27

Answer: c

Watch Video Solution

13. Let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $f(x) = \sqrt{g(x)}$, $f(x)$ have its non-zero local minimum and maximum values at -3 and 3 respectively. If $a_3 \in$ the domain of the function $h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$

The value of a_0 is

A. equal to 50

B. greater than 54

C. less than 54

D. less than 50

Answer: b

Watch Video Solution

14. Let $g(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ and $f(x) = \sqrt{g(x)}$, f(x) has its non-zero local minimum and maximum values at -3 and 3, respectively.

If $a_3 \in$ the domain of the function

$h(x) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$.

f(10) is defined for

A. $a_0 > 830$

B. $a_0 < 830$

C. $a_0 = 830$

D. None of these

Answer: d

View Text Solution

15. Let $f: [2, \infty) \to \{1, \infty)$ defined by $f(x) = 2^{x^4 - 4x^3}$ and $g: \left[\frac{\pi}{2}, \pi\right] \to A$ defined by $g(x) = \frac{\sin x + 4}{\sin x - 2}$ be two invertible functions, then

$f^{-1}(x)$ is equal to

A. $\sqrt{2 + \sqrt{4 - \log_2 x}}$

B. $\sqrt{2+\sqrt{4+\log_2 x}}$

C. $\sqrt{2-\sqrt{4+\log_2 x}}$

D. None of these

Answer: b

Watch Video Solution

16. Let $f:[2,\infty)\to\{1,\infty)$ defined by $f(x)=2^{x^4-4x^3}$ and $g:\left[\frac{\pi}{2},\pi\right]\to A$ defined by $g(x)=\frac{\sin x+4}{\sin x-2}$ be two invertible functions, then

The set "A" equals to

A. [-5,-2]

B. [2,5]

C. [-5,2]

D. [-3,-2]

Answer: a

Watch Video Solution

17. Let $f: [2, \infty) \rightarrow [1, \infty)$ defined by $f(x) = 2^{x^4 - 4x^2}$ and $g: \left[\frac{\pi}{2}, \pi\right] \rightarrow A$ defined by $g(x) = \frac{\sin x + 4}{\sin x - 2}$ be two invertible functions.

The domain of $f^{-1}g^{-1}(x)$ is

A. [-5, sin 1]

B. $\left[-5, \frac{\sin 1}{2 - \sin 1}\right]$

C. $\left[-5, -\frac{(4 + \sin 1)}{2 - \sin 1}\right]$

D. $\left[-\frac{(4 + \sin 1)}{2 - \sin 1}, -2\right]$

Answer: c

View Text Solution

18. p(x) be a polynomial of degree at most 5 which leaves remainder - 1 and 1 upon division by $(x-1)^3$ and $(x+1)^3$respectively, the number of real roots of $P(x)=0$ is

A. 1

B. 3

C. 5

D. 2

Answer: a

Watch Video Solution

19. Let P(x) be polynominal of degree atmost 5 which leaves remainders -1 and 1 upon division by $(x-1)^3$ and $(x+1)^3$, respectively.

The maximum value of y=p''(x) can be obtained at x is equal to

A. $-\frac{1}{\sqrt{3}}$

B. 0

C. $\frac{1}{\sqrt{3}}$

D. 1

Answer: c

View Text Solution

20. p(x) be a polynomial of degree at most 5 which leaves remainder - 1 and 1 upon division by $(x-1)^3$ and $(x+1)^3$respectively, the number of real roots of $P(x) = 0$ is

A. $-\frac{5}{3}$

B. $-\frac{10}{3}$

C. 2

D. -5

Answer: b

Watch Video Solution

21. Consider $\alpha > 1$ and $f: \left[\frac{1}{\alpha}, \alpha\right] \to \left[\frac{1}{\alpha}, \alpha\right]$ be bijective function. Suppose that $f^{-1}(x) = \frac{1}{f(x)}$, for all $\in \left[\frac{1}{\alpha}, \alpha\right]$.

Then f(1) is equal to

A. 1

B. 0

C. -1

D. does'nt attain a unique value

Answer: a

View Text Solution

22. Consider $\alpha > 1$ and $f: \left[\frac{1}{\alpha}, \alpha\right] \to \left[\frac{1}{\alpha}, \alpha\right]$ be bijective function. Suppose that $f^{-1}(x) = \frac{1}{f(x)}$, for all $\in \left[\frac{1}{\alpha}, \alpha\right]$.

Which of the following statements can be concluded about (f(x))?

A. f(x) is discontinuous in $\left[\frac{1}{\alpha}, \alpha\right]$

B. f(x) is increasing in $\left[\frac{1}{\alpha}, \alpha\right]$

C. f(x) is decreasing in $\left[\frac{1}{\alpha}, \alpha\right]$

D. None of the above

Answer: b

View Text Solution

23. Consider $\alpha > 1$ and $f\colon \left[\frac{1}{\alpha}, \alpha\right] \to \left[\frac{1}{\alpha}, \alpha\right]$ be bijective function. Suppose that $f^{-1}(x) = \frac{1}{f(x)}, \quad \text{for all} \quad \in \left[\frac{1}{\alpha}, \alpha\right].$

Which of the following statements can be concluded about f(f(x))?

A. f(f(x)) is discontinuous in $\left[\frac{1}{\alpha}, \alpha\right]$

B. f(f(x)) is increasing in $\left[\frac{1}{\alpha}, \alpha\right]$

C. f(f(x)) is decreasing in $\left[\frac{1}{\alpha}, \alpha\right]$

D. None of the above

Answer: b

View Text Solution

24. Let f be real valued function from N to N satisfying. The relation f(m+n)=f(m)+f(n) for all $m, n \in N$.

The range of f contains all the even numbers, the value of f(1) is

A. 1

B. 2

C. 1 or 2

D. 4

Answer: a

Watch Video Solution

25. Let f be real valued function from N to N satisfying. The relation f(m+n)=f(m)+f(n) for all $m, n \in N$.

If domain of f is first 3m natural numbers and if the number of elements common in domain and range is m, then the value of f(1) is

A. 2

B. 3

C. 6

D. Can't say

Answer: B

Watch Video Solution

Exercise (Matching Type Questions)

1. Match the statements of Column I with values of Column II.

Column I	Column II
(A) $\sqrt{\sin(\cos x)}$ has domain	(p) $x \in R$
(B) $(\sqrt{\cos(\sin x)})^{-1}$ has domain	(q) $R - \left\{n\pi \pm \frac{\pi}{6}\right\}$
(C) $\tan(\pi \sin x)$ has domain	(r) $x \in \left(n\pi, n\pi + \frac{\pi}{2}\right)$
(D) $\ln(\tan x)$ has domain	(s) $x \in \left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2}\right]$

Watch Video Solution

2. Match the statements of Column I with values of Column II.

Column I	Column II
(A) $\lvert 4\sin x - 1 \rvert < \sqrt{5}, x \in [0, \pi]$, the domain is	(p) $\left[0, \frac{\pi}{4}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$
(B) $4\sin^2 x - 8$ $\sin x + 3 \le 0, [0, 2\pi]$, the domain is	(q) $\left[\frac{3\pi}{2}, 2\pi\right] \cup \{0\}$
(C) $\lvert \tan x \rvert \le 1$ and $x \in [0, \pi]$, the domain is	(r) $\left[0, \frac{3\pi}{10}\right)$
(D) $\cos x - \sin x \ge 1$ and $[0, 2\pi]$, the domain is	(s) $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$

Watch Video Solution

Exercise (Single Integer Answer Type Questions)

1. A function $f(x)$ is defined for all $x \in R$ and satisfies, $f(x+y) = f(x) + 2y^2 + kxy \forall x, y \in R$, where k is a given constant. If $f(1) = 2 \text{ and } f(2) = 8,$ find $f(x)$ and show that $f(x+y).\, f\left(\frac{1}{x+y}\right) = k, x + y \neq 0.$

Watch Video Solution

2. If $f: R \to R$ satisfying f(x-f(y))=f(f(y))+xf(y)+f(x)-1, for all $x, y \in R$, then $\frac{-f(10)}{7}$ is ....

Watch Video Solution

3. Let $f: N \to R$ be such that f(1)=1 and f(1)+2f(2)+3f(3)+... +nf(n)=n(n+1)f(n), for $n \geq 2, \quad \text{then}$ /(2010f(2010))` is ....

Watch Video Solution

Watch Video Solution

4. If $f(x) = \frac{2010x + 165}{165x - 2010}, x > 0$ and $x \neq \frac{2010}{165}$, the least value of $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ is ....

Watch Video Solution

5. If $\alpha, \beta, \gamma \in R, \alpha + \beta + \gamma = 4$ and $\alpha^2 + \beta^2 + \gamma^2 = 6$, the number of integers lie in the exhaustive range of α is ....

Watch Video Solution

6. The number or linear functions f satisfying $f(x + f(x)) = x + f(x) \forall x \in \mathbb{R}$ is

Watch Video Solution

7. If A={1,2,3}, B={1,3,5,7,9}, the ratio of number of one-one functions to the number of strictly monotonic functions is ....

Watch Video Solution

8. If n(A)=4, n(B)=5 and number of functions from A to B such that range contains exactly 3 elements is k, $\frac{k}{60}$ is ....

Watch Video Solution

9. If a and b are constants, such that

$f(x) = a\sin x + bx\cos x + 2x^2$ and f(2)=15, f(-2) is ....

Watch Video Solution

10. If the functions $f(x) = x^5 + e^{x/3}$ and $g(x) = f^{-1}(x)$, the value of g'(1) is ....

Watch Video Solution

11. If $f(x) = x^3 - 12x + p, p \in \{1, 2, 3, \ldots, 15\}$ and for each 'p', the number of real roots of equation f(x)=0 is denoted by θ, the $\frac{1}{5}\sum \theta$ is equal to ....

Watch Video Solution

12. Let f(x) denotes the number of zeroes in f'(x). If f(m)-f(n)=3, the value of $\frac{(m-n)_{\max} - (m-n)_{\min}}{2}$ is ....

View Text Solution

13. If $x^2 + yx^2 = 4$ then find the xamimum value of $\frac{x^3 + y^3}{x + y}$

Watch Video Solution

14. Let $f(n)$ denotes the square of the sum of the digits of natural number n, where $f^2(n)$ denotes $f(f(n))$. $f^3(n)$ denote $f(f(f(n)))$ and so on.the value of $\dfrac{f^{2011}(2011)-f^{2010}(2011)}{f^{2013}(2011)-f^{2012}(2011)}$ is....

Watch Video Solution

15. If $[\sin x]+\left[\dfrac{x}{2\pi}\right]+\left[\dfrac{2x}{5\pi}\right]=\dfrac{9x}{10\pi}$, where $[\cdot]$ denotes the greatest integer function, the number of solutions in the interval (30,40) is ....

View Text Solution

16. The number of integral solutions of $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{6}$ with $x\leq y$ is $'\alpha'$. The value of $'\alpha-6'$ is ....

Watch Video Solution

17. If f(x) is a polynominal of degree 4 with leading coefficient '1' satisfying f(1)=10,f(2)=20 and f(3)=30, then $\left(\frac{f(12)+f(-8)}{19840}\right)$ is ....

Watch Video Solution

18. If $a+b=3-\cos 4\theta$ and $a-b=4\sin 2\theta$, then ab is always less than or equal to

Watch Video Solution

19. Let 'n' be the number of elements in the domain set of the function $f(x)=\left|\ln\sqrt{{}^{x^2+4x}C_{2x^2+3}}\right|$ and 'Y' be the global maximum value of f(x), then [n+[Y]] is .... (where $[\cdot]$=greatest integer function).

Watch Video Solution

20. If $f(x)$ is a function such that $f(x-1)+f(x+1)=\sqrt{3}f(x)$ and $f(5)=10,$ then the sum of digit of the value of $\sum_{r=0}^{19} f(15+12r)$ is

Watch Video Solution

21. If $2f(x)=f(xy)+f\left(\frac{x}{y}\right)$ for all positive values of x and y, $f(1)=0$ and $f'(1)=1,$ then $f(e)$ is.

Watch Video Solution

22. Let f be a function from the set of positive integers to the set of real number such that f(1)=1 and $\sum_{r=1}^{n} rf(r)=n(n+1)f(n), \forall n \geq 2$ the value of 2126 f(1063) is ....

Watch Video Solution

23. If $f(x)=\dfrac{x^4+x^2+1}{x^2-x+1}$, the value of $f(\omega^n)$ (where $'\omega'$ is the non-real root of the equation $z^3=1$ and 'n' is a multiple of 3), is ....

Watch Video Solution

24. If $f^2(x)\cdot f\left(\dfrac{1-x}{1+x}\right)=x^3, [x\neq -1, 1 \text{ and } f(x)\neq 0]$, then find $|[f(-2)]|$ (where [] is the greatest integer function).

Watch Video Solution

25. An odd function is symmetric about the vertical line $x=a, (a>0), and$ if $\sum_{r=0}^{\infty}[f(1+4r]^r=8$, then find the value of $f(1)\cdot$

Watch Video Solution

26. $Let\dfrac{e^x-e^{-x}}{e^x+e^{-x}}=\ln\sqrt{\dfrac{1+x}{1-x}}$,then find x.

Watch Video Solution

27. If x is real , the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ i s

Watch Video Solution

28. If $f(x)$ satisfies the relation $f(x)+f(x+4)=f(x+2)+f(x+6)$ for all x, then prove that $f(x)$ is periodic and find its period.

Watch Video Solution

29. 83. A non-zero function f (x) is symmetrical about the line $y=x$ then the value of λ (constant) such that $f^2(x)=\left(f^{-1}(x)\right)^2-\lambda x f(x) f^{-1}(x)+3x^2 f(x)$ where all x $\in R^+$

Watch Video Solution

30. Let $f: R \to R$ and $f(x) = \frac{3x^2 + mx + n}{x^2 + 1}$. If the range of this function is [-4,3], then the value of $\frac{m^2 + n^2}{4}$ is ....

Watch Video Solution

31. Let f(x)be a monotic ploynomial of degree (2m-1) where $m \in N$ Then the equation

$f(x) - f(3x) + f(5x) + ... + f((2m-1)$ has

Watch Video Solution

Exercise (Subjective Type Questions)

1. Let x be a real number , [x] denotes the greatest integer function, {x} denotes the fractional part and (x) denotes the least integer function, then solve the following:

$$\text{(i)}(x)^2 = [x]^2 + 2x$$

(ii) [2x]--2x=[x+1]

$(\text{iii})\left[x^2\right]+2[x]=3x, 0 \leq x \leq 2$

$(\text{iv}) y=4-[x]^2$ and [y]+y=6

$(\text{v})[x]+|x-2| \leq 0 \quad \text{and} \quad -1 \leq x \leq 3$

View Text Solution

2. Let n be a positive integer with $f(n)=1!+2!+3!+\ldots\ldots\ldots+n!$ and $p(x), Q(x)$ be polynomial in x such that $f(n+2)=P(n)f(n+1)+Q(n)f(n)$ for all $n \geq$, Then

Watch Video Solution

3. If $f(x)=\dfrac{a^x}{a^x+\sqrt{a}}(a>0), g(n)=\sum_{r=1}^{2n-1} 2f\left(\dfrac{r}{2n}\right)$. Find te value $g(4)$

Watch Video Solution

4. Find the domain of the function,

$$f(x) = \log\left\{\log_{|\sin x|}\{x^2 - 8x + 23) - \frac{3}{\log_2|\sin x|}\right\}.$$

Watch Video Solution

5. Let S(n) denotes the number of ordered pairs (x,y) satisfying $\frac{1}{x} + \frac{1}{y} = \frac{1}{n}, \quad \text{where } n > 1 \text{ and } x, y, n \in N.$

(i) Find the value of S(6).

(ii) Show that, if n is prime, then S(n)=3, always.

Watch Video Solution

6. Solve $\frac{1}{x} + \frac{1}{[2x]} = \{x\} + \frac{1}{3}$where [.] denotes the greatest integers function and{.} denotes fractional part function.

Watch Video Solution

7. Let $f(x) = x^2 + 3x - 3, x \leq 0$. If n points $x_1, x_2, x_3, \ldots .., x_n$ are so chosen on the x-axis such that

Watch Video Solution

8. Let $f(x) = x^2 - 2x, x \in R, and g(x) = f(f(x) - 1) + f(5 - (x))$. Show that $g(w) \geq o \forall x \in R$.

Watch Video Solution

9. If f is polynomial function satisfying $2 + f(x)f(y) = f(x) + f(y) + f(xy) \forall x, y \in R$ and if $f(2) = 5$, then find the value of $f(f(2))$.

Watch Video Solution

10. If a+b+c=abc,a,b and c $\in R^+$, prove that $a + b + c \geq 3\sqrt{3}$.

Watch Video Solution

11. Consider the function $f(x) = \begin{cases} x - [x] - \frac{1}{2} & x \notin \\ 0 & x \in I \end{cases}$ where [.] denotes the fractional integral function and I is the set of integers. Then find $g(x)$ $\max . \left[x^2, f(x), |x|\right\}, -2 \leq x \leq 2.$

Watch Video Solution

12. Let $g(t) = |t-1| - |t| + |t+1|, \forall \quad t \in R.$

Find $f(x) = \max\left\{g(t) : -\frac{3}{2} \leq t \leq x\right\}, \forall x \in \left(\frac{-3}{2}, \infty\right)$.]

Watch Video Solution

13. Find the integral solution for $n_1 n_2 = 2n_1 - n_2, \quad \text{where} \quad n_1, n_2 \in \text{integer}.$

Watch Video Solution

Exercise (Questions Asked In Previous 13 Years Exam)

1. If function $f(x) = x^2 + e^{x/2}$ and $g(x) = f^{-1}(x)$, then the value of g'(x) is

Watch Video Solution

2. Let F(x) be an indefinite integral of $\sin^2 x$

Statement-1: The function F(x) satisfies $F(x + \pi) = F(x)$ for all real x. because

Statement-2: $\sin^3(x + \pi) = \sin^2 x$ for all real x.

A) Statement-1: True , statement-2 is true, Statement -2 is not a correct explanation for statement -1

c) Statement-1 is True, Statement -2 is False.

D) Statement-1 is False, Statement-2 is True.

A. Statement I is true, Statement II is also true, Statement II is the correct explanation of Statement I.

B. Statement I is true, Statement II is also true, Statement II is not the correct explanation of Statement I.

C. Statement I is true, Statement II is false.

D. Statement is false, Statement II is true.

Answer: D

Watch Video Solution

3. Find the range of values of t for which $2\sin t = \frac{1-2x+5x^2}{3x^2-2x-1}$

Watch Video Solution

4. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where $x \in \mathbb{R}$ and $k \geq 1$. Then $f_4(x) - f_6(x)$ equals

A. $1/6$

B. $1/3$

C. $1/4$

D. $1/12$

Answer: D

Watch Video Solution

5. The function $f:[0,3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$ is

A. one-one and onto

B. onto but not one-one

C. one-one but not onto

D. neither one-one nor onto

Answer: D

Watch Video Solution

6. Let $f(x) = x^2 and g(x) = \sin x f$ or $all x \in R \cdot$ Then the set of all x satisfying $(fogogof)(x) = (gogof)(x), where(fog)(x) = f(g(x)),$ is

$\pm\sqrt{n\pi}, n \in \{0, 1, 2, .\}$ $\pm\sqrt{n\pi}, n \in \{1, 2, .\}$

$\frac{\pi}{2} + 2n\pi, n \in \{, -2, -1, 0, 1, 2\}$ $2n\pi, n \in \{, -2, -1, 0, 1, 2, \}$

A. $\pm\sqrt{n\pi}, n \in \{0, 1, 2, ...\}$

B. $\pm\sqrt{n\pi}, n \in \{1, 2, ...\}$

C. $\pi/2 + 2n\pi, n \in \{..., -2, -1, 0, 1, 2, ...\}$

D. $2n\pi, n \in \{..., -2, -1, 0, 1, 2, ...\}$

Answer: A

Watch Video Solution

7. Let $f: (0, 1) \to R$ be defined by $f(x) = \frac{b - x}{1 - bx}$, where b is constant such that $0 < b < 1$ .then ,

A. f is not invertible on (0,1)

B. $f \neq f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$

C. $f = f^{-1}$ on (0,1) and $f'(b) = \frac{1}{f'(0)}$

D. f^{-1} is differentiable on (0,1)

Answer: B

Watch Video Solution

8. Let f be a real-valued function defined on the inverval $(-1, 1)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1} dt,$ for all, $x \in (-1, 1) and let f^{-1}$ be the inverse function of f. Then $\left(f^{-1}\right)'(2)$ is equal to 1 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{e}$

A. 1

B. $1/3$

C. $1/2$

D. $1/e$

Answer: B

Watch Video Solution

9. If X and Y are two non-empty sets where $f: X \to Y$,is function is defined such that $f(c) = \{f(x): x \in C\}$ for $C \subseteq X$ and $f^{-1}(D) = \{x: f(x) \in D\}$ for $D \subseteq Y$,for any $A \subseteq Y$ and $B \subseteq Y$, then

A. $f^{-1}\{f(A)\} = A$

B. $f^{-1}\{f(A)\} = A$, only if f(X)=Y

C. $f^{-1}\{f(B)\} = B$, only if B $\subseteq$ f(x)

D. $f^{-1}\{f(B)\} = B$

Answer: C

Watch Video Solution

10. If $f(x) = \{x,$ when x is rational and $0,$ when x is irrational $g(x) = \{0,$ when x is rational and $x,$ when x is irrational then $(f - g)$ is

A. one-one and into

B. neither one-one nor onto

C. many one and onto

D. one-one and onto

Answer: D

Watch Video Solution

11. If f(x)=sinx+cosx, g(x)=$x^2 - 1$, then g{f(x)} is invertible in the domain

A. $\left[0, \frac{\pi}{2}\right]$

B. $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D. $[0, \pi]$

Answer: B

Watch Video Solution

12. Domain of definition of the function

$f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ for real valued of x, is

A. $\left[-\frac{1}{4}, \frac{1}{2}\right]$

B. $\left[-\frac{1}{2}, \frac{1}{2}\right]$

C. $\left(-\frac{1}{2}, \frac{1}{9}\right)$

D. None of these

Answer: A

Watch Video Solution

13. The range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}, x \in R,$ is $(1, \infty)$ (b) $\left(1, \frac{11}{7}\right)$ $\left(1, \frac{7}{3}\right)$ (d) $\left(1, \frac{7}{5}\right)$

A. $(1, \infty)$

B. (1,11/7)

C. (1,7/3]

D. (1,7/5)

Answer: C

Watch Video Solution

14. If $f:[0, \infty) \rightarrow [0, \infty) \text{ and } f(x) = \frac{x}{1+x}$, then f is

A. one-one and onto

B. one-one but not onto

C. onto but not one-one

D. neither one-one nor onto

Answer: B

Watch Video Solution

15. If $f: R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$, then check the nature of the function.

A. one-to-one and onto

B. one-to-one but not onto

C. onto but not one-to-one

D. neither one-to-one nor onto

Answer: A

Watch Video Solution

16. Let $E = \{1, 2, 3, 4\} and F - \{1, 2\}$. If N is the number of onto functions from$E \to F$, then the value of $N/2$ is

A. 14

B. 16

C. 12

D. 8

Answer: A

Watch Video Solution

17. Suppose $f(x) = (x+1)^2 f$ or $x \geq -1$. If $g(x)$ is the function whose graph is the reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equal. $a - \sqrt{x} - 1, x \geq 0$ (b) $\frac{1}{(x+1)^2}, x \succ 1$ $\sqrt{x+1}, x \geq -1$ (d) $\sqrt{x} - 1, x \geq 0$

A. $1 - \sqrt{x} - 1, x \geq 0$

B. $\frac{1}{(x+1)^2}, x > -1$

C. $\sqrt{x+1}, x \geq -1$

D. $\sqrt{x} - 1, x \geq 0$

Answer: D

Watch Video Solution

18. If $f:[1,\infty)\rightarrow[2,\infty)$ is given by $f(x)=x+\frac{1}{x}$, then $f^{-1}(x)$ equals

A. $\frac{x+\sqrt{x^2-4}}{2}$

B. $\frac{x}{1+x^2}$

C. $\frac{x-\sqrt{x^2-4}}{2}$

D. $1+\sqrt{x^2-4}$

Answer: A

Watch Video Solution

19. Let $f(x0=(1+b^2)x^2+2bx+1$ and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is

A. [0,1]

B. $\left[0,\frac{1}{2}\right]$

C. $\left[\frac{1}{2}, 1\right]$

D. (0,1]

Answer: D

Watch Video Solution

20. The domain of definition of function of $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is

A. $R/\{-1, -2\}$

B. $(-2, \infty)$

C. $R/\{-1, -2, -3\}$

D. $(-3, \infty)/\{-1, -2\}$

Answer: D

Watch Video Solution

21. Let $f(x) = \frac{\alpha x}{x+1}, x \neq -1$. Then, for what values of α is f[f(x)]=x?

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. 1

D. -1

Answer: D

Watch Video Solution

22. Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$, then for all x, $f[g(x)]$ is equal to

A. x

B. 1

C. $f(x)$

D. $g(x)$

Answer: B

Watch Video Solution

23. The domain of definition of the function y(x) is given by the equation $2^x + 2^y = 2$, is

A. $0 < x \leq 1$

B. $0 \leq x \leq 1$

C. $-\infty < x \leq 0$

D. $-\infty < x < 1$

Answer: D

Watch Video Solution

24. Let $f(\theta) = \sin\theta(\sin\theta + \sin 3\theta)$. then

A. $\geq$ 0, only when θ ge 0

B. $\leq$ 0, for all real θ

C. $\geq$ 0, for all real θ

D. $\leq$ 0, only when $\theta \leq 0$

Answer: C

Watch Video Solution