MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

MATHEMATICAL INDUCTION

Examples

1. $1^3 + 2^2 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$

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2. $1.2.3 + 2.3.4 + \ldots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

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3. Using the principle of mathematical induction, prove that : $1.2.3+2.3.4+\ +n(n+1)(n+2)=\frac{n(n+1)(n+2)(n+3)}{4}$ for all $n\in N$.

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4. Prove by mathematical induction that $\sum_{r=0}^{n} r^{n}C_{r}=n.2^{n-1},\ \forall n\in N.$

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5. Use the principle of mathematical induction to show that $5^{2n+1}+3^{n+2}.2^{n-1}$ divisible by 19 for all natural numbers n.

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6. Use the principle of mathematical induction to show that $a^{n}-b^{9}n)$ is divisble by $a-b$ for all natural numbers n.

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7. Using problems are of the Inequality Type. Examples of this type are as follows:

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8. Show using mathematical induciton that $n! < \left(\frac{n+1}{2}\right)^n$. Where $n \in N$ and $n > 1$.

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9. if $a + b = c + d$ and $a^2 + b^2 = c^2 + d^2$, then show by mathematical induction $a^n + b^n = c^n + d^n$

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10. Let $I_m = \int_0^{\pi} \left(\frac{1 - \cos mx}{1 - \cos x} \right) dx$ use mathematical induction to prove that $l_m = m\pi, m = 0, 1, 2.....$

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11. Given that $u_{n+1} = 3u_n - 2u_{n-1}$, and $u_0 = 2, u_1 = 3$, then prove that $u_n = 2^n + 1$ for all positive integer of n

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12. Let $u_1 = 1, u_2 = 2, u_3 = \frac{7}{2}$ and $u_{n+3} = 3u_{n+2} - \left(\frac{3}{2}\right) u_{n+1} - u_n$. Use the principle of mathematical induction to show that $u_n = \frac{1}{3}\left[2^n + \left(\frac{1+\sqrt{3}}{2}\right)^n + \left(\frac{1-\sqrt{3}}{2}\right)^n\right] \forall n \geq 1.$

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13. If p is a fixed positive integer, prove by induction that $p^{n+1} + (p+1)^{2n-1}$ is divisible by $P^2 + p + 1$ for all $n \in N$.

A. P

B. $P^2 + P$

C. $P^2 + P + 1$

D. $P^2 - 1$

Answer:

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14. Let P(n) denote the statement that $n^2 + n$ is odd . It is seen that $P(n) \Rightarrow P(n+1)$, $P(n)$ is true for all

A. $n > 1$

B. n

C. $n > 2$

D. None of these

Answer:

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15. Let $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{2^n - 1}$. Then

A. $a(100) > 100$

B. $a(100) < 200$

C. $a(200) \leq 100$

D. $a(200) > 100$

Answer: D

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16. Let $S(k) = 1 + 3 + 5 + \ldots + (2k - 1) = 3 + k^2$. Then which of the following is true ?

A. Principle of mathematical induction can be used to prove the formula

B. $S(k) \Rightarrow S(k+1)$

C. $S(k) \not\Rightarrow S(k+1)$

D. S(1) is correct

Answer:

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17. $10^n + 3\left(4^{n+2}\right) + 5$ is divisible by $(n \in N)$

A. 7

B. 5

C. 9

D. 7

Answer:

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18. Statement-1 For all natural number n, $1+2+\ldots.+n<(2n+1)^2$

Statement -2 For all natural numbers , $(2n+3)^2-7(n+1)<(2n+3)^3$.

A. Statement -1 is true , Statement -2 is true Statement -2 is correct explanation for Statement -1.

B. Statement -1 is true , Statement -2 is true , Statement -2 is not the correct explanation for Statement -1

C. Statement-1 is true , Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer: B

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19. prove that $7+77+777+.....+777......_{n-digits}7=\frac{7}{81}\left(10^{n+1}-9n-10\right)$ for all $n\in N$

A. Statement -1 is true , Statement -2 is true Statement -2 is correct explanation for Statement -1.

B. Statement -1 is true , Statement -2 is true , Statement -2 is not the correct explanation for Statement -2

C. Statement-1 is true , Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer: C

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20. Prove by induction that the integer next greater than $\left(3+\sqrt{5}\right)^n$ is divisible by 2^n for all $n \in N$.

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21. Using mathematical induction , show that $\left(1-\frac{1}{2^2}\right)\left(1-\frac{2}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\ldots\left(1-\frac{1}{(n+1)^2}\right)=\frac{n+2}{2(n+1)}, \forall n \in$.

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22. Using the principle of mathematical induction to show that $\tan^{-1}(n+1)x-\tan^{-1}x, \forall x \in N.$

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23. Use the principle of mathematical induction to prove that for all $n \in N$

$$\sqrt{2+\sqrt{2+\sqrt{2}+\ldots+\ldots+\sqrt{2}}} = 2\cos\left(\frac{\pi}{2^{n+1}}\right)$$

When the LHS contains n radical signs.

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24. Prove by mathematical induction that

$$\frac{1}{1+x}+\frac{2}{1+x^2}+\frac{4}{1+x^4}+\ldots\ldots+\frac{2^n}{1+x^{2^n}}=\frac{1}{x-1}+\frac{2^{n+1}}{1-x^{2^{n+1}}}$$

where , $|x| \neq 1$ and n is non - negative integer.

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25. Using the principle of mathematical induction to prove that

$$\int_0^{\pi/2}\frac{\sin^2 nx}{\sin x}dx = 1+\frac{1}{3}+\frac{1}{5}+\ldots\ldots+\frac{1}{2n-1}$$

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26. Use induction to show that for all $n \in N$.

$$\sqrt{a+\sqrt{a+\sqrt{a+\ldots.+\sqrt{a}}}} < \frac{1+\sqrt{(4a+1)}}{2}$$

where'a' is fixed positive number and n radical signs are taken on LHS.

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27. Prove by induction that

$$\left\{\prod_{r=0}^{n} f_r(x)\right\}' = \sum_{i=1}^{n} \{f_1(x) f_2(x) \ldots. f_i'(x) \ldots. f_n(x)\},$$

where dash denotes derivative with respect to x.

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Mathematical Induction Exercise 1 Single Option Correct Tpye Questions

1. If $a_n = \sqrt{7+\sqrt{7+\sqrt{7}+\ldots\ldots}}$) having n radical signs then by methods of mathematical induction which is true

A. $a_n > 7, \forall n \geq 1$

B. $n_n > 3, \forall n \geq 1$

C. $a_n < 4, \forall n \geq 1$

D. $a_n < 3, \forall n \geq 1$

Answer:

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2. If $P(n) = 2 + 4 + 6 + ... + 2n, n \in N.$ Then $P(k) = k(k+1)$ $\Rightarrow P(k+1) = (k+1)(k+2), \forall k \in N$, So , we can conclude that $P(n) = n(n+1)$ for

A. all $n \in N$

B. $n > 1$

C. $n > 2$

D. Nothing can be said

Answer:

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3. Show by using the principle of mathematical induction that for all natural number $n > 2$, $2^n > 2n + 1$

A. for $n \geq 3$

B. for $n < 3$

C. for all n

D. for mn

Answer:

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Exercise Statement I And Ii Type Questions

1. If $a_1 = 1, a_2 = 5$ and $a_{n+2} = 5a^{n+1} - 6a_n, n \geq 1$. Show by using mathematical induction that $a_n = 3^n - 2^n$

A. Statement -1 is true , Statement -2 is true, Statement -2 is correct explanation for Statement -1

B. Statement -1 is true , Statement -2 is true , Statement -2 is not correct explanation for Staement -1

C. Statement -1 is true , Statement -2 is false

D. Statement -1 is false , Statement - 2 is true.

Answer:

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2. Statement -1 for all natural numbers n , $2.7^n + 3.5^n - 5$ is divisible by 24.

Statement -2 if f(x) is divisible by x, then $f(x+1)-f(x)$ is divisible by $x+1, \forall x \in N$.

A. Statement -1 is true , Statement -2 is true, Statement -2 is correct explanation for Statement -2

B. Statement -1 is true , Statement -2 is true , Statement -2 is not correct explanation for Staement -2

C. Statement -1 is true , Statement -2 is false

D. Statement -1 is false , Statement - 2 is true.

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3. Statement -1 For all natural numbers n , $0.5+0.55+0.555+.....$ upto n terms $=\frac{5}{9}\left\{n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right\}$ Statement-2 $a+ar+ar^2+....+ar^{n-1}=\frac{a(1-r^n)}{(1-r)}$, for $0<r<1$.

A. Statement -1 is true , Statement -2 is true, Statement -2 is correct explanation for Statement -3

B. Statement -1 is true , Statement -2 is true , Statement -2 is not correct explanation for Staement -3

C. Statement -1 is true , Statement -2 is false

D. Statement -1 is false , Statement - 2 is true.

Answer:

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Exercise Subjective Type Questions

1. Prove the following by the principle of mathematical induction: $11^{n+2}+12^{2n+1}$ is divisible 133 for all $n \in N$.

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2. $n^7 - n$ is divisible by 42 .

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3. $3^{2n} + 24n - 1$ is divisible by 32 .

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4. prove using mathematical induction:$-n(n+1)(n+5)$ is divisible by 6 for all natural numbers

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5. Prove that $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$ for all $n = 1, 2,$

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6. $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$

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7. Prove by induction that if n is a positive integer not divisible by 3. then $3^{2n} + 3^n + 1$ is divisible by 13.

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8. prove that the product of three consecutive positive integers is divisible by 6.

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9. Prove by induction that the sum of the cubes of three consecutive natural numbers is divisible by 9.

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10. When the square of any odd number, greater than 1, is divided by 8, it always leaves remainder 1 (b) 6 (c) 8 (d) Cannot be determined

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11. Prove the following by using iduction for all $n \in N$. $1 + 2 + 3 + \ldots\ldots + n = \frac{n(n+1)}{2}$

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12. $1^2 + 2^2 + 3^2 + \ + n^2 = \frac{n(n+1)(2n+1)}{6}$

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13. $1.3 + 3.5 + 5.7 + \ldots\ldots + (2n-1)(2n+1) = \frac{n\left(4n^2 + 6n - 1\right)}{3}$

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14. Prove the following by the principle of mathematical induction:

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}++\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

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15. Prove $1.4.7+2.5.8+3.6.9+\ldots\ldots.$ upto n terms $=\frac{n}{4}(n+1)(n+6)(n+7)$

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16. $\frac{1^2}{1.3}+\frac{2^2}{3.5}+\frac{3^2}{5.7}+\ldots..+\frac{n^2}{(2n-1)(2n+1)}=\frac{(n)(n+1)}{(2(2n+1))}$

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17. Let $a_0=2, a_1=5$ and for $n\geq 2, a_n=5a_{n-1}-6a_{n-2}$, then prove by induction that $a_n=2^n+3^n, \forall n\geq 0, n\in N.$

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18. If $a_1 = 1, a_{n+1} = \frac{1}{n+1} a_n, a \geq 1$, then prove by induction that $a_{n+1} = \frac{1}{(n+1)!} n \in N.$

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19. if a,b,c,d,e and f are six real numbers such that $a + b + c = d + e + f$ $a^2 + b^2 + c^2 = d^2 + e^2 + f^2$ and $a^3 + b^3 + c^3 = d^3 + e^3 + f^3$, prove by mathematical induction that $a^n + b^n + c^n = d^n + e^n + f^n \forall n \in N.$

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20. Prove that $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \ldots\ldots\ldots + \tan^{-1}\left(\frac{1}{n^2 + n +}\right.$

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Exercise Questions Asked In Previous 13 Years Exam

1. Statement-1: For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$$

Statement-2: For every natural number $n \geq 2$,

$$\sqrt{n(n+1)} < n+1$$

A. Statement-1 is true , Statement-2 is true, Statement-2 is correct explanation for Statement-1

B. Statement-1 is true , Statement-2 is true , Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true , Statement-2 is false

D. Statement-1 is false , Statement -2 is true .

Answer:

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2. Statement -1 For each natural number n, $(n+1)^7 - n^7 - 1$ is divisible by 7.

Statement -2 For each natural number n, $n^7 - n$ is divisible by 7.

A. Statement-1 is false , Statement-2 is true

B. Statement-1 is true , Statement-2 is true , Statement-2 is correct explanation for Statement-1

C. Statement-1 is true , Statement-2 is true , Statement-2 is not a correct explanation for Statement-1

D. Statement-1 is true , Statement-2 is false

Answer:

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