



MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

LOGARITHM AND THEIR PROPERTIES



1. Find the value of the following

 $\log_9 27$

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2. $\log_{3\sqrt{2}} 324$

3. Find the value of the following

 $\log_{1/9}\!\left(27\sqrt{3}\right)$

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$$\log_{\left(5+2\sqrt{6}
ight)}\left(5-2\sqrt{6}
ight)$$

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5. Find the value of the following

 $\log_{0.2} 0.008$



6. Find the value of the following

 $2^{\log_4 5}$







11. Find the value of the following



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12. Find the value of the following

 $\log_{30} 1$

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13. If log2=0.301 and log3=0.477, find the number of integers in

(ii) 6^{20}



14. Find the numbers of zeroes between the decimal point and first significant digit of $(0.036)^{16}$, where $\log 2 = 0.301$ and $\log 3 = 0.477$.



15. Solve the equation
$$3 \cdot x^{\log 5^2} + 2^{\log 5^x} = 64.$$

16. If
$$4^{\log 16^4} + 9^{\log 3^9} = 10^{\log x^{83}}$$
 , find x.

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17. Prove that
$$a\sqrt{\log_a b} - b\sqrt{\log_b a} = 0$$

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18. Prove that
$$rac{\log_2(24)}{\log_{96}2} - rac{\log_2(192)}{\log_{12}2} = 3$$

19. Solve for a, λ if $\log_{\lambda} a \cdot \log_{5} \lambda \cdot \log_{\lambda} 25 = 2$.



20. Arrange in ascending order

 $\log_2(x), \log_3(x), \log_e(x), \log_{10}(x),$ if

 $\mathbf{I}.x > 1$

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21. Arrange in ascending order

 $\log_2(x), \log_3(x), \log_e(x), \log_{10}(x),$ if

 $\mathsf{II.0} < x < 1.$

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22. If log 11=1.0414, prove that $10^{11} > 11^{10}$.

23. If $\log_2(x-2) < \log_4(x-2)$, find the interval in which x lies.



24. Prove that $\log_n(n+1) > \log_{n+1}(n+2)$ for any natural number

n > 1.

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25. For x>1, the minimum value of $2\log_{10}(x)-\log_x(0.01)$ is



26. Which is smaller 2 or $(\log_{\pi} 2 + \log_2 \pi)$

27. The expression $\log_2 5 - \sum_{k=1}^4 \log_2 \left(\sin \left(\frac{k\pi}{5} \right) \right)$ reduces to $\frac{p}{q},$ where

p and q are co-prime, the value of p^2+q^2 is

A. 13

B. 17

C. 26

D. 29

Answer: B

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28. If $3\leq a\leq 2015, 3\leq b\leq 2015$ such that $\log_a b+6\log_b a=5$, the

number of ordered pairs (a,b) of integers is

A. 48

B. 50

C. 52

D. 54

Answer: C

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29. The lengths of the sides of a traingle are $\log_{10} 12$, $\log_{10} 75$ and $\log_{10} n$, where $n \in N$. If a and b are the least ad greatest values of n respectively. The value of b-a is divisible by

A. 221

B. 222

C. 223

D. 224

Answer: C

30. Let
$$5\log_{abc}(a^3 + b^3 + c^3) = 3\lambda\left(\frac{1 + \log_3(abc)}{\log_3(abc)}\right)$$
 and $(abc)^{a+b+c} = 1$ and $\lambda = \frac{m}{n}$ m, n are relatively prime , find $|m+n| + |m-n|$

A. 8

B. 10

C. 12

D. 14

Answer: B

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31. If
$$a^{\log b^c} = 3.3^{\log 4^3}.3^{\left(\log 4^3\right)^{\log 4^3}}.3^{\left(\log 4^3\right)^{\left(\log 4^3\right)^{\log 4^{-3}}}}$$
 ...oo

where a,b,c $\in~\mathsf{Q}$ the value of abc is -+

B. 12

C. 16

D. 20

Answer: C

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32. Number of real roots of equation

$$3^{\log 3^{\left(x^2-4x+3
ight)}}=(x-3)$$
 is

A. 0

B. 1

C. 2

D. infinite

Answer: A

33. If $\log_6 a + \log_6 b + \log_6 c = 6$, where a,b,c, $\in N$ and a,b,c are in GP and b-a is a square of an integer, then the value of a+b-c is

A.	21
В.	15
C.	9
D.	3

Answer: B

34. If

$$x = \log_{2a}\left(\frac{bcd}{2}\right), y = \log_{3b}\left(\frac{acd}{3}\right), z = \log_{4c}\left(\frac{abd}{4}\right) \text{ and } w = \log_{5d}\left(\frac{abd}{5}\right)$$
and $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1} + \frac{1}{w+1} = \log_{abcd} N + 1$, then value of $\frac{N}{40}$ is

A. 40

B. 80

C. 120

D. 160

Answer: A::B::C

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35. The equation

$$\left(\log_{10}x+2
ight)^3+\left(\log_{10}x-1
ight)^3=\left(2\log_{10}x+1
ight)^3$$
 has

A. no natural solution

B. two rational solutions

C. no prime solutions

D. one irritional solution

Answer: A

36. The value of
$$\frac{\log_5 9 \cdot \log_7 5 \cdot \log_3 7}{\log_3 \sqrt{6}} + \frac{1}{\log_4 \sqrt{6}}$$
 is co-prime with
A. 1
B. 3
C. 4
D. 5

Answer: A::C::D



37. Consider the quadratic equation
$$(\log_{10} 8)x^2 - (\log_{10} 5)x = 2(\log_2 10)^{-1} - x$$
. Which of the following quantities are irrational.

A. sum of roots

B. product of roots

C. sum of coefficients

D. discriminant

Answer: A::B

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38. Find the real solutions to the system of equations $\log_{10}(2000xy) - \log_{10} x \cdot \log_{10} y = 4$, $\log_{10}(2yz) - \log_{10} y \log_{10} z = 1$ and $\log_{10} zx - \log_{10} z \log_{10} x = 0$ A. $x_1 + x_2 = 101$ B. $y_1 + y_2 = 25$ C. $x_1 + x_2 = 100$ D. $z_1 + z_2 = 100$

Answer: A



39. Suppose that
$$\log_{10}(x-2) + \log_{10} y = 0$$
 and $\sqrt{x} + \sqrt{y-2} = \sqrt{x+y}$. Then the value of $(x + y)$ is
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40. Suppose that $\log_{10}(x-2) + \log_{10} y = 0$ and $\sqrt{x} + \sqrt{y-2} = \sqrt{x+y}$. Then the value of $(x + y)$ is
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41. Suppose that $\log_{10}(x-2) + \log_{10} y = 0$ $\sqrt{x} + \sqrt{y-2} = \sqrt{x+y}$.

If $x^{2t^2-6}+y^{6-2t^2}=6$, the value of t_1,t_2,t_3,t_4 is

42. IF $10^{\log p \{\log q (\log r^x)\}}$ =1 and $\log_q \{\log_r (\log_p x)\} = 0.$

The value of x is

A. q^r B. r^q

 $\mathsf{C}.r^p$

D. rq

Answer: B

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43. $10^{\log_p\left(\log_q\left(\log_r\left(x\right)\right)\right)} = 1$ and $\log_q\left(\log_r\left(\log_p(x)\right)\right) = 0$, then 'p' is equals a. $r^{\frac{q}{r}}$ b. rq c. 1 d. $r^{\frac{r}{q}}$

A.
$$\log_q \{\log_r (\log_p x)\} = 0.$$

B. rq

C. 1

D. $r^{r/p}$

Answer: A



44. IF $10^{\log_p \{\log_q (\log_r x)\}}$ =1 and $\log_q \{\log_r (\log_p x)\} = 0$. The value of q is

A. $r^{p \, / \, r}$

 $\mathsf{B.}\,p\log_p r$

 $\mathsf{C}. r \log_r p$

D. $r^{r/p}$

Answer: C



45. If x_1 and x_2 are the solutions of the equation $x^{\log_{10} x} = 100x$ such that

$$x_1>1$$
 and $x_2<1$, the value of $rac{x_1x_2}{2}$ is

46. If
$$(31.6)^a = (0.0000316)^b = 100$$
 , the value of $rac{1}{a} - rac{1}{b}$ is

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47. Statement-1(Assertion and Statement -2 (reason) Each of these examples also has four alternative choices, only one of which is the correct answer. You have select the correct choice as given below.

Statement -1 If $N = \left(\frac{1}{0.4}\right)^{20}$, then N contains 7 digit before decimal. Statement -2 Characteristic of the logarithm of N to the base 10 is 7.

A. Statement -1 if true, Statement-2 is true, Statement-2 is a correct

explanation for Statement -1

B. Statement -1 if true, Statement-2 is true, Statement-2 is not a

correct explanation for Statement -1

C. Statement -1 if true, Statement-2 is false

D. Statement -1 if false , Statement-2 is true

Answer: D

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48. Statement-1(Assertion and Statement -2 (reason) Each of these examples also has four alternative choices, only one of which is the correct answer. You have select the correct choice as given below. Statement-1 If $p,q \in N$ satisfy the equation $x^{\sqrt{x}} = (\sqrt{x})^x$ and q > p, then q is a perfect number . Itbgt If a number is equal to the sum of its factor, then number is known as perfect number.

A. Statement -1 if true, Statement-2 is true, Statement-2 is a correct

explanation for Statement -2

B. Statement -1 if true, Statement-2 is true, Statement-2 is not a

correct explanation for Statement -2

C. Statement -1 if true, Statement-2 is false

D. Statement -1 if false , Statement-2 is true

Answer: D

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49. Prove $\log_3(5)$ is irrational.

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50. Find the value of the expressions $(\log 2)^3 + \log 8. \log(5) + (\log 5)^3$.





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54. Given $a^2+b^2=c^2$. Prove that $\log_{b+c}a+\log_{c-b}a=2\log_{c+b}a.\log_{c-b}a,\ orall a>0,a
eq 1$ c-b>0,c+b>0c-b
eq 1,c+b
eq 1.



55. Let
$$a > 0, c > 0, b = \sqrt{a}c, a, c$$
 and $ac \neq 1, N > 0$.
Prove that $\frac{\log_a N}{\log_c N} = \frac{\log_a N - \log_b N}{\log_b N - \log_c N}$

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56.

$$a^x = b, b^y = c, c^z = a, x = (\log_b a)^{k1}, y = (\log_c b)^{k2}, z = (\log_a c)^{k3},$$

IF

find the minimum value of $3k_1 + 6k_2 + 12k_3$.

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57. IF
$$x = 1 + \log_a bc, y = 1 + \log_b ca, z = 1 + \log_c ab$$
, prove that

xyz=xy+yz+zx.

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58. If
$$\frac{Ina}{(b-c)}=\frac{Inb}{(c-a)}=\frac{Inc}{(a-b)}$$
 ,Prove that a^{b+c} . b^{c+a} . $c^{a+b}=1$

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59. Simplify
$$5^{\log 1/5^{(1/2)}} + \log_{\sqrt{2}} \left(\frac{4}{\sqrt{7} + \sqrt{3}}\right) + \log_{1/2} \left(\frac{1}{10 + 2\sqrt{21}}\right).$$

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60. Find the square of the sum of the roots of the equation $\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_3 x \cdot \log_5 x$

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61. Given that
$$\log_2 a = \lambda$$
, $\log_4 b = \lambda^2$ and $\log_{c^2}(8) = \frac{2}{\lambda^3 + 1}$ write $\log_2\left(\frac{a^2b^5}{5}\right)$ as a function of λ , $(a, b, c > 0, c \neq 1)$.

62. Given that $\log_2(3) = a, \log_3(5) = b, \log_7(2) = c$, express the

logrithm of the number 63 to the base 140 in terms of a,b & c.



64. Solve the equations
$$\log_{1000} |x+y| = rac{1}{2} . \log_{10} y - \log_{10} |x| = \log_{100} 4$$

for x and y

Match Mid





66. The value of x satisfying $|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$ is

67. Find all real numbers x which satisfy the equation.
$$2\log_2\log_2 x + \log_{\frac{1}{2}}\log_2(2\sqrt{2}x) = 1$$

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$$\textbf{68.} \log_{\frac{3}{4}} \log_8 \big(x^2 + 7 \big) + \log_{\frac{1}{2}} \log_{\frac{1}{4}} \big(x^2 + 7 \big)^{-1} = -2.$$

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69. Prove that: `2(sqrt((log)_a4sqrt(a b)+(log)_b4sqrt(a b))-(log)_a4sqrt(2/b)+(log)_b4sqrt(a/b))dotsqrt((log)_a b)={2ifbgeqa >1
2^((log)_a bif1



A. 6 B. 8 C. 3 D. 5

Answer: A

2. The value of
$$\log_{\left(8-3\sqrt{7}
ight)}\left(8+3\sqrt{7}
ight)$$
 is

B. -1

C. 0

D. Not defined

Answer: B

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3. The value of
$$(0.16)^{\log_{2.5} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \cos \right)}$$
, is

A. 2

B. 4

C. 6

D. 8

Answer: B

4. If log2=0.301, the number of integers in the expansion of 4^{17} is

B. 11

A. 9

C. 13

D. 15

Answer: B

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5. If log2=0.301, the number of zeroes between the decimal point and the

first significant figure of 2^{-34} is

A. 9

B. 10

C. 11

D. 12

Answer: B

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Exercise For Session 2

1. If $a = \log_{24} 12$, $b = \log_{36} 24$, $c = \log_{48} 36$, then show that 1 + abc = 2bcA. 2ab B. 2bc C. 2ca D. ba+bc Answer: B

2. The value of $\log_4[~|~\log_2\{\log_2(\log_3)81)\}]$ is equal to

A. -1 B. O C. 1 D. 2

Answer: B

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3.
$$\frac{\log_2 \log_2 \left(\sqrt{\sqrt{\dots \sqrt{\sqrt{2}}}}\right)}{nTimes}$$
 is equal to

A. 0

B. 1

C. n

 $\mathsf{D}.-n$

Answer: D



4. If x = $\log_3(5)$ and y = $\log_{17}(25)$, which one of the following is correct?

A. a < b

B. a=b

 $\mathsf{C}.\,a>b$

D. None of these

Answer: C

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5. The value of
$$\log_{0.75}\log_2\sqrt{\sqrt[-2]{(0.125)}}$$
 is equal to

A. -1

B. 0

C. 1

D. None of these

Answer: C

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Exercise For Session 3

1. If $\log_{0.16}(a+1) < \log_{0.4}(a+1)$, then a satisfies

A. a > 0

 ${\sf B}.\, 0 < a < 1$

C. -1 < a < 0

D. None of these

Answer: C

- **2.** The value of x satisfying the inequation $x^{rac{1}{\log 10^x}}$. $\log_{10} x < 1$, is
 - A. 0 < x < 10
 - B. $0 < x < 10^{10}$
 - C. $0 < x < 10^{1/10}$
 - D. None of these

Answer: C

(D) Watch Video Solution

- 3. IF $\log_{\cos ecx} \sin x > 0$ then,
 - A. x > 0
 - B. x < 0

 $\mathsf{C}.-1 < x < 1$

D. None of these

Answer: D



4. The value of $\log_{10} 3$ lies in the interval

A.
$$\left(\frac{2}{5}, \frac{1}{2}\right)$$

B. $\left(0, \frac{1}{2}\right)$
C. $\left(0, \frac{2}{5}\right)$

D. None of these

Answer: A



5. The least value of n in order that the sum of first n terms of the infinite

series $1+rac{3}{4}+\left(rac{3}{4}
ight)^2+\left(rac{3}{4}
ight)^3+...$, should differ from the sum of the

series by less than 10^{-6} , is $(given \log 2 = 0.30103, \log 3 = 0.47712)$

A. 14

B. 27

C. 53

D. 57

Answer: C

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Exercise Single Option Correct Type Questions

1. If $\log_{10}2=0.3010...,~$ the number of digits in the number 2000^{2000} is

A. 6601
B. 6602

C. 6603

D. 6604

Answer: C

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2. There exists a positive number k such that $\log_2 x + \log_4 x + \log_8 x = \log_k x$, for all positive real no x. If $k=a^{\frac{1}{b}}$ where (a,b) ε N, the smallest possible value of (a+b)= (C) 12 A) 75 (B) 65 (D) 63_

A. 12

B. 63

C. 65

D. 75

Answer: D



3. If $x_1, x_2 \& x_3$ are the three real solutions of the equation $x^{\log_{10}^2 x + \log_{10} x^3 + 3} = rac{2}{\left(rac{1}{\sqrt{x+1-1}} - rac{1}{\sqrt{x+1+1}}
ight)}, ext{ where } x_1 > x_2 > x_3,$

then

A. AP

B. GP

C. HP

D.
$$a^{-1} + b^{-1} = c^{-1}$$

Answer: B

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4. If
$$f(n) = \prod_{i=2}^{n-1} \log_i (i+1)$$
 , the value of $\sum_{k=1}^{100} f\left(2^k
ight)$ equals

A. 5010

B. 5050

C. 5100

D. 5049

Answer: B

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5. If $\log_3 27. \, \log_x 7 = \log_{27} x. \, \log_7 3$, the least value of x is

A. 7^{-3}

B. 3^{-7}

 $C. 7^{3}$

 $D.3^7$

Answer: A

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6. If $x = \log_5(1000)$ and $y = \log_7(2058)$,then

A. x > y

 $\mathsf{B.}\, x < y$

 $\mathsf{C}.\,x=y$

D. None of these

Answer: A

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7. Solve for
$$x$$

 $\log_5 120 + (x - 3) - 2 \cdot \log_5 (1 - 5^{x-3}) = -\log_5 (0.2 - 5^{x-4})$
A. 1
B. 2
C. 3
D. 4

Answer: A



8. If $x_n > x_{n-1} > \dots > x_3 > x_1 > 1$. then the value of $\log_{x1} \left[\log_{x2} \left\{ \log_{x3} \dots \log_{x4} \left(x_n
ight)^{x_{r=i}}
ight\}
ight]$

A. 0

B. 1

C. 2

D. undefined

Answer: B



9. If
$$rac{x(y+z-x)}{\log x}=rac{y(z+x-y)}{\log y}rac{z(x+y-z)}{\log z}, prove that x^yy^x=z^xy^z=x^z$$

A.
$$z^x x^z$$

 $\mathsf{B.}\, x^z y^x$

 $\mathsf{C}.\, x^y y^z$

D. $x^x y^y$

Answer: A

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10. If
$$y=a^{rac{1}{1-\log_a x}}$$
 , $z=a^{rac{1}{1-\log_a y}}$ then prove that $x=a^{rac{1}{1-\log_a x}}$

A.
$$a^{\frac{1}{1+\log_a z}}$$

B. $a^{\frac{1}{2+\log_a z}}$

$$\mathsf{C}.\,a^{\frac{1}{1-\log_a z}}$$

D.
$$a^{rac{1}{2-\log_a z}}$$

Answer: C

11. If $(\log)_{0.3}(x-1) < (\log)_{0.09}(x-1)$, then x lies in the interval $(2,\infty)$ (b) (1,2) (-2, -1) (d) None of these

A. $(-\infty.1)$

B. (1,2)

 $\mathsf{C}.(2,\infty)$

D. None of the above

Answer: C



B. 2

C. 0

D. -1

Answer: C



13. If
$$x = 1 + \log_a bc$$
, $y = 1 + \log_b ca$, $z = 1 + \log_c ab$, then

$$\frac{xyz}{xy + yz + zx}$$
 is equal to
A. 0
B. 1
C. -1
D. 2

Answer: B

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14. The value of $a^{rac{\log_b(\log_b x)}{\log_b a}}, \,$ is

A. $\log_a N$

 $\operatorname{B.log}_b N$

 $\mathsf{C}.\log_N a$

 $D. \log_N b$

Answer: B

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15. Find the value of $49^{(1-\log_7(2))} + 5^{-\log_5(4)}$ is

A. 10.5

B. 11.5

C. 12.5

D. 13.5

Answer: C



16. The number of real values of the parameter k for which the equation $(\log_{16}x)^2 - \log_{16}x + \log_{16}k = 0$ with real coefficients will have exactly one solution, is

A. 1

B. 2

C. 3

D. 4

Answer: B

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17. The number of roots of the equation $x^{\log_X (x+3)^2} = 16$ is

B. 0

- C. 2
- D. 4

Answer: B

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18. The point on the graph $y=\log_2\log_6\left\{2^{\sqrt{2x+1}}+4
ight\}$ whose y coordinate is 1 is

A. (1,1)

B. (6,1)

C. (8,1)

D. (12,1)

Answer: D

19. Given ,log2=0.301 and log3=0.477, then the number of digits before decimal in $3^{12} imes 2^8$ is

A. 7 B. 8

C. 9

D. 11

Answer: C

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20. The values of x, satisfying the equation for $orall a>0, 2\log_x a+\log_{ax} a+3\log_{a^2x} a=0$ are

B. two

C. three

D. four

Answer: B

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Exercise More Than One Correct Option Type Questions

1. If
$$x^{\left(\log_2 x\right)^2-6\log_2 x+11}=64$$
 then x is equal to

A. 2

B.4

C. 6

D. 8

Answer: A::B::D

2. If $\log_\lambda x. \log_5 \lambda = \log_x 5, \lambda
eq 1, \lambda > 0$, then x is equal to

A. λ

B. 5

C.
$$\frac{1}{5}$$

D. None of these

Answer: B::C



3. If
$$S = \left\{x\!:\!\sqrt{\log_x\sqrt{3x}},$$
 where $\log_3x > -1\}$, then

A. S is a finite set

 $\mathsf{B}.\,S\not\in\emptyset$

 $\mathsf{C}.\,S\subset(0,\infty)$

D. S properly contains
$$\left(rac{1}{3},\infty
ight)$$

Answer: C::D

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4. If x satisfies
$$\log_2(9^{x-1}+7) = 2 + \log_2(3^{x-1}+1)$$
, then

A. $x
ot \in Q$

 $\mathsf{B}.\, x \not\in N$

 $\mathsf{C}.\, x \notin \{x \notin Q\!:\! x < 0\}$

D. $x
ot\in N_e$ (set of even natural number)

Answer: A::B





- $\mathsf{C}.\,\frac{1}{n}$
- $\operatorname{\mathsf{D.}}\log_{1/p}(p^n)$

Answer: B::D

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6. If $\log_a x = lpha, \log_b x = eta, \log_c x = \gamma$ and $\log_d x = \delta, x
eq 1$ and a,b,c,

d
eq 1,then \log_{abcd} x equals

$$\begin{array}{l} \mathsf{A.} & \leq \displaystyle \frac{\alpha+\beta+\gamma+\delta}{16} \\ \mathsf{B.} & \geq \displaystyle \frac{\alpha+\beta+\gamma+\delta}{16} \\ \mathsf{C.} \displaystyle \frac{1}{\alpha^{-1}+\beta^{-1}+\gamma^{-1}+\delta^{-1}} \end{array}$$

D.
$$rac{1}{lphaeta\gamma\delta}$$

Answer: A::C



7. If
$$\log_{10}5=a$$
 and $\log_{10}3=b$,then

A.
$$\log_{10}8 = 3(1-a)$$

B. $\log_{40}15 = rac{(a+b)}{(3-2a)}$
C. $\log_{243}32 = \left(rac{1-a}{b}
ight)$

D. All of these

Answer: A::B::C::D



8. If x is a positive real number different from 1 such that $\log_a x$, $\log_b x$, $\log_c x$ are in A.P then

A.
$$\log b = 2rac{(\log a)(\log c)}{(\log a + \log c)}$$

B. $b = rac{a+c}{2}$
C. $b = \sqrt{a}c$

D.
$$c^2 = (ac)^{\log_a b}$$

Answer: A::D

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9. If |a| < |b|, b-a < 1 and a, b are the real roots of the equation $x^2 - |\alpha|x - |\beta| = 0$, then the equation $\log_{|b|} \left| \frac{x}{a} \right| - 1 = 0$ has

A. one root lying in interval $(-\infty, a)$

B. one root lying in interval (b,∞)

C. one positive root

Answer: C::D



Exercise Passage Based Questions

1. Let $\log_2 N=a_1+b_1, \log_3 N=a^2+b^2$ and $\log_5 N=a_3+b_3$, where $a_1,a_2,a_3
ot\in 1$ and $b_1,b_2b_3
ot\in [0,1].$

If $a_1 = 5 \,$ and $\, a_2 = 3$, the number of integral values of N is

A. 16

B. 32

C. 48

D. 64

Answer: B

2. Let $\log_2 N=a_1+b_1, \log_3 N=a_2+b_2$ and $\log_5 N=a_3+b_3$, where $a_1,a_2,a_3
ot\in 1$ and $b_1,b_2b_3\in [0,1).$

If $a_1 = 6, a_2 = 4$ and $a_3 = 3$,the difference of largest and smallest integral values of N, is

A. 124

B. 63

C. 624

D. 127

Answer: D

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3. Let $\log_2 N = a_1 + b_1, \log_3 N = a_2 + b_2$ and $\log_5 N = a_3 + b_3$, where

$$a_1,a_2,a_3
ot\in 1$$
 and $b_1,b_2b_3
ot\in [0,1).$

If $a_1 = 6, a_2 = 4$ and $a_3 = 3$,the difference of largest and smallest integral values of N, is

A. 2 B. 8 C. 14

D. 20

Answer: A

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4. Let S denotes the antilog of 0.5 to the base 256 and K denotes the number of digits in 6^{10} (given $\log_{10} 2 = 0.301$, $\log_{10} 3 = 0.477$) and G denotes the number of positive integers, which have the characteristic 2, when the base of logarithm is 3.

The value of SKG is

B. 24

C. 30

D. 36

Answer: A

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5. Let S denotes the antilog of 0.5 to the base 256 and K denotes the number of digits in 6^{10} (given $\log_{10} 2 = 0.301$, $\log_{10} 3 = 0.477$) and G denotes the number of positive integers, which have the characteristic 2, when the base of logarithm is 3.

The value of SKG is

A. 72

B. 144

C. 216

D. 288

Answer: B

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6. Let S denotes the antilog of 0.5 to the base 256 and K denotes the number of digits in 6^{10} (given $\log_{10} 2 = 0.301$, $\log_{10} 3 = 0.477$) and G denotes the number of positive integers, which have the characteristic 2, when the base of logarithm is 3.

The value of SKG is

A. 1440

B. 17280

C. 2016

D. 2304

Answer: D

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7. Suppose U denotes the number of digits in the number $(60)^{100}$ and M denotes the number of cyphers after decimal, before a significant figure comes in $(8)^{-296}$. If the fraction U/M is expressed as rational number in the lowest term as p/q (given $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.477$). The value of p is

- A. 1
- B. 2
- C. 3
- D. 4

Answer: B



8. Suppose U denotes the number of digits in the number $(60)^{100}$ and M denotes the number of cyphers after decimal, before a significant figure comes in $(8)^{-296}$. If the fraction U/M is expressed as rational number in

the lowest term as p/q (given $\log_{10}2=0.301$ and $\log_{10}3=0.477$) .

The value of q is

A. 5 B. 2 C. 3

D. 4

Answer: C

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9. Suppose U denotes the number of digits in the number $(60)^{100}$ and M denotes the number of cyphers after decimal, before a significant figure comes in $(8)^{-296}$. If the fraction U/M is expressed as rational number in the lowest term as p/q (given $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.477$). The equation whose roots are p and q, is

A.
$$x^2 - 3x + 2 = 0$$

C.
$$x^2 - 7x + 12 = 0$$

D. $x^2 - 9x + 20 = 0$

B. $x^2 - 5x + 6 = 0$

Answer: B

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10. Let G,O,E and L be positive real numbers such that log(G.L)+log(G.E)=3,log(E.L)+log(E.O)=4, log(O.G)+log(O.L)=5 (base of the log is 10)

If the value of the product (GOEL) is λ , the value of $\sqrt{\log \lambda \sqrt{\log \lambda \sqrt{\log \lambda \ldots}}}$ is

A. 3

B. 4

C. 5

D. 7

Answer: B



11. Let G,O,E and L be positive real numbers such that log(G.L)+log(G.E)=3,log(E.L)+log(E.O)=4, log(O.G)+log(O.L)=5 (base of the log is 10)

If the minimum value of 3G+2L+2O+E is $2^{\lambda}3^{\mu}5^{\nu}$,Where λ , μ ,and ν are whole numbers, the value of $\sum (\lambda^{\mu} + \mu^{\nu})$ is

A. 7

B. 13

C. 19

D. None of these

Answer: A

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12. Let G,O,E and L be positive real numbers such that $\log(G.L)+\log(G.E)=3,\log(E.L)+\log(E.O)=4, \log(O.G)+\log(O.L)=5$ (base of the log is 10) If $\log\left(\frac{G}{O}\right)$ and $\log\left(\frac{O}{E}\right)$ are the roots of the equation A. $x^2 + x = 0$ B. $x^2 - x = 0$ C. $x^2 - 2x + 3 = 0$ D. $x^2 - 1 = 0$

Answer: D

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Exercise Single Integer Answer Type Questions

1. If
$$x,y\in R^+$$
 and $\log_{10}(2x)+\log_{10}y=2$, $\log_{10}x^2-\log_{10}(2y)=4$ and $x+y=rac{m}{n}$,Where m and n are relative prime , the value of $m-3n^6$ is

2. A line $x = \lambda$ intersects the graph of $y = \log_5 x$ and $y = \log_5 (x + 4)$.The distance between the point of intersection is 0.5. Given $\lambda = a + \sqrt{b}$,where a and b are integers, the value of (a+b) is

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3. If the left hand side of the equation

$$a(b-c)x^2+b(c-a)xy+c(a-b)y^2=0$$
 is a perfect square , the

value of

$$\left\{rac{\log(a+c)+\log(a-2b+c)^2}{\log(a-c)}
ight\}^2$$
, $ig(a,b,c\in R^+,a>cig)$ is

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4. Number of integers satisfying the inequality

$$\left(rac{1}{3}
ight)^{rac{|x+2|}{2-|x|}} < 9$$
 is

5. If x>2 is a solution of the equation

 $\left|\log_{\sqrt{3}}x-2
ight|+\left|\log_{3}x-2
ight|=2$, then the value of x is

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6. Number of integers satisfying the inequality

$$\log_2\sqrt{x}-2\log_{1/4}^2x+1>0$$
,is

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8. The value of
$$rac{2^{\log_{rac{1}{24}}2}-3^{\log_{27}125}-4}{\left(7^{4\log_{49}2}
ight)-3}$$
 is

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9. If x_1 and x_2 $(x_2 > x_1)$ are the integral solutions of the equation

$$\left(\log_5 x
ight)^2 + \log_{5x}\!\left(rac{5}{x}
ight) = 1$$
, , the value of $|x_2 - 4x_1|$ is

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10. If
$$x=\log_\lambda a=\log_a b=rac{1}{2}{\log_b c}$$
 and $\log_\lambda c=nx^{n+1}$, the value of n is

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Exercise Matching Type Questions

1. Match the following Column I and Column II

	Column I		Column II
(A)	$\frac{\log_{3} 243}{\log_{2} \sqrt{32}}$	(p)	positive integer
(B)	$\frac{2 \log 6}{(\log 12 + \log 3)}$	(q)	negative integer
(C)	$\log_{1/3}\left(\frac{1}{9}\right)^{-2}$	(r)	rational but not integer
(D)	$\frac{\log_5 16 - \log_5 4}{\log_5 128}$	(s)	prime

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2. Match the following Column I and Column II

Column I		
If $\log_{1/x} \left\{ \frac{2(x-2)}{(x+1)(x-5)} \right\} \ge 1$, then x can	(p)	$\left(0,\frac{1}{3}\right]$
belongs to		
If $\log_3 x - \log_3^2 x \le \frac{3}{2} \log_{(1/2\sqrt{2})} 4$, then x	(q)	(1, 2]
can belongs to		
If $\log_{1/2}(4 - x) \ge \log_{1/2} 2 - \log_{1/2}(x - 1)$, then x belongs to	(r)	[3, 4)
Let α and β are the roots of the quadratic equation $(\lambda^2 - 3\lambda + 4)x^2 - 4(2\lambda - 1)x + 16 = 0$,	(s)	(3, 8)
if α and β satisfy the condition $\beta > 1 > \alpha$, then <i>p</i> can lie in		
	Column I If $\log_{1/x} \left\{ \frac{2(x-2)}{(x+1)(x-5)} \right\} \ge 1$, then x can belongs to If $\log_3 x - \log_3^2 x \le \frac{3}{2} \log_{(1/2\sqrt{2})} 4$, then x can belongs to If $\log_{1/2}(4-x) \ge \log_{1/2} 2 - \log_{1/2}(x-1)$, then x belongs to Let α and β are the roots of the quadratic equation $(\lambda^2 - 3\lambda + 4)x^2 - 4(2\lambda - 1)x + 16 = 0$, if α and β satisfy the condition $\beta > 1 > \alpha$, then p can lie in	Column ICoIf $\log_{1/x} \left\{ \frac{2(x-2)}{(x+1)(x-5)} \right\} \ge 1$, then x can(p)belongs toIf $\log_3 x - \log_3^2 x \le \frac{3}{2} \log_{(1/2\sqrt{2})} 4$, then x(q)can belongs toIf $\log_{1/2}(4-x) \ge \log_{1/2}2 - \log_{1/2}(x-1)$, (r)(r)then x belongs toLet α and β are the roots of the quadratic equation(s) $(\lambda^2 - 3\lambda + 4)x^2 - 4(2\lambda - 1)x + 16 = 0$, if α and β satisfy the condition $\beta > 1 > \alpha$, then p can lie in(s)

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Exercise Statement I And Ii Type Questions

1. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 $\log_{10} x < \log_3 x < \log_e x < \log_2 x \ (x > 0, x
eq 1)$

Statment If 0 < x < 1, then $\log_x a > \log_x b \Rightarrow 0 < a < b$.

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2. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.
(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1
Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 The equation $7^{\log_7\left(x^3+1
ight)}-x^2=1$ has two distinct real roots .

Statement -2 $a^{\log_a N} = N$, where a > 0, a
eq 1 and N > 0

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3. Statement -1
$$\left(\frac{1}{2}\right)^{\prime} < \left(\frac{1}{3}\right)^{4}$$

 $\Rightarrow 7 \log\left(\frac{1}{2}\right) < 4 \log\left(\frac{1}{3}\right) \Rightarrow 7 < 4$

Statement-2 If ax < ay, where a < 0,x,y > 0, then x > y.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct

explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true.



4. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.
(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 The equation $x^{\log_x(1-x)^2} = 9$ has two distinct real solutions.

Statement -2 $a^{\log_a b}=b$, when a<0, a
eq 1, b>0.

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5. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct
explanation for Statement-1

Statement-1 is true, Statement-2 is true, Statement-2 is not a correct

explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 The equation $(\log x)^2 + \log x^2 - 3 = 0$ has two distinct solutions.

Statement-2 $\log x^2$ =2logx.

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6. Statement-1(Assertion) and Statement-2 (reason) Each of these question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice as given below.
(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1
Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement -1

(c) Statement -1 is true, Statement -2 is false

(d) Statement -1 is false, Statement -2 is true

Statement -1 $\log_x 3$. $\log_{x/9} 3 = \log_{81}(3)$ has a solution.

Statement-2 Change of base in logarithms is possible .



Exercise Subjective Type Questions

1. If $\log_7 12 = a$, $\log_{12} 24 = b$, then find value of $\log_{54} 168$ in terms of a

and b.

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2. If $\log_3 4 = a$, $\log_5 3 = b$, then find the value of $\log_3 10$ in terms of a and

b.

3. If
$$\frac{Ina}{b-c} = \frac{Inb}{c-a} = \frac{Inc}{a-b}$$
, prove the following .



4. If
$$\frac{Ina}{b-c} = \frac{Inb}{c-a} = \frac{Inc}{a-b}$$
, prove the following . $a^a. b^b. c^c = 1$

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5. If
$$\frac{Ina}{b-c}=\frac{Inb}{c-a}=\frac{Inc}{a-b}$$
, prove the following . $a^{b^2+bc+c^2}$. $b^{c^2+ca+a^2}$. $c^{a^2+ab+b^2}=1$

6. If
$$\displaystyle \frac{Ina}{b-c} = \displaystyle \frac{Inb}{c-a} = \displaystyle \frac{Inc}{a-b},$$
 prove the following . $a+b+c \geq 3$

7. If
$$rac{Ina}{b-c}=rac{Inb}{c-a}=rac{Inc}{a-b}$$
, prove the following . $a^a+b^b+c^c\geq 3$



8. If
$$\frac{Ina}{b-c} = \frac{Inb}{c-a} = \frac{Inc}{a-b}$$
, prove the following .
 $a^{b^2+bc+c^2} \cdot b^{c^2+ca+a^2} \cdot c^{a^2+ab+b^2} = 1$

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9. Prove that
$$\log_{10} 2$$
 lies between $\frac{1}{4}$ and $\frac{1}{3}$.

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10. If log2=0.301 and log3=0.477, find the number of integers in 5^{200}



11. If log2=0.301 and log3=0.477, find the number of integers in 6^{20}





 ${\rm (ii)}{\log_2}(9+2^x)=3$

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18. Solve the following equations.

(iii) 2.
$$x^{\log_4 3} + 3^{\log_4 x} = 27$$

(iv) $\log_4 \log_3 \log_2 x = 0$



20. Solve the following equations.

(v) $x^{rac{\log_{10}x+5}{3}}=10^{5+\log_{10}x}$

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21. Solve the following equations.

(vi)
$$\log_3\left(\log_9 x + rac{1}{2} + 9^x
ight) = 2x$$

(vii)
$$4^{\log_{10}x+1} - 6^{\log_{10}x} - 2.3^{\log_{10}x^2+2} = 0$$

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(viii)
$$rac{\log 10(x-3)}{\log_{10}(x^2-21)} = rac{1}{2}$$

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24. Solve the following equations. $x^{\log_2 x + 4} = 32$



25. Solve the following equations.

(x)
$$\log_a x = x$$
, where $a = x^{\log_a x}$

(xi)
$$\log_{\sqrt{2}\sin x}(1+\cos x)=2$$



27. A rational number which is 50 times its own logarithm to the base 10,

is

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$$\mathbf{28.} \left[\frac{2}{\log_4 \left(2000 \right)^6} + \frac{3}{\log_5 \left(2000 \right)^6} \right]$$

29. Find the value of x satisfying

$$\log_aig\{1+\log_big\{1+\log_cig(1+\log_p xig)ig\}ig\}=0.$$

30. Find the value of
$$4^{5 \log_{4\sqrt{2}} (3-\sqrt{6}) - 6 \log_8 (\sqrt{3}-\sqrt{2})}$$
.



(i)
$$\log_{(2x+3)} x^2 < 1$$

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32. Solve the following inequation .

(ii)
$$\log_{2x} ig(x^2-5x+6ig) < 1$$

(iii) $\log_2(2-x) < \log_{1/2}(x+1)$

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34. Solve the following inequation .

(iv) $\log_{x^2}(x+2) < 1$

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35. Solve the following inequation .

(v)
$$3^{\log_3 \sqrt{(x-1)}} < 3^{\log_3 (x-6)} + 3$$

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36. Solve the following inequation .

(vi)
$$\log_{1/2}\left(3x-1
ight)^2 < \log_{1/2}\left(x+5
ight)^2$$

(vii) $\log_{10}x+2\leq \log_{10}^2x$

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38. Solve the following inequation .

(viii)
$$\log_{10} ig(x^2-2x-2ig) \leq 0$$

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39. Solve the following inequation .

(ix)
$$\log x \left(2x - \frac{3}{4}\right) > 2$$

40. Solve the following inequation . $\log_{1/3} x < \log_{1/2} x$



42. Solve the following inequation .

(xii)
$$\log_2^2 x + 3 \log_2 x \geq rac{5}{2} \log_{4\sqrt{2}} 16$$

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43. Solve the following inequation .

(xiii)
$$\left(x^2+x+1
ight)^x < 1$$

(xiv) $\log_{(3x^2+1)} 2 < rac{1}{2}$

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45. Solve the following inequation .

(xv) $x^{\left(\log_{10}x
ight)^2 - 3\log_{10}x + 1} > 1000$

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46. Solve the following inequation .

(xvi) $\log_4ig\{14+\log_6ig(x^2-64ig)ig\}\leq 2$



47. Solve the following inequation .

(xvii)
$$\log_2(9-2^x) \le 10^{\log_{10}(3-x)}$$





(xviii)
$$\log_{a}\!\left(rac{2x+3}{x}
ight) \geq 0$$
 for (a) $a > 1$,(b) $0 < a < 1$

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49. Solve the following inequation .

(xix)
$$1+\log_2(x-1)\leq \log_{x-1}4$$

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50. Solve the following inequation .

(xx)
$$\log_{5x+4} x^2 \leq \log_{5x+4} (2x+3)$$

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51. Solve
$$\sqrt{\log_x (ax)^{\frac{1}{5}} + \log_a (ax)^{\frac{1}{5}}} + \sqrt{\log_a \left(\frac{x}{a}\right)^{\frac{1}{5}} + \log_x \left(\frac{a}{x}\right)^{\frac{1}{5}}} = a$$

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52. It is known that x=9 is root of the equation. $\log_{\lambda} \left(x^2 + 15a^2\right) - \log_{\lambda}(a-2) = \log_{\lambda} \frac{8ax}{a-2}$ find the other roots of this equation.

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53. Solve
$$\log_4(\log_3 x) + \log_{1/4} \Bigl(\log_{1/3} y \Bigr) = 0$$
 and $x^2 + y^2 = rac{17}{4}.$

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54. Find the real values (s) of x satisfying the equation $\log_{2x}(4x) + \log_{4x}(16x) = 4.$

55. Find the sum and product of all possible values of x which makes the

following statement true.

$$\log_6 54 + \log_x 16 = \log_{\sqrt{2}} x - \log_{36}\!\left(rac{4}{9}
ight).$$

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56. Solve :
$$rac{3}{2} \log_4 \left(x+2
ight)^2 + 3 = \log_4 \left(4-x
ight)^3 + \log_4 \left(6+x
ight)^3.$$

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57. Solve
$$\log_2(4^{x+1}+4)$$
. $\log_2(4^x+1) = \log_{1/\sqrt{2}}\left(\frac{1}{\sqrt{8}}\right)$.

58. Solve the system of equation
$$2^{\sqrt{x}+\sqrt{y}} = 256$$
 and $\log_{10}\sqrt{xy} - \log_{10}\left(rac{3}{2}
ight) = 1.$



61. Solve
$$\log_3ig(\sqrt{x}+ig|\sqrt{x}-1ig)=\log_9ig(4\sqrt{x}-3+4ig|\sqrt{x}-1ig).$$

62. In the equality

$$\left(\log_2 x
ight)^4 - \left(\log_{1/2} rac{x^5}{4}
ight)^2 - 20 \log_2 x + 148 < 0$$

holds true in (a,b), where a,b \in N. Find the value of ab (a+b).



64. If P is the number of natural numbers whose logarithms to the base 10 have the characteristic pandQ is the number of natural numbers logarithms of whose reciprocals to the base 10 have the characteristic -q, then find the value of $\log_{10} P - (\log)_{10} Q$. 1. Let $a = (\log)_3 (\log)_3 2$. An integer k satisfying $1 < 2^{-k+3^{(-a)}} < 2$, must be less than

B. 1 C. 2 D. 3

A. 0

Answer: B

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2. Let (x_0, y_0) be the solution of the following equations

 $\left(2x
ight)^{In2}=\left(3y
ight)^{In3}3^{Inx}=2^{Iny}$, then x_{0} is

A. $\frac{1}{6}$

B.
$$\frac{1}{3}$$

C. $\frac{1}{2}$
D. 6

Answer: C

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3. The value of

$$6 + \log_{3/2} \left(\frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \dots}}} \right)$$
 is

4. If
$$3^x = 4^{x-1}$$
, then x equals

A.
$$rac{2 \log_3 2}{2 \log_3 2 - 1}$$

B. $rac{2}{2 - \log_2 3}$

C.
$$\frac{1}{1 - \log_4 3}$$

D. $\frac{2 \log_2 3}{2 \log_2 3 - 1}$

Answer: A::B::C