MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

HYPERBOLA

Examples

1. To find the equation of the hyperbola from the definition that hyperbola is the locus of a point which moves such that the difference of its

distances from two fixed points is constant with

the fixed point as foci

2. Find the equation of the hyperbola whose

diretrix is $2x + y = 1,$ focus (1, 2) and eccentricity

$\sqrt{3}.$

3. Find the lengths of the transvers and the

conjugate axis, eccentricity, the coordinates of foci,

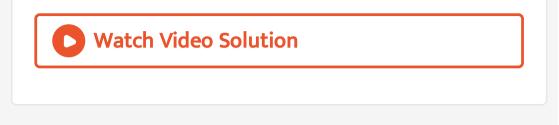vertices, the lengths of latus racta, and the equations of the directrices of the following hyperbola: $16x^2 - 9y^2 = -144$.

4. Find the eccentricity of the hyperbola whose latusrectum is half of its transverse axis.

A. $\dfrac{5}{2}$

B. $\dfrac{3}{2}$

C. $\sqrt{\dfrac{3}{2}}$

D. $\sqrt{\dfrac{5}{2}}$

Answer: C

5. Prove that the point $\left\{ \frac{a}{2}\left(t + \frac{1}{t}\right), \frac{b}{2}\left(t - \frac{1}{t}\right) \right\}$ lies on the hyperbola for all values of $t(t \neq 0)$.

6. The equation of the hyperbola, whose foci are $(6, 4)$ and $(-4, 4)$ and eccentricity is 2, is

7. Obtain the equation of a hyperbola with coordinate axes as principal axes given that the distances of one of its vertices from the foci are 9 and 1 units.

> ▶ **View Text Solution**

8. The foci of a hyperbola coincide with the foci of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$. Find the equation of the hyperbola, if its eccentricity is 2.

> ▶ **Watch Video Solution**

9. If two points $P \& Q$ on the hyperbola ,
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$ whose centre is C be such that CP is perpendicularal to CQ and $a < b1$,then prove that $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2}$.

▶ **Watch Video Solution**

10. Find the position of the point (5, -4) relative to the hyperbola $9x^2 - y^2 = 1$.
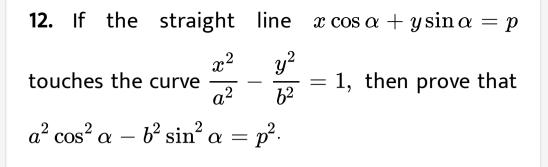
A. inside

B. outside

C. on the hyperbola

D. none of the above

Answer: A

▶ **Watch Video Solution**

---

11. If the line $lx + my + n = 0$ touches the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. Then

▶ **Watch Video Solution**

12. If the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1,$ then prove that $a^2\cos^2\alpha - b^2\sin^2\alpha = p^2.$

13. For what value of λ does the line $y = 2x + \lambda$ touches the hyperbola $16x^2 - 9y^2 = 144$?

14. If it is possible to draw the tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ having slope 2, then find its range of eccentricity.

15. Find the equation of the tagent to the hyperbola $x^2 - 4y^2 = 36$ which is perpendicular to the line $x - y + 4 = 0$.

16. Find length of common tangents to the hyperbolas $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$.

17. PQ is a chord joining the points ϕ_1 and ϕ_2 on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. If ϕ_1 and $\phi_2 = 2\alpha$, where $alha$ is constant, prove that PQ touches the hyperbola $\dfrac{x^2}{a^2}\cos^2\alpha - \dfrac{y^2}{b^2} = 1$

18. If the line $y = mx + \sqrt{a^2m^2 - b^2}$ touches the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at the point $(a \sec \phi, b \tan \phi)$, show that $\phi = \sin^{-1}\left(\dfrac{b}{a}m\right)$.

19. A normal to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ meets the axes in M and N and lines MP and NP are drawn perpendicular to the axes meeting at P. Prove that the locus of P is the hyperbola $a^2x^2 - b^2y^2 = \left(a^2 + b^2\right)^2$

20. The line $lx + my + n = 0$ is a normal to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. then prove that $\dfrac{a^2}{l^2} + \dfrac{b^2}{m^2} = \dfrac{\left(a^2 - b^2\right)^2}{n^2}$

21. If the normal at $P(\theta)$ on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{2a^2} = 1$ meets the transvers axis at G, then prove that $AG\dot{A}'G = a^2\left(e^4 \sec^2 \theta - 1\right)$, where $A and A'$ are the vertices of the hyperbola.

22. Find the locus of the foot of perpendicular from the centre upon any normal to line hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

23. The locus of the poles of the chords of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ which subtend a right angle at its centre is
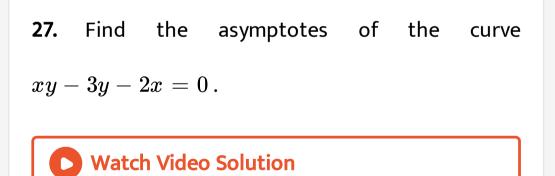
24. From the points on the circle $x^2 + y^2 = a^2$, tangents are drawn to the hyperbola $x^2 - y^2 = a^2$: prove that the locus of the middle-points $\left(x^2 - y^2\right)^2 = a^2\left(x^2 + y^2\right)$

25. Prove that the locus of the middle-points of the chords of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ which pass through a fixed point (α, β) is a hyperbola whose centre is $\left(\dfrac{\alpha}{2}, \dfrac{\beta}{2}\right)$.

26. Find the conditions for the lines $Ax^2 + 2Hxy + By^2 = 0$ to the conjugate diameter of $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1.$

27. Find the asymptotes of the curve $xy - 3y - 2x = 0$.

28. The asymptotes of a hyperbola are parallel to lines $2x + 3y = 0$ and $3x + 2y = 0.$ The hyperbola has its centre at (1, 2) and it passes through (5, 3). Find its equation.

▶ Watch Video Solution

29. If the normal to the rectangular hyperbola $xy = c^2$ at the point $'t'$ meets the curve again at t_1 then $t^3 t_1,$ has the value equal to

▶ Watch Video Solution

30. A triangle has its vertices on a rectangular hyperbola. Prove that the orthocentre of the triangle also lies on the same hyperbola.

31. A ray emanating from the point (5, 0) is meident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8. Find the equation of the reflected ray after the first reflection if point P lies in the first quadrant.

32. The equation of the transverse and conugate axes of a hyperbola are respectively $3x + 4y - 7 = 0$ and $4x - 3y + 8 = 0$ and their respective lengths are 4 and 6. Find the equation of the hyperbola.

▶ Watch Video Solution

33. If the eccentricity of the hyperbola $x^2 - y^2 \left(\sec^2(\alpha) \right) = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2 \left((\sec)^2 \alpha \right) + y^2 = 25$, then a value of α is :

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{3}$

D. $\dfrac{\pi}{2}$

Answer: B

▶ **Watch Video Solution**

34. Find the area of the triangle formed by any tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ with its asymptotes.

A. $\sec \lambda$

B. $\cos ec\lambda$

C. $\sec^2 \lambda$

D. $\cos ec^2\lambda$

Answer: A

▶ **Watch Video Solution**

35. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$, is

A. $\dfrac{x}{x_1 + x_2} + \dfrac{y}{y_1 + y_2} = 1$

B. $\dfrac{x}{x_1 - x_2} + \dfrac{y}{y_1 - y_2} = 1$

C. $\dfrac{x}{y_1 + y_2} + \dfrac{y}{x_1 + x_2} = 1$

D. $\dfrac{x}{y_1 - y_2} + \dfrac{y}{x_1 - x_2} = 1$

Answer: A

▶ **Watch Video Solution**

36. Area of the quadrilateral formed with the foci of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = -1$ (a)

$4(a^2 + b^2)$ (b) $2(a^2 + b^2)$ (c) $(a^2 + b^2)$ (d)
$\frac{1}{2}(a^2 + b^2)$

A. $4(a^2 + b^2)$

B. $2(a^2 + b^2)$

C. $(a^2 + b^2$

D. $\frac{1}{2}(a^2 + b^2)$

Answer: B

▶ **Watch Video Solution**

37. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec c\phi, b\tan\phi)$ (where $\theta + \phi = \dfrac{\pi}{2}$ be two points on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ If (h, k) is the point of intersection of the normals at P and Q then k is equal to (A) $\dfrac{a^2 + b^2}{a}$ (B) $-\left(\dfrac{a^2 + b^2}{a}\right)$ (C) $\dfrac{a^2 + b^2}{b}$ (D) $-\left(\dfrac{a^2 + b^2}{b}\right)$

A. $\left(\dfrac{a^2 + b^2}{a}\right)$

B. $-\left(\dfrac{a^2 + b^2}{a}\right)$

C. $\left(\dfrac{a^2 + b^2}{b}\right)$

D. $-\left(\dfrac{a^2 + b^2}{b}\right)$

Answer: D

38. Let the major axis ofa standard ellipse equals the transverse axis of a standard hyperbola and their director circles have radius equal to 2R and R respectively. If e, and e, are the eccentricities ofthe ellipse and hyperbola then the correct relation is

(a) $4e_1^2 - e_2^2 = 6$ (b) $e_1^2 - 4e_2^2 = 2$ (c) $4e_2^2 - e_1^2 = 6$

(d) $2e_1^2 - e_2^2 = 4$

A. $4e_1^2 - e_2^2 = 6$

B. $e_1^2 - 4e_2^2 = 2$

C. $4e_2^2 - e_1^2 = 6$

D. $e_2^2 - 4e_1^2 = 2$

Answer: C

▶ **Watch Video Solution**

39. The tangent to the hyperbola $xy = c^2$ at the point P intersects the x-axis at T and y- axis at T'.The normal to the hyperbola at P intersects the x-axis at N and the y-axis at N' . The areas of the triangles PNT and PN'T' are Δ and Δ' respectively, then $\frac{1}{\Delta} + \frac{1}{\Delta'}$ is (A) equal to 1 (B) depends on t (C) depends on c D) equal to 2

A. equal to 1

B. depends on t

C. depends on c

D. equal to 2

Answer: C

▶ **Watch Video Solution**

---

40. Let any double ordinate PNP^1 of the hyperbol $\frac{x^2}{9} - \frac{y^2}{4} = 1$ be produced both sides to meet the asymptotes in Q and Q', then $PQ. \, P'Q$ is equal to

A. 9

B. 16

C. 25

D. 41

Answer: B

41. Find the point on the hyperbola $\dfrac{x^2}{24} - \dfrac{y^2}{18} = 1$ which is nearest to the line $3x + 2y + 1 = 0$ and compute the distance between the point and the line.

A. $(6, 3)$

B. $(-6, -3)$

C. $(6, -3)$

D. $(-6, 3)$

Answer: D

▶ **Watch Video Solution**

---

42. For each positive integer consider the point P with abscissa n on the curve $y^2 - x^2 = 1$. If d_n represents the shortest distance from the point P

to the line $y = x$ then $\underset{n \to \infty}{\text{Lim}}$ $(n.\ d_n)$ as the value equal to (A) $\dfrac{1}{2\sqrt{2}}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{1}{\sqrt{2}}$ (D) 0

A. $\dfrac{1}{2\sqrt{2}}$

B. $\dfrac{1}{2}$

C. $\dfrac{1}{\sqrt{2}}$

D. 0

Answer: A

▶ **Watch Video Solution**

43. If two tangents can be drawn the different branches of hyperbola $\dfrac{x^2}{1} - \dfrac{y^2}{4} = 1$ from $\left(\alpha, \alpha^2\right)$, then

A. $\alpha \in (-\infty, -2)$

B. $\alpha \in (-2, 0)$

C. $\alpha \in (0, 2)$

D. $\alpha \in (2, \infty)$

Answer: A::D

> ▶ **Watch Video Solution**

44. If the ellipse $x^2 + \lambda^2 y^2 = \lambda^2 a^2$, $\lambda^2 > 1$ is confocal with the hyperbola $x^2 - y^2 = a^2$, then

A. ratio of eccentricities of ellipse and hyperbola

   is $1 : \sqrt{3}$

B. ratio of major axis of ellipse and transverse

   axis of hyperbola is $\sqrt{3} : 1$

C. The ellipse and hyperbola cuts each other

   orthogonally

D. ratio of length of latusrectumof ellipse and

   hyperbola is $1 : 3$

Answer: A::B::C

45. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, $S(x_4, y_4)$, then :

(A) $x_1 + x_2 + x_3 + x_4 = 0$ (B) $y_1 + y_2 + y_3 + y_4 = 0$ (C) $x_1 x_2 x_3 x_4 = c^4$ (D) $y_1 y_2 y_3 y_4 = c^4$

A. $\sum x_1 = 0$

B. $\sum y_1 = 0$

C. $\prod x_1 = 0$

D. $\prod y_1 = 0$

46. . A straight line touches the rectangular hyperbola $9x^2 - 9y^2 = 8$ and the parabola $y^2 = 32x$. An equation of the line is

A. $9x + 3y - 8 = \infty$

B. $9x - 3y + 8 = 0$

C. $9x + 3y + 8 = 0$

D. $9x - 3y - 8 = 0$

47. The differential equation $\dfrac{dy}{dx} = \dfrac{3y}{2x}$ represents a family of hyperbolas (except when it represents a pair of lines) with eccentricity. $\sqrt{\dfrac{3}{5}}$ (b) $\sqrt{\dfrac{5}{3}}$ $\sqrt{\dfrac{2}{5}}$ (d) $\sqrt{\dfrac{5}{2}}$

A. $\sqrt{\dfrac{7}{3}}$

B. $\sqrt{\dfrac{5}{3}}$

C. $\sqrt{\dfrac{3}{2}}$

D. $\sqrt{\dfrac{5}{2}}$

Answer: B::D

48. A conic C satisfies the differential equation, $(1 + y^2)dx - xy\,dy = 0$ and passes through the point $(1, 0)$.An ellipse E which is confocal with C having its eccentricity equal to $\sqrt{\dfrac{2}{3}}$.

A. 1

B. 2

C. 3

D. 4

Answer: B

49. A conic C satisfies the differential equation, $(1 + y^2) dx - xy dy = 0$ and passes through the point $(1, 0)$.An ellipse E which is confocal with C having its eccentricity equal to $\sqrt{\dfrac{2}{3}}$.

A. $\dfrac{x^2}{3} + \dfrac{y^2}{1} = 1$

B. $\dfrac{x^2}{1} + \dfrac{y^2}{3} = 1$

C. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

D. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$
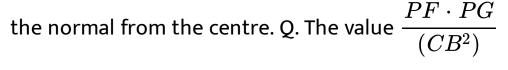
Answer: A

▶ **Watch Video Solution**

50. A conic C satisfies the differential equation,

$(1 + y^2) dx - xy\,dy = 0$ and passes through the

point $(1, 0)$.An ellipse E which is confocal with C

having its eccentricity equal to $\sqrt{\dfrac{2}{3}}$.

A. $x^2 + y^2 = 4$

B. $x^2 + y^2 = 8$

C. $x^2 + y^2 = 10$

D. $x^2 + y^2 = 13$

Answer: A

▶ **Watch Video Solution**

51. For the hyperbola $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, the normal at point P meets the transverse axis AA' in G and the connjugate axis BB' in g and CF be perpendicular to

the normal from the centre. Q. The value $\dfrac{PF \cdot PG}{(CB^2)}$ is equal to

A. 4

B. 3

C. 2

D. 1

Answer: D

52. For the hyperbola $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, the normal at point P meets the transverse axis AA' in G and the connjugate axis BB' in g and CF be perpendicular to the normal from the centre. Q. The value $PF \cdot Pg$ is equal to

A. $(CA)^2$

B. $(CF)^2$

C. $(CB)^2$

D. $CA \cdot CB$

Answer: A

▶ Watch Video Solution

53. For the hyperbola $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, the normal at point P meets the transverse axis AA' in G and the connjugate axis BB' in g and CF be perpendicular to the normal from the centre. Q. Locus of middle-point of G and g is a hyperbola of eccentricity

A. $\dfrac{1}{\sqrt{e^2 - 1}}$

B. $\dfrac{e}{\sqrt{e^2 - 1}}$

C. $2\left(\sqrt{e^2 - 1}\right)$

D. $\dfrac{e}{2}$

Answer: B

54. The equation of transverse axis of hyperbola (passing through origin) having asymptotes

$$3x - 4y = 1 \text{ and } 4x - 3y = 6$$ is

$$ax + by - c = 0, , a, b \in N \text{ and } g, c, d(a, b, c) = 1$$

then the value of $a + b + c$ is

55. If a variable line has its intercepts on the coordinate axes e and e', where $\frac{e}{2}$ and $e^{\prime}_{\square} 2$ are the eccentricities of a hyperbola and its conjugate

hyperbola, then the line always touches the circle

$x^2 + y^2 = r^2$, where $r = $ 1 (b) 2 (c) 3 (d) cannot

be decided

> ▶ **Watch Video Solution**

56. Statement-I Director circle of hypebola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} + 1 = 0$ is defined only when $b \geq a$.

Statement-II Director circle of hyperbola

$\dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$ is $x^2 + y^2 = 16$.

A. Statement-I is true, Statement-II is also true,

Statement-II is the correct explanation of

Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: B

▶ **Watch Video Solution**

57. Statement-I If a circle S=0 intersect hyperbola $xy = 4$ at four points, three of them being $(2, 2), (4, 1)$ and $\left(6, \dfrac{2}{3}\right)$, then the coordinate of the fourth point are $((1)(4), 16)$.

Statement-II If a circle S=0 intersects a hyperbola $xy = c^2$ at t_1, t_2, t_3 and t_4, then $t_1 t_2 t_3 t_4 = 1$

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of

Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: D

▶ **Watch Video Solution**

58. Prove that the perpendicular focal chords of a rectangular hyperbola are equal.

▶ **Watch Video Solution**

59. The normal at three points P, Q, R on a rectangular hyperbola intersect at a point T on the curve. Prove that the centre of the hyperbola is the centroid of the triangle PQR.

60. Find the equation of the hyperbola, whose asymptotes are the straight lines $(x + 2y + 3) = 0, (3x + 4y + 5) = 0$ and which passes through the point (1-1).

61. In both an ellipse and hyperbola , prove that the focal distance of any point and the perpendicular from the centre upon the tangent at it meet on a circle whose centre is the focus and whose radius is the semi-transverse axis.

62. A circle with centre $(3\alpha, 3\beta)$ and of variable radius cuts the rectangular hyperbola $x^2 - y^2 = 9a^2$ at the points P, Q, S, R. Prove that the locus of the centroid of triangle PQR is $(x - 2\alpha)^2 - (y - 2\beta)^2 = a^2$.

Jee Type Solved Examples Subjective Type Questions

1. If the normals at four points $P(x_i y_i), i = 1, 2, 3, 4$ on the rectangular hyperbola $xy = c^2$, meet at the point Q(h, k), prove that

Exercise For Session 1

1. The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$ is 1 (b) $\sqrt{2}$ (c) 2 (d) $\dfrac{1}{2}$

A. 1

B. $\dfrac{1}{2}$

C. -1

D. $\sqrt{2}$

Answer: D

▶ **Watch Video Solution**

2. If e_1 and e_2 represent the eccentricity of the curves $6x^2 - 9y^2 = 144$ and $9x^2 - 16y^2 = 144$ respectively . Then $\dfrac{1}{e_1^2} + \dfrac{1}{e_2^2}$ is equal to

A. $e_1^2 - e_2^2 = 1$

B. $e_1^2 - e_2^2 < 3$

C. $e_1^2 - e_2^2 = 3$

D. $e_1^2 - e_2^2 > 3$

Answer: B

▶ Watch Video Solution

3. The transverse axis of a hyperbola is of length $2a$ and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio $2:1$. The equation of the hyperbola is

A. $4x^2 - 5y^2 = 4a^2$

B. $4x^2 - 5y^2 = 5a^2$

C. $5x^2 - 4y^2 = 4a^2$

D. $5x^2 - 4y^2 = 5a^2$

Answer: D

▶ Watch Video Solution

4. The eccentricity of the hyperbola whose latuscrectum is 8 and conjugate axis is equal to half the distance between the foci, is

A. $\dfrac{2}{\sqrt{3}}$

B. $\dfrac{3}{\sqrt{3}}$

C. $\dfrac{4}{\sqrt{3}}$

D. $\dfrac{5}{\sqrt{3}}$

Answer: A

▶ Watch Video Solution

5. The straight line $x + y = \sqrt{2}p$ will touch the hyperbola $4x^2 - 9y^2 = 36$ if $2p^2$ is ?

A. $\dfrac{5}{2}$

B. 5

C. $\dfrac{2}{5}$

D. 2

Answer: B

6. The equation of the tangent parallel to $y - x + 5 = 0$ drawan to $\dfrac{x^2}{3} - \dfrac{y^2}{2} = 1$ is

A. $x - y - 1 = 0$

B. $x - y + 2 = 0$

C. $x + y - 1 = 0$

D. x+y+2=0`

Answer: A

7. If e and e' are the eccentricities of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$, then the point $\left(\dfrac{1}{e}, \dfrac{1}{e'}\right)$ lies on the circle (A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = 2$ (C) $x^2 + y^2 = 3$ (D) $x^2 + y^2 = 4$

A. x^(2)+y^(2)=1`

B. $x^2 + y^2 = 2$

C. $x^2 + y^2 = 3$

D. $x^2 + y^2 = 4$

Answer: A

Watch Video Solution

8. If e and e' are the eccentricities of the ellipse $5x^2 + 9y^2 = 45$ and the hyperbola `5x^(2) - 4y^(2) = 45 respectively , then ee' is equal to

A. -1

B. 1

C. -4

D. 9

Answer: B

9. The equation $\dfrac{x^2}{10 - \lambda} + \dfrac{y^2}{6 - \lambda} = 1$ represents

A. a hyperbola if $\lambda < 6$

B. an ellipse if $\lambda > 6$
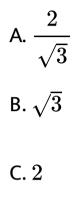
C. a hyperbola if $6 < \lambda < 10$

D. an ellipse if $0 < \lambda < 6$

Answer: C::D

▶ **Watch Video Solution**

10. Find the centre, eccentricity, foci and directrices of the hyperbola : $x^2 - 3y^2 - 2x = 8$.

A. $\dfrac{2}{\sqrt{3}}$

B. $\sqrt{3}$

C. 2

D. $\sqrt{2}$

Answer: (c)

▶ **Watch Video Solution**

11. For hyperbola $x^2 \sec^2 \alpha - y \cos ec^2 \alpha = 1,$ which of the following remains constant with change in $'\alpha'$ abscissa of vertices (b) abscissa of foci eccentricity (d) directrix

    A. abscissae of vertices

    B. abscissae of foci

    C. eccentricity

    D. directrix

Answer: B

12. If the foci of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$ coincide, then find the value of b^2

A. 1

B. 5

C. 7

D. 9

Answer: C

▶ Watch Video Solution

13. Find the standard equation of hyperbola in each of the following cases:

(i) Distance between the foci of hyperbola is 16 and its eccentricity is $\sqrt{2}$.

(ii) Vertices of hyperbola are $(\pm 4, 0)$ and foci of hyperbola are $(\pm 6, 0)$.

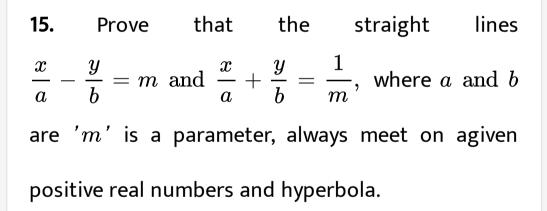(iii) Foci of hyperbola are $(0, \pm \sqrt{10})$ and it passes through the point (2,3).

(iv) Distance of one of the vertices of hyperbola from the foci are 3 and 1.

▶ Watch Video Solution

14. Find the equation of the hyperbola whose foaci are $(0, 5)$ and $(-2, 5)$ and eccentricity 3.

15. Prove that the straight lines $\dfrac{x}{a} - \dfrac{y}{b} = m$ and $\dfrac{x}{a} + \dfrac{y}{b} = \dfrac{1}{m}$, where a and b are $'m'$ is a parameter, always meet on agiven positive real numbers and hyperbola.

16. Find the centre, eccentricity and length of axes of the hyperbola $3x^2 - 5y^2 - 6x + 20y - 32 = 0$.

▶ **Watch Video Solution**

17. The eccentricity of the conjugate hyperbola of the hyperbola $x^2 - 3y^2 = 1$ is 2 (b) $2\sqrt{3}$ (c) 4 (d) $\dfrac{4}{5}$

▶ **Watch Video Solution**

18. If the line $y = 3x + \lambda$ touches the hyperbola $9x^2 - 5y^2 = 45$, then λ=

19. Find the equation of tangents to the curve

$4x^2 - 9y^2 = 1$ which are parallel to $4y = 5x + 7$.

Exercise For Session 2

1. The tangents from $(1, 2\sqrt{2})$ to the hyperbola

$16x^2 - 25y^2 = 400$ include between them an angle

equal to:

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{3}$

D. $\dfrac{\pi}{2}$

Answer: D

▶ **Watch Video Solution**

2. If $4x^2 + py^2 = 45$ and $x^2 - 4y^2 = 5$ cut orthogonally, then the value of p is

A. $\dfrac{1}{9}$

B. $\dfrac{1}{3}$

C. 9

D. 18

Answer: C

▶ **Watch Video Solution**

---

3. If the tangent at the point $(2\sec\theta, 3\tan\theta)$ to the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$ is parallel to $3x - 4y + 4 = 0$, then the value of θ, is

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{3}$

D. $\dfrac{5\pi}{12}$

Answer: A

4. If the line $2x + \sqrt{6}y = 2$ touches the hyperbola

$x^2 - 2y^2 = 4$, then the point of contact is

A. $\left(-2, \sqrt{6} \right)$

B. $\left(-5, 2\sqrt{6} \right)$

C. $\left(\dfrac{1}{2}, \dfrac{1}{\sqrt{6}} \right)$

D. $\left(4, -\sqrt{6} \right)$

Answer: D

▶ **Watch Video Solution**

---

5. The equation of that chord of hyperbola

$25x^2 - 16y = 400$, whose mid point is (5,3) is

    A. $115x - 47y = 434$

    B. $125x - 48y = 481$

    C. $127x - 49y = 488$

D. $155x - 67y = 574$

Answer: B

6. The value of m for which $y = mx + 6$ is a tangent to the hyperbola $\dfrac{x^2}{100} - \dfrac{y^2}{49} = 1$, is

A. $\sqrt{\dfrac{17}{20}}$

B. $-\sqrt{\dfrac{17}{21}}$

C. $\sqrt{\dfrac{20}{17}}$

D. $-\sqrt{\dfrac{21}{17}}$

Answer: A

7. P is a point on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, N is the foot of the perpendicular from P on the transverse axis. The tangent to the hyperbola at P meets the transverse axis at T. If O is the centre of the hyperbola, then $OT . ON$ is equal to

A. a^2

B. b^2

C. e^2

D. `b^(2)la

Answer: A

8. If $x = 9$ is the chord of contact of the hyperbola

$x^2 - y^2 = 9$ then the equation of the

corresponding pair of tangents is (A)

$9x^2 - 8y^2 + 18x - 9 = 0$ (B)

$9x^2 - 8y^2 - 18x + 9 = 0$ (C)

$9x^2 - 8y^2 - 18x - 9 = 0$ (D) 9x^2-8y^2+18x+9=0`

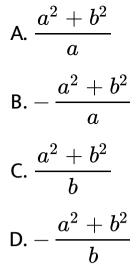A. $9x^2 - 8y^2 - 18x + 9 = 0$

B. $9x^2 - 8y^2 - 18x - 9 = 0$

C. $9x^2 - 8y^2 + 18x + 9 = 0$

D. $9x^2 - 8y^2 + 18x - 9 = 0$

Answer: B

▶ **Watch Video Solution**

9. Let $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ (where $\theta + \phi = \dfrac{\pi}{2}$ be two points on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ If (h, k) is the point of intersection of the normals at P and Q then k is equal to

A. $\dfrac{a^2 + b^2}{a}$

B. $-\dfrac{a^2 + b^2}{a}$

C. $\dfrac{a^2 + b^2}{b}$

D. $-\dfrac{a^2 + b^2}{b}$

Answer: D

▶ **Watch Video Solution**

10. The tangent at a point P on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ passes through the point $(0, \, -b)$ and the normal at P passes through the point

$\left(2a\sqrt{2}, 0\right)$. Then the eccentricity of the hyperbola

is 2 (b) $\sqrt{2}$ (c) 3 (d) $\sqrt{3}$

A. $\dfrac{5}{4}$

B. $\dfrac{3}{2}$

C. $\sqrt{2}$

D. $2\sqrt{2}$

Answer: C

> ▶ Watch Video Solution

11. A tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ cuts the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at $P and Q$. Show that the locus of the midpoint of PQ is

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 = \frac{x^2}{a^2} - \frac{y^2}{b^2}.$$

> ▶ **Watch Video Solution**

12. A line through the origin meets the circle $x^2 + y^2 = a^2$ at P and the hyperbola $x^2 - y^2 = a^2$ at Q. Prove that the locus of the point of intersection of tangent at P to the circle with the tangent at Q to the hyperbola is a straight line.
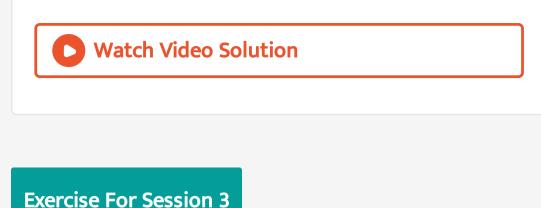
13. Normal are drawn to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at point $\theta_1 and th\eta_2$ meeting the conjugate axis at $G_1 and G_2$, respectively. If $\theta_1 + \theta_2 = \dfrac{\pi}{2}$, prove that $CG_1 \dot{C}G_2 = \dfrac{a^2 e^4}{e^2 - 1}$, where C is the center of the hyperbola and e is the eccentricity.

14. Chords of the hyperbola, $x^2 - y^2 = a^2$ touch the parabola, $y^2 = 4ax$. Prove that the locus of

their middlepoints is the curve, $y^2(x - a) = x^3$.

Exercise For Session 3

1. The diameter of $16x^2 - 9y^2 = 144$ which is conjugate to $x = 2y$ is

A. $y = \dfrac{16}{9}x$

B. $y = \dfrac{32}{9}x$

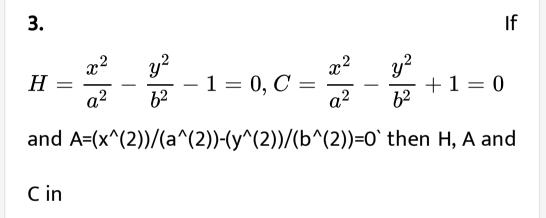C. $x = \dfrac{16}{9}y$

D. $x = \dfrac{32}{9}y$

Answer: B

2. Tangents drawn from a point on the circle $x^2 + y^2 = 9$ to the hyperbola $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$, then tangents are at angle (A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{2}$ (C) $\dfrac{\pi}{3}$ (D) $\dfrac{2\pi}{3}$

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{3}$

D. $\dfrac{\pi}{2}$

Answer: D

3. If

$$H = \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0, C = \frac{x^2}{a^2} - \frac{y^2}{b^2} + 1 = 0$$

and A=(x^(2))/(a^(2))-(y^(2))/(b^(2))=0` then H, A and

C in

A. AP

B. GP

C. HP
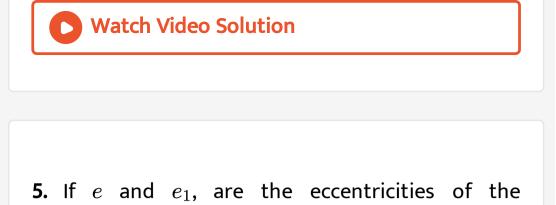
D. AGP

4. The angle between the asymptotes of the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$, is

A. $\tan^{-1}\left(\dfrac{2}{3}\right)$

B. $\tan^{-1}\left(\dfrac{3}{2}\right)$

C. $2\tan^{-1}\left(\dfrac{2}{3}\right)$

D. $2\tan^{-1}\left(\dfrac{3}{2}\right)$

Answer: D

5. If e and e_1, are the eccentricities of the hyperbolas $xy = c^2$ and $x^2 - y^2 = c^2$, then $e^2 + e_1^2$ is equal to

A. 2

B. 4

C. 6

D. 8

Answer: D

6. Find the product of the length of perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 - 2 = 0$ to its asymptotes.

A. $\dfrac{1}{2}$

B. 2

C. $\dfrac{2}{3}$

D. $\dfrac{3}{2}$

Answer: C

▶ Watch Video Solution

7. The number of points on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 3$ from which mutually perpendicular tangents can be drawn to the circle $x^2 + y^2 = a^2$

is/are 0 (b) 2 (c) 3 (d) 4

A. 0

B. 2

C. 3

D. 4

Answer: A

▶ **Watch Video Solution**

8. If the sum of the slopes of the normal from a point P to the hyperbola $xy = c^2$ is equal to $\lambda(\lambda \in R^+)$, then the locus of point P is $x^2 = \lambda c^2$ (b) $y^2 = \lambda c^2$ $xy = \lambda c^2$ (d) none of these

A. $x^2 = \lambda c^2)$

B. $y^2 = \lambda c^2)$

C. $xy = \lambda c^2$

D. None of these

Answer: A

9. If $S = 0$ is the equation of the hyperbola $x^2 + 4xy + 3y^2 - 4x + 2y + 1 = 0$, then the value of k for which $S + K = 0$ represents its asymptotes is 20 (b) -16 (c) -22 (d) 18

A. 20

B. 18

C. -16

D. -22

Answer: D

10. A ray emanating from the point $(dqrt(41), 0)$ is incident on the hyperbola $16x^2 - 25y^2 = 400$ at the point P with abscissa10.Find the equation of a reflected ray after first reflection and point P lies in 2nd quadrant is

A. $4\sqrt{3} - (10 - \sqrt{41})y + 4\sqrt{123} = 0$

B. $4\sqrt{3} + (10 - \sqrt{41})y - 4\sqrt{123} = 0$

C. $4\sqrt{3} + (10 - \sqrt{41})y + 4\sqrt{123} = 0$

D. $4\sqrt{3} - (10 - \sqrt{41})y - 4\sqrt{123} = 0$

Answer: B

▶ Watch Video Solution

11. A ray of light incident along the line $3x + (5 - 4\sqrt{2})y = 15$ gets reflected from the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$, then its reflected ray goes along the line

A. $x\sqrt{2} - y + 5 = 0$

B. $y\sqrt{2} - x + 5 = 0$

C. $y\sqrt{2} - x - 5 = 0$

D. $3x - y(4\sqrt{2} + 5) + 15 = 0$

Answer: D

12. The equation of the transvers and conjugate axes of a hyperbola are, respectively, $x + 2y - 3 = 0$ and $2x - y + 4 = 0$, and their respective lengths are $\sqrt{2}$ and $2\sqrt{3}$. The equation of the hyperbola is

$$\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$$

$$\frac{2}{5}(x - y - 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$$

$$2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$$

$$2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$$

A. $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 5$

B. $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 5$

C. $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

D. $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$

Answer: B

▶ **Watch Video Solution**

13. Find the equation of that diameter which bisects the chord $7x + y - 2 = 0$ of the hyperbola $\frac{x^2}{3} - \frac{y^2}{7} = 1$.

▶ **Watch Video Solution**

14. Find the equation of the hyperbola which has $3x - 4y + 7 = 0$ and $4x + 3y + 1 = 0$ as its asymptotes and which passes through the origin.

15. The asymptotes of a hyperbola are parallel to lines $2x + 3y = 0$ and $3x + 2y = 0$. The hyperbola has its centre at (1, 2) and it passes through (5, 3). Find its equation.

16. If the pair of straight lines $Ax^2 + 2Hxy + By^2 = 0$ be conjugate diameters of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then prove that $Aa^2 = Bb^2$.

17. A circle cuts the rectangular hyperbola $xy = 1$ in the points (x_1, y_1), $r = 1, 2, 3, 4$.

Prove that $x_1 x_2 x_3 x_4 = y_1 y_2 y_3 y_4 = 1$

1. P is any point on the hyperbola $x^2 - y^2 = a^2$. If F_1 and F_2 are the foci of the hyperbola and $PF_1 \cdot PF_2 = \lambda(OP)^2$. Where O is the origin, then λ is equal to

A. 1

B. $\sqrt{2}$

C. 2

D. 3

Answer: A

2. If the sum of the slopes of the normal from a point P to the hyperbola $xy = c^2$ is equal to $\lambda(\lambda \in R^+)$, then the locus of point P is $x^2 = \lambda c^2$ (b) $y^2 = \lambda c^2$ $xy = \lambda c^2$ (d) none of these

A. $x^2 - y^2 = \lambda c^2$

B. $y^2 = \lambda c^2$)

C. $xy = \lambda c^2$

D. $x^2 = \lambda c^2$

Answer: D

3. If $xy = \lambda^2 - 9$ be a rectangular hyperbola whose branches lie only in the second and fourth quadrant, then

A. $|\lambda| \geq 3$

B. $|\lambda| < 3$

C. $\lambda \in R - \{-3, 3\}$

D. None of these

Answer: B

4. If there are two points A and B on rectangular hyperbola $xy = c^2$ such that abscissa of $A =$ ordinate of B, then locusof point of intersection of tangents at A and B is

A. $y^2 = x^2 + 2c^2$

B. $y^2 = x^2 + \dfrac{c^2}{2}$

C. $y = x$

D. $y = 3x$

Answer: C

▶ Watch Video Solution

5. A series of hyperbola are drawn having a common transverse axis of length 2a. Prove that the locus of point P on each hyperbola, such that its distance from the transverse axis is equal to its distance from an asymptote, is the curve $\left(x^2 - y^2\right)^2 = \lambda x^2 \left(x^2 - a^2\right)$, then λ equals

A. $\left(x^2 - y^2\right)^2 = 4x^2 \left(x^2 - a^2\right)$

B. $\left(x^2 - y^2\right)^2 = x^2 \left(x^2 - a^2\right)$

C. $\left(x^2 - y^2\right)^2 = 4y^2 \left(x^2 - a^2\right)$

D. $\left(x^2 - y^2\right)^2 = y^2 \left(x^2 - a^2\right)$

6. If a rectangular hyperbola $(x - 1)(y - 2) = 4$

cuts a circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at

points $(3, 4), (5, 3), (2, 6)$ and $(-1, 0)$, then the

value of (g+f) is equal to

A. -3

B. -9

C. 8

D. 9

7. If p, q, r, s ae rational numbers and the roots of

$f(x) = 0$ are eccentricities of a parabola and a

rectangular hyperbola, where

$f(x0 = px^3 + qx^2 + rx + s, then p + q + r + s =$

p b. $-p$ c. $2p$ d. 0

A. -1

B. 0

C. 1

D. data inadequate

Answer: B

▶ Watch Video Solution

---

8. From a point on the line $y = x + c$, c(parameter), tangents are drawn to the hyperbola $\frac{x^2}{2} - \frac{y^2}{1} = 1$ such that chords of contact pass through a fixed point (x_1, y_1). Then , $\frac{x_1}{y_1}$ is equal to

A. 2

B. 3

C. 4

D. None of these

Answer: A

▶ **Watch Video Solution**

---

9. Two conics $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $x^2 = -\dfrac{a}{b}y$

intersect, if

A. $0 < b \le \dfrac{1}{2}$

B. $0 \le a \le \dfrac{1}{2}$

C. $a^2 < b^2$

D. $a^2 > b^2$

Answer: B

10. the number of points outside the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ from where two perpendicular tangents can be drawn to the hyperbola are: (a) 0 (b) 1 (c) 2 (d) non of these
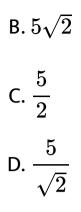
A. 0

B. 1

C. 2

D. None of these

Answer: A

▶ **Watch Video Solution**

11. Let $A = (-3, 4)$ and $B = (2, -1)$ be two fixed points. A point C moves such that $\tan\left(\frac{1}{2}\angle ABC\right) : \tan\left(\frac{1}{2}\angle BAC\right) = 3:1$ Thus, locus of C is a hyperbola, distance between whose foci is

A. 5

B. $5\sqrt{2}$

C. $\dfrac{5}{2}$

D. $\dfrac{5}{\sqrt{2}}$

Answer: B

▶ **View Text Solution**

---

12. A point P is taken on the right half of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ having its foci as S_1 and S_2. If the internal angle bisector of the

angle $\angle S_1 P S_2$ cuts the x-axis at poin $Q(\alpha, 0)$ then range of α is

A. $[-a, a]$

B. $[0, a]$

C. $[0, a]$

D. $[-a, 0]$

Answer: C

13. If the angle between the asymptotes of hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is 120^0 and the product of perpendiculars drawn from the foci upon its any tangent is 9, then the locus of the point of intersection of perpendicular tangents of the hyperbola can be $x^2 + y^2 = 6$ (b) $x^2 + y^2 = 9$ $x^2 + y^2 = 3$ (d) $x^2 + y^2 = 18$

A. $x^2 + y^2 = 3$

B. $x^2) + y^2 = 6$

C. $x^2 + y^2 = 9$

D. $x^2 + y^2 = 18$

Answer: D

14. If $\alpha + \beta = 3\pi$, then the chord joining the points α and β for the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ passes through which of the following points? Focus (b) Center One of the endpoints of the transverse exis. One of the endpoints of the conjugate exis.

A. focus

B. centre

C. one of the end point of the transverse axis

D. one of the end of the conjugate axis

Answer: B

15. If $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 (a > b)$ and $x^2 - y^2 = c^2$ cut at right angles, then:

A. $a^2 + b^2 = 2c^2$

B. $b^2 - a^2 = 2c^2$

C. $a^2 - b^2 = 2c^2$

D. $a^2b^2 = 2c^2$

Answer: C

> ▶ **Watch Video Solution**

---

16. Chords of the hyperbola, $x^2 - y^2 = a^2$ touch the parabola, $y^2 = 4ax$. Prove that the locus of their middlepoints is the curve, $y^2(x - a) = x^3$.

A. $y^2(x + a) = x^3$

B. $y^2(x - a) = x^3$

C. $y^2(x + 2a) = 3x^3$

D. $y^2(x - 2a) = 2x^3$

Answer: B

17. An ellipse has eccentricity $\dfrac{1}{2}$ and one focus at the point $P\left(\dfrac{1}{2}, 1\right)$. Its one directrix is the comionand tangent nearer to the point the P to the hyperbolaof $x^2 - y^2 = 1$ and the circle $x^2 + y^2 = 1$.Find the equation of the ellipse.

A. $9x^2 + 12y^2 = 108$

B. $9\left(x - \dfrac{1}{3}\right)^2 + 12(y - 1)^2 = 1$

C. $9\left(x - \dfrac{1}{3}\right)^2 + 4(y - 1)^2 = 36$

D. None of these

Answer: B

▶ **Watch Video Solution**

18. The equation of the line passing through the centre of a rectangular hyperbola is $x - y - 1 = 0$. If one of its asymptotoes is $3x - 4y - 6 = 0$, the equation of the other asymptote is

A. $4x - 3y + 8 = 0$

B. $4x + 3y + 17 = 0$

C. $3x - 2y + 15 = 0$

D.

Answer: B

▶ **Watch Video Solution**

19. The condition that a straight line with slope m

will be normal to parabola $y^2 = 4ax$ as well as a

tangent to rectangular hyperbola $x^2 - y^2 = a^2$ is

A. $m^6 - 4m^2 + 2m - 6y = 0$

B. $m^4 + 3m^3 + 2m + 1 = 0$

C. $m^6 - 2m = 0$

D. $m^6 + 4m^4 + 3m^2 + 1 = 0$

Answer: D

> ▶ **Watch Video Solution**

20. Find the locus of the midpoints of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to y = 2x.

A. $3x - 4y = 4$

B. $3y - 4x + 4 = 0$

C. $4x - 4y = 3$

D. $3x - 4y = 2$

Answer: A

21. The co-ordinates of the centre of the hyperbola, $x^2 + 3xy + 3y^2 + 2x + 3y + 2 = 0$ is $(-1, 0)$ (b) $(1, 0)$ $(-1, 1)$ (d) $(1, -1)$

A. $(-1, 0)$

B. $(1, 0)$

C. $(-1, 1)$

D. $(1, -1)$

Answer: A

▶ **Watch Video Solution**

---

22. Let F_1, F_2 are the foci of the hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ and F_3, F_4 are the foci of its conjugate hyperbola. If e_H and e_C are their eccentricities respectivley then the statement

which holds true is: A) Their equations of the asymptotes are different B) $e_H > e_C$ C) Area of the quadrilateral formed by their foci is 50 sq. units D) Their auxillary circles will have the same equation

A. `their equations of their asymptots are different

B. $e_h > e_c$

C. `area of the quadrilateral formed by their foci is 50sq. Units

D. their auciliary circles will have the same equation

Answer: C

23. Locus of the point of intersection of the tangents at the points with eccentric angles ϕ and $\dfrac{\pi}{2}$ on the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is

A. $x = a$

B. $y = b$

C. $x = ab$

D. $y = ab$

Answer: B

24. Latusrectum of the conic satisfying the differential equation $xdy + ydx = 0$ and passing through the point $(2, 8)$ is

A. $4\sqrt{2}$

B. 8

C. $8\sqrt{2}$

D. 16

Answer: C

25. The point of intersection of the curve whose parametrix equations are $x = t^2 + 1, y = 2t$ and $x = 2s, y = \dfrac{2}{s}$, is given by

A. $(1, \ -3)$

B. $(2, 2)$

C. $(\ -2, 4)$

D. $(1, 2)$

Answer: B

26. If the tangent and normal to a rectangular hyperbola cut off intercepts a_1 and a_2 on one axis and b_1 and b_2 on the other, then
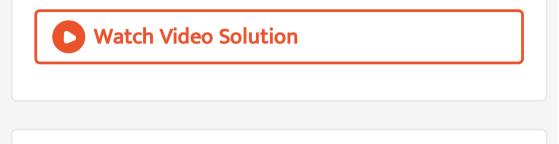
A. $x_1 y_1 + x_2 y_2 = 0$

B. $x_1 y_2 + x_2 y_1 = 0$

C. $x_1 x_2 + y_1 y_2 = 0$

D. None of these

Answer: C

---

27. The focus of rectangular hyperbola

$(x - a) \cdot (y - b) = c^2$ is

    A. $(h - p, k - p)$

    B. $(h - p, k + p)$

    C. $(h + p, k - p)$

    D. None of these

Answer: A

28. The equation of a hyperbola conjugate to the hyperbola $x^2 + 3xy + 2y^2 + 2x + 3y = 0$ is

A. $x^2 + 3xy + 2y^2 + 2x + 3y + 1 = 0$

B. $x^2 + 3xy + 2y^2 + 2x + 3y + 2 = 0$

C. $x^2 + 3xy + 2y^2 + 2x + 3y + 3 = 0$

D. $x^2 + 3xy + 2y^2 + 2x + 3y + 4 = 0$

Answer: B

29. If values of a, for which the line $y = ax + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of the equation $x^2 - (a_1 + b_1)x - 4 = 0$, then the values of $a_1 + b_1$ is

A. -2

B. 0

C. 2

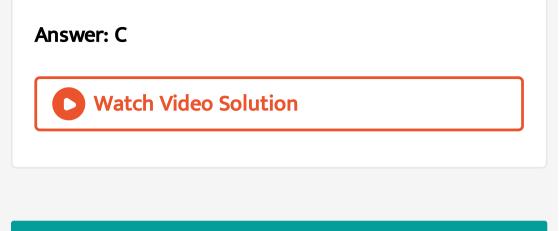D. 4

Answer: B

 Watch Video Solution

30. Let C be a curve which is the locus of the point of intersection of lines $x = 2 + m$ and $my = 4 - m.$ A circle $s \equiv (x - 2)^2 + (y + 1)^2 = 25$ intersects the curve C at four points: $P, Q, R, and S$. If O is center of the curve C, then $OP^2 + OP^2 + OR^2 + OS^2$ is 50 (b) 100 (c) 25 (d) $\dfrac{25}{2}$

A. 25

B. 50

C. 100

D. 200

Answer: C

Exercise More Than One Correct Option Type Questions

1. The equation of common tangent to the parabola

$y^2 = 8x$ and hyperbola $3x^2 - y^2 = 3$ is

A. $2x - y + 1 = 0$

B. $2x - y - 1 = 0$

C. $2x + y + 1 = 0$

D. $2x + y - 1 = 0$

Answer: A::C

2. If the length of minor axis of the ellipse $\dfrac{x^2}{k^2 a^2} + \dfrac{y^2}{b^2} = 1$ is equal to the length of transverse axis of hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, and the equation of ellipse is confocal with hyperbola then the value k is equal to

A. $-\sqrt{2}$

B. $\sqrt{2}$

C. $-\sqrt{3}$

D. $\left(\sqrt{3}\right)$

Answer: C::D

▶ **Watch Video Solution**

---

3. If $(a\sec\theta;\, b\tan\theta)$ and $(a\sec\phi;\, b\tan\phi)$ are the ends of the focal chord of $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ then prove that $\tan\left(\dfrac{x}{a}\right)\tan\left(\dfrac{\phi}{2}\right) = \dfrac{1-e}{1+e}$

A. $\dfrac{e-1}{e+1}$

B. $\dfrac{1-e}{1+e}$

C. $\dfrac{1+e}{1-e}$

D. $\dfrac{e+1}{e-1}$

Answer: B::C

> ▶ **Watch Video Solution**

4. If foci of $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ coincide with the foci of

$\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ and eccentricity of the hyperbola is

3. then

A. $a^2 + b^2 = 9$
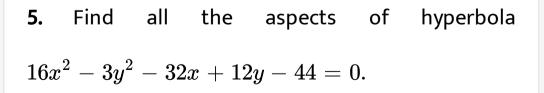
B. there is no directrix circle to the hyperbola

C. centre of the directrix circle is (0, 0)

D. Length of the latusrecum of the hyperbola

=16

Answer: A::B::D

▶ **Watch Video Solution**

5. Find all the aspects of hyperbola

$16x^2 - 3y^2 - 32x + 12y - 44 = 0.$

A. length of the transverse axis=$2\sqrt{3}$

B. length of the conjugate axis =8

C. centre at$(1, \; - \; 2)$

D. eccentricity $= \sqrt{19}$

Answer: A::B::C

6. If $ax + by + c = 0$ is a normal to hyperbola $xy = 1$, then (A) $a < 0, b < 0$ (B) $a < 0, b > 0$ (C) $a > 0, b > 0$ (D) $a > 0, b < 0$

A. $a > 0, b \geq 0$

B. $a > 0, b < 0$

C. $a < 0, b > 0$

D. $a < 0, b < 0$

Answer: B::C

▶ **Watch Video Solution**

7. If $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$ and $S(x_4, y_4)$ are 4 concyclic points on the rectangular hyperbola $xy = c^2$ the coordinates of the orthocentre of the $\triangle PQR$ are

A. $(x_4, \ -y_4)$

B. (x_4, y_4)

C. $(-x_4, y_4)$

D. $(-x_4, \ -y_4)$

Answer: D

▶ **Watch Video Solution**

8. The line $y = x + 5$ touches
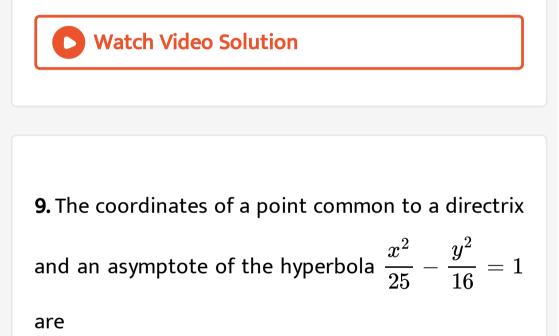
A. the parabola $y^2 = 20x$

B. the ellipse $9x^2 + 16y^2 = 144$

C. the hyperbola $\dfrac{x^2}{29} - \dfrac{y^2}{4} = 1$

D. the circle $x^2 + y^2 = 25$

Answer: A::B::C

▶ **Watch Video Solution**

9. The coordinates of a point common to a directrix and an asymptote of the hyperbola $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$ are

A. $\left(\dfrac{25}{\sqrt{41}}, \dfrac{20}{\sqrt{41}} \right)$

B. $\left(\dfrac{-25}{\sqrt{41}}, \dfrac{-20}{\sqrt{41}} \right)$

C. $\left(\dfrac{25}{3}, \dfrac{20}{3}\right)$

D. $\left(\dfrac{-25}{3}, \dfrac{-20}{3}\right)$

Answer: A::B

▶ **Watch Video Solution**

10. If $(5, 12) and (24, 7)$ are the foci of a hyperbola passing through the origin, then $e = \dfrac{\sqrt{386}}{12}$ (b) $e = \dfrac{\sqrt{386}}{13}$ $LR = \dfrac{121}{6}$ (d) $LR = \dfrac{121}{3}$

A. $e = \dfrac{\sqrt{386}}{12}$

B. $e = \dfrac{\sqrt{386}}{13}$

C. latusrectum $= \dfrac{121}{3}$

D. latusrectum $= \dfrac{121}{6}$

Answer: A::D

11. For hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, let n be the number of points on the plane through which perpendicular tangents are drawn.

A. If n=1, then e=$\sqrt{2}$

B. if $n > 1$, then $0 < e < \sqrt{2}$

C. if n=0, then $e > \sqrt{2}$

D. None of these

Answer: A::B::C

12. Which of the following equations in parametric form can represent a hyperbola, where t'isa parameter (B) Xt-0 & y -1-0 t/ (D) none of these

Wehsite: www.raoiit.com

A. $x = \dfrac{a}{2}\left(t + \dfrac{1}{t}\right)$ and $y = \dfrac{b}{2}\left(t - \dfrac{1}{t}\right)$

B. $\dfrac{tx}{a} - \dfrac{y}{b} + t = 0$ and $\dfrac{x}{a} + \dfrac{ty}{b} - 1 = 0$

C. $x = e^t + e^{-t}$ and $y = e^t - e^{-t}$

D. $x^2 - 6 = 2\cos t$ and $y^2 + 2 = 4\cos^2\left(\dfrac{t}{2}\right)$

Answer: A::C::D

> ▶ **Watch Video Solution**

13. Length of common tangents to the hyperbolas

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$ is

   A. $y = x + \sqrt{a^2 - b^2}$

   B. $y = x - \sqrt{a^2 - b^2}$

C. $y = -x + \sqrt{a^2 - b^2}$

D. $y = -x - \sqrt{a^2 - b^2}$

Answer: A::B::C::D

14. Given ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{17} = 1$ and the hyperbola $\dfrac{x^2}{144} - \dfrac{y^2}{81} = \dfrac{1}{25}$, if the ordinate of one of the points of intersection is produced to cut asymptote at P, then which of the following is true?

A. They have the same foci

B. Square of the ordinate of point of intersection is $\dfrac{63}{25}$

C. Sum of the squares of coordinate of P is 16

D. P lies on the auxiliary circle formed by ellipse

Answer: A::B::C::D

▶ **Watch Video Solution**

---

15. Solutions of the differential equation $\left(1 - x^2\right) \dfrac{dy}{dx} + xy = ax$ where a $\in$ R is

A. a conic which is an ellipse

B. centre of the conic is $(0, a)$

C. length of one of the principal axes is 1

D. length of one of the principal axes is equal to

2

Answer: A::B::D

> ▶ **Watch Video Solution**

Exercise Passage Based Questions

1. the graph of the conic $x^2 - (y - 1)^2 = 1$ has one tangent line with positive slope that passes

through the origin . The point of the tangency being (a, b) then find the value of $\sin^{-1}\left(\dfrac{a}{b}\right)$

A. $\dfrac{5\pi}{12}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{\pi}{4}$

D. $\dfrac{\pi}{3}$

Answer: C

▶ **Watch Video Solution**

2. The graph of the conic $x^2 - (y - 1)^2 = 1$ has one tangent line with positive slope that passes through the origin. The point of tangency being (a, b).

Q. Length of the latusrectum of the conic is

   A. 1

   B. $\sqrt{2}$

   C. 2

   D. 4

Answer: C

3. The graph of the conic $x^2 - (y-1)^2 = 1$ has one tangent line with positive slope that passes through the origin. The point of tangency being (a, b).

Q. If e be the eccentricity of the conic, then the value of $(1 + e^2 + e^4)$ is

A. 3

B. 7

C. $\dfrac{7}{4}$

D. 21

Answer: B

4. If the sum of slopes of concurrent normals to the curve xy = 4 is equal to the sum of ordinates of conormal points then locus of P is

A. $x^2 = 2y$

B. $x^2 = 4y$

C. $x^2 = 6y$

D. $x^2 = 8y$

Answer: B

5. A point P moves such that the sum of the slopes of the normals drawn from it to the hyperbola $xy = 4$ is equal to the sum of the ordinates of feet of normals. The locus of P is a curve C.

Q.If the tangent to the curve C cuts the coordinate axes at A and B, then , the locus of the middle point of AB is

A. $x^2 + 2y = 0$

B. $x^2 = y$

C. $2x^2 + y = 0$

D. $x^2 = 2y$

Answer: C

▶ **Watch Video Solution**

6. A point P moves such that the sum of the slopes of the normals drawn from it to the hyperbola $xy = 4$ is equal to the sum of the ordinates of feet of normals. The locus of P is a curve C.

Q. The area of the equilateral triangle inscribed in

the curve C having one vertex as the vertex of curve

C is

A. $8\sqrt{3}$ sq. units

B. $12\sqrt{3}$ sq. units

C. $27\sqrt{3}$ sq. units

D. $16\sqrt{3}$ sq. units

Answer: D

▶ **Watch Video Solution**

7. Let $P(x, y)$ is a variable point such that

$$\left| \sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2} \right| = 3$$

, which represents hyperbola. The eccentricity e' of the corresponding conjugate hyperbola is (A) $\dfrac{5}{3}$ (B) $\dfrac{4}{3}$ (C) $\dfrac{5}{4}$ (D) $\dfrac{3}{\sqrt{7}}$

A. $\dfrac{5}{4}$

B. $\dfrac{4}{3}$

C. $\dfrac{5}{3}$

D. $\dfrac{3}{2}$

Answer: C

▶ **Watch Video Solution**

8. Let P(x, y) be a variable point such that

$$\left|\sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2}\right| = 4$$

which represents a hyperbola.

Q. Locus of point of intersection of two perpendicular tangents to the hyperbola is

A. $(x-3)^2 + \left(y - \dfrac{7}{2}\right)^2 = \dfrac{1}{4}$

B. $(x-3)^2 + \left(y - \dfrac{7}{2}\right)^2 = \dfrac{3}{4}$

C. $(x-3)^2 + \left(y - \dfrac{7}{2}\right)^2 = \dfrac{5}{4}$

D. $(x-3)^2 + \left(y - \dfrac{7}{2}\right)^2 = \dfrac{7}{4}$

Answer: D

9. Let P(x, y) be a variable point such that

$$\left| \sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2} \right| = 4$$

which represents a hyperbola.

Q. If origin is shifted to point $\left(3, \dfrac{7}{2} \right)$ and axes are

rotated in anticlockwise through an angle θ, so that

the equation of hyperbola reduces to its standard

form $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then θ equals

A. $\tan^{-1}\left(\dfrac{4}{3} \right)$

B. $\tan^{-1}\left(\dfrac{3}{4} \right)$

C. $\tan^{-1}\left(\dfrac{5}{4} \right)$

D. $\tan^{-1}\left(\dfrac{4}{5}\right)$

Answer: B

10. Let $P(\theta_1)$ and $Q(\theta_2)$ are the extremities of any focal chord of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ whose eccentricity is e. Let θ be the angle between its asymptotes. Tangents are drawn to the hyperbola at some arbitrary points R. These tangent meet the coordinate axes at the points A and B respectively. The rectangle OABC (O being the origin) is

completedm, then

Q.Locus of point C is

A. $\dfrac{b^2}{x^2} - \dfrac{a^2}{y^2} = 1$

B. $\dfrac{b^2}{x^2} + \dfrac{a^2}{y^2} = 1$

C. $\dfrac{a^2}{x^2} - \dfrac{b^2}{y^2} = 1$

D. $\dfrac{a^2}{x^2} + \dfrac{b^2}{y^2} = 1$

Answer: D

▶ **View Text Solution**

11. Let $P(\theta_1)$ and $Q(\theta_2)$ are the extremities of any focal chord of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ whose eccentricity is e. Let θ be the angle between its asymptotes. Tangents are drawn to the hyperbola at some arbitrary points R. These tangent meet the coordinate axes at the points A and B respectively. The rectangle OABC (O being the origin) is completedm, then

Q. If $\cos^2\left(\dfrac{\theta_1 + \theta_2}{2}\right) = \lambda \cos^2\left(\dfrac{\theta_1 - \theta_2}{2}\right)$, then λ is equal to

A. $\dfrac{a^2 + b^2}{a^2}$

B. $\dfrac{a^2 + b^2}{b^2}$

C. $\dfrac{a^2 + b^2}{ab}$

D. $\dfrac{a^2 + b^2}{2ab}$
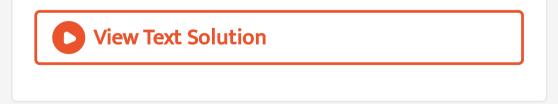
Answer: A

▶ **View Text Solution**

---

12. Let $P(\theta_1)$ and $Q(\theta_2)$ are the extremities of any focal chord of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ whose eccentricity is e. Let θ be the angle between its asymptotes. Tangents are drawn to the hyperbola at some arbitrary points R. These tangent meet the coordinate axes at the points A and B respectively.

The rectangle OABC (O being the origin) is completedm, then

Q. The value of $\cos\left(\dfrac{\theta}{2}\right)$ is

A. $\dfrac{1}{2e}$

B. $\dfrac{1}{e}$

C. $\dfrac{1}{e^2}$

D. $\dfrac{1}{2e^2}$

Answer: B

▶ View Text Solution

13. The vertices of $\triangle ABC$ lie on a rectangular hyperbola such that the orhtocentre of the triangle is (23) and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. The two perpendicular tangents of the hyperbola intersect at the point (1, 1). Q. The equation of the asymptotes is

A. $xy - 1 = y - x$

B. $xy + 1 = x + y$

C. $xy - 1 = x - y$

D. $xy + 1 = -x - y$

Answer: B

14. The vertices of $\triangle ABC$ lie on a rectangular hyperbola such that the orhtocentre of the triangle is (23) and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. The two perpendicular tangents of the hyperbola intersect at the point (1, 1). Q. The equation of the rectangular hyperbola is

A. $xy - 5 = y - x$

B. $xy - 1 = x + y$

C. $xy = x + y + 1$

D. $xy - 11 = -x - y$

Answer: C

▶ View Text Solution

15. The vertices of $\triangle ABC$ lie on a rectangular hyperbola such that the orhtocentre of the triangle is (23) and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. The two perpendicular tangents of the hyperbola

intersect at the point (1, 1). Q. The number of real tangents that can be drawn from the point (1, 1) to the rectangular hyperbola is

A. 0

B. 2

C. 3

D. 4

Answer: B

▶ View Text Solution

Exercise Single Integer Answer Type Questions

1. The ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and the hyperbola $\dfrac{x^2}{A^2} - \dfrac{y^2}{B^2} = 1$ are given to be confocal and length of mirror axis of the ellipse is same as the conjugate axis of the hyperbola. If e_1 and e_2 represents the eccentricities of ellipse and hyperbola respectively, then the value of $e_1^{-2} + e_1^{-2}$ is

2. If abscissa of orthocentre of a triangle inscribed in a rectangular hyperbola $xy = 4$ is $\dfrac{1}{2}$, then the ordinate of orthocentre of triangle is

3. Normals drawn to the hyperbola $xy = 2$ at the point $P(t_1)$ meets the hyperbola again at $Q(t_2)$, then minimum distance between the point P and Q is

4. The normal at P to a hyperbola of eccentricity $\dfrac{3}{2\sqrt{2}}$ intersects the transverse and conjugate axes

at M and N respectively. The locus of mid-point of MN is a hyperbola, then its eccentricity.

5. If radii of director circle of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ are in the ratio $1:3$ and $4e_1^2 - e_2^2 = \lambda$, where e_1 and e_2 are the eccetricities of ellipse and hyperbola respectively, then the value of λ is

6. The shortest distance between the curves $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ and $4x^2 + 4y^2 = a^2 (b > a)$ is f(a, b), then the value of $f(4, 6) + f(2, 3)$ is

7. ABC is a triangle such that $\angle ABC = 2\angle BAC$. If AB is fixed and locus of C is a hyperbola, then the eccentricity of the hyperbola is

8. Point P lie on $2xy = 1$. A triangle is contructed by P, S and S' (where S and S' are foci). The locus of ex-centre opposite S (S and P lie in first quandrant) is $(x + py)^2 = (\sqrt{2} - 1)^2 (x - y)^2 + q$, then the value of $p + q$ is

9. Chords of the circle $x^2 + y^2 = 4$, touch the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{16} = 1$.The locus of their middle-points is the curve $(x^2 + y^2)^2 = \lambda x^2 - 16y^2$, then the value of λ is

10. nd are inclined at avgicsTangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined atv angle θ and ϕ to the x-axis.If $\tan\theta . \tan\phi = 2,$ prove that $\beta^2 = 2\alpha^2 - 7.$

> ▶ **Watch Video Solution**

Exercise Statement I And Ii Type Questions

1. Statement-I $\dfrac{5}{3}$ and $\dfrac{5}{4}$ are the eccentricities of two conjugate hyperbolas.

Statement-II If e_1 and e_2 are the eccentricities of two conjugate hyperbolas, then $e_1 e_2 > 1$.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: B

▶ **Watch Video Solution**

2. Statement-I A hyperbola and its conjugate hyperbola have the same asymptotes.

Statement-II The difference between the second degree curve and pair of asymptotes is constant.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of

Statement-I.

B. Statement-I is true, Statement-II is also true,

Statement-II is not the correct explanation of

Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: A

▶ **Watch Video Solution**

3. Statement-I The equation of the directrix circle to the hyperbola $5x^2 - 4y^2 = 20$ is $x^2 + y^2 = 1$.

Statement-II Directrix circle is the locus of the point of intersection of perpendicular tangents.

  A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

  B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

  C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: D

▶ Watch Video Solution

---

4. Statement-I Two tangents are drawn from a point on the circle $x^2 + y^2 = 9$ to the hyperbola $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$, then the angle between tangnets is $\dfrac{\pi}{2}$.

Statement-II $x^2 + y^2 = 9$ is the directrix circle of $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: A

▶ Watch Video Solution

5. Statement-I If eccentricity of a hyperbola is 2, then eccentricity of its conjugate hyperbola is $\dfrac{2}{\sqrt{3}}$.

Statement-II if e and e_1 are the eccentricities of two conjugate hyperbolas, then $ee_1 > 1$.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: B

▶ **Watch Video Solution**

6. Statement-I The line $4x - 5y = 0$ will not meet the hyperbola $16x^2 - 25y^2 = 400$.

Statement-II The line $4x - 5y = 0$ is an asymptote ot the hyperbola.

A. Statement-I is true, Statement-II is also true, Statement-II is the correct explanation of

Statement-I.

B. Statement-I is true, Statement-II is also true,

Statement-II is not the correct explanation of

Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: A

▶ **Watch Video Solution**

7. Statement-I The point (5, -3) inside the hyperbola

$3x^2 - 5y^2 + 1 = 0.$

Statement-II The point (x_1, y_1) inside the

hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, then $\dfrac{x_1^2}{a^2} + \dfrac{y_1^2}{b^2} - 1 < 0.$

A. Statement-I is true, Statement-II is also true,

Statement-II is the correct explanation of

Statement-I.

B. Statement-I is true, Statement-II is also true,

Statement-II is not the correct explanation of

Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: C

▶ **Watch Video Solution**

8. Statement-I A hyperbola whose asymptotes include $\dfrac{\pi}{3}$ is said to be equilateral hyperbola.

Statement-II The eccentricity of an equilateral hyperbola is $\sqrt{2}$.

A. Statement-I is true, Statement-II is also true,

Statement-II is the correct explanation of

Statement-I.

B. Statement-I is true, Statement-II is also true, Statement-II is not the correct explanation of Statement-I.

C. Statement-I is true, Statement-II is false.

D. Statement-I is false, Statement-II is true

Answer: D

▶ **Watch Video Solution**

Exercise Subjective Type Questions

1. Given the base of a triangle and the ratio of the tangent of half the base angles .Show that the vertex moves on a hyperbola whose foci are the extremities of a diameter

2. A, B, C are three points on the rectangular hyperbola $xy = c^2$, The area of the triangle formed by the tangents at A, B and C.

3. If a hyperbola be rectangular, and its equation be $xy = c^2$, prove that the locus of the middle points of chords of constant length 2d is $(x^2 + y^2)(xy - c^2) = d^2 xy.$

4. If four points be taken on a rectangular hyperbola such that the chord joining any two is perpendicular to the chord joining the other two, and if $\alpha, \beta, \gamma, \delta$ be the inclinations to either asymptotes of the straight lines joining these

points to the centre, then $\tan\alpha\tan\beta\tan\gamma\tan\delta$ is

equal to

▶ **Watch Video Solution**

5. A circle cuts two perpendicular lines so that each

intercept is of given length. The locus of the centre

of the circle is conic whose eccentricity is

▶ **Watch Video Solution**

6. Let the tangent at a point P on the ellipse meet

the major axis at B and the ordinate from it meet

the major axis at A. If Q is a point on the AP such that $AQ = AB$, prove that the locus of Q is a hyperbola. Find the asymptotes of this hyperbola.

7. A series of hyperbola are drawn having a common transverse axis of length 2a. Prove that the locus of point P on each hyperbola, such that its distance from the transverse axis is equal to its distance from an asymptote, is the curve $\left(x^2 - y^2\right)^2 = \lambda x^2 \left(x^2 - a^2\right)$, then λ equals

1. The locus a point $P(\alpha, \beta)$ moving under the condition that the line $y = \alpha x + \beta$ is a tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is (A) a parabola (B) an ellipse (C) a hyperbola (D) a circle

A. an ellipse

B. a circle

C. a parabola

D. a hyperbola

Answer: D

2. Let a hyperbola passes through the focus of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1.$ The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then

A. the equation of hyperbola is $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

B. the equation of hyperbola is $\dfrac{x^2}{9} - \dfrac{y^2}{25} = 1$

C. focus of hyperbola is $(5, 0)$

D. vertex of hyperbola is $(5\sqrt{3}, 0)$

Answer: A::C

▶ **Watch Video Solution**

3. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

A. $x^2 \cos ec^2\theta - y^2 \sec^2\theta = 1$

B. $x^2 \sec^2\theta - y^2 \cos ec^2\theta = 1$

C. $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$

D. $x^2 \sin^2 \theta - y^2 \cos^2 \theta = 1$

Answer: A

▶ **Watch Video Solution**

4. Two braches of a hyperbola

    A. have a common tangent

    B. have a common normal

    C. do not have a common tangent

    D. do not have a common normal

Answer: B::C

5. For the hyperbola $\dfrac{x^2}{\cos^2 \alpha} - \dfrac{y^2}{\sin^2 \alpha} = 1$, which of the following remains constant when α varies?

(1) eccentricity (2) directrix (3) abscissae of vertices

(4) abscissae of foci

A. abscissae of vertices

B. abscissae of foci

C. eccentricity

D. directrix

Answer: B

6. Consider a branch of the hypebola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is (A) $1 - \sqrt{\dfrac{2}{3}}$ (B) $\sqrt{\dfrac{3}{2}} - 1$ (C) $1 + \sqrt{\dfrac{2}{3}}$ (D) $\sqrt{\dfrac{3}{2}} + 1$

A. $1 - \sqrt{\dfrac{2}{3}}$

B. $\sqrt{\dfrac{3}{2}} - 1$

C. $1 + \sqrt{\dfrac{2}{3}}$

D. $\sqrt{\dfrac{3}{2}} + 1$

Answer: B

> ▶ **Watch Video Solution**

7. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then

A. equation of ellipse $x^2 + 2y^2 = 2$

B. the foci of the ellipse are $(\pm 1, 0)$

C. equation of ellipse is $x^2 + 2y^2 = 4$

D. the foci of ellipse are $(\pm \sqrt{2}, 0)$

Answer: A::B

▶ **Watch Video Solution**

---

8. The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ intersect at the points A and B. The equation of a common tangent with positive slope to the circle as well as to the hyperbola, is

A. $2x - \sqrt{5}y - 20 = 0$

B. $2x - \sqrt{5}y + 4 = 0$

C. $3x - 4y + 8 = 0$

D. $4x - 3y + 4 = 0$

Answer: B

▶ **Watch Video Solution**

9. The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ intersect at the points A and B Equation of the circle with AB as its diameter is

A. $x^2 + y^2 - 12x + 24 = 0$

B. $x^2 + y^2 + 12x + 24 = 0$

C. $x^2 + y^2 + 24x - 12 = 0$

D. $x^2 + y^2 - 24x - 12 = 0$

Answer: A

▶ **Watch Video Solution**

10. The line $2x + y = 1$ is tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is

11. Let P(6,3) be a point on the hyperbola parabola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ If the normal at the point intersects the x-axis at (9,0), then the eccentricity of the hyperbola is

A. $\sqrt{\frac{5}{2}}$

B. $\sqrt{\frac{3}{2}}$

C. $\sqrt{2}$

D. $\sqrt{3}$

Answer: B

12. let the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be reciprocal to that of the ellipse $x^2 + 4y^2 = 4$. if the hyperbola passes through a focus of the ellipse then: (a) the equation of the hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$ (b) a focus of the hyperbola is $(2, 0)$ (c) the eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$ (d) the equation of the hyperbola is $x^2 - 3y^2 = 3$

A. the equation of hyperbola is $\frac{x^2}{3} - \frac{y^2}{2} = 1$

B. a focus of the hyperbola is $(2, 0)$

C. the eccentricity of the hyperbola is $\sqrt{\dfrac{5}{2}}$

D. the equation of the hyperbola is

$$x^2 - 3y^2 = 3$$

Answer: B::D

▶ **Watch Video Solution**

---

13. Tangents are drawn to the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ parallet to the sraight line $2x - y = 1$. The points of contact of the tangents on the hyperbola are (A) $\left(\dfrac{2}{2\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ (B)

$$\left(-\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$ (C) $$\left(3\sqrt{3}, -2\sqrt{2}\right)$$ (D)
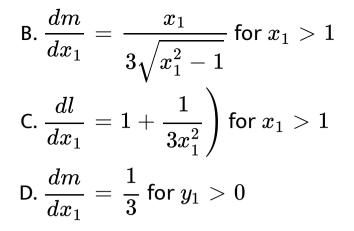
$$\left(-3\sqrt{3}, 2\sqrt{2}\right)$$

A. $$\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

B. $$\left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

C. $$\left(3\sqrt{3}, -2\sqrt{2}\right)$$

D. $$\left(-3\sqrt{3}, 2\sqrt{2}\right)$$

Answer: A::B

▶ **Watch Video Solution**

14. Consider the hyperbola $H: x^2 - y^2 = 1$ and a circle S with centre $N(x_2, 0)$ Suppose that H and S touch each other at a point $(P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$ The common tangent to H and S at P intersects the x-axis at point M. If (l,m) is the centroid of the triangle $\triangle PMN$ then the correct expression is (A) $\dfrac{dl}{dx_1} = 1 - \dfrac{1}{3x_1^2}$ for $x_1 > 1$ (B)

$\dfrac{dm}{dx_1} = \dfrac{x_!}{3\left(\sqrt{x_1^2} - 1\right)}\Bigg)$ f or $x_1 > 1$ (C)

$\dfrac{dl}{dx_1} = 1 + \dfrac{1}{3x_1^2}$ f or $x_1 > 1$ (D)

$\dfrac{dm}{dy_1} = \dfrac{1}{3}$ f or $y_1 > 0$

A. $\dfrac{dl}{dx_1} = 1 - \dfrac{1}{3x_1^2}$ for $x_1 > 1$

B. $\dfrac{dm}{dx_1} = \dfrac{x_1}{3\sqrt{x_1^2 - 1}}$ for $x_1 > 1$

C. $\dfrac{dl}{dx_1} = 1 + \dfrac{1}{3x_1^2}$ for $x_1 > 1$

D. $\dfrac{dm}{dx_1} = \dfrac{1}{3}$ for $y_1 > 0$

Answer: A::B::D

> ▶ **Watch Video Solution**

---

15. The eccentricity of the hyperbola whose latuscrectum is 8 and conjugate axis is equal to half the distance between the foci, is

A. $\dfrac{2}{\sqrt{3}}$

B. $\sqrt{3}$

C. $\dfrac{4}{3}$

D. $\dfrac{4}{\sqrt{3}}$

Answer: A

16. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point : $(\sqrt{3}, \sqrt{2})$ (2) $(-\sqrt{2}, -\sqrt{3})$ (3) $(3\sqrt{2}, 2\sqrt{3})$ (4) $(2\sqrt{2}, 3\sqrt{3})$

A. $\left(-\sqrt{2}, -\sqrt{3}\right)$

B. $\left(3\sqrt{2}, 2\sqrt{3}\right)$

C. $\left(2\sqrt{2}, 3\sqrt{3}\right)$

D. $\left(\sqrt{3}, \sqrt{2}\right)$

Answer: C

17. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{16} = 1$ then which of the following CANNOT be sides of a right angled triangle? $a, 4, 2$

(b) $a, 4, 1$ $2a, 4, 1$ (d) $2a, 8, 1$

A. $2a, 8, 1$

B. $a, 4, 1$

C. $a, 4, 2$

D. $2a, 4, 1$

Answer: A::B::C

▶ **Watch Video Solution**