MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

LIMITS

Examples

1. $\lim_{x \to 2} \dfrac{x^6 - 24x - 16}{x^3 + 2x - 12}$

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2. $\lim_{x \to 0} \dfrac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$, is

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3. Evaluate $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x - 1}$

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4. Evaluate $\lim_{x \to 1} \frac{x^3 - x^2 \log x + \log x - 1}{x^2 - 1}$

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5. Evaluate $\lim_{h \to 0} \frac{\sqrt{x + h} - \sqrt{x}}{h}$

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6. The value of $\lim_{x \to 2a} \frac{\sqrt{x - 2a} + \sqrt{x} = \sqrt{2a}}{\sqrt{x^2 - 4a^2}}$ is

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7. Evaluate $\lim_{x \to 2} \frac{x^3 - 2^3}{x - 2}$

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8. Evaluate $\lim_{x \to 1} \frac{x + x^2 + \ldots\ldots + x^n - n}{x - 1}$

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9. The value of $\lim_{x \to 1} \frac{y^3}{x^3 - y^2 - 1}$ as $(x, y) \to (1, 0)$ along the line $y = x - 1$ is

A. 1

B. -1

C. 0

D. Doesn't exist

Answer:

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10. Evaluate $\lim_{x \to \infty} \frac{x^2+5}{x^2+4x+3}$

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11. Evaluate $\lim_{n \to \infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$

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12. $\lim_{x \to \infty} \left(x-\sqrt{x^2+x}\right)$

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13. $\lim_{x \to \infty} \left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right) =$

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14. Evaluate $\lim_{x \to \infty} \frac{ax^2 + b}{x + 1}$ when $a \geq 0$

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15. If $\lim_{x \to \infty} \left(\frac{x^2 + 1}{x + 1} - ax - b \right) = 0$, find the values of a and b.

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16. If $\lim_{x \to \infty} \left\{ \frac{x^2 + 1}{x + 1} - ax - b \right\} = 2$, then

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17. If $\lim_{x \to \infty} \left(\frac{x^2 + 1}{x + 1} - ax - b \right) = \infty$ find a and b.

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18. Let $S_n = 1 + 2 + 3 + \quad + n$ and $P_n = \frac{S_2}{S_2 - 1}\frac{\dot{S}_3}{S_3 - 1}\frac{\dot{S}_4}{S_4 - 1}\frac{S_n}{S_n - 1}$

Where $n \in N, (n \geq 2)\cdot$ Then $(\lim)_{x \overrightarrow{\infty}} P_n =$_ __ _

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19. If $\lim_{n \to \infty} \frac{1}{(\sin^{-1} x)^n + 1} = 1,$ x lies in the interval

A. $(-\sin 1, \sin 1)$

B. $(-1, 1)$

C. $(0, 1)$

D. $(-1, 0)$

Answer: A::C

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20. Evaluate $\lim_{x \to o} \frac{1 - \cos x}{x^2}$

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21. Solve $\lim_{x \to 0} \frac{1 - \cos(1 - \cos x)}{\sin^4 x}$

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22. Evaluate $\lim_{x \to \infty} \left(2^{-x} \sin(2^x)\right)$

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23. Evaluate $\lim_{x \to \infty} e^x \sin(d / e^x)$.

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24. Evaluate $\lim_{x \to \infty} \sqrt{\frac{x - \sin x}{x + \cos^2 x}}$

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25. the value of $\lim_{x \to y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$ equals

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26. $\lim_{x \to \infty} ((x+5)\tan^{-1}(x+5) - (x+1)\tan^{-1}(x+1)$ is equal to

A. π

B. 2π

C. $\frac{\pi}{2}$

D. None of these

Answer: B

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27. Evaluate $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$

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28. Let $a = \min\left\{x^2 + 2x + 3, x \in R\right\}$ and $b = \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2}$ then the value of $\sum_{r=0}^{n} a^r \cdot b^{n-r}$ is :

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29. $\lim_{n \to \infty} cos\frac{x}{2} \cdot cos\frac{x}{4} \cdot cos\frac{x}{8} \ldots\ldots cos\frac{x}{2^n}$

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30. Evaluate $\lim_{x \to a} \frac{\log\{1 + (x - a)\}}{(x - a)}$

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31. Evaluate $\lim_{h \to 0} \frac{\log_{10}(1 + h)}{h}$

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32. Evaluate $\lim_{x \to 0} \frac{\log(5+x) - \log(5-x)}{x}$

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33. The value of $\lim_{h \to 0} \frac{In(1+2h) - 2\ln(1+h)}{h^2}$, is

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34. Solve $\lim_{x \to \infty} \left\{x - x^2 . \log\left(1 + \frac{1}{x}\right)\right\}$

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35. Evaluate $\lim_{x \to 0} \frac{a^x - b^x}{x}$

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36. Evaluate $\lim_{x \to 0} \frac{(ab)^x - a^x - b^x + 1}{x^2}$

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37. $\lim_{x \to 0} \frac{e^{\tan x} - e^x}{\tan x - x} =$

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38. Evaluate $\lim_{x \to 0} \frac{ae^x - b}{x} = 2$. Find a and b

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39. If $L = \lim_{x \to 0} \frac{a \sin x - bx + cx^2 + x^3}{2x^2 \log(1 + x) - 2x^3 + x^4}$ exists and is finie then a=, b=, c= L=

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40. Find the constant a, b and c such that $\lim_{x \to 0} \frac{axe^x - b\log(1 + x) + cxe^{-x}}{x^2 \sin x} = 2$

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41. Evaluate $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^{x}$

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42. $\lim_{x \to 1} \left(\log_3 3x\right)^{\log_x 3} =$

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43. Evaluate $\lim_{x \to a} \left(2 - \frac{a}{x}\right)^{\tan \frac{\pi x}{2a}}$

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44. Evaluate $\lim_{x \to \infty} \left(\frac{x + 6}{x + 1}\right)^{x + 4}$

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45. Evaluate $\lim_{x \to 0} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$

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46. Evaluate $\lim_{x \to 0} \left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{\frac{1}{x}}$

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47. Evaluate $\lim_{x \to 0} \left(\frac{1 + 5x^2}{1 + 3x^2}\right)^{1/x^2}$

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48. Evaluate $\lim_{x \to \infty} \left(\frac{x - 3}{x + 2}\right)^x$

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49. The value of $\lim_{x \to 0} \left\{\sin^2\left(\frac{\pi}{2 - ax}\right)\right\}^{\sec^2 \frac{\pi}{2 - bx}}$ is equal to

A. $e^{-a/b}$

B. a^{-a^2/b^2}

C. $a^{2a/b}$

D. $e^{4a/b}$

Answer: B

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50. Evaluate: $(\lim)_{x \to \frac{7}{2}} \left(2x^2 - 9x + 8\right)^{\cot(2x - 7)}$

A. $e^{5/2}$

B. $e^{-5/2}$

C. $e^{7/2}$

D. $e^{3/2}$

Answer: A::C

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51. The value of $\lim_{x \to 1} \left(\tan\left(\frac{\pi}{4} + \log x\right)\right)^{\frac{1}{\log x}}$ is equal to

A. e

B. e^{-1}

C. e^{2}

D. e^{-2}

Answer: A::C

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52. $\lim_{x \to 0} \left(\sin\frac{x}{m} + \cos\frac{3x}{m}\right)^{(2m)/x}$ is

A. e

B. 1

C. e^{-1}

D. e^2

Answer: A::C::D

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53. The value of $\lim_{n \to \infty} \left(\frac{a - 1 + \sqrt[n]{b}}{a}\right)^n (a > 0, b > 0)$ is equal to

A. $\sqrt[a]{b}$

B. $\sqrt[b]{a}$

C. $\sqrt{b}$

D. $\sqrt{a}$

Answer: A::C

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54. Evaluate $\lim_{x \to 0} |x|^{\sin x}$

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55. Evaluate $\lim_{n \to \infty} (\pi n)^{2/n}$

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56. evaluate $\lim_{n \to \infty} \left(\frac{e^n}{\pi}\right)^{\frac{1}{n}}$

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57. Evaluate $\lim_{x \to 0} (\cos ecx)^x$.

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58. Slove $\lim_{x \to 0} \frac{(1+x)^{1/x} - e}{x}$

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59. Evaluate the left-and right-hand limits of the function

$f(x) = \left\{ \frac{|x-4|}{x-4}, x \neq 40, x = 4 at x = 4 \right.$

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60. Let $f(x) = \begin{cases} 5x - 4, & 0 < x \leq 1 \\ 4x^3 - 3x, & 1 < x < 2. \end{cases}$

Find $\lim_{x \to 1} f(x)$.

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61. Show that $\lim_{x \to 0^-} \left(\frac{e^{1/x} - 1}{e^{1/x} + 1} \right)$ does not exist.

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62. $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$, is

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63. Solve (i) $\lim_{x \to 1} \sin x$ (ii) $\lim_{x \to 0^+} \left[\frac{\sin x}{x}\right]$

(iii) $\lim_{x \to 0^-} \left[\frac{\sin x}{x}\right]$

(where [.] denotes greatest integer function)

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64. Solve (i) $\lim_{x \to \infty} \tan^{-1} x$

(ii) $\lim_{x \to -\infty} \tan^{-1} x$

(where [.] denotes greatest integer function)

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65. Solve (i) $\lim_{x \to 0^+} \left[\frac{\tan x}{x}\right]$

(ii) $\lim_{x \to 0^-} \left[\frac{\tan x}{x}\right]$

(where [.] denotes greatest integer function)

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66. Solve (i) $\lim_{x \to 1^-} \sin\left(\sin^{-1} x\right)$ (ii) $\lim_{x \to \pi/2} \sin^{-1}(\sin x)$

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67. Solve (i) $\lim_{x \to 0} \cot x$

(ii) $\lim_{x \to +\infty} \cot^{-1} x$

(where $[.]$ denotes greatest integer function)

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68. Solve $\lim_{x \to 0} \left[\sin\frac{|x|}{x}\right]$, where $e[.]$ denotes greatest integer function.

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69. Solve $\lim_{x \to 0}\left[\frac{\sin|x|}{|x|}\right]$, where [.] denotes greatest integer function.

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70. $\lim_{x \to 0}\left[\frac{-2x}{\tan x}\right]$, where [.] denotes greatest integer function is

A. -1

B. 4

C. 5

D. None of these

Answer: A::C::D

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71. Evaluate $\lim_{x \to 0} x^3 \cos \frac{2}{x}$.

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72. Evaluate $\lim_{x \to \infty} \frac{x^2(2 + \sin^2 x)}{x + 100}$

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73. $\lim_{n \to \infty} \left(\frac{n}{n^2 + 1} + \frac{n}{n^2 + 2} + \frac{n}{n^2 + 3} + \dots \frac{n}{n^2 + n} \right)$

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74. The value of the $\lim_{x \to 0} \frac{x}{a} \left[\frac{b}{x} \right] (a \neq 0)(where[\cdot]$ denotes the greatest integer function) is

A. a

B. b

C. $\dfrac{b}{a}$

D. $1 - \dfrac{b}{a}$

Answer: A::C

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75. If [x] denotes the greatest integer less than or equal to x then

$$\lim_{n \to \infty} \frac{[x] + [2x] + [3x] + \ldots\ldots + [nx]}{n^2}$$

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76. $\lim (x \to 0) \left(\frac{1}{x^5} \int_0^5 e^{-t^2} dt - \frac{1}{x^4} + \frac{1}{3x^2} \right)$ is equal to

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77. Evaluate $\lim_{x \to 0} \frac{x - \left(\int_0^x \cos t dt \right)}{x^3 - 6x}$

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78. Evaluate $\lim_{x \to 0} \frac{\int_0^{x^2} \cos^2 t dt}{x \sin x}$

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79. $\lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2 + r^2}}$ equals

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80. The value of

$\lim_{n \to \infty} \left[\frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \ldots\ldots\ldots\ldots + \frac{n}{n^2 + n^2}\right]$, is

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81. $\lim_{n \to \infty} \frac{(n!)^{1/n}}{n}$ equals

A. $\frac{1}{e}$

B. e

C. e^2

D. $\frac{1}{e^2}$

Answer: A::C

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82. The value of $\lim_{x \to 0} \frac{x \sin(\sin \times) - \sin^2 x}{x^6}$ equals

A. $\frac{1}{6}$

B. $\frac{1}{12}$

C. $\frac{1}{18}$

D. $\frac{1}{24}$

Answer: A::B::C

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83. If $\lim_{x \to 0} \frac{\log_e \cot\left(\frac{\pi}{4} - K_1 x\right)}{\tan K_2 x} = 1$, then

A. $K_1 = K_2$

B. $2K_1 = K_2$

C. $K_1 = 2K_2$

D. $K_1 = 4K_2$

Answer: A::B

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84. Let

$f(\theta) = \frac{1}{\tan^2 \theta}\left\{(1 + \tan\theta)^3 + (2 + \tan\theta)^3 + \ldots\ldots + (10 + \tan\theta)^3\right\} - 10\,t$

Then, $\lim_{\theta \to \frac{\pi}{2}} f(\theta)$ is equal to

A. 170

B. 166

C. 165

D. None of these

Answer: A

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85. For positive integers k=1,2,3,....n,, let S_k denotes the area of $\triangle AOB_k$ such that $\angle AOB_k = \frac{k\pi}{2n}$, OA=1 and $OB_k = k$ The value of the $\lim_{n \to \infty} \frac{1}{n^2} \sum_{k-1}^{n} S_k$ is

A. $\frac{2}{\pi^2}$

B. $\frac{4}{\pi^2}$

C. $\frac{8}{\pi^2}$

D. $\frac{1}{(2\pi)^2}$

Answer: B

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86. If $S_n = \left(1 - \tan^4 \frac{\pi}{2^3}\right)\left(1 - \tan^4 \frac{\pi}{2^4}\right)\ldots\ldots\ldots\left(1 - \tan^4 \frac{\pi}{2^n}\right)$.

The value of $\lim_{n \to \infty} S_n$ is

A. $\frac{\pi^3}{4}$

B. $\frac{\pi^3}{16}$

C. $\frac{\pi^3}{32}$

D. $\frac{\pi^3}{256}$

Answer: B::C

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87. Evaluate: $(\lim)_{x \to 2} \frac{\sqrt{(x+7)} - 3\sqrt{(2x-3)}}{(x + 63 - 2(3x - 5)3)}$

A. $\frac{33}{23}$

B. $\frac{34}{23}$

C. $\frac{54}{25}$

D. None of these

Answer: B::C::D

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88. Let

$g(x) = \mid (f(x+\alpha), f(x+2a), f(x+3\alpha)), f(\alpha), f(2\alpha), f(3\alpha), (f'(\alpha), ($

, where alpha is a constant then $Lt_{x \to 0 \frac{g(x)}{x}} =$ (A) 0 (B) 1 (C) -1 (D) none of these

A. 0

B. 1

C. 2

D. 3

Answer: A::C::D

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89. Let $\lim_{x \to 1} \frac{x^a - ax + a - 1}{(x-1)^2} = f(a)$. The value of $f(101)$ equals

A. 5050

B. 5151

C. 4950

D. 101

Answer: A::B

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90. $(\lim)_{x \to 1} \frac{nx^{n+1} - (n+1)x^n + 1}{(e^x - e)\sin \pi x}, where n = 100,$ is equal to : $\frac{5050}{\pi e}$ (b) $\frac{100}{\pi e}$ (c) $-\frac{5050}{\pi e}$ (d) $-\frac{4950}{\pi e}$

A. $\frac{5050}{\pi e}$

B. $\frac{100}{\pi e}$

C. $-\dfrac{5050}{\pi e}$

D. $-\dfrac{4950}{\pi e}$

Answer: A

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91. The value of $\lim_{n \to \infty} \dfrac{1^2 . n + 2^2 . (n-1) + \ldots\ldots + n^2 . 1}{1^3 + 2^3 + \ldots\ldots + n^3}$ is equal to

A. $\dfrac{1}{3}$

B. $\dfrac{2}{3}$

C. $\dfrac{1}{2}$

D. $\dfrac{1}{6}$

Answer: A

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92. The $\lim_{x \to \infty} \frac{\cot^{-1}(x^{-a}\log_a x)}{\sec^{-1}(a^x \log_x a)}, a > 0, a \neq 1$ is equal to

A. 1

B. 0

C. $\frac{\pi}{2}$

D. Doesn't exist

Answer: A::B

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93. Suppose that a and $b(b \neq a)$ are real positive numbers, the value of

$\lim_{t \to 0}\left(\frac{b^{t+1} - a^{t+1}}{b - a}\right)^{1/t}$ has the is equal to

A. $\frac{aInb - bIna}{b - a}$

B. $\frac{bInb - aIna}{b - a}$

C. $bInb - aIna$

D. $\left(\frac{b^b}{a^a}\right)\left(\frac{1}{b-a}\right)$

Answer: D

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94. $\lim_{x \to \infty} \frac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^x\right)}$ is equal to

A. 1

B. 0

C. $\frac{\pi}{2}$

D. None of these

Answer: A

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95. $\lim_{n \to \infty} \cos\left(\pi\sqrt{n^2+n}\right)$ (where n is an integer) is equal to

A. 1

B. -1

C. 0

D. Doesn't exist

Answer:

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96. The value of $\lim_{x \to 0} \frac{(\tan(\{x\}-1))\sin\{x\}}{\{x\}(\{x\}-1)}$ is where $\{x\}$ denotes the fractional part function

A. 1

B. $\tan 1$

C. $\sin 1$

D. Doesn't exist

Answer: C::D

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97. $\lim_{x \to 0^-} \left(- In(\{x\} + |[x]|) \right)^{\{x\}}$ is equal to

A. 0

B. 1

C. In2

D. $In\frac{1}{2}$

Answer: A::B

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98. $\lim_{x \to \infty} \frac{2 + 2x + \sin 2x}{(2x + \sin 2x)e^{\sin x}}$ is :

A. 0

B.

C. -1

D. Non- existent

Answer: A

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99. The value of $\lim P(x \rightarrow 0)(\cos ax)^{\cos ec^2 bx}$ is

A. $e^{\left(-\frac{8b^2}{a^2}\right)}$

B. $e^{\left(-\frac{8a^2}{b^2}\right)}$

C. $e^{-\frac{a^2}{2b^2}}$

D. $e^{-\frac{b^2}{2a^2}}$

Answer: C

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100. $\lim_{n \rightarrow \infty} \sum_{r=1}^{n} \frac{r}{n^2+n+4}$ equals

A. 0

B. $1/3$

C. $1/2$

D. 1

Answer: A::B::C::D

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101. The value of $\lim_{n \to \infty}\left(\sqrt{n^2+n+1}-\left[\sqrt{n^2+n+1}\right]\right)$ where [.] denotes the greatest integer function is

A. 0

B. $1/2$

C. $2/3$

D. $1/4$

Answer: A::B

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102. $\lim_{x \to 1} \frac{\sin^2(x^3 + x^2 + x - 3)}{1 - \cos(x^2 - 4x + 3)}$ has the value equal to

A. 18

B. $9/2$

C. 9

D. None of these

Answer: A

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103. The graph of the function $y = f(x)$ has a unique tangent at the point $(e^a, 0)$ through which the graph passes then $\underline{\lim} \ (x \to e^a) \frac{\log_e\{1 + 7f(x)\} - \sin f(x)}{3f(x)}$

A. 1

B. 2

C. 7

D. None of these

Answer: B

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104. If $f(x) = \frac{3x^2 + ax + a + 1}{x^2 + x - 2}$, then which of the following can be correct $(\lim)_{x \to 1} f(x) e\xi sts a = -2$ $(\lim)_{x \to -2} f(x) e\xi sts a = 13$ $(\lim)_{x \to 1} f(x) = \frac{4}{3}$ $(\lim)_{x \to -2} f(x) = -\frac{1}{3}$

A. $\lim_{x \to 1} f(x) \text{exists} \Rightarrow a = -2$

B. $\lim_{x \to -2} f(x)$ exists $\Rightarrow a = 13$

C. $\lim_{x \to 1} f(x) = 4/3$

D. $\lim P(x \to -2) f(x) = -1/3$

Answer: A::C

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105. Let $f(x) = \begin{cases} \frac{\tan^2[x]}{x^2 - [x]^2} & \text{for} \quad x > 0 \\ 1 & \text{for} \quad x = 0 \\ \frac{1}{\sqrt{|x|\cot|x|}} & \text{for} \quad x < 0 \end{cases}$ where$[x]$ and $\{x\}$ denote respectively the greatest integer less than equal to x and the fractional part of x, then

A. $\lim_{x \to 0^+} f(x) = 1$

B. $\lim_{x \to 0^-} f(x) = 1$

C. $\cot^{-1}\left(\lim_{x \to 0^-} f(x)\right)^2 = 1$

D. None of the above

Answer: A

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106. Given that the derivative $f'(a)$ exists. Indicate which of the following statements(s) is/are always true?

A. $f'(a) = \lim_{h \to a} \frac{f(h) - f(a)}{h - a}$

B. $f'(a) = \lim_{h \to 0} \frac{f(a) - f(a-h)}{h}$

C. $f'(a) = \lim_{t \to 0} \frac{f(a + 2t) - f(a)}{t}$

D. $f'(a) = \lim_{t \to 0} \frac{f(a + 2t) - t(a + t)}{2t}$

Answer: A::C::D

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107. Let $f(x) = \lim_{n \to \infty} \sum_{r=0}^{n-1} \frac{x}{(rx + 1)\{(r + 1)x + 1\}}$,then

A. $f(0) = 0$

B. $f(0) = x$

C. $f(0^{+}) = 1$

D. $f(0^{-}) = 1$

Answer: A

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108. Let $f(x) = \begin{cases} e^{\{x^2\}} - 1 & x > 1 \\ \frac{\sin x - \tan x + \cos x - 1}{2x^2 + In(2+x) + \tan x} & x < 0 \\ 0 & x = 0 \end{cases}$

where { } represents fractional part function. Suppose lines L_1 and L_2 represent tangent and normal to curve $y = f(x)$ at $x = 0$. Consider the family of circles touching both the lines L_1 and L_2

Ratio of radii of two circles belonging to his family cutting each other orthogonally is

A. $2 + \sqrt{3}$

B. $\sqrt{3}$

C. $2 + \sqrt{2}$

D. $2 - \sqrt{2}$

Answer: B::C

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109. Let $f(x)=\begin{cases} e^{\{x^2\}}-1 & x>1 \\ \frac{\sin x-\tan x+\cos x-1}{2x^2+In(2+x)+\tan x} & x<0 \\ 0 & x=0 \end{cases}$

where { } represents fractional part function. Suppose lines L_1 and L_2 represent tangent and normal to curve $y=f(x)$ at $x=0$. Consider the family of circles touching both the lines L_1 and L_2

A circle having radius unity is inscribed in the triangle formed by L_1 and L_2 and a tangent to it. Then the minimum area of the triangle possible is

A. $3+\sqrt{2}$

B. $2+\sqrt{3}$

C. $3+2\sqrt{2}$

D. $3-2\sqrt{2}$

Answer: B::C

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110. Let $f(x)=\begin{cases} e^{\{x^2\}}-1 & x>1 \\ \frac{\sin x-\tan x+\cos x-1}{2x^2+In(2+x)+\tan x} & x<0 \\ 0 & x=0 \end{cases}$

where { } represents fractional part function. Suppose lines L_1 and L_2 represent tangent and normasl to curve $y=f(x)$ at $x=0$. Consider the family of circles touching both the lines L_1 and L_2

If centres of circles belonging to family having equal radii r joined, the area of figure formed is

A. $2r^2$

B. $4r^2$

C. $8r^2$

D. r^2

Answer: A::B::D

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111. Let A be $n \times n$ matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13}......a_{1n} \\ a_{21} & a_{22} & a_{23}...a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3}.\ a_{\text{nn}} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical column is a geometrical progression. It is known that each column in geometric progression have the same common ratio. Given that $a_{24} = 1, a_{42} = \frac{1}{8}$ and $a_{43} = \frac{3}{16}$

Let $S_n = \sum_{j=1}^{n} a_{4j}, \lim_{n \to \infty} \frac{S_n}{n^2}$ is equal to

A. $\frac{1}{4}$

B. $\frac{1}{8}$

C. $\frac{1}{16}$

D. $\frac{1}{32}$

Answer: D

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112. Let A be $n \times n$ matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13}\ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23}\ldots a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3}.\, a_{\text{nn}} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical column is a geometrical progression. It is known that each column in geometric progression have the same common ratio. Given that $a_{24} = 1, a_{42} = \frac{1}{8}$ and $a_{43} = \frac{3}{16}$

Let d_i be the common difference of the elements in with row then $\sum_{i=1}^{n} d_i$ is

A. n

B. $\frac{1}{2} - \frac{1}{2^{n+1}}$

C. $1 - \frac{1}{2^n}$

D. $\frac{n+1}{2^n}$

Answer: C

Watch Video Solution

113. Let A be $n \times n$ matrix given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13}\ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23}\ldots a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3}.\ a_{\text{nn}} \end{bmatrix}$$

Such that each horizontal row is arithmetic progression and each vertical column is a geometrical progression. It is known that each column in geometric progression have the same common ratio. Given that $a_{24} = 1, a_{42} = \frac{1}{8}$ and $a_{43} = \frac{3}{16}$

The value of $\lim_{n \to \infty} \sum_{i=1}^{n} a_{ii}$ is equal to

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

Answer: D

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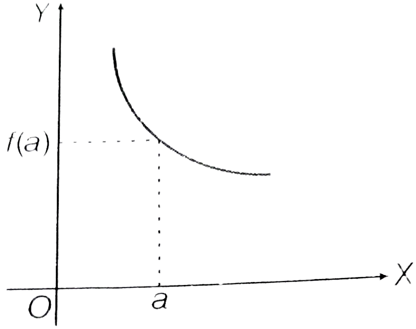
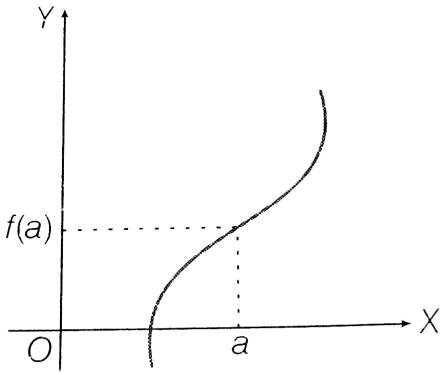
114. To evaluate $\lim_{x \to a} [f(x)]$, we must analyse the $f(x)$ in right hand neighbourhood as well as in left hand neighbourhood of $x = a$. E.g. In case of continuous function, we may come across followign cases.

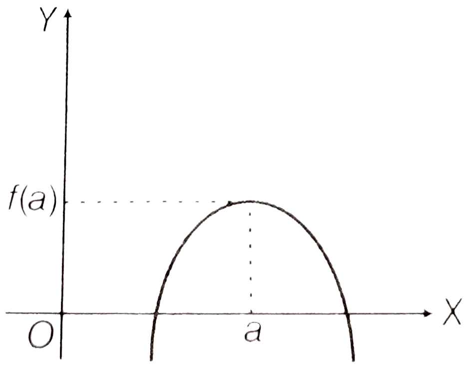
***Case* I**



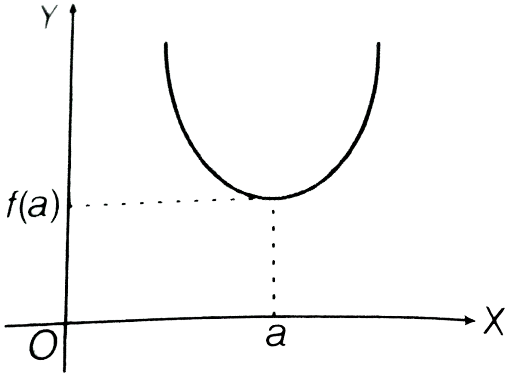
***Case* II**



***Case* III**



Case IV



If f(a) is an integer, the limit will exist in Case III and Case IV but not in Case I and Case II. $\lim_{x \to 0}\left[(1-e^x).\frac{\sin x}{|x|}\right]$ (where [.] denotes the greatest integer function) equals

A. 0

B. 1

C. -1

D. Doesn't exist

Answer: C

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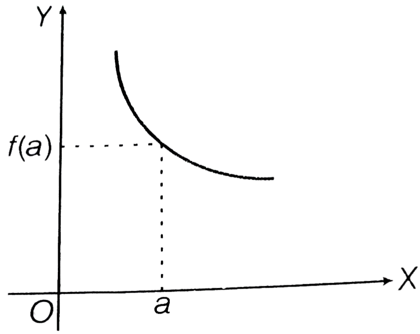
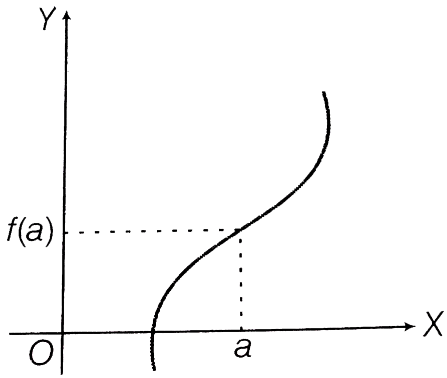
115. To evaluate $\lim_{x \to a} [f(x)]$, we must analyse the $f(x)$ in right hand neighbourhood as well as in left hand neighbourhood of $x = a$. E.g. In case of continuous function, we may come across followign cases.

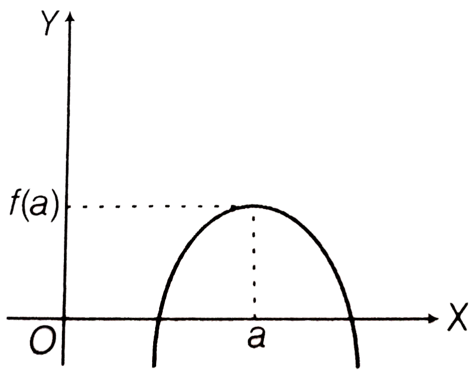
***Case* I**



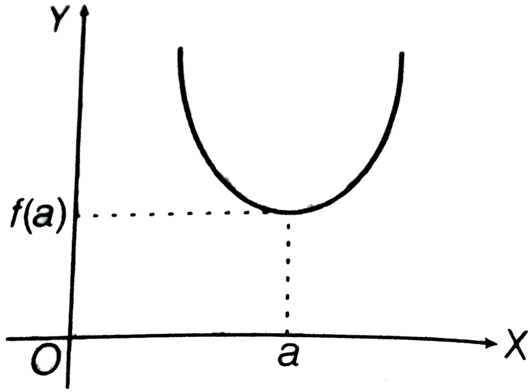
***Case* II**



***Case* III**



Case IV



If f(a) is an integer, the limit will exist in Case III and Case IV but not in Case I and Case II. $\lim_{x \to 1} \left[\operatorname{cosec}\frac{\pi x}{2}\right]^{-1/(1-x)}$ is equal to (where [.] denotes the greatest integer function).

A. 0

B. 1

C. ∞

D. Doesn't exist

Answer: B

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116. Statement 1 $\lim_{x \top i/2} \frac{\sin(\cot^2 x)}{(\pi - 2x)^2} = \frac{1}{2}$

Statement 2: $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{\tan \theta}{\theta} = 1$, where θ is measured in radians.

A. Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: A::B::D

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117. Statement 1 $\lim_{m \to \infty} \lim_{n \to \infty} \{\sin^{2m} n!\pi n\} = 0m, n\varepsilon N$,when x is rational.

Statement 2 When $n \rightarrow \infty$ and x is rational$n!x$ is integer.

A. Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer:

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118. Statement 1: If $\lim_{x \rightarrow 0}\left\{f(x)+\frac{\sin x}{x}\right\}$ does not exist then $\lim_{x \rightarrow 0} f(x)$ does not exist.

Statement 2: $\lim_{x \rightarrow 0}\left(\frac{e^{1/x}-1}{e^{1/x}+1}\right)$ does not exist.

A. Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation for statement 1.

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: B

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119. Match the following

	Column I	Column II	
(A)	$\lim_{x \to \infty} \left(\frac{x}{1+x}\right)^x$ equals	(p)	e^2
(B)	$\lim_{x \to \infty} \left(\sin\frac{1}{x} + \cos\frac{1}{x}\right)^x$ equals	(q)	$e^{-1/2}$
(C)	$\lim_{x \to 0} (\cos x)^{\cot^2 x}$ equals	(r)	e
(D)	$\lim_{x \to 0} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{1/x}$ equals	(s)	e^{-1}

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120. Match the following

	Column I		Column II
(A)	$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}\right)$ equals	(p)	-2
(B)	The value of the $\lim_{x\to 0}\dfrac{\sin 2x - 2\tan x}{\ln(1+x^3)}$ is	(q)	-1
(C)	$\lim_{x\to 0^+}\left(\ln \sin^3 x - \ln(x^4+ex^3)\right)$ equals	(r)	0
(D)	Let $\tan(2\pi\lvert\sin\theta\rvert)=\cot(2\pi\lvert\cos\theta\rvert)$, where $\theta\in R$ and $f(x)=(\lvert\sin\theta\rvert+\lvert\cos\theta\rvert)^x$. The value of $\lim_{x\to\infty}\left[\dfrac{2}{f(x)}\right]$ equals (here, [·] represents greatest integer function)	(s)	1

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121. If $\lim_{x\to 0}\left(x^{-3}\sin 3x + ax^{-2}+b\right)$ exists and is equal to zero the value of $a+2b$ is..........

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122. For a certain value of 'c' $\lim_{x \to \infty}\left[\left(x^5+7x^4+2\right)^c-x\right]$ is finite and non-zero. Then the vlaue oimit is :

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123. Consider the curve $y=\tan^{-1}x$ and a point $A\left(1, \frac{\pi}{4}\right)$ on ti if the variable pont $P_i(x_i, y_i)$ moves on the curve for $i=1, 2, 3, . n(n\varepsilon N)$ such that

$y_r=\sum_{m=1}^{r}\tan^{-1}\left(\frac{1}{2m^2}\right)$ and $B(x, y)$ be the limiting position of variable point P_n as $n \to \infty$, the value of reciprocal of the slope of AB will be.

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124. If $\lim_{x \to 0}\int_0^x \frac{2tdt}{\left(e^x-1-x\right)\sqrt{\frac{2a}{3}-\frac{t}{2}+104}}=\frac{1}{19}$, then $\frac{a}{2010}$ equals.....

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125. Evaluate $\lim_{x \to 0^+} \log_{\sin x} \sin 2x$.

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126.

$$\lim_{x \to \infty} \left(\sqrt{x^4 + ax^3 + 3x^2 + bx + 2} - \sqrt{x^4 + 2x^3 + cx^2 + 3x - d}\right) = 4$$

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127. If x is a real number in $[0, 1]$. Find the value oif

$\lim_{m \to \infty} \lim_{n \to \infty} \left[1 + \cos^{2m}(n!\pi x)\right]$

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128. If $a_1 = 1$ and $a_{n+1} = \dfrac{4 + 3a_n}{3 + 2a_n}$, $n \geq 1$, show that $a_{n+2} \geq a_{n+1}$ and if a lim l as $n \to \infty$ the evaluate $\lim_{n \to \infty} a_n$`

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129. Let a_1, a_2, a_n be sequence of real numbers with $a_{n+1} = a_n + \sqrt{1 + a_n^2}$ and $a_0 = 1$. Prove that $\lim_{x \to \infty} \left(\frac{a_n}{2^n}\right) = \frac{4}{\pi}$

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130. A square is inscribed in a circle of radius R, a circle is inscribed in this square then a square in this circle and so on n times. Find the limit of the sum of areas of all the squares as $n \to \infty$.

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Exercise Single Option Correct Type Questions

1. $\lim_{x \to 0} \frac{\sin(\pi \cos^2(\tan(\sin x)))}{x^2}$ is equal to

A. π

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. None of these

Answer: A

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2. $\lim_{t \to 0} \frac{1-(1+t)^t}{In(1+t)-t}$ is equal to

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 2

D. -2

Answer: C

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3. If $I_1 = \lim_{x \to \infty} \left(\tan^{-1} \pi x - \tan^{-1} x\right) \cos x$ and $I_2 = \lim_{x \to 0} \left(\tan^{-1} \pi x - \tan^{-1} x\right) \cos x$ then (I_1, I_2) is

A. $(0, 0)$

B. $(0, 1)$

C. $(1, 0)$

D. None of the above

Answer: A

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4. If f(x)=0 be a quadratic equation such that $f(-\pi)$=$f(\pi)$=0 and $f\left(\frac{\pi}{2}\right) = \frac{-3\pi^2}{4}$,then $\lim_{x \to \pi} \frac{f(x)}{\sin(\sin x)}$ is equal to:

A. 0

B. π

C. 2π

D. None of these

Answer: C

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5. $\lim_{x \to 0} \frac{\sqrt[3]{1+\sin^2 x} - \sqrt[4]{1-2\tan x}}{\sin x + \tan^2 x}$ is equal to:

A. -1

B. 1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: C

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6. If $x_{n+1}=\sqrt{\frac{1+x_n}{2}}$ and $|x_0|<1\ n\in W$ then $\lim_{n\to\infty}\frac{\sqrt{1-x_0^2}}{x_1x_2x_3\ldots x_{n+1}}$

=

A. -1

B. 1

C. $\cos^{-1}(x_0+1)$

D. $\cos^{-1}(x_0)$

Answer: D

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7. For $n\in N$, let

$f_n(x)=\tan\frac{x}{2}(1+\sec x)(1+\sec 2x)(1+\sec 4x)\ldots\ldots(1+\sec 2^n x)$,

the $\lim_{x\to 0}\frac{f_n(x)}{2x}$ is equal to :

A. 0

B. 2^n

C. 2^{n-1}

D. 2^{n+1}

Answer: C

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8. Let $f(x)$ be a real valued function defined for all $x \geq 1$, satistying $f(1) = 1$ and $f'(x) = \frac{1}{x^2 + (f(x))}$, then $\lim_{x \to \infty} f(x)$

A. doesn't exist

B. exists and less than $\frac{\pi}{4}$

C. exists and less than $1 + \frac{\pi}{4}$

D. exists and equal to 0

Answer: C

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9. The quadratic equation whose roots are the minimum value of $\sin^2\theta - \sin\theta + \frac{1}{2}$ and $\lim_{x \to \infty} \sqrt{(x+1)(x+2)} - x$ is

A. $3x^2 - 7x + 3 = 0$

B. $8x^2 - 14x + 3 = 0$

C. $x^2 - 7x + 3 = 0$

D. $2x^2 - 8x + 3 = 0$

Answer: B

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10. If $x_1 = \sqrt{3}$ and $x_{n+1} = \frac{x_n}{1 + \sqrt{1 + x_n^2}}, \forall n \in N$ then $\lim_{n \to \infty} 2^n x_n$ is equal to

A. $\frac{3}{2\pi}$

B. $\frac{2}{3\pi}$

C. $\frac{3\pi}{3}$

D. $\frac{3\pi}{2}$

Answer: C

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11. $\lim_{x \to a^-} \frac{\sqrt{x-b}-\sqrt{a-b}}{(x^2-a^2)}, (a > b)$ is

A. $\frac{1}{4a}$

B. $\frac{1}{a\sqrt{a-b}}$

C. $\frac{1}{2a\sqrt{a-b}}$

D. $\frac{1}{4a\sqrt{a-b}}$

Answer: D

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12. $\lim_{n \to \infty} (\sin^n 1 + \cos^n 1)^n$ is equal to

A. $\cot 1$

B. $\tan 1$

C. $\cos 1$

D. $\sin 1$

Answer: D

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13. The value of $\lim (x \to 0) \frac{2}{x^3} \left(\sin^{-1} x - \tan^{-1} x\right)^{2/x^2}$ equals

A. e

B. $\sqrt{e}$

C. $\frac{1}{e}$

D. $\frac{1}{\sqrt{e}}$

Answer: D

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14. The value of $\lim \sum_{k=1}^{n} \frac{n-k}{n} \cos\left(\frac{4k}{n}\right)$ equals to

A. $\frac{1}{4}\sin 4 + \frac{1}{15}\cos 4 - \frac{1}{16}$

B. $\frac{1}{4}\sin 4 - \frac{1}{16}\cos 4 + \frac{1}{16}$

C. $\frac{1}{16}(1 - \sin 4)$

D. $\frac{1}{16}(1 - \cos 4)$

Answer: D

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15. $\lim_{x \to 0} \frac{1 - \sqrt{\cos 2x} \cdot \sqrt[3]{\cos 3x} \ldots . \sqrt[n]{\cos nx}}{x^2}$ has value 10 then value of n equal to

A. 6

B. 7

C. 8

D. 9

Answer: A::B::D

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16. $\lim_{z \to \infty} \frac{\int_{1/2}^{2} \left[\cot^{-1} x\right] dx}{\int_{1/2}^{z} \left[1 + \frac{1}{x}\right] dx}$, where [.] denotes the greatest integer function equals

A. 0

B. 1

C. $\cot 1$

D. not defined

Answer: A::B::D

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17. If α and β are roots of $x^2 - \left(\sqrt{1 - \cos 2\theta}\right)x + \theta = 0$, where $0 < \theta < \frac{\pi}{2}$. Then $\lim_{\theta \to 0^+} \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is

A. $\frac{1}{\sqrt{2}}$

B. $-\sqrt{2}$

C. $\sqrt{2}$

D. None of these

Answer: C

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18. If $f(x) = \frac{1}{3}\left(f(x+1) + \frac{5}{f(x+2)}\right)$ and $f(x) > 0, \forall x \varepsilon R,$ then $\lim_{x \to \infty} f(x)$ is

A. 0

B. $\sqrt{\frac{2}{5}}$

C. $\sqrt{\frac{5}{2}}$

D. ∞

Answer: C

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19. Let $f:(1, 2)\vec{R}$ satisfies the inequality `(cos(2x-4)-33)/2

A. 16

B. -16

C. does'nt exist

D. θ

Answer: B

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20. Let $f(x)$ be polynomial of degree 4 with roots 1,2,3,4 and leading coefficient 1 and $g(x)$ be the polynomial of degree 4 with roots $1, \frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ with leading coefficient 1. Then $\lim_{x \to 1} \frac{f(x)}{g(x)}$ equals

A. $\frac{1}{24}$

B. -24

C. $\frac{1}{12}$

D. $-\frac{1}{12}$

Answer: B

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21. The value of $\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\cos x + \sin x)^5}{1 - \sin 2x}$ is

A. $\sqrt{2}$

B. $3\sqrt{5}$

C. $5\sqrt{2}$

D. $-5\sqrt{2}$

Answer: C

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22. $I\frac{f(\lim)_{x \xrightarrow{} \infty}(n.3^n)}{n(x-2)^n + n.3^{n+1} - 3^n} = \frac{1}{3}, thenthe$ range of x is (where $n \in N)\cdot$ (2,5) (b) (1,5) (c) $-5, 5)$ (d) $(-\infty, \infty)$

A. 3

B. 4

C. 5

D. infinite

Answer: C

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23. Let $f(x) = \lim_{n \to \infty} \frac{1}{\left(\frac{3}{\pi}\tan^{-1} 2x\right)^{2n} + 5}$ then the set of value of x for which $f(x) = 0$ is

A. $|2x| > \sqrt{3}$

B. $|2x| < \sqrt{3}$

C. $|2x| \geq \sqrt{3}$

D. $|2x| \leq \sqrt{3}$

Answer: A::B::D

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24. The integer n for which the

$\lim_{x \to 0} \frac{\cos^2 x - \cos x - e^x \cos x + e^x - \frac{x^3}{2}}{x^n}$ is a finite non-zero number is

A. 2

B. 3

C. 4

D. None of these

Answer: C

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25. If $I_1 = \lim_{x \to 0} \sqrt{\frac{\tan^{-1} x}{x} - \frac{\sin^{-1} x}{x}}$ and $I_2 = \lim_{x \to 0} \sqrt{\frac{\sin^{-1} x}{x} - \frac{\tan^{-1} x}{x}}$, where $|x| < 1$, which of the following statement is true?

A. Neither I_1 nor I_2 exist

B. I_1 exists and I_2 doesn't exist

C. I_1 doesn't exist and I_2 exists

D. None of the above

Answer: C

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26. $\lim_{x \to \frac{\pi}{2}} \frac{\left[\frac{x}{2}\right]}{\log_e(\sin x)}$ ([.] denotes greatest integer function)

A. 0

B. 1

C. -1

D. Doesn't exist

Answer: A::B::D

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27. Let $a_1 = 1$, $a_n = n(a_{n-1} + 1$ for $n = 2, 3, \ldots$ where $P_n = \left(1 + \frac{1}{a_1}\right)\left(1 + \frac{1}{a_2}\right)\left(1 + \frac{1}{a_3}\right)\ldots\left(1 + \frac{1}{a_n}\right)$ then $Lt_{n \to \infty} P_n =$

A. e

B. $\frac{e}{2}$

C. $2e$

D. $3e$

Answer: A::B::D

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28. If $f(x+y) = f(x) + f(y)$ for $x, y \in R$ and $f(1) = 1$, then find the value of $\lim_{x \to 0} \frac{2^{f(\tan x)} - 2^{f(\sin x)}}{x^2 \cdot f(\sin x)}$

A. $\log 2$

B. $\log 4$

C. $\log \sqrt{2}$

D. $\log 8$

Answer: C

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29. $\lim_{n \to \infty} n^{-n^2}\left((n+1)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{2^2}\right)....\left(n+\frac{1}{2^{n-1}}\right)\right)^n$

A. e

B. e^2

C. e^3

D. e^4

Answer: B

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30. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & [x] \neq 0 \\ 0 & [x] = 0 \end{cases}$

where [.] denotes the greatest integer less than or equal to x then

A. 1

B. 0

C. -1

D. does

Answer: D

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31. Let $f(x) = \dfrac{|x^3 - 6x^2 + 11x - 6|}{x^3 - 6x^2 + 11x - 6}$

then the number of solutions of a where $\lim_{x \to a} f(x)$ does not exist is

A. 3

B. 2

C. 1

D. 4

Answer: A::B::D

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32. Let the rth term t_r of a series is given by $t_r = \frac{r}{1 + r^2 + r^4}$, the value of $\lim_{n \to \infty} \sum_{r=1}^{n} t_r$ is

A. 2

B. $\frac{1}{2}$

C. 1

D. $\frac{1}{4}$

Answer: B

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33. The value of $\lim_{n \to \infty} \sum_{r=1}^{n} \cot^{-1}\left(\frac{r^3 - r + \frac{1}{r}}{2}\right)$ is

A. π

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. π

Answer: C

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34. Let $\tan\alpha. x + \sin\alpha \cdot y = \alpha$ and $\alpha. \cos ec\alpha \bullet x + \cos\alpha \cdot y = 1$ be two variable straight lines, α being the parameter. Let P be the point of intersection of the lines. In the limiting position when $a \to 0$, the point P lies on the line :

A. $(2, \ -1)$

B. $(2, 1)$

C. $(-2, 1)$

D. $(-2, \ -1)$

Answer: A::B::D

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35. The polynomial of least degree such that $\lim_{x \to 0} \left(1 + \frac{x^2 + f(x)}{x^2}\right)^{1/x} = e^2$ is

A. x^2

B. $x^3 + 2x^2$

C. $-x^2 + 2x^3$

D. None of these

Answer: C

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36. If n is a non zero integer and [*] denotes the greatest integer function then $\lim_{x \to 0} \left[\left(n\frac{\sin x}{x}\right] + \lim_{x \to} \left[n\frac{\tan x}{x}\right]$ equals

A. $n + 1$

B. $2n$

C. $n-1$

D. $2n-1$

Answer: D

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37. The value of $\lim_{x \to a} \left[\sqrt{2-x} + \sqrt{1+x}\right]$, where $a \varepsilon \left[0, \frac{1}{2}\right]$ and $[.]$ denotes the greatest integer function is

A. 1

B. 2

C. 3

D. 4

Answer: B

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38. The value of

$$\lim_{x \to 0^+} \frac{-1+\sqrt{(\tan x - \sin x)+\sqrt{(\tan x - \sin x)+\sqrt{(\tan x - \sin x)+}}}}{-1+\sqrt{x^3+\sqrt{x^3+\sqrt{x^3+\ldots\infty}}}}$$

is

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. 1

Answer: A::B::D

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39. Evaluate: $(\lim)_{n \to 0} \frac{\cos^2(1-\cos^2(1-\cos^2(\cos^2(\theta))))}{s \in \left(\pi \frac{\sqrt{(\theta+4)}-2}{\theta}\right)}$

A. 2

B. $\sqrt{2}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

Answer: B

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40. The value of $\lim_{n \to \infty} a_n$ when $a_{n+1} = \sqrt{2 + a_n}, n = 1, 2, 3,$ ..... is

A. 1

B. 2

C. 3

D. 4

Answer: B

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Exercise More Than One Correct Option Type Questions

1. If $\lim_{x \to \infty} 4x\left(\frac{\pi}{4} - \tan^{-1}\left(\frac{x+1}{x+2}\right)\right) = y^2 + 4y + 5$ then y can be equal to

A. 1

B. -1

C. -4

D. -3

Answer: B::D

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2. $\lim_{x \to 0} \frac{1 - \cos\left(x^2\right)}{x^3\left(4^x - 1\right)}$ is equal to

A. $\frac{1}{2} In2$

B. $In2$

C. $In4$

D. $1 - \frac{1}{2}In\left(\frac{e^2}{4}\right)$

Answer: null

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3. If $f(x) = e^{[\cot x]}$, where $[y]$ represents the greatest integer less than or equal to y then

A. $\lim_{x \to \frac{\pi}{2}^+} f(x) = 1$

B. $\lim_{x \to \frac{\pi}{2}^+} f(x) = \frac{1}{e}$

C. $\lim_{x \to \frac{\pi}{2}^-} f(x) = \frac{1}{e}$

D. $\lim_{x \to \frac{\pi}{2}^-} f(x) = 1$

Answer: B::D

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4. $\lim_{x \to 0} \left[m \frac{\sin x}{x}\right]$ is equal to (where $m \varepsilon I$ and $[.]$ denotes greatest integer function)

A. m if $m < 0$

B. $m - 1$ if $m > 0$

C. $m - 1$, if $m < 0$

D. m if $m > 0$

Answer: A::B

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5. If $\lim_{x \to 0} \left(1 + ax + bx^2\right)^{\frac{2}{x}} = e^3$, then

A. $a = 3, b = 0$

B. $a = \frac{3}{2}, b = 1$

C. $a = \frac{3}{2}, b = 4$

D. $a = 2, b = 3$

Answer: B::C

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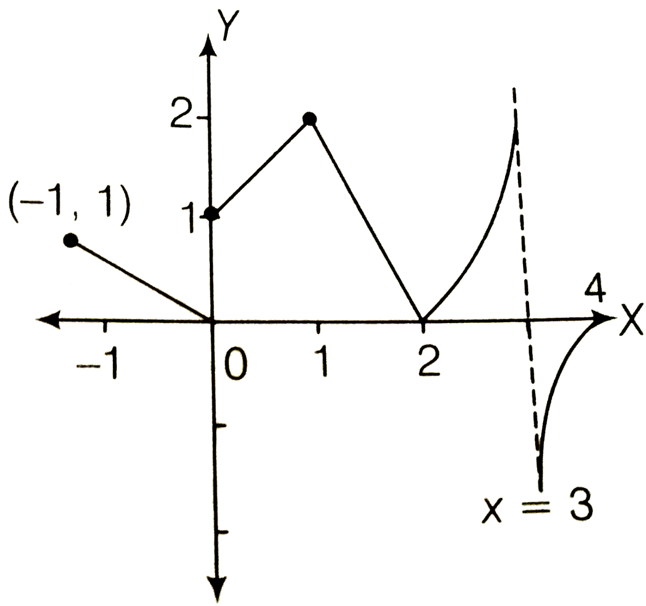
6. The graph of the function $y = f(x)$ is shown in the adjacent figure then correct statement is



A. $\lim_{x \to 0^+} f(x) = 1$

B. $\lim_{x \to 1} f(x) = 2$

C. $\lim_{x \to 3} f(x) =$ does not exist

D. $\lim_{x \to 4} f(x) = 0$

Answer: A::B::C::D

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7. Find $\lim_{x \to 0} \frac{\cot^{-1}\left(\frac{1}{x}\right)}{x}$

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8. If $l = \lim_{x \to \infty} \left(\frac{x+1}{x-1}\right)^x$, the value of $\{l\}$ and $[l]$ are

A. 7

B. $7 - e^2$

C. -7

D. $e^2 - 7$

Answer: A::D

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9. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]} & [x] \neq 0 \\ 0 & [x] = 0 \end{cases}$

where [.] denotes the greatest integer less than or equal to x then

A. $\lim_{x \to 0^-} f(x) = \sin 1$

B. $\lim_{x \to 0^+} f(x) = 0$

C. limt does not exist at $x = 0$

D. limt exists at $x = 0$

Answer: A::B::C::D

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10. $\lim_{x \to c} f(x)$ does not exist when

where $[.]$ and $\{.\}$ denotes greatest integer and fractional part of x

A. $f(x) = [x] - [2x - 1], c = 3$

B. $f(x) = [x] = -x, c = 1$

C. $f(x) = \{x\}^2 - \{-x\}^2, c = 0$

D. $f(x) = \dfrac{\tan(sgnx)}{(sgnx)}, c = 0$

Answer: B::C

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11. Identify the correct statement

A. $\lim_{x \to \infty} \left[\sum_{r=1}^{n} \frac{1}{2^r}\right] = 1$

B. If $f(x) = (x - 1)\{x\}$, where $[.]$ and $\{.\}$ denotes greatest integer function and fractional part of x respectively, the limit of $f(x)$ does not exist at $x = 1$

C. $\lim_{x \to 0^+} \left[\frac{\tan x}{x}\right] = 1$

D. $\left[\lim_{x \to 0^+} \frac{\tan x}{x}\right] = 1$

Answer: C::D

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12. For $a > 0$, let $l = \lim_{x \to \frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos c}$ and $m = \lim_{x \to -\infty} \left(\sqrt{x^2 + ax}\right) - \left(\sqrt{x^2 - ax}\right)$ then

A. $l > m$ for all $a > 0$

B. $l > m$ only when $a \geq 1$

C. $l > m$ for all $a > e^{-a}$

D. $e^{l} + m = 0$

Answer: B::C

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13. Consider the function $f(x) = \left(\frac{ax + 1}{bx + 2}\right)^{x}$, where $a, b > 0$, the $\lim_{x \to \infty} f(x)$ is

A. exists for all values of a and b

B. zer for $a < b$

C. non existent for $a > b$

D. $e^{-(1/a)}$ or $e^{-(1/b)}$ if $a = b$

Answer: B::C::D

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14. If $f(x) = \frac{x.2^x - x}{1 - \cos x}$ and $g(x) = 2^x . \sin\left(\frac{\log 2}{2^x}\right)$ then

A. $\lim_{x \to 0} f(x) = \log 2$

B. $\lim_{x \to \infty} g(x) = \log 4$

C. $\lim_{x \to 0} f(x) = \log 4$

D. $\lim_{x \to \infty} g(x) = \log 2$

Answer: C::D

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15. If $\lim_{x \to 3} \frac{x^3 + cx^2 + 5x + 12}{x^2 - 7x + 12} = l$ (finite real number), ten

A. $l = 4$

B. $c = -6$

C. $c = 4$

D. $x \varepsilon R$

Answer: A::B

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Exercise Passage Based Questions

1. If $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$ then $\lim_{x \to a} \{f(x)\}^{g(x)} = e^{\lim_{x \to a} (f(x) - 1) x g(x)}$

$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{\sin x}{x - \sin x}}$ is equal to

A. $1/e$

B. $-1/e$

C. e

D. $-e$

Answer: A

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2. If $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$ then $\lim_{x \to a} \{f(x)\}^{g(x)} = e^{\lim_{x \to a} (f(x) - 1) g(x)}$ $\lim_{x \to 0} \left(\frac{x - 1 + \cos x}{x}\right)^{\frac{1}{x}}$ is equal to

A. $e^{1/2}$

B. $e^{-1/2}$

C. e^{1}

D. $\frac{1}{e}$

Answer: B

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3. If $\lim_{x \to a} f(x) = 1$ and $\lim_{x \to a} g(x) = \infty$ then $\lim_{x \to a} \{f(x)\}^{g(x)} = e^{\lim_{x \to a} (f(x)-1)g(x)}$ $\lim_{x \to 0} \left(\frac{a^x + b^x + c^x}{3}\right)^{\frac{2}{x}}$ is equal to

A. $a^{2/3} + b^{2/3} + c^{2/3}$

B. abc

C. $(abc)^{2/3}$

D. 1

Answer: C

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4. Let $f(x) = \lim_{n \to \infty} \left(\cos\sqrt{\frac{x}{n}}\right)^n, g(x) = \lim_{n \to \infty} \left(1 + x + x\sqrt[n]{e}\right)^n$

Now consider the function $y = h(x)$ where $h(x) = \tan^{-1}\left(g^{-1}f^{-1}(x)\right)$.

$\lim_{x \to 0} \frac{In(f(x))}{In(g(x))}$ is equal to

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. 0

D. 1

Answer: B

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5. Let $f(x) = \lim_{n \to \infty} \left(\cos\sqrt{\frac{x}{n}}\right)^n, g(x) = \lim_{n \to \infty} \left(1 + x + x\sqrt[n]{e}\right)^n$

Now consider the function $y = h(x)$ where $h(x) = \tan^{-1}\left(g^{-1}f^{-1}(x)\right)$.

Domain of the function $y = h(x)$ is

A. $(0, \infty)$

B. R

C. $(0, 1)$

D. $[0, 1]$

Answer: C

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6. Let $f(x) = \lim_{n \to \infty} \left(\cos\sqrt{\frac{x}{n}}\right)^n$, $g(x) = \lim_{n \to \infty} (1 + x + x\sqrt[n]{e})^n$

Now consider the function $y = h(x)$ where $h(x) = \tan^{-1}\left(g^{-1}f^{-1}(x)\right)$.

Range of the function $y = h(x)$ is

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(-\frac{\pi}{2}, 0\right)$

C. R

D. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Answer: D

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7. Let $K_1 =$ Total number of ways of selecting of ball from a bag which contains n balls of first colour $(n+1)$ balls of second colour, $(n+2)$ balls of third colour ,........,$(2n-1)$ balls of n colour.

$K_2 =$ number of n -digit numbers using the digit 1,2,3,n and

$K_3 =$ number of ways of arranging $(n+1)$ objects on a circle.

The value of $\lim_{n \to \infty} \left(\frac{K_1}{K_2}\right)^{1/n}$, is

A. e^2

B. $\log 4 - 1$

C. $\frac{4}{e}$

D. $\frac{1}{e}$

Answer: C

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8. Let $K_1 =$ Total number of ways of selecting of ball from a bag which contains n balls of first colour $(n+1)$ balls of second colour, $(n+2)$

balls of third colour ,........,$(2n-1)$ balls of n colour.

$K_2=$ number of n -digit numbers using the digit 1,2,3,n and

$K_3=$ number of ways of arranging $(n+1)$ objects on a circle.

The value of $\lim_{n\to\infty}\left(\frac{K_1}{K_2+K_3}\right)$ is

A. e

B. 1

C. 0

D. does not exist

Answer: B

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9. Let $f: N\to R$ and $g: N\to R$ be two functions and $f(1)=08, g(1)=0.6,$

$f(n+1)=f(n)\cos(g(n))-g(n)\sin(g(n))$ and $g(n+1)=f(n)\sin(g(n))$

for $n\geq 1$. $\lim_{n\to\infty} f(n)$ is equal to

A. -1

B. 0

C. 1

D. does not exist

Answer: A

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10. Let $f: N \to R$ and $g: N \to R$ be two functions and $f(1) = 08, g(1) = 0.6,$

$f(n+1) = f(n)\cos(g(n)) - g(n)\sin(g(n))$ and $g(n+1) = f(n)\sin(g(n))$

for $n \geq 1$. $\lim_{n \to \infty} f(n)$ is equal to

A. -1

B. 0

C. 1

D. does not exist

Answer: B

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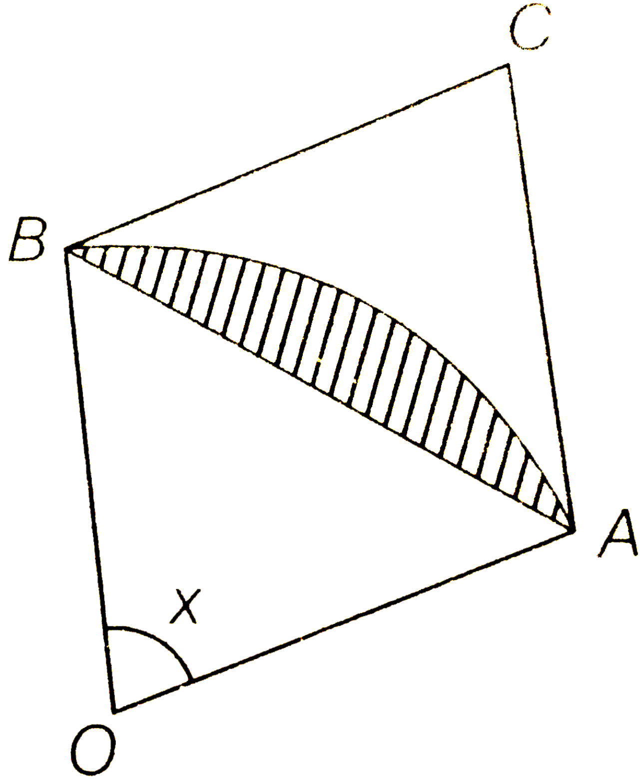
11. A circle arc of radius 1 subtends an angle of x radians as shown in figure. The centre of the circle is O and the point C is the intersection of two tangent lines at A and B. Let $T(x)$ be the area of ΔABC and $S(x)$ be the area of shaded region.



$$\lim_{x \to 0} \frac{T(x)}{x^3}$$ is

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Answer: D

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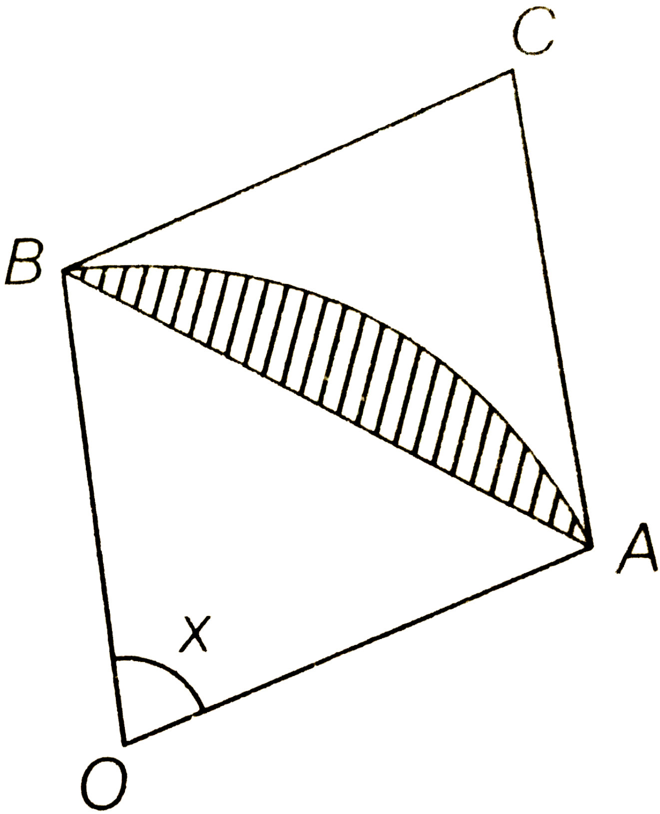
12. A circle arc of radius 1 subtends an angle of x radians as shown in figure. The centre of the circle is O and the point C is the intersection of two tangent lines at A and B. Let $T(x)$ be the area of ΔABC and $S(x)$ be the area of shaded region.



$\lim_{x \to 0} \frac{S(x)}{x}$ is

A. 0

B. $\frac{1}{2}$

C. 1

D. None of these

Answer: A

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13. A circular are of radius 1 subtends an angle of x radians $0 < x < \frac{\pi}{2}$ as shown in the figure . the point r is the intersection of the two tangent line at P and Q. let $T(x)$ be the area of triangle PQR and $S(x)$ be the area of the shaded region then find $T(x) \& S(x) \& \lim_{x \to 0} \frac{T(x)}{S(x)}$

A. $\frac{1}{4}$

B. $\frac{3}{4}$

C. $\frac{3}{2}$

D. 0

Answer: C

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Exercise Matching Type Questions

1. Match the statements of Column I with values of Column II.

Column I	Column II
(A) $\lim_{x \to \frac{\pi^+}{2}} \tan^{-1}(\tan x)$	(p) 0
(B) $\lim_{n \to \infty} \left[\sum_{r=1}^{n} \frac{1}{2^r}\right]$ ([$\cdot$] denotes the greatest integer function)	(q) Doesn't exist
(C) $\lim_{x \to \infty} \sec^{-1}\left(\frac{x}{x+1}\right)$	(r) $-\frac{\pi}{2}$
(D) $\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{(1-\sin x)^{2/3}}$	(s) $\frac{\pi}{2}$

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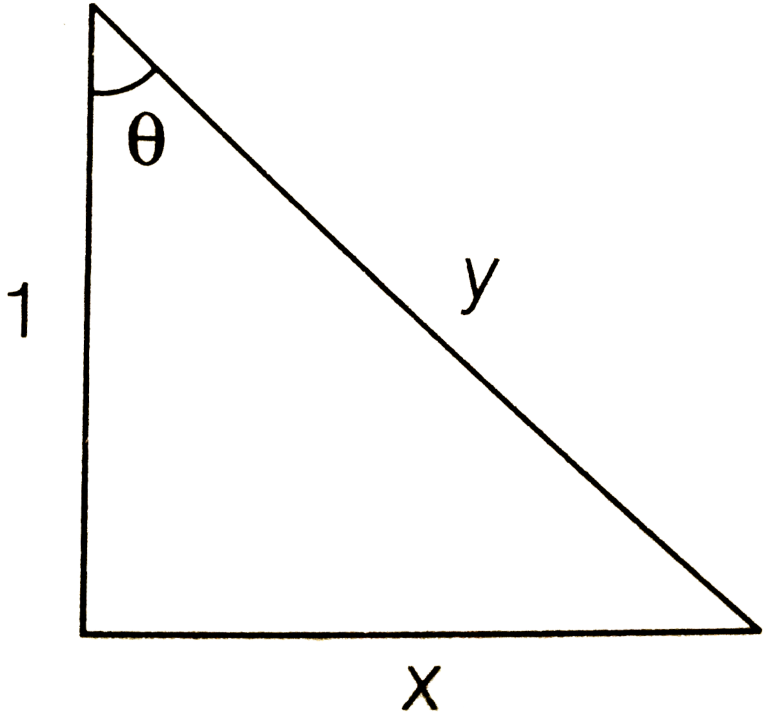
2. A right angled triangle has legs 1 and x. The hypotenuse is y and angle opposite to the side x is θ. Shown as



	Column I		Column II
(A)	$\lim_{\theta \to \frac{\pi}{2}} (\sqrt{y} - \sqrt{x})$	(p)	0
(B)	$\lim_{\theta \to \frac{\pi}{2}} (y - x)$	(q)	$\frac{1}{2}$
(C)	$\lim_{\theta \to \frac{\pi}{2}} (y^2 - x^2)$	(r)	1
(D)	$\lim_{\theta \to \frac{\pi}{2}} (y^3 - x^3)$	(s)	∞

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3. Match the column.

	Column I	Column II	
(A)	$\lim_{x \to 0} \frac{1 - \cos 2x}{e^{x^2} - e^x + x}$ equals	(p)	1
(B)	If the value of $\lim_{x \to 0^+} \left(\frac{(3/x) + 1}{(3/x) - 1}\right)^{1/x}$ can be expressed in the form of $e^{p/q}$, where p and q are relative prime, then $(p + q)$ is equal to	(q)	2
(C)	$\lim_{x \to 0} \frac{\tan^3 x - \tan x^3}{x^5}$ equals	(r)	4
(D)	$\lim_{x \to 0} \frac{x + 2 \sin x}{\sqrt{x^2 + 2 \sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ equals	(s)	5

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Exercise Single Integer Answer Type Questions

1. Let $L = \lim_{x \to \infty} \left(x \log x + 2x \cdot \log \sin\left(\frac{1}{\sqrt{x}}\right)\right)$, then value of $\left(-\frac{2}{L}\right)$ is .....

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2. For $n \varepsilon N$ let x_n be defined as $\left(1+\frac{1}{n}\right)^{(n+x_n)}=e$ then $\lim_{n\to\infty}(2x_n)$ equals.....

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3. Let $f(x)=\left[\sqrt{n}+\frac{1}{2}\right]$ where [.] denotes greatest integer function $\forall, n \varepsilon N$

Then $\sum_{n=1}^{\infty}\frac{2^{f(n)}+2^{-f(n)}}{2^n}$ is equal to ....

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4. If the arithmetic mean of the product of all pairs of positive integers whose sum is n is A_n then $\lim_{n\to\infty}\frac{n^2}{A_n}$ equals to

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5. The value of $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{6^k}{\left(3^k - 2^k\right)\left(3^{k+1} - 2^{k+1}\right)}$ is equal to

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6. The value of $\lim_{x \to \frac{\pi}{2}} \sqrt{\frac{\tan x - \sin\{\tan^{-1}(\tan x)\}}{\tan x + \cos^2(\tan x)}}$ is ....

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7. The figure shows two regions in the first quadrant. $P\left(t, \sin t^2\right)$ the curve $B(t)$, $A(t)$ is the area under the curve $y = \sin x^2$ from 0 to t and $B(t)$ is the area of the triangle with vertices P and $M(t, 0)$. Find $\operatorname{Lim}_{t \to 0} A\frac{t}{B}(t)$

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8. If the two $AB: \left(\int_0^{2t} \left(\frac{\sin x}{x} + 1\right) dx\right) x + y = 3t$ and $AC: 2tx + y = 0$ intersect at a point A the x-coordinate of a point A as $t \to 0$, is equal to $\frac{p}{q}$ (p and q are in their lowest form) the $(p + q)$ is ....

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9. Consider a parabola $y = \frac{x^2}{4}$ and the point $F(0, 1)$. Let $A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3), . . , A_n(x_n, y_n)$ are n points on the parabola such that $x_k > 0$ and $\angle OFA_k = \frac{k\pi}{2n}(k = 1, 2, \ldots\ldots n)$ If the value of $\lim_{x \to \infty} \left(\frac{1}{n}\right). \sum_{k=1}^{n} FA_k = \frac{m}{\pi}$ then m is ....

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10. If $L = \lim_{x \to \frac{\pi}{2}^+} \frac{\cos \tan^{-1}(\tan x)}{x - \pi/2}$ then $\cos(2\pi L)$ is

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11. Number of solutions of the equation $\cos ec\theta = k$ in $[0, \pi]$ where $k = \lim_{n \to \infty} \pi_{r=2}^{n}\left(\frac{r^3 - 1}{r^3 + 1}\right)$ is

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12. If C satisfies the equation $\lim_{x \to \infty}\left(\frac{x + c}{x - c}\right)^x = 4$ then $\left|\frac{e^c}{2}\right|$ is ....

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13. If $\lim_{x \to -\infty} \frac{(3x^4 + 2x^2).\sin\left(\frac{1}{x}\right) + |x|^3 + 5}{|x|^3 + |x^2| + |x| + 1} = k$ then $\left|\frac{k}{2}\right|$ is ....

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14. If $f(x) = \lim_{t \to 0}\left[\frac{2x}{\pi}.\tan^{-1}\left(\frac{x}{t^2}\right)\right]$,then $f(1)$ is ....

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15. The value of $\lim_{n \to \infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+\ldots\ldots+\frac{1}{\sqrt{n^2+2n}}\right)$ is

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16. If $l=\lim_{x \to 1^+} 2^{-2^{\frac{1}{1-x}}}$ and $m=\lim_{x \to 1^+}\frac{x\sin(x-[x])}{x-1}$ (where [.] denotes greatest integer function). Then $(l+m)$ is ....

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17. The value of $\lim_{x \to 0}\left[\frac{\sin x.\tan x}{x^2}\right]$ is ....

(where [.] denotes greatest integer function).

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18. If

$$\lim +(n \to \infty)\sum_{r=1}^{n}\frac{kr}{1\times 3\times 5\times \ldots.\times(2r-1)\times(2r+1)}=1$$

then k^2 is ......

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19. If $f(x) = \lim_{n \to \infty} \sin^4 x + \frac{1}{4}\sin^4 2x + .. = \frac{1}{4^n}. \sin^4(2^n x)$ and $g(x)$ is a differentiable function satisfying $g(x) + f(x) = 1$, then the maximum value of $\left(\sqrt{f(x)} + \sqrt{g(x)}\right)^4$ is ....

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20. If $f(x+y+z) = f(x) + f(y) + f(z)$ with $f(1) = 1$ and $f(2) = 2$ and $x, y, z \varepsilon R$ the value of $\lim_{x \to \infty} \sum_{r=1}^{n} \frac{(4r)f(3r)}{n^3}$ is ....

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21. If $\lim_{x \to \infty} \frac{n.3^n}{n.(x-1)^n + n.3^{n+1} - 3^n} = \frac{1}{3}$, the number of the integral values of x is ....

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22. The value of $\lim_{x \to \infty} \left(((x-1)(x-2)(x+3)(x+10)(x+15))^{\frac{1}{5}} - x\right)$ is ....

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23. If $\lim_{x \to \infty} \left([f(x)] + x^2\right)\{f(x)\} = k$, where $f(x) = \frac{\tan x}{x}$ and $[.], \{.\}$ denote geatest integer and fractional part of x respectively, the value of $[k/e]$ is ....

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24. The value of $\lim_{n \to \infty} \left\{\left(\sqrt{3}+1\right)^{2n}\right\}$ is........(where {.} denotes fractional part of x).

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25. Let $p(x)$ be a polynomial of degree 4 having extremum at $x = 1, 2$ and $\lim_{x \to 0}\left(1 + \frac{p(x)}{x^2}\right) = 2$. Then find the value of $p(2)$.

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26. If α is the number of solution of $|x| = \log(x - [x])$, (where [.] denotes greatest integer function) and

$\lim_{x \to \alpha} \frac{xe^{ax} - b\sin x}{x^3}$ is finite, the value of $(a - b)$ is

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27. Suppose $x_1 = \tan^{-1} 2 > x_2 > x_3 > \ldots.$ are the real numbers satisfying $\sin(x_{n+1} - x_n) + 2^{-(n+1)} \cdot \sin x_n \cdot \sin x_{n+1}$ for all $n > 1$ and $l = \lim_{x \to \infty} x_n$, the value of $[4l]$ is...where [t] denotes greatest integer function.

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Exercise Questions Asked In Previous 13 Years Exam

1. Let $\alpha, \beta \in R$ such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

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2. Let m and n be two positive integers greater than 1.If $\lim_{\alpha \to 0} \frac{e^{\cos \alpha^n} - e}{\alpha^m} = -\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is

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3. Let $f : R\overrightarrow{R}$ be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t) dt f \text{ or } all x \in [-1, 2] and G(x) = \int_{-1}^{x} t|f(f(t))| dt f \text{ or } a$

Then the value of $f\left(\frac{1}{2}\right)$ is

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4. The largets value of non negative integer for which

$$\lim_{x \to 1} \frac{(-ax + \sin(x-1) + a]1 - \sqrt{x}}{x + \sin(x-1) - 1}\Bigg\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

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5. Let $L = (\lim)_{\underset{x\,0}{\rightarrow}} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0. If Li \in ite, then\ a = 2$

(b) $a = 1$ $L = \frac{1}{64}$ (d) $L = \frac{1}{32}$

A. $a = 2$

B. $a = 1$

C. $L = \frac{1}{64}$

D. $L = \frac{1}{32}$

Answer: C

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6. The $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals

A. $\frac{1}{4}$

B. $\frac{1}{24}$

C. $\frac{1}{16}$

D. $\frac{1}{8}$

Answer: C

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7. Let $p = \lim_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to`

A. 2

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: B

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8. $\lim_{n \to \infty} \left(\frac{(n+1)(n+2)(n+3).......3n}{n^{2n}} \right)^{\frac{1}{n}}$ is equal to

A. $\frac{18}{e^4}$

B. $\frac{27}{e^2}$

C. $\frac{9}{e^2}$

D. $3 \log 3 - 2$

Answer: D

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9. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to

A. $\frac{\pi}{2}$

B. 1

C. $-\pi$

D. π

Answer: D

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10. $\lim_{x \to 0} \frac{(1 - \cos x)(3 + \cos x)}{x \tan 4x}$ is equal to

A. 4

B. 3

C. 2

D. $\frac{1}{2}$

Answer: B

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11. If $\lim_{x \to \infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

A. $a=1, b=4$

B. $a=1, b=\,-4$

C. $a=2, b=\,-3$

D. $a=2, b=3$

Answer: D

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12. let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $\left((1+a)^{\frac{1}{3}}-1\right)x^2+\left((1+a)^{\frac{1}{2}}-1\right)x+\left((1+a)^{\frac{1}{6}}-1\right)=0$ where $a>\,-1$ then, $\lim_{a \to 0^+}\alpha(a)$ and $\lim_{a \to 0^+}\beta(a)$

A. $-\frac{5}{2}$ and 1

B. $-\frac{1}{2}$ and -1

C. $-\frac{7}{2}$ and 2

D. $-\frac{9}{2}$ and 3

Answer: C

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13. If $(\lim)_{x \to 0}\left[1+x 1n\left(1+b^2\right)\right]^{\frac{1}{x}}=2b\sin^2\theta, b>0, smf\theta\in(-\pi, \pi]$, then the value of θ is $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

A. $\pm\frac{\pi}{4}$

B. $\pm\frac{\pi}{3}$

C. $\pm\frac{\pi}{6}$

D. $\pm\frac{\pi}{2}$

Answer: D

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14. For $x > 0$, $\lim_{x \to 0} \left\{ (\sin x)^{1/x} + \left(\frac{1}{x}\right)^{\sin x} \right\}$, is

A. 0

B. -1

C. 1

D. 2

Answer: D

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15. $\lim_{h \to 0} \frac{f(2h + 2 + h^2)}{f(h - h^2 + 1) - f(1)}$ given that $f'(2) = 6$ and $f'(1) = 4$

A. does not exist

B. is equal to $-3/2$

C. is equal to $3/2$

D. is equal to 3

Answer: C

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16. $\lim_{x \to 0} \frac{\sin(nx)((a-n)nx - \tan x)}{x^2} = 0$, when n is a non-zero positive integer, then a is equal to

A. 0

B. $\frac{n+1}{n}$

C. n

D. $n + \frac{1}{n}$

Answer: C

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17. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number is :

A. 1

B. 2

C. 3

D. 4

Answer: C

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18. Let $f: R \rightarrow R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then $\lim_{x \to 0} \left[\frac{f(1+x)}{f(1)}\right]^{1/x}$ equals

A. 1

B. $e^{\frac{1}{2}}$

C. e^2

D. None of these

Answer: C

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19. For $x \varepsilon R$, $\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x$ is equal to

A. e

B. e^{-1}

C. e^{-5}

D. e^5

Answer: C

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Exercise For Session 1

1. If $f(a) = f'(a) = g(a) = -g'(a) = -$ then $\lim_{x \to a} \frac{g(x)f(a) - g(a)f(x)}{x-a}$

A. -5

B. 3

C. -3

D. 5

Answer: C

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2. The value of $\lim_{x \to 0} \frac{x \cos x - \log(1+x)}{x^2}$ is

A. 1

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. None of these

Answer: C

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3. $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$ is equal to

A. $\frac{3}{2}$

B. $-\frac{3}{2}$

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

Answer: C

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4. The value of $\lim_{x \to a} \frac{\cos x - \cos a}{\cot x - \cot a}$ is

A. $-\sin^3 a$

B. $\cos^2 a$

C. $\sin^3 a$

D. $\cot a$

Answer: A

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5. The value of $\lim_{x \to 0}\left(\frac{1}{x^2} - \cot x\right)$ is

A. 0

B. 1

C. $\frac{1}{4}$

D. None of these

Answer: C

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6. The value of $\lim_{x \to \infty}\left(\sqrt{a^2x^2 + ax + 1} - \sqrt{a^2x^2 + 1}\right), (a > 0)$ is

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. does'nt exist

D. None of these

Answer: A

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7. The value of $\lim_{x \to \infty} \frac{1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + n^3}{(n^2+1)^2}$

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. $\frac{1}{2\sqrt{2}}$

D. None of these

Answer: A

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8. The value of $\lim_{n \to \infty} \frac{1.\,n + 2.\,(n-1) + 3.\,(n-2) + \ldots + n.1}{1^2 + 2^2 + \ldots + n^2}$

A. 1

B. -1

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer:

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9. The value of $\lim_{n \to \infty} \frac{a^n + b^n}{a^n - b^n}$, (where $a > b >$ is

A. 1

B. -1

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

Answer:

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Exercise For Session 2

1. If $\lim_{x \to 0} \left(x^{-3} \sin 3x + ax^{-2} + b\right)$ exists and is equal to 0 then

A. $a = -3, b = \frac{9}{2}$

B. $a = 3, b = \frac{9}{2}$

C. $a = -3, b = \frac{-9}{2}$

D. None of these

Answer: D

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2. The value of $\lim_{x \to a} \dfrac{x \sin a - a \sin x}{x - a}$ is

A. $a \sin a - \cos a$

B. $\sin a - a \cos a$

C. $\cos a + a \sin a$

D. $\sin a + a \cos a$

Answer: B

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3. Evaluate the following limits:

$$\lim_{x \to \pi} \frac{\sqrt{2 + \cos x} - 1}{(\pi - x)^2}$$

A. $\dfrac{1}{4}$

B. $\dfrac{1}{2}$

C. 2

D. Doesn't exist

Answer: A

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4. evaluate $\lim_{\theta \to \frac{\pi}{4}} \frac{\sqrt{2} - \cos\theta - \sin\theta}{(4\theta - \pi)^2}$

A. $\frac{1}{16\sqrt{2}}$

B. $\frac{1}{16}$

C. $\frac{1}{8\sqrt{2}}$

D. $\frac{1}{2\sqrt{2}}$

Answer: D

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5. The value of $\lim_{x \to \pi/4} \frac{(\cos x + \sin x)^3 - 2\sqrt{2}}{1 - \sin 2x}$ is

A. $\frac{\sqrt{3}}{2}$

B. $-\dfrac{3}{\sqrt{2}}$

C. $\dfrac{1}{2}$

D. $-\dfrac{1}{\sqrt{2}}$

Answer: B

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Exercise For Session 3

1. The value of $\lim_{x \to 0} \dfrac{e^x - e^{x \cos x}}{x + \sin x}$ is

A. 0

B. 1

C. -1

D. None of these

Answer: A

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2. the value of $\lim_{x \to y} \frac{x^y - y^x}{x^x - y^y}$ is:

A. $\frac{1 - \log x}{1 + \log x}$

B. $\frac{1 - \log y}{1 + \log y}$

C. $\frac{\log x - \log y}{\log x + \log y}$

D. None of these

Answer: B

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3. The value of $\lim_{x \to 0} \frac{p^x - q^x}{r^x - s^x}$ is

A. $\frac{1 - \log p}{1 + \log p}$

B. $\frac{\log p - \log q}{\log r - \log s}$

C. $\frac{\log p . \log q}{\log r . \log s}$

D. None of these

Answer: B

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4. The value of $\lim_{x \to \infty} (x+2)\tan^{-1}(x+2) - (x\tan^{-1}x)$ is

A. $\frac{\pi}{2}$

B. Doesn't exist

C. $\frac{\pi}{4}$

D. None of these

Answer: D

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5. The value of $\lim_{x \to 4} \frac{(\cos\alpha)^x - (\sin\alpha)^x - \cos 2\alpha}{x-4}, \alpha\varepsilon\left(0, \frac{\pi}{2}\right)$ is

A. $\log(\cos\alpha) + (\sin\alpha)^4 \log(\sin\alpha)$

B. $(\cos^4\alpha)\log(\cos\alpha) - (\sin\alpha)^4\log(\sin\alpha)$

C. $(\cos^4\alpha)\log(\cos\alpha)$

D. None of these

Answer: B

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6. The value of $\lim_{x\to 0}\left(\frac{1^x + 2^x + 3^x + \ldots\ldots\ldots\ldots + n^x}{n}\right)^{a/x}$ is

A. $(n!)^{a/n}$

B. $n!$

C. $a^{n!}$

D. Doesn't exist

Answer:

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7. If $\lim_{x \to 0} \left(1 + ax + bx^2\right)^{\frac{2}{x}} = e^3$,the value of a and b are

A. $a = \frac{3}{2}, b\varepsilon R$

B. $a = \frac{1}{2}, b\varepsilon R$

C. $a\varepsilon R, b\varepsilon R$

D. None of these

Answer:

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8. If α an β are roots of $ax^2 + bx + c = 0$ the value for $\lim_{x \to \alpha} \left(1 + ax^2 + bx + c\right)^{2/x - \alpha}$ is

A. $e^{2a(\alpha - \beta)}$

B. $e^{a(\alpha - \beta)}$

C. $e^{\frac{2a}{3}}(\alpha-\beta)$

D. None of these

Answer:

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9. The value of $\lim (n \to \infty)\left((1.5)^n + \left[(1+0.0001)^{10000}\right]^n\right)^{\frac{1}{n}}$, where [.] denotes the greatest integer function is:

A. 1

B. $\frac{1}{2}$

C. does'nt exist

D. 2

Answer:

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10. The value of $\lim_{x \to 0} |x|^{[\cos x]}$, [.] denotes greatest integer function is

A. 0

B. Doesn't exist

C. 1

D. None of these

Answer:

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Exercise For Session 4

1. Evaluate $\lim_{x \to 0^+} (\sin x)^x$

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2. Evaluate $\lim_{x \to 0^+} (\sin x)^{\tan x}$

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3. evaluate $\lim_{n \to \infty} \left(\frac{e^n}{\pi}\right)^{\frac{1}{n}}$

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4. The value of $\lim_{x \to 0} \frac{e - (1+x)^{1/x}}{\tan x}$ is

A. e

B. $\frac{11e}{24}$

C. $\frac{e}{2}$

D. None of these

Answer: D

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Exercise For Session 5

1. The value of $\lim_{x \to 1} (\{1 - x + [x] + [1 - x]\}$ (where [.] donetes the greatest integral function) is

A. -1

B. Doesn't exist

C. 1

D. None of these

Answer:

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2. The value of $\lim_{x \to 0} \frac{\sin[x]}{[x]}$ (where [.] denotes the greatest integer function) is

A. 1

B. $\sin 1$

C. does'nt exist

D. None of these

Answer:

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3. The value of $\lim_{x \to 0} \sin^{-1}\{x\}$ (where {.} denotes fractional part of x) is

A. 0

B. $\frac{\pi}{2}$

C. does'nt exist

D. None of these

Answer:

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4. If [.] denotes the greatest integer function, then $\lim_{x \to 0} \left[\frac{x^2}{\tan x \sin x}\right]$, is

A. 0

B. 1

C. does'nt exist

D. None of these

Answer:

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Exercise For Session 6

1. The value o $\lim_{x \to \infty} \left(\frac{1}{n^3}\right)\left(\left[1^2x + 1^2\right] + \left[2^2x + 2^2\right] + \ldots\ldots + \left[n^2x + n^2\right]\right)$ is where [.] denotes the greatest integer function.

A. $\frac{x}{3}$

B. $x + \frac{1}{3}$

C. $\frac{x}{3} + \frac{1}{3}$

D. None of these

Answer:

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2. The value of $\lim_{x \to 1^+} \frac{\int_1^x |t-1| dt}{\sin(x-1)}$ is

A. 0

B. 1

C. does'nt exist

D. None of these

Answer:

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3. The value of $\lim_{n \to \infty} \sum_{k=1}^{n} \log\left(1 + \frac{k}{n}\right)^{\frac{1}{n}}$,is

A. $\log_e\left(\frac{e}{4}\right)$

B. $\log_e\left(\frac{4}{e}\right)$

C. $\log_e 4$

D. None of these

Answer:

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4. Evaluate: $(\lim)_{n \to \infty} n\left[\frac{1}{na} + \frac{1}{na+1} + \frac{1}{na+2} + + \frac{1}{nb}\right]$

A. $\log\left(\frac{a}{b}\right)$

B. $\log\left(\frac{b}{a}\right)$

C. $\log(ab)$

D. None of these

Answer:

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