MATHS

NCERT - NCERT MATHEMATICS (ENGLISH)

LIMITS AND DERIVATIVES

Exercise 13 1

1. Suppose

$f(x) = [a + bx, x < 1, 4, x = 1, b - ax, x > 1]$ and $\lim f(x) wherexten$

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2. Let $a_1, a_2, \ldots . , a_n$ be fixed real numbers and define a function f(x) = $(x - a_1)(x - a_2) \ldots . (x - a_n)$. What is $(\lim)_{x \to a_1} f(x)$? For some $a \neq a_1, a_2, \ldots . , a_n$, compute $(\lim)_{x \to a} f(x)$

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3. $\left(\lim\right)_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

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4. $\left(\lim\right)_{x \to 0} \frac{\sin ax + bx}{ax + \sin bx} a, b, a + b \neq 0$

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5. Evaluate $(\lim)_{x \to 0} f(x),\ where\ f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0, \\ 0, & x = 0 \end{cases}$

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6. Find $\left(\lim\right)_{x \to 5} f(x)$, where $f(x) = |x| - 5$

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7. Find $\left(\lim\right)_{x \to 1} f(x)$,where $f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ -x^2 - 1 & x > 1 \end{cases}$

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8. Evaluate $(\lim)_{x \to 0} f(x),\ where\ f(x) = \left\{ \frac{|x|}{x},\ x \neq 0, 0,\ x = 0 \right.$

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9. If the function f(x) satisfies $\left(\lim\right)_{x \to 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $\left(\lim\right)_{x \to 1} f(x)$

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10. If $f(x) = [|x| + 1, x < 00, x = 0|x| - 1, x > 0$

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11. If $f(x) = \begin{bmatrix} mx^2 + n, x < 0 \\ nx + m, x \leq x \leq 1 \\ nx^3 + m, x > 1 \end{bmatrix}$. For what integers m and n does both $\left(\lim \right)_{x \to 1} f(x)$.

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12. Find $f(x) and \left(\lim \right)_{x \to 1} f(x), where f(x) = \begin{bmatrix} 2x + 3, x \leq 0 \\ 3(x + 1), x > 0 \end{bmatrix}$

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13. $\left(\lim \right)_{x \to 0} (\cos ecx - \cot x)$

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14. $\left(\lim \right)_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

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15. $\left(\lim\right)_{x \to 0} \frac{ax + b}{cx + 1}$

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16. Evaluate: $\left(\lim\right)_{x \to 4} \frac{4x + 3}{x - 2}$

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17. $\left(\lim\right)_{x \to -1} \frac{x^{10} + x^5 + 1}{x - 1}$

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18. $\left(\lim\right)_{x \to 0} \frac{(x + 1)^5 - 1}{x}$

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19. $\left(\lim\right)_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$

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20. $\left(\lim\right)_{x \to 3} x + 3$

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21. $\left(\lim\right)_{x \to \pi}\left(x - \frac{22}{7}\right)$

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22. Find $\left(\lim\right)_{r \to 1} \pi r^2$

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23. $\left(\lim\right)_{x \to 0} x \sec x$

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24. $\left(\lim\right)_{x \to 0} \frac{ax + x\cos x}{b\sin x}$

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25. $\left(\lim\right)_{x \to 0} \frac{\cos 2x - 1}{\cos x - 1}$

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26. $\left(\lim\right)_{x \to 0} \frac{\cos x}{\pi - x}$

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27. $\left(\lim\right)_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

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28. $\left(\lim\right)_{x \to 0} \frac{sinax}{\sin bx} a, b, \neq 0$

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29. $\left(\lim\right)_{x \to 0} \frac{\sin ax}{bx}$

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30. $\left(\lim\right)_{x \to 2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$

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31. $\lim_{x \to 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$

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32. $\left(\lim\right)_{z \to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$

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Solved Examples

1. Find the derivative of f (x) from the first principles, where f(x) is(i) $s \in x + \cos x$ (ii) $xs \in x$

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2. Find the derivative of(i) $\frac{x^5 - \cos x}{\sin x}$ (ii) $\frac{x + \cos x}{\tan x}$

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3. Compute derivative of (i) $f(x) = \sin 2x$ (ii) $g(x) = \cot x$

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4. Find the derivative of $f(x) = x^2$

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5. Find the derivative of the constant function $f(x) = a$for a fixed real number a.

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6. Find the derivative of $f(x) = \frac{1}{x}$

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7. Compute the derivative of $6x^{100} - x^{55} + x$.

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8. Find the derivative of $f(x) = 1 + x + x^2 + x^3 + \vdots + x^{50}$at $x = 1$.

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9. Find the derivative of $f(x) = \dfrac{x+1}{x}$.

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10. Compute the derivative of sin x.

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11. Compute the derivative of tan x.

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12. Compute the derivative of $f(x) = \sin^2 x$.

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13. Find the derivative of f from the first principles, where f is given by(i) $f(x) = \frac{2x+3}{x-2}$ (ii) $f(x) = x + \frac{1}{x}$

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14. Evaluate(i) $\left(\lim\right)_{x \to 1} \frac{x^{15}-1}{x^{10}-1}$ (ii) $\left(\lim\right)_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

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15. Find the limits (i) $\left(\lim\right)_{x \to 1}\left[\frac{x^2+1}{x+100}\right]$ (ii) $\left(\lim\right)_{x \to 2}\left[\frac{x^3-4x^2+4x}{x^2-4}\right]$

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16. Find the limits: (i) $\left(\lim\right)_{x \to 1}\left[x^3 - x^2 + 1\right]$ (iii) $\left(\lim\right)_{x \to 3}[x(x+1)]$ (iii) $\left(\lim\right)_{x \to 1}\left[1 + x + x^2 + \ldots\ldots + x^{10}\right]$

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17. Find the derivative of sin x at $x = 0$.

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18. Find the derivative of the function $f(x) = 2x^2 + 3x - 5$at $x = 1$.

Also prove that $f'(0) + 3f'(-1) = 0$.

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19. Find the derivative at $x = 2$of the function $f(x) = 3x$.

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20. Evaluate (i) $\left(\lim\right)_{x \to 0} \frac{\sin 4x}{\sin 2x}$ (ii) $\left(\lim\right)_{x \to 0} \frac{\tan x}{x}$

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21. Find the derivative of $f(x) = 10x$.

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22. Find the derivative of $f(x) = 3$at $x = 0$and at $x = 3$.

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23. Evaluate $\left(\lim\right)_{x \to 0} \frac{(\log)_e x}{x - 1}$

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24. Compute $\lim_{x \to 0} \frac{e^{3x} - \sin x - 1}{x}$

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25. Compute $\left(\lim\right)_{x \to 0} \frac{e^{3x} - 1}{x}$

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Miscellaneous Exercise

1. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{x}{\sin^n x}$

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2. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x + a)$

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3. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(px+q)\left(\frac{r}{x}+s\right)$

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4. Find the derivative of the following functions from first principle:

(i) -x (ii) $(-x)^{-1}$ (iii) $\sin(x+1)$ (iv) $\cos\left(x-\frac{\pi}{8}\right)$

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5. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{1+\frac{1}{x}}{1-\frac{1}{x}}$

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6. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{1}{ax^2+bx+c}$

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7. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax+b)(cx+d)^2$

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8. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{ax+b}{cx+d}$

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9. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{ax+b}{px^2+qx+r}$

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10. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{px^2+qx+r}{ax+b}$

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11. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\sin^n x$

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12. Find derivative of the following functions: $\frac{\sec x - 1}{\sec x + 1}$

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13. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\cos ecx \cot x$

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14. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): sin (x + a)

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15. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\sin x + \cos x}{\sin x - \cos x}$

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16. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\cos x}{1 + \sin x}$

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17. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $4\sqrt{x} - 2$

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18. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$

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19. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax+b)^n(cx+d)^m$

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20. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax+b)^n$

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21. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\left(ax^2+\sin x\right)(p+q\cos x)$

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22. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x+\cos x)(xtanx)$

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23. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{4x+5\sin x}{3x+7\cos x}$

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24. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$

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25. Find derivative of the following functions (it is to be understood that a, b, c and d are fixed non-zero constants): $\frac{a + b \sin x}{c + d \cos x}$

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26. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\sin(x + a)}{\cos x}$

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27. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $x^4(5\sin x - 3\cos x)$

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28. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\left(x^2+1\right)\cos x$

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29. Find derivative of the following functions: $\frac{x}{1+\tan x}$

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30. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x + secx)(x - tanx)$

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Exercise 13 2

1. Find the derivative of cos x from first principle.

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2. Find the derivative of the following functions:(i) $\sin x \cos x$ (ii) $\sec x$ (iii) $5 \sec x + 4 \cos x$(iv) $\cos ecx$ (v) $3 \cot x + 5 \cos ecx$

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3. Find the derivative of (i) $2x - \frac{3}{4}$ (ii) $\left(5x^3 + 3x - 1\right)(x - 1)$ (iii) $x^{-3}(5 + 3x)$

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4. Find the derivative of $\frac{x^n - a^n}{x - a}$ for some constant a.

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5. Find the derivative of $x^2 - 2$at $x = 10$.

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6. Find the derivative of x at $x = 1$.

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7. Find the derivative of 99x at $x = 100$.

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8. For the function

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1$$

. Prove that $f'(1) = 100f'(0)$.

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9. Find the derivative of the following functions from first principle.(i) $x^3 - 27$ (ii) $(x-1)(x-2)$ (iii) $\frac{1}{x^2}$ (iv) $\frac{x+1}{x-1}$

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10. For some constants a and b, find the derivative of(i) $(x-a)\cdot(x-b)$ (ii) $\left(ax^2+b\right)^2$ (iii) $\frac{x-a}{x-b}$

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11. Find the derivative of $x^n + ax^{n-1}1 + a^2x^{n-2} + \vdots + a^{n-1}x + a^n$ for some fixed real number a.

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Exercise 13 3

1. Evaluate the limits, if exist $\lim_{x \to 0} \left(x \frac{e^x - 1}{1 - \cos x} \right)$

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2. Evaluate the limits, if exist $\lim_{x -> 0} \frac{\log_e(1 + 2x)}{x}$

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3. Evaluate the limits, if exist $\left(\lim\right)_{x \to 0} \dfrac{e^{\sin x} - 1}{x}$

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4. Evaluate the limits, if exist $\left(\lim\right)_{x \to 0} \dfrac{e^{2+x} - e^2}{x}$

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5. Evaluate the limits, if exist $\left(\lim\right)_{x \to 5} \dfrac{e^x - e^5}{x - 5}$

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6. Evaluate the limits, if exist $\left(\lim\right)_{x \to 0} \dfrac{e^{4x} - 1}{x}$

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7. Evaluate the limits, if exist $\lim\limits_{x \to 3} \dfrac{e^x - e^3}{x - 3}$

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8. Evaluate the limits, if exist$\left(\lim\right)_{x \to 0} \frac{\log\left(1+x^3\right)}{\sin^3 x}$

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