MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

PRODUCT OF VECTORS

Example

1. If θ is the angle between the vectors $a = 2\hat{i} + 2\hat{j} - \hat{k}$ and $b = 6\hat{i} - 3\hat{j} + 2\hat{k}$, then

A. $\cos\theta = \frac{4}{21}$

B. $\cos\theta = \frac{3}{19}$

C. $\cos\theta = \frac{2}{19}$

D. $\cos\theta = \frac{5}{21}$

Answer: A

Watch Video Solution

2. $\left(a \cdot \hat{i}\right)\hat{i} + \left(a \cdot \hat{j}\right)\hat{j} + \left(a \cdot \hat{k}\right)\hat{k}$ is equal to

A. a

B. 2a

C. 3a

D. 0

Answer: A

Watch Video Solution

3. If $|a| = 3, |b| = 4$, then a value of λ for which $a + \lambda b$ is perpendicular to $a - \lambda b$ is

A. $9/16$

B. $3/4$

C. $3/2$

D. $4/3$

Answer: B

Watch Video Solution

4. Find the projection oif $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector \`vecb=hati+2hatj+hatk.

A. $\frac{1}{\sqrt{14}}$

B. $\frac{2}{\sqrt{14}}$

C. $\sqrt{14}$

D. $\frac{-2}{\sqrt{14}}$

Answer: B

Watch Video Solution

5. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \lambda\hat{k}, f \in d\lambda$ such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal

Watch Video Solution

6. If $\vec{a}, \vec{b}, and\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then find the value of $\vec{a}\dot{\vec{b}} + \vec{b}\dot{\vec{c}} + \dot{\vec{}}\vec{a}$

Watch Video Solution

7. If $\vec{a}, \vec{b}, and\vec{c}$ are mutually perpendicular vectors of equal magnitudes, then find the angle between vectors $\vec{a}and\vec{a} + \vec{b} + \dot{\vec{}}$

Watch Video Solution

8. Find the value of c for which the vectors $\vec{a} = \left(c\log_2 x\right)\hat{i} - 6\hat{j} + 3\hat{k}and\vec{b} = \left((\log)_2 x\right)\hat{i} + 2\hat{j} + \left(2c(\log)_2 x\right)\hat{k}$ make an obtuse angle for any $x \in (0, \infty)$.

Watch Video Solution

9. If $a + 2b + 3c = 4,$ then find the least value of $a^2 + b^2 + c^2$.

Watch Video Solution

10. Find the unit vector which makes an angle of 45° with the vector $2\hat{i} + 2\hat{j} - \hat{k}$ and an angle of 60° with the vector $\hat{j} - \hat{k}$.

Watch Video Solution

11. Prove using vectors: The median to the base of an isosceles triangle is perpendicular to the base.

Watch Video Solution

12. In ΔABC, prove that $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ by vector method.

Watch Video Solution

13. In any triangle ABC, prove the projection formula$a = b\cos C + os B$ using vector method.

Watch Video Solution

14. If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{i} + 4\hat{k}$ find the vector component of $\vec{a} alond \vec{b}$.

Watch Video Solution

15. Expressthe vector $\vec{=} \left(5\hat{i} - 2\hat{j} + 5\hat{k}\right)$ as sum of two vectors such that one is paralle to the vector s $\vec{=} \left(3\hat{i} + \hat{k}\right)$ and the other is perpendicular to $\vec{b}$.

Watch Video Solution

16. Two forces $f_1 = 3\hat{i} - 2\hat{j} + \hat{k}$ and $f_2 = \hat{i} + 3\hat{j} - 5\hat{k}$ acting on a particle at A move it to B. find the work done if the position vector of A and B are $-2\hat{i} + 5\hat{k}$ and $3\hat{i} - 7\hat{j} + 2\hat{k}$.

Watch Video Solution

17. Forces of magnitudes 5 and 3 units acting in the directions $6\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - \hat{i} + 6\hat{k}$ respectively act on a particle which is displaced from the point (2,2,-1) to (4,3,1) . The work done by the forces, is

Watch Video Solution

18. If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\vec{a} \times \vec{b} = \vec{0}$ then find (m,n)

Watch Video Solution

19. Show that $\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right) = 2\left(\vec{a} \times \vec{b}\right)^{\cdot}$

Watch Video Solution

20. If $\vec{a}$ is any vector, then $\left(\vec{a} \times \hat{i}\right)^2 + \left(\vec{a} \times \hat{j}\right)^2 + \left(\vec{a} \times \hat{k}\right)^2 = \ \vec{a}^2$ b. $2\vec{a}62$ c. $3\vec{a}^2$ d. $4\vec{a}^2$

A. $|a|^2$

B. 0

C. $3|a|^2$

D. $2|a|^2$

Answer: D

Watch Video Solution

21. If $\vec{a}.\vec{b} = 0$ and $\vec{a} \times \vec{b} = 0$, prove that $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$.

Watch Video Solution

22. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}\dot{\ }\vec{b} = \vec{a}\dot{\ }\vec{c}$ and $\vec{a}\ x\ \vec{b} = \vec{a}\ x\ \vec{c},\ \vec{a}\ \neq 0$, then show that $\vec{b} = \vec{c}$.

Watch Video Solution

23. If a,b and c are three non-zero vectors such that $a \cdot (b \times c) = 0$ and b and c are not parallel vectors, prove that $a = \lambda b + \mu c$ where λ and μ are scalar.

Watch Video Solution

24. If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq \vec{0} and \vec{b} \neq \vec{c}$, show that $\vec{b} = \vec{c} + t\vec{a}$ for some scalar $\dot{t}$

Watch Video Solution

25. For any two vectors $\vec{u} and \vec{v}$ prove that $\left(\vec{u}\dot{}\vec{v}\right)^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2|\vec{v}|^2 and$

$$\left(\vec{1} + |\vec{u}|^2\right)\left(\vec{1} + |\vec{v}|^2\right) = \left(1 - \vec{u}\dot{}\vec{v}\right)^2 + |\vec{u} + \vec{v} + (\vec{u} \times \vec{v})|^2$$

Watch Video Solution

26. The sine of the angle between the vector $a = 3\hat{i} + \hat{j} + \hat{k}$ and $b = 2\hat{i} - 2\hat{j} + \hat{k}$ is

A. $\sqrt{\frac{74}{99}}$

B. $\sqrt{\frac{55}{99}}$

C. $\sqrt{\frac{37}{99}}$

D. $\frac{5}{\sqrt{41}}$

Answer: A

Watch Video Solution

27. If $\left|\vec{a}\right| = 2\left|\vec{b}\right| = 5$ and $\left|\vec{a} \times \vec{b}\right| = 8$, then find the value of $\vec{a} \cdot \vec{b}$

Watch Video Solution

28. The unit vector perpendicular to the vectors $6\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 6\hat{j} - 2\hat{k}$, is

A. $\dfrac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$

B. $\dfrac{2\hat{i} - 3\hat{j} - 6\hat{k}}{7}$

C. $\dfrac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$

D. $\dfrac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7}$

Answer: C

Watch Video Solution

29. Find the vector perpendicular to the plne determined by the points P(1,-1,2), Q(2,0,-1) and R(0,2,1)

Watch Video Solution

30. Let A,B and C be unit vectors. Suppose $A \cdot B = A \cdot C = 0$ and the angle betweenn B and C is $\frac{\pi}{4}$. Then,

A. $A = \pm 2(B \times C)$

B. $A = \pm \sqrt{2}(B \times C)$

C. $A = \pm 3(B + C)$

D. $A = \pm \sqrt{3}(B \times C)$.

Answer: b

Watch Video Solution

31. If the vectors $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ form a right handedsystem, then $\vec{c}$ is

A. $z\hat{i}$ - $x\hat{k}$

B. 0

C. $y\hat{j}$

D. -$z\hat{i} + x\hat{k}$

Answer: A

Watch Video Solution

32. Let $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a} \times \vec{b} = \vec{c} and \vec{b} \times \vec{c} = \vec{a}$; prove that $\vec{a}, \vec{b}, \vec{c}$ are mutually at righ angles such that $\left|\vec{b}\right| = 1 and \left|\vec{c}\right| = \left|\vec{a}\right|^{\cdot}$

Watch Video Solution

33. If$\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices A, B, C of a triangle ABC, show that the area triangle $ABC is \frac{1}{2}\left|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right|$. Deduce the condition for points $\vec{a}, \vec{b}, \vec{c}$ to be collinear.

Watch Video Solution

34. Show that distance of the point $\vec{c}$ from the line joining $\vec{a} and \vec{b}$ is

$$\frac{|\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}}{\left|\vec{b} - \vec{a}\right|}$$

Watch Video Solution

35. find the area of a parallelogram whose diagonals are $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$.

Watch Video Solution

36. Find the moment about (1,-1,-1) of the force $3\hat{i} + 4\hat{j} - 5\hat{k}$ acting at (1,0,-2)

Watch Video Solution

37. Three forces $\hat{i} + 2\hat{j} - 3\hat{k}, 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ acting on a particle at the point (0,1,2) the magnitude of the moment of the forces about the point (1,-2,0) is

A. $2\sqrt{35}$

B. $6\sqrt{10}$

C. $4\sqrt{7}$

D. none of these

Answer: B

Watch Video Solution

38. The moment about a line through the origin having the direction of 1

12hati -4hatj -3hatk` is

Watch Video Solution

39. The moment of the couple formed by the forces $5\hat{i} + \hat{j}$ and $-5\hat{i} - \hat{k}$

acting at the points (9 ,-1,2) and (3,-2,1) respectively, is

A. $-\hat{i} + \hat{j} + 5\hat{k}$

B. $\hat{i} - \hat{j} - 5\hat{k}$

C. $2\hat{i} - 2\hat{j} - 10\hat{k}$

D. $-2\hat{i} + 2\hat{j} + 10\hat{k}$

Answer: B

Watch Video Solution

40. A particle has an angular speed of 3 rad/s and the axis of rotation passes through the points $(1, 1, 2) and (1, 2, -2)^{\cdot}$ Find the velocity of the particle at point $P(3, 6, 4)^{\cdot}$

Watch Video Solution

41. A rigid body is spinning about a fixed point (3,-2,-1) with an angular velocity of 4 rad/s, the axis of rotation being in the direction of (1,2,-2). Find the velocity of the particle at point (4,1,1).

Watch Video Solution

42. Find the volume of the parallelopiped whose edges are represented by $a = 2\hat{i} - 3\hat{j} + 4\hat{k}, b = \hat{i} + 2\hat{j} - \hat{k}$ and $c = 3\hat{i} - \hat{j} + 2\hat{k}$.

Watch Video Solution

43. Let $a = x\hat{i} + 12\hat{j} - \hat{k}, b = 2\hat{i} + 2x\hat{j}j + \hat{k}$ and $c = \hat{i} + \hat{k}$. If b,c,a in that order form a left handed system, then find the value of x.

$$\left[x_1 a + y_1 b + z_1 c, x_2 a + y_2 b + z_2 c, x_3 a + y_3 b + z_3 c\right]$$

$$= \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} [abc].$$

Watch Video Solution

44. For any three vectors $a,\ b,\ c$ prove that $\left[\vec{a} + \vec{b}\ \vec{b} + \vec{c}\ \vec{c} + \vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]^{.}$

Watch Video Solution

45. Show that vectors $\vec{a},\ \vec{b},\ \vec{c}$ are coplanar if $\vec{a} + \vec{b},\ \vec{b} + \vec{c},\ \vec{c} + \vec{a}$ are coplanar.

Watch Video Solution

46. For any three vectors a,b and c prove that

$$[\text{a b c}]^2 = \begin{vmatrix} a \cdot a & a \cdot b & a \cdot c \\ b \cdot a & b \cdot b & b \cdot c \\ c \cdot a & c \cdot b & c \cdot c \end{vmatrix}$$

Watch Video Solution

47. If a,b,c,l and m are vectors, prove that

$$[\text{a b c}]\,(l \times m) = \begin{vmatrix} a & b & c \\ a \cdot l & b \cdot l & c \cdot l \\ a \cdot m & b \cdot m & c \cdot m \end{vmatrix}$$

Watch Video Solution

48. If a and b are non-zero and non-collinear vectors, then show that

$$a \times b = [\text{a b i}]\hat{i} + [\text{a b j}]\hat{j} + [\text{a b k}]\hat{k}$$

Watch Video Solution

49. If a,b and c are any three vectors in space, then show that

$(c + b) \times (c + a) \cdot (c + b + a) =$ [a b c]

Watch Video Solution

50. If $\vec{u}, \vec{v} and \vec{w}$ are three non-cop0lanar vectors, then prove that

$\left(\vec{u} + \vec{v} - \vec{w}\right)\dot{\vec{u}} - \vec{v} \times \left(\vec{v} - \vec{w}\right) = \dot{\vec{u}}\dot{\vec{v}} \times \vec{w}$

A. 0

B. $u \cdot (v \times w)$

C. $u \cdot (w \times v)$

D. $3u \cdot (v \times w)$

Answer: B

Watch Video Solution

51. If a,b,c ar enon-coplanar vectors and λ is a real number, then the vectors $a + 2b + 3c, \lambda b + 4c$ and $(2\lambda - 1)c$ are non-coplanar for

A. no value of λ

B. all except one value of λ

C. all except two values of λ

D. all values of λ

Answer: C

Watch Video Solution

52. If the vectors $\vec{a}, \vec{b}, \vec{c}$ are non -coplanar and l, m, n are distinct scalars such that

$\left[l\vec{a} + m\vec{b} + n\vec{c} \quad l\vec{b} + m\vec{c} + n\vec{a} \quad l\vec{c} + m\vec{a} + n\vec{b} \right] = 0$ then

A. $x + y + z = 0$

B. $xy + yz + zx = 0$

C. $x^3 + y^3 + z^3 = 0$

D. $x^2 + y^2 + z^2 = 0$

Answer: A

Watch Video Solution

53. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-coplanar unit vectors each inclined with other at an angle of 30°, then the volume of tetrahedron whose edges are $\vec{a}, \vec{b}, \vec{c}$ is (in cubic units)

A. $\dfrac{\sqrt{3\sqrt{3} - 5}}{12}$

B. $\dfrac{3\sqrt{3} - 5}{12}$

C. $\dfrac{5\sqrt{2} + 3}{12}$

D. none of these

Answer: A

Watch Video Solution

Watch Video Solution

54. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j}, \vec{c} = \hat{i}$ and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \lambda \vec{a} + \mu \vec{b}$, then $\lambda + \mu =$

A. 0

B. 1

C. 2

D. 3

Answer: A

Watch Video Solution

55. Q8) Ifa, b, c (b, care non-parallel) are unit vectors such that ax (b×c) = (1/2) then the angle which a makes with b and are en the angle which a makes with b and c are A. 30, 60 B. 600, 90° C. 90, 60 D. 60°, 30° 0 c00 0 300

Watch Video Solution

Watch Video Solution

56. If $\vec{a} = -\hat{i} + \hat{j} + \hat{k}, \hat{b} = 2\hat{i} + 0\hat{j} + \hat{k},$ then a vector $\vec{X}$ satisfying the conditions:

(i) that it is coplanar with $\vec{a}$ and $\vec{b}$. (ii) that is perpendicular to $\vec{b}$

(iii) that $\vec{a}.\vec{X} = 7$, is

Watch Video Solution

57. Prove that

$a \times (b \times c) + b \times (c \times a) + c \times (a \times b) = 0$

Watch Video Solution

58. Show that the vectors $\vec{a} \times \left(\vec{\times}\vec{c}\right)\vec{x}\left(\vec{c} \times \vec{a}\right)$ and $\vec{c} \times \left(\vec{a} \times \vec{b}\right)$ are coplanar.

Watch Video Solution

59. If $[a \times bb \times cc \times a] = \lambda[abc]^2$, then λ is euqual to

Watch Video Solution

60. If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then show that $\vec{\times} \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ are also coplanar.

Watch Video Solution

61. If $\vec{A}, \vec{B}$ and $\vec{C}$ are vectors such that $|\vec{B}|$ - $|\vec{C}|$. Prove that $\left[\left(\vec{A}+\vec{B}\right) \times \left(\vec{A}+\vec{C}\right)\right] \times \left(\vec{B}+\vec{C}\right). \left(\vec{B}+\vec{C}\right) = 0$

Watch Video Solution

62. If $\vec{b}$ and $\vec{c}$ are two non-collinear such that $\vec{a} \mid \ \mid \left(\vec{b} \times \vec{c}\right)$. Then prove that $\left(\vec{a} \times \vec{b}\right). \left(\vec{a} \times \vec{c}\right)$ is equal to $|\vec{a}|^2\left(\vec{b}. \vec{c}\right)$`

Watch Video Solution

63. Find the set of vector reciprocal to the set off vectors

$2\hat{i} + 3\hat{j} - \hat{k}, \hat{i} - \hat{j} - 2\hat{k}, \ - \hat{i} + 2\hat{j} + 2\hat{k}.$

Watch Video Solution

64. Find a set of vector reciprocal to the vectors a,b and $a \times b$.

Watch Video Solution

65. If $a' = \dfrac{b \times c}{[\text{a b c}]}, b' = \dfrac{c \times a}{[\text{a b c}]}, c' = \dfrac{a \times b}{[\text{a b c}]}$

then show that

$a \times a' + b \times b' + c \times c' = 0$, where a,b and c are non-coplanar.

Watch Video Solution

66. If (e_1, e_2, e_3) and (e'_1, e'_2, e'_3) are two sets of non-coplanar vectors such that $i = 1, 2, 3$ we have $e_i.e'_j = \{1, \quad \text{if} \quad i = j \mid 0, \quad \text{if} \quad i \neq j\}$ then show that $[e_1 e_2 e_3][e'_1 e'_2 e'_3] = 1$

Watch Video Solution

67. Solve the vector equation $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}, \vec{r}.\vec{c} = 0$ provided that $\vec{c}$ is not perpendicular to $\vec{b}$

Watch Video Solution

68. Solve for x, such that $A \cdot X = C$ and $A \times X = B$ with $C \neq 0.$

Watch Video Solution

69. Solve the vector equation $\vec{r} \times \vec{a} + k\vec{r} = \vec{b}$, where $\vec{a}, \vec{b}$ are two non-collinear vectors and k is any scalar.

Watch Video Solution

70. Solve for vectors A and B, where

$A + B = a, A \times B = b, A \cdot a = 1.$

Watch Video Solution

71. If $\left|\vec{a}\right| = 5, \left|\vec{a} - \vec{b}\right| = 8$ and $\left|\vec{a} + \vec{b}\right| = 10$, then $\left|\vec{b}\right|$ is equal to

A. 1

B. $\sqrt{57}$

C. 3

D. none of these

Answer: B

Watch Video Solution

72. Angle between diagonals of a parallelogram whose side are represented by $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} - \hat{k}$

A. $\cos^{-1}\left(\frac{1}{3}\right)$

B. $\cos^{-1}\left(\frac{1}{2}\right)$

C. $\cos^{-1}\left(\frac{4}{9}\right)$

D. $\cos^{-1}\left(\frac{5}{9}\right)$

Answer: A

Watch Video Solution

73. Let $\vec{a}, \vec{b}, \vec{c}$ be vectors of length $3, 4, 5$ respectively. Let $\vec{a}$ be perpendicular to $\vec{b} + \vec{c}$, $\vec{b}$ to $\vec{c} + \vec{a}$ and $\vec{c}$ to $\vec{a} + \vec{b}$. Then $\left|\vec{a} + \vec{b} + \vec{c}\right|$ is :

A. $2\sqrt{5}$

B. $2\sqrt{2}$

C. $10\sqrt{5}$

D. $5\sqrt{2}$

Answer: D

Watch Video Solution

74. Let $a, b > 0$ and $\alpha = \frac{\hat{i}}{a} + \frac{4\hat{j}}{b} + b\hat{k}$ and $\beta = b\hat{i} + a\hat{j} + \frac{1}{b}\hat{k}$, then the maximum value of $\frac{10}{5 + \alpha \cdot \beta}$ is

A. 1

B. 2

C. 4

D. 8

Answer: A

Watch Video Solution

75. If unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1$ and $0 \leq \theta \leq \pi$, then θ lies in the interval

A. $\left[0, \frac{\pi}{6}\right)$

B. $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

C. $\left(\frac{5\pi}{6}, \pi\right]$

D. $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$.

Answer: A

Watch Video Solution

76. If $\vec{a} = 3\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ are given vectors. A vector $\vec{c}$ which is perpendicular to z-axis satisfying $\vec{c}.\vec{a} = 9$ and $\vec{c}.\vec{b} = -4$. If inclination of $\vec{c}$ with x-axis and y-axis and y-axis is α and β respectively, then which of the following is not true?

A. $\alpha > \frac{\pi}{4}$

B. $\beta > \frac{\pi}{2}$

C. $\alpha > \frac{\pi}{2}$

D. $\beta < \frac{\pi}{2}$

Answer: C

Watch Video Solution

77. If A is 3×3 matrix and u is a vector. If Au and u are thogonal for all real u, then matrix A is a

A. singular

B. non-singular

C. symmetric

D. skew-symmetric

Answer: A

Watch Video Solution

78. Let the cosine of angle between the vectors p and q be λ such that $2p + q = \hat{i} + \hat{j}$ and $p + 2q = \hat{i} - \hat{j}$, then λ is equal to

A. $\frac{5}{9}$

B. $-\frac{4}{5}$

C. $\frac{3}{9}$

D. $\frac{7}{9}$

Answer: B

Watch Video Solution

79. The three vectors a, b and c with magnitude 3, 4 and 5 respectively and $a + b + c = 0$, then the value of $a.b + b.c + c.a$ is

A. 47

B. 25

C. 50

D. -25

Answer: D

Watch Video Solution

80. Let $\vec{u}, \vec{v} and \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2 and |\vec{w}| = 3.$ If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v} and \vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

A. $\sqrt{14}$

B. $\sqrt{7}$

C. 2

D. 14

Answer: A

Watch Video Solution

81. If $\vec{a}, \vec{b}, \vec{c} areunit \vec{\rightarrow} rs, then$|veca-vecb|^2+|vecb-vec|^2+|vecc^2-veca^2|^2` does not exceed (A) 4 (B) 9 (C) 8 (D) 6

A. 4

B. 9

C. 8

D. 6

Answer: B

Watch Video Solution

82. The value of x for which the angle between $\vec{a} = 2x^2\hat{i} + 4x\hat{j} = \hat{k} + \hat{k}$ and $\vec{b} = 7\hat{i} - 2\hat{j} = x\hat{k}$, is obtuse and the angle between $\vec{b}$ and the z-axis is acute and less than $\pi/6$, are

A. $0 < \lambda < \frac{1}{2}$

B. $\lambda > \sqrt{159}$

C. $-\frac{1}{2} < \lambda < 0$

D. null set

Answer: D

Watch Video Solution

83. The locus of a point equidistant from two points with position vectors $\vec{a}$ and $\vec{b}$ is

A. $\left[r - \frac{1}{2}(a+b)\right] \cdot (a+b) = 0$

B. $\left[r - \frac{1}{2}(a+b)\right] \cdot (a-b) = 0$

C. $\left[r - \frac{1}{2}(a+b)\right] \cdot a = 0$

D. $[r - (a+b)] \cdot b = 0$

Answer: B

Watch Video Solution

84. If A is $\left(x_1, y_1\right)$ where $x_1 = 1$ on the curve $y = x^2 + x + 10$.the tangent at Acuts the x-axisat B. The value of $\overline{OA}.\overline{AB}$ is

A. - $\dfrac{520}{3}$

B. - 148

C. 140

D. 12

Answer: B

Watch Video Solution

85. In a tetrahedron OABC, the edges are of lengths, $|OA| = |BC| = a, |OB| = |AC| = b, |OC| = |AB| = c.$ Let G_1 and G_2 be the centroids of the triangle ABC and AOC such that $OG_1 \perp BG_2$, then the value of $\dfrac{a^2 + c^2}{b^2}$ is

A. 2

B. 3

C. 6

D. 9

Answer: B

Watch Video Solution

86. If OABC is a tetrahedron such that $OA^2 + BC^2 = OB^2 + CA^2 = OC^2 + AB^2$ then

A. $OA \perp BC$

B. $OB \perp AC$

C. $OC \perp AB$

D. $AB \perp AC$

Answer: D

Watch Video Solution

87. If a,b,c and A,B,C $\in R$ - $\{0\}$ such that $aA + bB + cC + \sqrt{(a^2 + b^2 + c^2)(A^2 + B^2 + C^2)} = 0,$ then value of $\frac{aB}{bA} + \frac{bC}{cB} + \frac{cA}{aC}$ is

A. 3

B. 4

C. 5

D. 6

Answer: A

Watch Video Solution

88. The unit vector in XOZ plane and making angles $45°$ and $60°$ respectively with $\vec{a} = 2i + 2j - k$ and $\vec{b} = 0i + j - k$, is

A. $\frac{1}{\sqrt{2}}\left(-\hat{i} + \hat{k}\right)$

B. $\frac{1}{\sqrt{2}}\left(\hat{i} - \hat{k}\right)$

C. $\frac{\sqrt{3}}{2}\left(\hat{i} + \hat{k}\right)$

D. none of these

Answer: B

Watch Video Solution

89. The units vectors orthogonal to the vector $-\hat{i} + 2\hat{j} + 2\hat{k}$ and making equal angles with the X and Y axes islare) :

A. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

B. $\frac{1}{3}\left(2\hat{i} - 2\hat{j} - \hat{k}\right)$

C. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$

D. $\frac{1}{3}\left(2\hat{i}+2\hat{j}+\hat{k}\right)$

Answer: A

Watch Video Solution

90. If $(a+3b)\cdot(7a-5b)=0$ and $(a-4b)\cdot(7a-2b)=0$. Then, the angle between a and b is

A. $60°$

B. $30°$

C. $90°$

D. none of these

Answer: A

Watch Video Solution

91. Let two non-collinear vectors $\vec{a}$ and $\vec{b}$ inclined at an angle $\frac{2\pi}{3}$ be such that $|\vec{a}| = 3$ and $|\vec{b}| = 2$. If a point P moves so that at any time t its position vector $\overrightarrow{OP}$ (where O is the origin) is given as $\overrightarrow{OP} = \left(t + \frac{1}{t}\right)\vec{a} + \left(t - \frac{1}{t}\right)\vec{b}$ then least distance of P from the origin is

A. $\sqrt{2\sqrt{133} - 10}$

B. $\sqrt{2(133) + 10}$

C. $\sqrt{5 + \sqrt{133}}$

D. none of these

Answer: B

Watch Video Solution

92. If a,b,c be non-zero vectors such that a is perpendicular to b and c and $|a| = 1, |b| = 2, |c| = 1, b \cdot c = 1$ and there is a non-zero vector d coplanar with a+b and 2b-c and $d \cdot a = 1$, then minimum value of |d| is

A. $\frac{2}{\sqrt{13}}$

B. $\frac{3}{\sqrt{13}}$

C. $\frac{4}{\sqrt{5}}$

D. $\frac{4}{\sqrt{13}}$.

Answer: D

Watch Video Solution

93. A groove is in the form of a broken line ABC and the position vectors fo the three points are respectively $2\hat{i} - 3\hat{j} + 2\hat{k}, 3\hat{i} - \hat{k}, \hat{i} + \hat{j} + \hat{k}$, A force of magnitude $24\sqrt{3}$ acts on a particle of unit mass kept at the point A and moves it angle the groove to the point C. If the line of action of the force is parallel to the vector $\hat{i} + 2\hat{j} + \hat{k}$ all along, the number of units of work done by the force is

A. $144\sqrt{2}$

B. $144\sqrt{3}$

C. $72\sqrt{2}$

D. $72\sqrt{3}$

Answer: C

Watch Video Solution

94. For any vectors $a, b, |a \times b|^2 + (a \cdot b)^2$ is equal to

A. $|a|^2|b|^2$

B. $|a + b|$

C. $|a|^2 - |b|^2$

D. 0

Answer: A

Watch Video Solution

95. If $a = \hat{i} + \hat{j} + \hat{k}, b = \hat{i} + \hat{j} - \hat{k},$ then vectors perpendicular to a and b is/are

A. $\lambda\left(\hat{i} + \hat{j}\right)$

B. $\lambda\left(\hat{i} + \hat{j} + \hat{k}\right)$

C. $\lambda\left(\hat{i} + \hat{k}\right)$

D. none of these

Answer: C

Watch Video Solution

96. If $a \times b = b \times c \neq 0$, then the correct statement is

A. $b \mid \mid c$

B. $a \mid \mid b$

C. $(a + c) \mid \mid b$

D. none of these

Answer: C

Watch Video Solution

97. If $a = \hat{i} + 2\hat{j} + 3\hat{k}, b = -\hat{i} + 2\hat{j} + \hat{k}$ and $c = 3\hat{i} + \hat{j}$. If $(a + tb) \perp c$, then t is equal to

A. 5

B. 4

C. 3

D. 2

Answer: A

Watch Video Solution

98. If $a = 2\hat{i} - 3\hat{j} + \hat{k}, b = -\hat{i} + \hat{k}, c = 2\hat{k}j - \hat{k}$, then the area (in sq units) of parallelogram with diagonals $a + b$ and $b + c$ will be

A. $\sqrt{21}$

B. $2\sqrt{21}$

C. $\frac{1}{2}\sqrt{21}$

D. none of these

Answer: C

Watch Video Solution

99. The coordinates of the mid-points of the sides of ΔPQR, are $(3a, 0, 0), (0, 3b, 0)$ and $(0, 0, 3c)$ respectively, then the area of ΔPQR is

A. $18\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

B. $9\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

C. $\frac{9}{2}\sqrt{b^2c^2 + c^2a^2 + a^2b^2}$

D. $18\sqrt{ab + bc + ca}$

Answer: A

Watch Video Solution

100. In a parallelogram $ABCD, AB = \hat{i} + \hat{j} + \hat{k}$ and diagonal $AC = \hat{i} - \hat{j} + \hat{k}$ and area of parallelogram is $\sqrt{8}sq$ units, $\angle BAC$ is equal to

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\sin^{-1}\left(\frac{\sqrt{8}}{3}\right)$

D. $\cos^{-1}\left(\frac{\sqrt{8}}{3}\right)$

Answer: C

Watch Video Solution

101. Let ΔABC be a given triangle, if $|BA - tBC| \geq |AC|$ for any $t \in R$, then ΔABC is

A. Equilateral

B. Right angled

C. Isosceles

D. none of these

Answer: B

View Text Solution

102. If $a^2 + b^2 + c^2 = 1$ where, a,b,$c \in R$, then the maximum value of $(4a - 3b)^2 + (5b - 4c)^2 + (3c - 5a)^2$ is

A. 25

B. 50

C. 144

D. none of these

Answer: B

Watch Video Solution

103. If a, b, c are then $p^{th}, q^{th}, r^{th},$ terms of an HP and $\vec{u} = (q - r)\hat{i} + (r - p)\hat{j} + (p - q)\hat{k}$ and $\vec{v} = \frac{\hat{i}}{a} + \frac{\hat{j}}{b} + \frac{\hat{k}}{c}$ then

A. u and v are parallel vectors

B. u and v are orthogonal vectors

C. $u \cdot v = 1$

D. $u \times v = \hat{i} + \hat{j} + \hat{k}.$

Answer: B

Watch Video Solution

104. If the vector product of a constant vector $\vec{OA}$ with a variable vector $\vec{OB}$ in a fixed plane OAB be a constant vector, then the locus of B is a straight line perpendicular to $\vec{O}A$ b. a circle with centre O and radius equal to $\left|\vec{O}A\right|$ c. a straight line parallel to $\vec{O}A$ d. none of these

A. a straight line perpendicular to OA

B. a circle with centre O radius equal to |OA|

C. a straight line parallel to OA

D. none of these

Answer: C

Watch Video Solution

105. Unit vector perpendicular to the plane of ΔABC with position vectors a,b,c of the vertices A,B,C is

A. $\dfrac{a \times b + b \times c + c \times a}{\Delta}$

B. $\dfrac{a \times b + b \times c + c \times a}{2\Delta}$

C. $\dfrac{a \times b + b \times c + c \times a}{4\Delta}$

D. none of these

Answer: B

Watch Video Solution

106. The vector r satisfying the conditions that I. it is perrpendicular to $3\hat{i} + 2\hat{j} + 2\hat{k}$ and $18\hat{i} - 22\hat{j} - 5\hat{k}$ II. It makes an obtuse angle with Y-axis III. $|r| = 14$.

A. $2\left(-2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

B. $2\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$

C. $4\hat{i} + 6\hat{j} - 12\hat{k}$

D. none of these

Answer: A

Watch Video Solution

107. Let a,b,c denote the lengths of the sides of a triangle such that

$$(a - b)\vec{u} + (b - c)\vec{v} + (c - a)\left(\vec{u} \times \vec{v}\right) = \vec{0}$$

For any two non-collinear vectors $\vec{u}$ and $\vec{u}$,then the triangle is

A. right angled

B. equilateral

C. isosceles

D. scalene

Answer: B

Watch Video Solution

108. $\hat{i}.\left(\hat{j} \times \hat{k}\right) + \hat{j}.\left(\hat{i} \times \hat{k}\right) + \hat{k}.\left(\hat{i} \times \hat{j}\right)$ is equal to

A. 3

B. 2

C. 1

D. 0

Answer: C

Watch Video Solution

109. For non zero vectors $\vec{a}, \vec{b}, \vec{c} \left|\left(\vec{a} \times \vec{b}\right). \vec{c}\right| = \left|\vec{a}\right|\left|\vec{b}\right| \mid \vec{|}$ holds if and only if (A) $\vec{a}. \vec{b} = 0, \vec{b}. \vec{c} = 0$ (B) $\vec{b}. \vec{c} = 0, \vec{c}. \vec{a} = 0$ (C) $\vec{c}. \vec{a} = 0, \vec{a}. \vec{b} = 0$ (D) $\vec{a}. \vec{b} = \vec{b}. \vec{c} = \vec{c}. \vec{a} = 0$

A. $a \cdot b = 0, b \cdot c = 0$

B. $b \cdot c = 0, c \cdot a = 0$

C. $c \cdot a = 0, a \cdot b = 0$

D. $a \cdot b = b \cdot c = c \cdot a = 0$

Answer: D

Watch Video Solution

110. The position vectors of three vertices A,B,C of a tetrahedron OABC with respect to its vertex O are $\hat{i}, 6\hat{j}, \hat{k}$, then its volume (in cu units) is

A. 3

B. $\frac{1}{3}$

C. $\frac{1}{6}$

D. 6

Answer: D

Watch Video Solution

111. A parallelepiped is formed by planes drawn parallel to coordinate axes through the points A=(1,2,3) and B=(9,8,5). The volume of that parallelepiped is equal to (in cubic units)

A. 192

B. 48

C. 32

D. 96

Answer: D

Watch Video Solution

112. If $|a| = 1, |b| = 3$ and $|c| = 5$, then the value of $[a - b \quad b - c \quad c - a]$ is

A. 0

B. 1

C. -1

D. none of these

Answer: A

Watch Video Solution

113. If a,b,c are three non-coplanar vectors, then $3a - 7b - 4c, 3a - 2b + c$ and $a + b + \lambda c$ will be coplanar, if λ is

A. -1

B. 1

C. 3

D. 2

Answer: D

Watch Video Solution

114. Let $\vec{r} = (\vec{a} \times \vec{b})\sin x + (\vec{b} \times \vec{c})\cos y + (\vec{c} \times \vec{a})$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are non-zero non-coplanar vectors, If $\vec{r}$ is orthogonal to $3\vec{a} + 5\vec{b} + 2\vec{c}$, then the value of $\sec^2 y + \operatorname{cosec}^2 x + \sec y \ \operatorname{cosec}\ x$ is

A. 3

B. 4

C. 5

D. 6

Answer: A

Watch Video Solution

115. Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lies in a plane then c is

A. HM of a and b

B. 0

C. AM of a and b

D. GM of a and b

Answer: D

Watch Video Solution

116. If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number, then $\left[\lambda\left(\vec{a}+\vec{b}\right) \quad \lambda^2\vec{b} \quad \lambda\vec{c}\right] = \left[\vec{a} \quad \vec{b}+\vec{c} \quad \vec{b}\right]$ for

A. exactly two values of λ

B. exactly one value of λ

C. exactly three values of λ.

D. no value of λ

Answer: C

Watch Video Solution

117. In a regular tetrahedron, let θ be angle between any edge and a face not containing the edge. Then the value of $\cos^2\theta$ is

A. $1/6$

B. $1/9$

C. $1/3$

D. none of these

Answer: C

Watch Video Solution

118. DABC be a tetrahedron such that AD is perpendicular to the base ABC and $\angle ABC = 30^\circ$. The volume of tetrahedron is 18. if value of $AB + BC + AD$ is minimum, then the length of AC is

A. $6\sqrt{2 - \sqrt{3}}$

B. $3\left(\sqrt{6} - \sqrt{2}\right)$

C. $6\sqrt{2 + \sqrt{3}}$

D. $3\left(\sqrt{6} + \sqrt{2}\right)$.

Answer: A

Watch Video Solution

119. If $a = \hat{i} + \hat{j} + \hat{k}, b = \hat{i} - \hat{j} + \hat{k}, c = \hat{i} + 2\hat{j} - \hat{k},$ then the value of

$$\begin{vmatrix} a \cdot a & a \cdot b & a \cdot c \\ b \cdot a & b \cdot b & b \cdot c \\ c \cdot a & c \cdot b & c \cdot c \end{vmatrix}$$ is

A. 2

B. 4

C. 16

D. 64

Answer: C

Watch Video Solution

120. The value of a so that the volume of the paralelopiped formed by $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum is

A. -3

B. 3

C. $1/\sqrt{3}$

D. $\sqrt{3}$

Answer: C

Watch Video Solution

121. If a,b and c be any three non-zero and non-coplanar vectors, then any vector r is equal to

where, $x = \frac{[rbc]}{[abc]}, y = \frac{[rca]}{[abc]}, z = \frac{[rab]}{[abc]}$

A. $za + xb + yc$

B. $xz + yb + zc$

C. $ya + zb + xc$

D. none of these

Answer: B

Watch Video Solution

122. The position vectors of vertices of ΔABC are a,b,c and $a \cdot a = b \cdot b = c \cdot c = 3.$ if [a b c]=0, then the position vectors of the orthocentre of ΔABC is

A. $a + b + c$

B. $\frac{1}{3}(a + b + c)$

C. 0

D. none of these

Answer: A

Watch Video Solution

123. If α and β are two mutaully perpendicular unit vectors $\{r\alpha + r\beta + s(\alpha \times \beta\}, [\alpha + (\alpha \times \beta)]$ and $\{s\alpha + s\beta + t(\alpha \times \beta)\}$ are coplanar, then s is equal to

A. AM of r and t

B. HM of r and t

C. GM of r and t

D. none of these

Answer: C

Watch Video Solution

124. Let $\vec{b} = -\vec{i} + 4\vec{j} + 6\vec{k}, \vec{c} = 2\vec{i} - 7\vec{j} - 10\vec{k}.$ If $\vec{a}$ be a unit vector and the scalar triple product $\left[\vec{a}\vec{b}\vec{c}\right]$ has the greatest value then $\vec{a}$ is

A. $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

B. $\frac{1}{\sqrt{5}}\left(\sqrt{2}\hat{i} - \hat{j} - \sqrt{2}\hat{k}\right)$

C. $\frac{1}{3}\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$

D. $\frac{1}{\sqrt{59}}\left(3\hat{i} - 7\hat{j} - \hat{k}\right)$

Answer: C

Watch Video Solution

125. Prove that vectors $\vec{u} = \left(al + a_1 l_1\right)\hat{i} + \left(am + a_1 m_1\right)\hat{j} + \left(an + a_1 n_1\right)\hat{k}$

$\vec{v} = \left(bl + b_1 l_1\right)\hat{i} + \left(bm + b_1 m_1\right)\hat{j} + \left(bn + b_1 n_1\right)\hat{k}$

$\vec{w} = \left(bl + b_1 l_1\right)\hat{i} + \left(bm + b_1 m_1\right)\hat{j} + \left(bn + b_1 n_1\right)\hat{k}$

A. form an equilateral triangle

B. are coplanar

C. are collinear

D. are mutually perpendicular

Answer: B

Watch Video Solution

126. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\left[\vec{a} \ \vec{b} \ \vec{c}\right] = 2$. If $\vec{r} = l\left(\vec{b} \times \vec{c}\right) + m\left(\vec{c} \times \vec{a}\right) + n\left(\vec{a} \times \vec{b}\right)$ be perpendicular to $\vec{a} + \vec{b} + \vec{c}$, then the value of $l + m + n$ is

A. 2

B. 1

C. 0

D. none of these

Answer: C

Watch Video Solution

127. If a,b and c are three mutually perpendicular vectors, then the projection of the vectors

$l\frac{a}{|a|}+m\frac{b}{|b|}+n\frac{(a\times b)}{|a\times b|}$ along the angle bisector of the vectors a and b is

A. $\frac{l+m}{\sqrt{2}}$

B. $\sqrt{l^2+m^2+n^2}$

C. $\frac{\sqrt{l^2+m^2}}{\sqrt{l^2+m^2+b^2}}$

D. none of these

Answer: A

Watch Video Solution

128. If the volume of the parallelopiped formed by the vectors $\vec{a}, \vec{b}, \vec{c}$ as three coterminous edges is 27 units, then the volume of the parallelopiped having $\vec{\alpha} = \vec{a} + 2\vec{b} - \vec{c}, \vec{\beta} = \vec{a} - \vec{b}$ and $\vec{\gamma} = \vec{a} - \vec{b} - \vec{c}$ as three coterminous edges, is

A. 27

B. 9

C. 81

D. none of these

Answer: C

Watch Video Solution

129. If V is the volume of the parallelopiped having three coterminus edges as a,b and c, then the volume of the parallelopiped having the edges as

$\alpha = (a.\,a)a + (a.\,b)b + (a.\,c)c; \beta = (a.\,b)a + (b.\,b)b + (b.\,c)b; \gamma = (a.\,c)a + (b.\,c)b +$

, is

A. V^3

B. $3V$

C. V^2

D. 2V

Answer: A

Watch Video Solution

130. Let $\vec{r}, \vec{a}, \vec{b} and \vec{c}$ be four nonzero vectors such that

$\vec{r}\cdot\vec{a} = 0, \left|\vec{r} \times \vec{b}\right| = \left|\vec{r}\right|\left|\vec{b}\right| and \left|\vec{r} \times \vec{c}\right| = \left|\vec{r}\right|\left|\vec{c}\right|$. Then $[abc]$ is equal to $|a||b||c|$

b. - $|a||b||c|$ c. 0 d. none of these

A. $|a||b||c|$

B. - $|a||b||c|$

C. 0

D. none of these

Answer: C

Watch Video Solution

131. If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors forming a linearly independent system, then $\forall \theta \in R$

$\vec{p} = \vec{a}\cos\theta + \vec{b}\sin\theta + \vec{c}(\cos 2\theta)$

$\vec{q} = \vec{a}\cos\left(\frac{2\pi}{3} + \theta\right) + \vec{b}\sin\left(\frac{2\pi}{3} + \theta\right) + \vec{c}(\cos 2)\left(\frac{2\pi}{3} + \theta\right)$

and $\vec{r} = \vec{a}\cos\left(\theta - \frac{2\pi}{3}\right) + \vec{b}\sin\left(\theta - \frac{2\pi}{3}\right) + \vec{c}\cos 2\left(\theta - \frac{2\pi}{3}\right)$

then $\left[\vec{p}\vec{q}\vec{r}\right]$

A. [a b c]$\cos\theta$

B. [a b c]$\cos 2\theta$

C. [a b c]$\cos 3\theta$

D. none of these

Answer: D

Watch Video Solution

132. Let $\bar{a}, \bar{b}, \bar{c}$ be three non-coplanar vectors and $\bar{d}$ be a non-zero vector, which is perpendicularto $\bar{a} + \bar{b} + \bar{c}$. Now, if $\bar{d} = (\sin x)\left(\bar{a} \times \bar{b}\right) + (\cos y)\left(\bar{b} \times \bar{c}\right) + 2\left(\bar{c} \times \bar{a}\right)$ then minimum value of $x^2 + y^2$ is equal to

A. π^2

B. $\frac{\pi^2}{2}$

C. $\frac{\pi^2}{4}$

D. $\frac{5\pi^2}{4}$

Answer: D

Watch Video Solution

133. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors having magnitudes 1,1 and 2 resectively. If $\vec{a} \times \left(\vec{a} \times \vec{c}\right) + \vec{b} = \vec{0}$ then the acute angel between $\vec{a}$ and $\vec{c}$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. none of these

Answer: C

Watch Video Solution

134. Let $a = 2\hat{i} + \hat{j} + \hat{k}, b = \hat{i} + 2\hat{j} - \hat{k}$ and c is a unit vector coplanar to them. If c is perpendicular to a, then c is equal to

A. $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$

B. $-\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

C. $\frac{1}{\sqrt{5}}\left(\hat{i} - 2\hat{j}\right)$

D. $\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$

Answer: A

Watch Video Solution

135. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}. \vec{=} \left|\vec{c}\right|, \left|\vec{c} - \vec{a}\right| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°, then $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right| = .$

A. $\frac{2}{3}$

B. $\frac{3}{2}$

C. 2

D. 3

Answer: B

Watch Video Solution

136. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\hat{a}.\hat{b} = \frac{1}{3}$ and $\vec{a} \times \vec{b} = \vec{c}$, Also

$\vec{F} = \alpha\hat{a} + \beta\hat{b} + \lambda\hat{c}$,

where, α, β, λ are scalars. If $\alpha = k_1\left(\hat{F}.\hat{a}\right) - k_2\left(\hat{F}.\hat{b}\right)$ then the value of $2\left(k_1 + k_2\right)$ is

A. $2\sqrt{3}$

B. $\sqrt{3}$

C. 3

D. 1

Answer: C

Watch Video Solution

137. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}, \vec{c} = \hat{k} - \hat{i}. If \hat{d}$ is a unit vector such that $\vec{a}.\hat{d} = 0 = \left[\vec{b}, \vec{c}, \vec{d}\right]$ then hat$d equals (A)$+-(hati+hatj-2hatk)/sqrt(6)(B)+-(hati+hatj-hatk)/sqrt(3)(C)+-(hati+hatj+hatk)/sqrt(3)(D)+-hatk`

A. $\pm\dfrac{\left(\hat{i} + \hat{j} + 2\hat{k}\right)}{\sqrt{6}}$

B. $\pm\dfrac{\left(\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{3}}$

C. $\pm\dfrac{\left(\hat{i} + \hat{j} + \hat{k}\right)}{\sqrt{3}}$

D. $\pm\hat{k}$

Answer: A

Watch Video Solution

138. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{\vec{b} + \vec{c}}{\sqrt{2}}$ then the angle between $\vec{a}$ and $\vec{b}$ is

A. $\dfrac{3\pi}{4}$

B. $\dfrac{\pi}{4}$

C. $\dfrac{\pi}{2}$

D. π

Answer: A

Watch Video Solution

139. The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with the vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is

A. $\dfrac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\dfrac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$

C. $\dfrac{3\hat{j} - \hat{k}}{\sqrt{10}}$

D. $\dfrac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$.

Answer: C

Watch Video Solution

140. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the non zero vectors such that $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. if theta is the acute angle between the vectors $\vec{b}$ and $\vec{a}$ then theta equals (A) $\frac{1}{3}$ (B) $\frac{\sqrt{2}}{3}$ (C) $\frac{2}{3}$ (D) $2\frac{\sqrt{2}}{3}$

A. $\dfrac{2\sqrt{2}}{3}$

B. $\dfrac{\sqrt{2}}{3}$

C. $\dfrac{2}{3}$

D. $\dfrac{1}{3}$

Answer: A

Watch Video Solution

141. The value for $[a \times (b + c), b \times (c - 2a), c \times (a + 3b)]$ is equal to

A. $[abc]^2$

B. $7[abc]^2$

C. $-5[a \times b \quad b \times c \quad c \times a]$

D. none of these

Answer: B

View Text Solution

142. If a,b,c and p,q,r are reciprocal systemm of vectors, then $a \times p + b \times q + c \times r$ is equal to

A. $[abc]$

B. $[p + q + r]$

C. 0

D. a+b+c

Answer: C

Watch Video Solution

143. Solve $\vec{a}.\vec{r}=x, \vec{b}.\vec{r}=y, \vec{c}.\vec{r}=z where \vec{a}, \vec{b}, \vec{c}$ are given non coplasnar vectors.

Watch Video Solution

144. If $z_1 = ai + bj$ and $z_2 = ci + dj$ are two vectors in i and j system, where $|z_1| = |z_2| = r$ and $z_1 . z_2 = 0$ then $w_1 = ai + cj$ and $w_2 = bi + dj$ satisfy

A. $|w_1| = r$

B. $|w_2| = r$

C. $w_1 \cdot w_2 = 0$

D. none of these

Answer: A::B::C

Watch Video Solution

145. If unit vectors $\hat{i}$ and $\hat{j}$ are at right angles to each other and $p = 3\hat{i} + 4\hat{j}, q = 5\hat{i}, 4r = p + q$ and $2s = p - q$, then

A. $|r + ks| = |r - ks|$ for all real k

B. r is perpendicular to s

C. $r + s$ is perpendicular to r-s

D. $|r| = |s| = |p| = |q|$

Answer: A::B::C

Watch Video Solution

146. Let $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a}.\vec{a} = \vec{b}.\vec{b} = \vec{c}.\vec{c} = 3$ and $\left|\vec{a} - \vec{b}\right|^2 + \left|\vec{b} - \vec{c}\right|^2 + \left|\vec{c} - \vec{a}\right|^2 = 27$, then

A. a,b and c are necessarily coplanar

B. a,b and c represent sides of a triangle in magnitude and direction

C. $a \cdot b + b \cdot c + c \cdot a$ has the least value $-9/2$

D. a,b and c represent orthogonal triad of vectors

Answer: A::B::C

Watch Video Solution

147. If $\vec{a}$ and $\vec{b}$ are non - zero vectors such that $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - 2\vec{b}\right|$ then

A. $2a \cdot b = |b|^2$

B. $a \cdot b = |b|^2$

C. Least value of $a \cdot b + \frac{1}{|b|^2 + 2}$ is $\sqrt{2}$

D. Least value of $a \cdot b + \frac{1}{|b| + 2}$ is $\sqrt{2} - 1$

Answer: A::D

Watch Video Solution

148. If vectos $b = \left(\tan\alpha, -1, 2\sqrt{\sin\alpha/2}\right)$ and $c = \left(\tan\alpha, \tan\alpha, -\frac{3}{\sqrt{\sin\alpha/2}}\right)$ are orthogonal and vectors $a = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the Z-axis, then the value of α is

A. $\alpha = (4n + 1)\pi + \tan^{-1}2$

B. $\alpha = (4n + 1)\pi - \tan^{-1}2$

C. $\alpha = (4n + 2)\pi + \tan^{-1}2$

D. $\alpha = (4n + 2)\pi - \tan^{-1}2$

Answer: B::D

Watch Video Solution

149. If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the greatest postive integer in the range of $\frac{3\left|\vec{a}+\vec{b}\right|}{2}+2\left|\vec{a}-\vec{b}\right|$

A. 2

B. 3

C. 4

D. 5

Answer: D

Watch Video Solution

150. Which of the following expressions are meaningful?

A. $u \cdot (v \times w)$

B. $(u \cdot v) \cdot w$

C. $(u \cdot v)w$

D. $u \times (v \cdot w)$

Answer: A::C

Watch Video Solution

151. If $a + 2b + 3c = 0$, then $a \times b + b \times c + c \times a$ is equal to

A. $2(a \times b)$

B. $6(b \times c)$

C. $3(c \times a)$

D. 0

Answer: A::B::C

Watch Video Solution

152. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}, \vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$ and $\vec{\gamma} = c\hat{i} + a\hat{j} + b\hat{k}$ be three coplnar vectors with $a \neq b$, and $\vec{v} = \hat{i} + \hat{j} + \hat{k}$. Then $\vec{v}$ is perpendicular to

A. α

B. β

C. γ

D. none of these

Answer: A::B::C

Watch Video Solution

153. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{r}$ is non-zero vector such that $p\vec{r} + \left(\vec{r} \cdot \vec{a}\right)\vec{b} = \vec{c}$, then $\vec{r} = \frac{\vec{c}}{p} - \frac{\left(\vec{a} \cdot \vec{c}\right)\vec{b}}{p^2}$ (b) $\frac{\vec{a}}{p} - \frac{\left(\cdot \vec{b}\right)\vec{a}}{p^2}$ $\frac{\vec{a}}{p} - \frac{\left(\vec{a} \cdot \vec{b}\right)\vec{c}}{p^2}$ (d)

$$\frac{\vec{c}}{p^2} - \frac{\left(\vec{a}\cdot\vec{c}\right)\vec{b}}{p}$$

A. $[rac] = 0$

B. $p^2 r = pa - (c \cdot a)b$

C. $p^2 r = pb - (a \cdot b)c$

D. $p^2 r = pc - (b \cdot c)a$

Answer: A::D

Watch Video Solution

154. If $\alpha(a \times b) + \beta(b \times c) + \gamma(c \times a) = 0$, then

A. a,b,c are coplanar if all of $\alpha, \beta, \gamma \neq 0$

B. a,b,c are coplanar if any one of $\alpha, \beta, \gamma \neq 0$

C. a,b,c are non-coplanar for any $\alpha, \beta, \gamma \neq 0$

D. none of these

Answer: A::B

Watch Video Solution

155. If $a = \hat{i} + \hat{j} + \hat{k}$ and $b = \hat{i} - \hat{j}$, then vectors $\left(a \cdot \hat{i} + \hat{i} + \left(a \cdot \hat{j} + \left(a \cdot \hat{k}\right)\hat{k}, \left(b \cdot \hat{i}\right)\hat{i} + \left(b\hat{j}\right)\hat{j} + \left(b \cdot \hat{k}\right)\hat{k}$ and $\hat{i} + \hat{j} - 2\hat{k}$

A. are mutually perpendicular

B. are coplanar

C. form a parallepiped of volume 6 units

D. form a parallelopiped of volume 3 units

Answer: A::C

Watch Video Solution

156. The volume of the parallelepiped whose coterminous edges are represented by the vectors $2\vec{b} \times \vec{c}, 3\vec{c} \times \vec{a}$ and $4\vec{a} \times \vec{b}$ where

$$\hat{i} = \sin\left(\theta + \frac{2\pi}{3}\right)\hat{i} + \cos\left(\theta + \frac{2\pi}{3}\right)\hat{j} + \sin\left(2\theta + \frac{4\pi}{3}\right)\hat{k},$$

$$\vec{c} = \sin\left(\theta - \frac{2\pi}{3}\right)\hat{i} + \cos\left(\theta - \frac{2\pi}{3}\right)\hat{j} + \sin\left(2\theta - \frac{4\pi}{3}\right)\hat{k}$$

is 18 cubic units, then the values of θ, in the interval $\left(0, \frac{\pi}{2}\right)$, is/are

A. $\frac{\pi}{9}$

B. $2\frac{\pi}{9}$

C. $\frac{\pi}{3}$

D. $4\frac{\pi}{9}$

Answer: A::B::D

Watch Video Solution

157. If $a = x\hat{i} + y\hat{j} + z\hat{k}, b = y\hat{i} + z\hat{j} + x\hat{k}$ and $c = z\hat{i} + x\hat{j} + y\hat{k}$, then $a \times (b \times c)$ is/are

A. parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$

D. parallel to $\hat{i}+\hat{j}+\hat{k}$

Answer: A::B::C

Watch Video Solution

158. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, then which of the following statement(s) is/are true?

A. $a \times (b \times c), b \times (c \times a), c \times (a \times b)$ from a right handed system.

B. $c, (a \times b) \times c, a \times b$ from a right handed system.

C. $a \cdot b + b \cdot c + c \cdot a < 0$, iff a+b+c=0

D. $\dfrac{(a \times b) \cdot (b \times c)}{(b \times c) \cdot (a \times c)} = -1$, if a+b+c=0.

Answer: B::C::D

Watch Video Solution

159. Let the unit vectors a and b be perpendicular and the unit vector c be inclined at an angle θ to both a and b. If $c = \alpha a + \beta b + \gamma(a \times b)$, then

A. $l = m$

B. $n^2 = 1 - 2l^2$

C. $n^2 = -\cos 2\alpha$

D. $m^2 = \dfrac{1 + \cos 2\alpha}{2}$

Answer: A::B::C::D

Watch Video Solution

160. If $a \times (b \times c)$ is perpendicular to $(a \times b) \times c$, we may have

A. $(a \cdot c)|b|^2 = (a \cdot b)(b \cdot c)$

B. $a \cdot b = 0$

C. $a \cdot c = 0$

D. $b \cdot c = 0$

Answer: A::C

Watch Video Solution

161. If $\left(\vec{a} \times v\vec{b}\right) \times \left(\vec{c} \times \vec{d}\right). \left(\vec{a} \times \vec{d}\right) = 0$ then which of the following may be true ?

A. a,b,c and d are necessarily coplanar

B. a lies in the plane of c and d

C. b lies in the plane o a and d

D. c lies in the plane of a and d

Answer: B::C::D

Watch Video Solution

162. The angles of a triangle, two of whose sides are represented by vectors $\sqrt{3}\left(\hat{a} \times \hat{b}\right)$ and $\hat{b}$ - $\left(a \cdot \hat{b}\right)\hat{a}$, where $\hat{b}$ is a non-zero vector and $\hat{a}$ is a unit vector in the direction of $\hat{a}$ are

A. $\tan^{-1}(\sqrt{3})$

B. $\tan^{-1}(1/\sqrt{3})$

C. $\cot^{-1}(0)$

D. $\tan^{-1}(1)$

Answer: A::B::C

Watch Video Solution

163. Let the vectors PQ,OR,RS,ST,TU and UP represent the sides of a regular hexagon.

Statement I: $PQ \times (RS + ST) \neq 0$

Statement II: $PQ \times RS = 0$ and $PQ \times ST \neq 0$

A. Both statement I and statement II are correct and statement II is the correct explanation of statement I

B. both statement I and statement II are correct but statement II is not the correct explanation of statement I

C. Statement I is correct but statement II is incorrect

D. Statement II is correct but statement I is incorrect

Answer: C

Watch Video Solution

164. p,q and r are three vectors defined by $p = a \times (b + c), q = b \times (c + a)$ and $r = c \times (a + b)$

Statement I: p,q and r are coplanar.

Statement II: Vectors p,q,r are linearly independent.

A. Both statement I and statement II are correct and statement II is the correct explanation of statement I

B. both statement I and statement II are correct but statement II is not the correct explanation of statement I

C. Statement I is correct but statement II is incorrect

D. Statement II is correct but statement I is incorrect

Answer: C

Watch Video Solution

165. Assertion : *If* $\in$ *a*/_\ABC, vec(BC)=vecp/|vecp|-vecq/|vecq| and vec(AC)= (2vecp)/|vecp|,|vecp|!=|veq|*thenthevalueof*cos2A+cos2B+cos2C *is* - 1. , *Reason*: *If* $\in$ /_\ABC, /_C=90^0 then cos2A+cos2B+cos2C=-1` (A) Both A and R are true and R is the correct explanation of A (B) Both A and R are true R is not te correct explanation of A (C) A is true but R is false. (D) A is false but R is true.

A. Both statement I and statement II are correct and statement II is the correct explanation of statement I

B. both statement I and statement II are correct but statement II is not the correct explanation of statement I

C. Statement I is correct but statement II is incorrect

D. Statement II is correct but statement I is incorrect

Answer: B

Watch Video Solution

166. Statement I: If a is perpendicular to b and c, then $a \times (b \times c) = 0$

Statement II: if a is perpendicular to b and c, then $b \times c = 0$

A. Both statement I and statement II are correct and statement II is the correct explanation of statement I

B. both statement I and statement II are correct but statement II is not the correct explanation of statement I

C. Statement I is correct but statement II is incorrect

D. Statement II is correct but statement I is incorrect

Answer: C

Watch Video Solution

167. Let $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{c} = -2\hat{i} + 3\hat{j} + 6\hat{k}$. Let $\vec{a}_1$ be the projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then $\vec{a}_2$ is equal to

A. $\frac{943}{49}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

B. $\frac{943}{49^2}\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)$

C. $\frac{943}{49}\left(-2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

D. $\frac{943}{49^2}\left(-2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

Answer: B

Watch Video Solution

168. Let $a = 2\hat{i} + 3\hat{j} - 6\hat{k}, b = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $c = -2\hat{i} + 3\hat{j} + 6\hat{k}$, Let a_1 be the projection of a on b and a_2 be the projection of a_1 and c. Then Q. a_1, b is equal to

A. -41

B. $-\frac{41}{7}$

C. 41

D. 287

Answer: A

Watch Video Solution

169. Let $a = 2\hat{i} + 3\hat{j} - 6\hat{k}, b = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $c = -2\hat{i} + 3\hat{j} + 6\hat{k}$, Let a_1 be the projection of a on b and a_2 be the projection of a_1 and c. Then Q. Which of the following is true?

A. a and a_2 are collinear

B. a_1 and c are collinear

C. a, a_1 and b are coplanar

D. a, a_1 and a_2 are coplanar

Answer: C

Watch Video Solution

170. Let a, b be two vectors perependicular to each other and $|a| = 2, |b| = 3$ and $c \times a = b$. Q. The least value of |c-a| is

A. 1

B. $\frac{1}{2}$

C. $\frac{1}{4}$

D. $\frac{3}{2}$

Answer: D

Watch Video Solution

171. Let a, b be two vectors perependicular to each other and $|a| = 2, |b| = 3$ and $c \times a = b$. Q. When |c-a| is least the value of α (when α

is angle between a and c) equals

A. $\tan^{-1}(2)$

B. $\dfrac{\tan^{-1}(3)}{4}$

C. $\cos^{-1}\left(\dfrac{2}{3}\right)$

D. None of these

Answer: B

Watch Video Solution

172. Let a, b be two vectors perependicular to each other and $|a| = 2, |b| = 3$ and $c \times a = b$. Q, When |c-a| attains least value, then the valuw of |c| is

A. $\dfrac{1}{2}$

B. $\dfrac{7}{2}$

C. $\dfrac{5}{2}$

D. 4

Answer: C

Watch Video Solution

173. Consider a triangular pyramid ABCD the position vectors of whone agular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 3, 2)$ and $D(0, -5, 4)$ Let G be the point of intersection of the medians of the triangle BCT. The length of the vector $\overline{AG}$ is

A. $\sqrt{17}$

B. $\dfrac{\sqrt{51}}{3}$

C. $\dfrac{3}{\sqrt{6}}$

D. $\dfrac{\sqrt{59}}{4}$

Answer: B

Watch Video Solution

174. Consider a triangulat pyramid ABCD the position vector of whose angular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 2, 3)$ and $D(0, -5, 4)$. Let G be the point of intersection of the medians of the $\triangle(BCD)$.

Q. Area of the $\triangle(ABC)$ (in sq. units) is

A. 24

B. $8\sqrt{6}$

C. $4\sqrt{6}$

D. None of these

Answer: C

Watch Video Solution

175. Consider a triangulat pyramid ABCD the position vector of whose angular points are $A(3, 0, 1), B(-1, 4, 1), C(5, 2, 3)$ and $D(0, -5, 4)$. Let G be the point of intersection of the medians of the $\triangle(BCD)$.

Q. The length of the perpendicular from the vertex D on the opposite face is

A. $\frac{14}{\sqrt{6}}$

B. $\frac{2}{\sqrt{6}}$

C. $\frac{3}{\sqrt{6}}$

D. None of these

Answer: A

Watch Video Solution

176. If AP, BQ and CR are the altitudes of acute $\triangle ABC$ and $9AP + 4BQ + 7CR = 0$ Q. $\angle ACB$ is equal to

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\cos^{-1}\left(\frac{1}{3\sqrt{7}}\right)$

D. $\cos^{-1}\left(\frac{1}{\sqrt{7}}\right)$

Answer: B

View Text Solution

177. If AP, BQ and CR are the altitudes of acute $\triangle ABC$ and $9AP + 4BQ + 7CR = 0$ Q. $\angle ABC$ is equal to

A. $\frac{\cos^{-1}(2)}{\sqrt{7}}$

B. $\frac{\pi}{2}$

C. $\cos^{-1}\left(\frac{\sqrt{7}}{3}\right)$

D. $\frac{\pi}{3}$

Answer: A

View Text Solution

178. Let a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non zero scalars satisfying $a \times b + b \times c = pa + qb + rc$

Q. Volume of parallelopiped with edges a, b, c is

A. $p + (q + r)\cos\theta$

B. $(p + q + r)\cos\theta$

C. $2p - (q + r)\cos\theta$

D. None of these

Answer: A

Watch Video Solution

179. Let a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non zero scalars satisfying $a \times b + b \times c = pa + qb + rc$

Q. The value of $\left(\frac{q}{p} + 2\cos\theta\right)$ is

A. 1

B. 0

C. $2[abc]$

D. None of these

Answer: B

Watch Video Solution

180. Let a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non zero scalars satisfying $a \times b + b \times c = pa + qb + rc$

Q. The value of $|(p + q)\cos\theta + r|$ is

A. $(1 + \cos\theta)\left(\sqrt{1 - 2\cos\theta}\right)$

B. $2\frac{\sin(\theta)}{2}\sqrt{(1 + 2\cos\theta)}$

C. $(1 - \sin\theta)\sqrt{1 + 2\cos\theta}\Big)$

D. None of these

Answer: B

Watch Video Solution

181. Given that

$\vec{u} = \hat{i} - 2\hat{j} + 3\hat{k}; \vec{v} = 2\hat{i} + \hat{j} + 4\hat{k}; \vec{w} = \hat{i} + 3\hat{j} + 3\hat{k} and \left(\vec{u}\dot{R} - 15\right)\hat{i} + \left(\vec{v}\dot{R} - 30\right)\hat{j} + ($

Then find the greatest integer less than or equal to $\left|\vec{R}\right|^{\cdot}$

Watch Video Solution

182. The position vector of a point P is $r = x\hat{i} + y\hat{j} + \hat{k}z,$ where $x, y, z \in N$ and $a = \hat{i} + 2\hat{j} + \hat{k}. If r \cdot a = 20$ and the number of possible of P is 9λ, then the value of λ is

Watch Video Solution

183. Let $\vec{u}$ be a vector on rectangular coordinate system with sloping angle $60^{0^{\cdot}}$ Suppose that $\left|\vec{u} - \hat{i}\right|$ is geometric mean of

$\left|\vec{u}\right| and \left|\vec{u} - 2\hat{i}\right|, where \hat{i}$ is the unit vector along the x-axis. Then find the value of $\left(\sqrt{2} + 1\right)\left|\vec{u}\right|^{.}$

Watch Video Solution

184. Let $A\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right), B\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$ and $C\left(\lambda\hat{i} + 5\hat{j} + \mu\hat{k}\right)$ are vertices of a triangle and its median through A is equally inclined to the positive directions of the axes, the value of $2\lambda - \mu$ is

Watch Video Solution

185. Three vectors $a(|a| \neq 0), b$ and c are such that $a \times b = 3a \times c$, also $|a| = |b| = 1$ and $|c| = \frac{1}{3}$. If the angle between b and c is 60° and $|b - 3x| = \lambda|a|$, then the value of λ is

Watch Video Solution

186. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}\dot{\vec{b}} = 0 = \vec{a}\dot{\vec{c}}$ and the angel between $\vec{b} and \vec{c}$ is $\frac{\pi}{3}$, then find the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$.

Watch Video Solution

187. the area of the triangle whose vertices are A (1,-1,2) , B (1,2, -1) ,C (3, -1, 2) is _______.

Watch Video Solution

188. Let $\vec{O}A - \vec{a}, \hat{O}B = 10\vec{a} + 2\vec{b} and \vec{O}C = \vec{b}, where O, A and C$ are non-collinear points. Let p denotes the areaof quadrilateral $OACB,$ and let q denote the area of parallelogram with $OA and OC$ as adjacent sides. If $p = kq,$ then find$k$.

Watch Video Solution

189. If $\vec{x}, \vec{y}$ are two non-zero and non-collinear vectors satisfying

$$\left[(a-2)\alpha^2+(b-3)\alpha+c\right]\vec{x}+\left[(a-2)\beta^2+(b-3)\beta+c\right]\vec{y}+\left[(a-2)\gamma^2+(b-3)\gamma+c\right.$$

are three distinct real numbers, then find the value of $\left(a^2+b^2+c^2-4\right)^{\cdot}$

Watch Video Solution

190. Let $\vec{V}=2\hat{i}+\hat{j}-\hat{k}and\vec{W}=\hat{i}+3\hat{k}^{\cdot}$ If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is -1 b. $\sqrt{10}+\sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

Watch Video Solution

191. Let $a=\alpha\hat{i}+2\hat{j}-3\hat{k}, b=\hat{i}+2\alpha\hat{j}-2\hat{k}$ and $c=2\hat{i}-\alpha\hat{j}+\hat{k}$. Then the value of 6α, such that $\{(a\times b)\times(b\times c)\}\times(c\times a)=a$, is

View Text Solution

192. If $\hat{a}$ and $\hat{b}$ are unit vectors such that $\left(\hat{a} + \hat{b}\right)$ is a unit vector, what is the angle between $\hat{a}$ and $\hat{b}$?

Watch Video Solution

193. Determine the value of c to that for all real x, the vectors $cx\hat{i} - 6\hat{j} + 3\hat{k}$ and $x\hat{i} + 2\hat{j} + 2cx\hat{k}$ make an obtuse angle with each other.

Watch Video Solution

194. A, B, C and D are four points in space. Using vector methods, prove that $AC^2 + BD^2 + AC^2 + BC^2 \geq AB^2 + CD^2$ what is the implication of the sign of equality.

Watch Video Solution

195. Prove that the perpendicular let fall from the vertices of a triangle to the opposite sides are concurrent.

Watch Video Solution

196. Using vector method, prove that the angel in a semi circle is a right angle.

Watch Video Solution

197. The corner P of the square OPQR is folded up so that the plane OPQ is perpendicular to the plane OQR, find the angle between OP and QR.

Watch Video Solution

198. In a $\triangle ABC$, prove by vector method that $\cos 2A + \cos 2B + \cos 2C \geq \frac{-3}{2}$.

Watch Video Solution

199. Let $\beta = 4\hat{i} + 3\hat{j}$ and $\vec{\gamma}$ be two vectors perpendicular to each other in the XY plane. Find all the vectors in the same plane having the projections 1 and 2 along $\vec{\beta}$ and $\vec{\gamma}$ respectively.

Watch Video Solution

200. If a, b and c are three coplanar vectors. If a is not parallel to b, show that $c = \dfrac{\begin{vmatrix} c\cdot a & a\cdot b \\ c\cdot b & b\cdot b \end{vmatrix} a + \begin{vmatrix} a\cdot a & c\cdot a \\ a\cdot b & c\cdot b \end{vmatrix} b}{\begin{vmatrix} a\cdot a & a\cdot b \\ a\cdot b & b\cdot b \end{vmatrix}}$.

Watch Video Solution

201. In $\triangle ABC$, D is the mid point of the side AB and E is centroid of $\triangle CDA$. If $OE \cdot CD = 0$, where O is the circumcentre of $\triangle ABC$, using vectors prove that AB=AC.

Watch Video Solution

202. Let I be the incentre of $\triangle ABC$. Using vectors prove that for any point P

$$a(PA)^2 + b(PB)^2 + c(PC)^2 = a(IA)^2 + b(IB)^2 + c(IC)^2 + (a + b + c)(IP)^2$$

where a, b, c have usual meanings.

Watch Video Solution

203. If two circles intersect in two points; prove that the line through the centres is the perpendicular bisector of the common chord.

Watch Video Solution

204. Using vector method prove that $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Watch Video Solution

205. A circle is inscribed in an n-sided regular polygon $A_1, A_2, \ldots. A_n$ having each side a unit for any arbitrary point P on the circle, pove that

$$\sum_{i=1}^{n}\left(PA_i\right)^2 = n\frac{a^2}{4}\frac{1+\cos^2\left(\frac{\pi}{n}\right)}{\sin^2\left(\frac{\pi}{n}\right)}$$

Watch Video Solution

206. If $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ are the position vectors of the vertices of a cyclic quadrilateral $ABCD,$ prove that

$$\frac{\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{d}+\overrightarrow{dxx\vec{a}}\right|}{\left(\vec{b}-\vec{a}\right)\cdot\vec{d}-\vec{a}}+\frac{\left|\vec{b}\times\vec{c}+\vec{c}\times\vec{d}+\overrightarrow{dxx\vec{b}}\right|}{\left(\vec{b}-\vec{c}\right)\cdot\vec{d}-\vec{c}}=\cdot$$

Watch Video Solution

207. In a $\triangle ABC$points D,E,F are taken on the sides BC,CA and AB respectively such that $\frac{BD}{DC} = \frac{CE}{EA} = \frac{AF}{FB} = n$ prove that $\triangle DEF = \frac{n^2 - n + 1}{(n+1)^2}$ /_\ABC`

Watch Video Solution

208. Let the area of a given triangle ABC be Δ. Points $A_1, B_1,$ and C_1 , are the mid points of the sides BC,CA and AB respectively. Point A_2 is the mid point of CA_1. Lines C_1A_1 and AA_2 meet the median BB_2 points E and D respectively. If Δ_1 be the area of the quadrilateral A_1A_2DE, using vectors or otherwise find the value of $\frac{\Delta_1}{\Delta}$

Watch Video Solution

209. If $\vec{a}, \vec{b}, and \vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}, \vec{b} \times \vec{c} = \vec{a}, \vec{c} \times \vec{a} = \vec{b},$ then prove that $|\vec{a}| = |\vec{b}| = |\vec{c}|$

Watch Video Solution

210. If a, b, c and d are four coplanr points, then prove that

$[abc] = [bcd] + [abd] + [cad]$.

Watch Video Solution

211. Let vecu and vecv$beunit \vec{\to} rs. If$vecw$isa \vec{\to} rsucht\hat{}$

vecw+vecwxxvecu=vecv, $thenprovet\hat{}$ |(vecuxxvecv).vecw|le1/2

and $\hat{t}theequalityholds$ if and $only$ if vecu$isperpendic\underline{a}r \to$ vecv`.

Watch Video Solution

212. Prove that

$$\vec{R} + \frac{\left[\vec{R}\vec{\beta} \times \left(\vec{\beta} \times \vec{\alpha}\right)\right]\vec{\alpha}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} + \frac{\left[\vec{R}\vec{\alpha} \times \left(\vec{\alpha} \times \vec{\beta}\right)\right]\vec{\beta}}{\left|\vec{\alpha} \times \vec{\beta}\right|^2} = \frac{\left[\vec{R}\vec{\alpha}\vec{\beta}\right]\left(\vec{\alpha} \times \vec{\beta}\right)}{\left|\vec{\alpha} \times \vec{\beta}\right|^2}$$

Watch Video Solution

213. Prove that the formula for the volume V of a tetrahedron, in terms of the lengths of three coterminous edges and their mutul inclinations is

$$V^2 = \frac{a^2b^2c^2}{36}\begin{vmatrix} 1 & \cos\phi & \cos\psi \\ \cos\phi & 1 & \cos\theta \\ \cos\psi & \cos\theta & 1 \end{vmatrix}$$

Watch Video Solution

214. A pyramid with vertex at point P has a regular hexagonal base ABCDEF. Position vectors of points A and B are $\hat{i}$ and $\hat{i} + 2\hat{j}$, respectively. The centre of the base has the position vector $\hat{i} + \hat{j} + \sqrt{3}\hat{k}$.

Altitude drawn from P on the base meets the diagonal AD at point G. Find all possible vectors of G. It is given that the volume of the pyramid is $6\sqrt{3}$ cubic units and AP is 5 units.

Watch Video Solution

215. Let $\hat{a}, \hat{b}$ and $\hat{c}$ be the non-coplanar unit vectors. The angle between $\hat{b}$ and $\hat{c}$ be α and angled between $\hat{c}$ and $\hat{a}$ be β and between $\hat{a}$ and $\hat{b}$ be γ. If $A\left(\hat{a}\cos\alpha, 0\right), B\left(\hat{b}\cos\beta, 0\right)$ and $C\left(\hat{c}\cos\gamma, 0\right)$, then show that in $\triangle ABC$.

$$\frac{\left|\hat{a} \times \left(\hat{b} \times \hat{c}\right)\right|}{\sin A} = \frac{\left|\hat{b} \times \left(\hat{c} \times \hat{a}\right)\right|}{\sin B} = \frac{\left|\hat{c} \times \left(\hat{a} \times \hat{b}\right)\right|}{\sin C}$$

Watch Video Solution

216. Let a and b be given non-zero and non-collinear vectors, such that $c \times a = b - c$. Express c in terms for a, b and aXb.

Watch Video Solution

Exercise For Session 1

1. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k} and 3\hat{i} - 2\hat{j} + \hat{k}$

Watch Video Solution

2. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ nd 2 respectively such that $\vec{a}.\vec{b} = \sqrt{6}$

Watch Video Solution

3. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 3\hat{j} - 5\hat{k}$ are at right angles.

Watch Video Solution

4. If $\vec{r}\dot{\hat{i}} = \vec{r}\dot{\hat{j}} = \vec{r}\dot{\hat{k}} and \left|\vec{r}\right| = 3,$ then find the vector $\dot{\vec{r}}$

Watch Video Solution

5. Find the anlge between the vectors a+b and a-b, if $a = 2\hat{i} - \hat{j} + 3\hat{k}$ and $b = 3\hat{i} + \hat{j} - 2\hat{k}$.

Watch Video Solution

6. Find the angle between the vectors $\hat{i}+3\hat{j}+7\hat{k}$ and $7\hat{i}-\hat{j}+8\hat{k}$.

Watch Video Solution

7. If the scalar projection of vector $x\hat{i}-\hat{j}+\hat{k}$ on vector $2\hat{i}-\hat{j}+5\hat{k} is \frac{1}{\sqrt{30}}$,

then find the value of $x^{\cdot}$

Watch Video Solution

8. If $\left|\vec{a}\right|+\left|\vec{b}\right|=\left|\vec{c}\right| and \vec{a}+\vec{b}=\vec{c}$, then find the angle between $\vec{a} and \vec{b}^{\cdot}$

Watch Video Solution

9. If three unit vectors $\vec{a}, \vec{b}, and \vec{c}$ satisfy $\vec{a}+\vec{b}+\vec{c}=0$, then find the angle between $\vec{a} and \vec{b}^{\cdot}$

Watch Video Solution

10. If $\vec{a} = x\hat{i} + (x - 1)\hat{j} + \hat{k} and \vec{b} = (x + 1)\hat{i} + \hat{j} + a\hat{k}$ make an acute angle $\forall x \in R$, then find the values of a

Watch Video Solution

11. Find the component of $\hat{i}$ in the direction of vector $\hat{i} + \hat{j} + 2\hat{k}$.

Watch Video Solution

12. Find the vector component of a vector $2\hat{i} + 3\hat{j} + 6\hat{k}$ along and perpendicular to the non-zero vector $2\hat{i} + \hat{j} + 2\hat{k}$.

Watch Video Solution

13. A particle acted by constant forces $4\hat{i} + \hat{j} - 3\hat{k} and 3\hat{i} + \hat{9} - \hat{k}$ is displaced from point $\hat{i} + 2\hat{j} + 3\hat{k}$ to point $5\hat{i} + 4\hat{j} + \hat{k}$ find the total work done by the forces in units.

Watch Video Solution

Exercise For Session 2

1. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k} and \vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}^{\cdot}$

Watch Video Solution

2. Find the values of γ and μ for which $\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) \times \left(\hat{i} + \gamma\hat{j} + \mu\hat{k}\right) = \vec{0}$

Watch Video Solution

3. If $a = 2\hat{i} + 3\hat{j} - \hat{k}, b = -\hat{i} + 2\hat{j} - 4\hat{k}, c = \hat{i} + \hat{j} + \hat{k}$, then find the value of $(a \times b) \cdot (a \times c)$.

Watch Video Solution

4. Prove that $\left(\vec{a}\dot{\hat{i}}\right)\left(\vec{a}\times\hat{i}\right)+\left(\vec{a}\dot{j}\right)\left(\vec{a}\times\hat{j}\right)+\left(\vec{a}\dot{\hat{k}}\right)\left(\vec{a}\times\hat{k}\right)=0.$

Watch Video Solution

5. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ then show that $\vec{a}$ - $\vec{d}$ is parallel to $\vec{b}$ - $\vec{c}$

Watch Video Solution

6. If $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\dot{\vec{b}}\right)^2=144and\left|\vec{a}\right|=4,$ then find the value of $\left|\vec{b}\right|^{\cdot}$

Watch Video Solution

7. If $|a|=2, |b|=7$ and $\vec{a}x\vec{b}=3\hat{i}+2\hat{j}+6\hat{k},$ find the angle between $\vec{a}$ and $\vec{b}$

.

Watch Video Solution

8. Let the vectors $\vec{a} and \vec{b}$ be such that $|\vec{a}| = 3|\vec{b}| = \frac{\sqrt{2}}{3}, then \vec{a} \times \vec{b}$ is a unit vector, if the angel between $\vec{a} and \vec{b}$ is?

Watch Video Solution

9. If $|\vec{a}| = \sqrt{26}, |\vec{b}| = 7 \text{ and } |\vec{a} \times \vec{b}| = 35, f \in d\vec{a}.\vec{b}$

Watch Video Solution

10. Find a unit vector perpendicular to the plane of two vectors $a = \hat{i} - \hat{j} + 2\hat{k} \text{ and } b = 2\hat{i} + 3\hat{j} - \hat{k}.$

Watch Video Solution

11. Find a vector of magnitude 15, which is perpendicular to both the vectors $\left(4\hat{i} - \hat{j} + 8\hat{k}\right)$ and $\left(-\hat{j} + \hat{k}\right)$.

Watch Video Solution

12. Let $\rightarrow a = \hat{i} + 4\hat{j} + 2\hat{k}$, $\rightarrow b = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\rightarrow c = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\rightarrow d$ which is perpendicular to both $\rightarrow a$ and $\rightarrow b$ and $\rightarrow c$. $\rightarrow d = 15$.

Watch Video Solution

13. Let A,B and C be the unit vectors . Suppose that A.B=A.C =0 and the angle between B and C is $\frac{\pi}{6}$ then prove that $A = \pm 2(B \times C)$

Watch Video Solution

14. Find the area of the triangle whose adjacent sides are determined by the vectors $\vec{a} = \left(-2\hat{i} - 5\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 2\hat{j} - \hat{k}\right)$.

Watch Video Solution

15. Find the area of parallelogram whose adjacent sides are represented by the vectors $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 2\hat{j} - \hat{k}$.

Watch Video Solution

16. What is the area of the parallelogram having diagonals $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$?

Watch Video Solution

17. A force $F = 2\hat{i} + \hat{j} - \hat{k}$ acts at point A whose position vector is $2\hat{i} - \hat{j}$. Find the moment of force F about the origin.

Watch Video Solution

18. Find the moment of $\vec{F}$ about point (2, -1, 3), where force $\vec{F} = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is acting on point (1, -1, 2).

Watch Video Solution

19. Forces $2\hat{i} + \hat{j}, 2\hat{i} - 3\hat{j} + 6\hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$ act at a point P, with position vector $4\hat{i} - 3\hat{j} - \hat{k}$. Find the moment of the resultant of these force about the point Q whose position vector is $6\hat{i} + \hat{j} - 3\hat{k}$.

Watch Video Solution

Exercise For Session 3

1. If $\vec{a} and \vec{b}$ are two vectors such that $\left|\vec{a} \times \vec{b}\right| = 2$, then find the value of $\left[\vec{a}\vec{b}\vec{a} \times \vec{b}\right]^{.}$

Watch Video Solution

2. If the vectors $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k} and 3\hat{i} - \hat{k}$ form three concurrent edges of a parallelepiped, then find the volume of the parallelepiped.

Watch Video Solution

3. If the volume of a parallelepiped whose adjacent edges are $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \vec{b} = \hat{i} + \alpha\hat{j} + 2\hat{k}, \vec{c} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ is 15, then find the value of α if $(\alpha > 0)^{\cdot}$

Watch Video Solution

4. The position vectors of the four angular points of a tetrahedron are $A\left(\hat{j} + 2\hat{k}\right), B\left(3\hat{i} + \hat{k}\right), C\left(4\hat{i} + 3\hat{j} + 6\hat{k}\right) and D\left(2\hat{i} + 3\hat{j} + 2\hat{k}\right)^{\cdot}$ Find the volume of the tetrahedron $ABCD^{\cdot}$

Watch Video Solution

5. Find the altitude of a parallelopiped whose three conterminous edges are verctors $A = \hat{i} + \hat{j} + \hat{k}, B = 2\hat{i} + 4\hat{j} - \hat{k}$ and $C = \hat{i} + \hat{j} + 3\hat{k}$ with A and B as the sides of the base of the parallelopiped.

Watch Video Solution

6. Examine whether the vectors $a = 2\hat{i} + 3\hat{j} + 2\hat{k}, b = \hat{i} - \hat{j} + 2\hat{k}$ and $c = 4\hat{i} + 2\hat{j} + 4\hat{k}$ form a left handed or a right handed system.

Watch Video Solution

7. Show that the vectors $\hat{i} - \hat{j} - 6\hat{k}, \hat{i} - 3\hat{j} + 4\hat{k}$ and $2\hat{i} - 5\hat{j} + 3\hat{k}$ are coplanar.

Watch Video Solution

8. Prove that $[abc][uvw] = \begin{vmatrix} a \cdot u & b \cdot u & c \cdot u \\ a \cdot v & b \cdot v & c \cdot v \\ a \cdot w & b \cdot w & c \cdot w \end{vmatrix}$

Watch Video Solution

9. If $[abc] = 2$, then find the value of $[(a + 2b - c)(a - b)(a - b - c)]$.

Watch Video Solution

10. If $\vec{a}, \vec{b}, and \vec{c}$ are three non-coplanar vectors, then find the value of

$$\frac{\vec{a} \overset{.}{\vec{b}} \times \vec{c}}{\vec{b} \overset{.}{\vec{c}} \times \vec{a}} + \frac{\vec{b} \overset{.}{\vec{c}} \times \vec{a}}{\vec{\because} (\vec{a} \times \vec{b})} + \frac{\vec{\because} (\vec{b} \times \vec{a})}{\vec{a} \overset{.}{\vec{b}} \times \vec{c}}.$$

Watch Video Solution

Exercise For Session 4

1. Find the value of $\alpha \times (\beta \times \gamma)$, where $\alpha = 2\hat{i} - 10\hat{j} + 2\hat{k}, \beta = 3\hat{i} + \hat{j} + 2\hat{k}, \gamma = 2\hat{i} + \hat{j} + 3\hat{k}$.

Watch Video Solution

2. Find the vector of length 3 unit which is perpendicular to $\hat{i} + \hat{j} + \hat{k}$ and lies in the plane of $\hat{i} + \hat{j} + \hat{k} and 2\hat{k} - 3\hat{j}$.

Watch Video Solution

3. Show that $(b \times c) \cdot (a \times d) + (a \times b) \cdot (c \times d) + (c \times a) \cdot (b \times d) = 0$

Watch Video Solution

4. Prove that $\hat{i} \times \left(\vec{a} \times \hat{i}\right)\hat{j} \times \left(\vec{a} \times \hat{j}\right) + \hat{k} \times \left(\vec{a} \times \hat{k}\right) = 2\vec{a}$

Watch Video Solution

5. Prove that $[a \times b, a \times c, d] = (a \cdot d)[a, b, c]$

Watch Video Solution

6. If $\vec{a}, \vec{b}, and \vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \dfrac{\vec{b} + \vec{c}}{\sqrt{2}}, \vec{b} and \vec{c}$ are non-parallel, then prove that the angel between $\vec{a} and \vec{b} is 3\pi/4.$

Watch Video Solution

7. Find a set of vectors reciprocal to the set $\hat{i} + \hat{j} + \hat{k}, \hat{i} - \hat{j} + \hat{k}, \hat{i} + j + \dot{\hat{k}}$

Watch Video Solution

8. If a, b, c and a', b', c' are recoprocal system of vectors, then prove that $a' \times b' + b' \times c' + c' \times a' = \dfrac{a + b + c}{[abc]}.$

Watch Video Solution

9. Solve: $\vec{r} \times \vec{b} = \vec{a}, where \vec{a}$ and $\vec{b}$ are given vectors such that $\vec{a}. \vec{b} = 0.$

Watch Video Solution

10. Find vector $\vec{r}$ if $\vec{r}. \vec{a} = m$ and $\vec{r} \times \vec{b} = \vec{c}, where \vec{a}. \vec{b} \neq 0$

Watch Video Solution

Exercise (Single Option Correct Type Questions)

1. If a has magnitude 5 and points North-East and vector b has magnitude 5 and point North-West, then |a-b| is equal to

A. 25

B. 5

C. $7\sqrt{3}$

D. $5\sqrt{2}$

Answer: D

Watch Video Solution

2. If $|a + b| > |a - b|$, then the angle between a and b is

A. acute

B. obtuse

C. $\frac{\pi}{2}$

D. π

Answer: A

Watch Video Solution

3. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} = \vec{b} + \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{2}$, then

A. $a^2 = b^2 + c^2$

B. $b^2 = a^2 + c^2$

C. $c^2 = a^2 + b^2$

D. $2a^2 - b^2 = c^2$

Answer: A

Watch Video Solution

4. If the angle between the vectors a and b be θ and $a \cdot b = \cos\theta$ then the true statement is

A. a and b are equal vectors

B. a and b are like vectors

C. a and b are unlike vectors

D. a and b are unit vectors

Answer: D

Watch Video Solution

5. If the vectors $\hat{i} + \hat{j} + \hat{k}$ makes angle α, β and γ with vectors $\hat{i}, \hat{j}$ and $\hat{k}$ respectively, then

A. $\alpha = \beta \neq \gamma$

B. $\alpha = \gamma \neq \beta$

C. $\beta = \gamma \neq \alpha$

D. $\alpha = \beta = \gamma$

Answer: D

Watch Video Solution

6. $\left(r\cdot\hat{i}\right)^2+\left(r\cdot\hat{j}\right)^2+\left(r\cdot\hat{k}\right)^2$ is equal to

A. $3r^2$

B. r^2

C. 0

D. None of these

Answer: B

Watch Video Solution

7. If $\hat{a}$ and $\hat{b}$ are two unit vectors inclined at an angle θ, then $\sin\left(\frac{\theta}{2}\right)$

A. $\frac{1}{2}|a-b|$

B. $\frac{1}{2}|a+b|$

C. $|a-b|$

D. $|a+b|$

Answer: A

Watch Video Solution

8. If $\vec{a} = 4\hat{i} + 6\hat{j} and \vec{b} = 3\hat{j} + 4\hat{k},$ then find the component of $\vec{a} and \vec{b}$

A. $\dfrac{18}{10\sqrt{3}}\left(3\hat{j} + 4\hat{k}\right)$

B. $\dfrac{18}{25}\left(3\hat{j} + 4\hat{k}\right)$

C. $\dfrac{18}{\sqrt{3}}\left(3\hat{j} + 4\hat{k}\right)$

D. $\left(3\hat{j} + 4\hat{k}\right)$

Answer: B

Watch Video Solution

9. If vectors $a = 2\hat{i} - 3\hat{j} + 6\hat{k}$ and vector $b = -2\hat{i} + 2\hat{j} - \hat{k}$, then (projection of vector a on b vectors)/(projection of vector b on a vector) is equal to

A. $\frac{3}{7}$

B. $\frac{7}{3}$

C. 3

D. 7

Answer: B

Watch Video Solution

10. If a and b are two vectors , then $(a \times b)^2$ is equal to

A. $\begin{vmatrix} a \cdot b & a \cdot a \\ b \cdot b & b \cdot a \end{vmatrix}$

B. $\begin{vmatrix} a \cdot a & a \cdot b \\ b \cdot a & b \cdot b \end{vmatrix}$

C. $\begin{vmatrix} a \cdot b \\ b \cdot a \end{vmatrix}$

D. None of these

Answer: b

Watch Video Solution

11. The moment of the force F acting at a point P, about the point C is

A. $F \times CP$

B. $CP \cdot F$

C. a vector having the same direction as F

D. $CP \times F$

Answer: D

Watch Video Solution

12. The moment of a force represented by $F = \hat{i} + 2\hat{j} + 3\hat{k}$ about the point $2\hat{i} - \hat{j} + \hat{k}$ is equal to

A. $5\hat{i} - 5\hat{j} + 5\hat{k}$

B. $5\hat{i} + 5\hat{j} - 6\hat{k}$

C. $-5\hat{i} - 5\hat{j} + 5\hat{k}$

D. $-5\hat{i} - 5\hat{j} + 2\hat{k}$

Answer: D

Watch Video Solution

13. A force of magnitude 6 acts along the vector $(9, 6, -2)$ and passes through a point $A(4, -1, -7)$. Then moment of force about the point $O(1, -3, 2)$ is

A. $\frac{150}{11}\left(2\hat{i} - 3\hat{j}\right)$

B. $\frac{6}{11}\left(50\hat{i} - 75\hat{j} + 36\hat{k}\right)$

C. $150\left(2\hat{i} - 3\hat{k}\right)$

D. $6\left(50\hat{i} - 75\hat{j} + 36\hat{k}\right)$

Answer: A

Watch Video Solution

14. A force $F = 2\hat{i} + \hat{j} - \hat{k}$ acts at point A whose position vector is $2\hat{i} - \hat{j}$. Find the moment of force F about the origin.

A. $\hat{i} + 2\hat{j} - 4\hat{k}$

B. $\hat{i} - 2\hat{j} - 4\hat{k}$

C. $\hat{i} + 2\hat{j} + 4\hat{k}$

D. $\hat{i} - 2\hat{j} + 4\hat{k}$

Answer: C

Watch Video Solution

15. If a, b and c are any three vectors and their inverse are a^{-1}, b^{-1} and c^{-1} and $[abc] \neq 0$, then $\left[a^{-1}b^{-1}c^{-1}\right]$ will be

A. zero

B. one

C. non-zero

D. [a b c]

Answer: C

Watch Video Solution

16. If a, b and c are three non-coplanar vectors, then find the value of $\dfrac{a \cdot (b \times c)}{c. (a \times b)} + \dfrac{b \cdot (c \times a)}{c \cdot (a \times b)}$.

A. 0

B. 2

C. -2

D. None of these

Answer: A

Watch Video Solution

17. $a \times (b \times c)$ is coplanar with

A. b and c

B. a and c

C. a and b are unlike vectors

D. a, b and c

Answer: A

Watch Video Solution

18. If $u = \hat{i}\left(a \times \hat{i}\right) + \hat{j}\left(a \times \hat{j}\right) + \hat{k}\left(a \times \hat{k}\right)$, then

A. $u = 0$

B. $u = \hat{i} + \hat{j} + \hat{k}$

C. $u = 2a$

D. $u = a$

Answer: a

Watch Video Solution

19. If $a = \hat{i} + 2\hat{j} - 2\hat{k}, b = 2\hat{i} - \hat{j} + \hat{k}$ and $c = \hat{i} + 3\hat{j} - \hat{k},$ then $a \times (b \times c)$ is equal to

A. $20\hat{i} - 3\hat{j} + 7\hat{k}$

B. $20\hat{i} - 3\hat{j} - 7\hat{k}$

C. $20\hat{i} + 3\hat{j} - 7\hat{k}$

D. None of these

Answer: A

Watch Video Solution

20. If $a \times (b \times c) = 0$, then

A. $|a| = |b| \cdot |c| = 1$

B. $b \mid \mid c$

C. $a \mid \mid b$

D. $b\dot{c}$

Answer: B

Watch Video Solution

21. A vectors which makes equal angles with the vectors $\frac{1}{3}\left(\hat{i} - 2\hat{j} + 2\hat{k}\right), \frac{1}{5}\left(- 4\hat{i} - 3\hat{k}\right), \hat{j}$ is:

A. $5\hat{i} + 5\hat{j} + \hat{k}$

B. $5\hat{i} + \hat{j} - 5\hat{k}$

C. $5\hat{i} + \hat{j} + 5\hat{k}$

D. $\pm\left(5\hat{i} - \hat{j} - 5\hat{k}\right)$

Answer: D

Watch Video Solution

22. [Find by vector method the horizontal force and the force inclined at an angle of 60° to the vertical whose resultant is a vertical force P.]

A. $P, 2P$

B. $P, P\sqrt{3}$

C. $2P, P\sqrt{3}$

D. None of these

Answer: D

Watch Video Solution

23. If $x + y + z = 0, |x| = |y| = |z| = 2$ and θ is angle between y and z, then the value of $\cos ec^2\theta + \cot^2\theta$ is equal to

A. $\frac{4}{3}$

B. $\frac{5}{3}$

C. $\frac{1}{3}$

D. 1

Answer: B

Watch Video Solution

24. The values of x for which the angle between the vectors $\vec{a} = x\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute and the angle between b and y-axis lies between $\frac{\pi}{2}$ and π are:

A. $x > 0$

B. $x < 0$

C. $x > 1 only$

D. $x < -1$ only

Answer: B

Watch Video Solution

25. If a, b and c are non-coplanar vectors and $d = \lambda a + \mu b + vc$, then λ is equal to

A. $\dfrac{[dbc]}{[bac]}$

B. $\dfrac{[bcd]}{[bca]}$

C. $\dfrac{[bdc]}{[abc]}$

D. $\dfrac{[cbd]}{[abc]}$

Answer: B

Watch Video Solution

26. If the vectors $3\vec{p} + \vec{q}; 5\vec{p}$ - $3\vec{q}$ and $2\vec{p} + \vec{q}; 4\vec{p}$ - $2\vec{q}$ are pairs of mutually perpendicular then sin($\vec{p}$ ^ $\vec{q}$) is :

A. $\dfrac{\sqrt{55}}{4}$

B. $\frac{\sqrt{55}}{8}$

C. $\frac{3}{16}$

D. $\frac{\sqrt{247}}{16}$

Answer: B

Watch Video Solution

27. Let $\vec{u} = \hat{i} + \hat{j}, \vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$.If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n}$ then $\left|\vec{w} \cdot \hat{n}\right|$ is equal to

A. 1

B. 2

C. 3

D. 0

Answer: C

Watch Video Solution

28. Given a parallelogram $ABCD$. If $\left|\overrightarrow{AB}\right| = a, \left|\overrightarrow{AD}\right| = b \& \left|\overrightarrow{AC}\right| = c$, then $\overrightarrow{DB}.\overrightarrow{AB}$ has the value

A. $\dfrac{3a^2 + b^2 - c^2}{2}$

B. $\dfrac{a^2 + 3b^2 - c^2}{2}$

C. $\dfrac{a^2 - b^2 + 3c^2}{2}$

D. None of these

Answer: A

Watch Video Solution

29. For two particular vectors $\vec{A}$ and $\vec{B}$ it is known that $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$.

What must be true about the two vectors?

A. Atleast one of the two vectors must be the zero vector

B. $A \times B = B \times A$ is true for any two vectors

C. One of the two vectors is a scalar multiple of the other vector

D. The two vectors must be perpendicular to each other

Answer: C

Watch Video Solution

30. For some non zero vector $\bar{V}$, if the sum of $\bar{V}$ and the vector obtained from $\bar{V}$ by rotating it by anangle 2α equals to the vector obtained from $\bar{V}$ by rotating it by α then the value of α, is

A. $2n\pi \pm \frac{\pi}{3}$

B. $n\pi \pm \frac{\pi}{3}$

C. $2n\pi \pm \frac{2\pi}{3}$

D. $n\pi \pm \frac{2\pi}{3}$

Answer: A

Watch Video Solution

31. In isosceles triangles ABC, $\left|\vec{AB}\right| = \left|\vec{BC}\right| = 8$, a point E divides AB internally in the ratio 1:3, then find the angle between $\vec{C}Eand\vec{C}A\left(where\left|\vec{C}A\right| = 12\right)^{.}$

A. $\frac{-3\sqrt{7}}{8}$

B. $\frac{3\sqrt{8}}{17}$

C. $\frac{3\sqrt{7}}{8}$

D. $\frac{-3\sqrt{8}}{17}$

Answer: C

Watch Video Solution

32. Given an equilateral triangle ABC with side length equal to 'a'. Let M and N be two points respectivelyABIn the side AB and AC such that

$\overrightarrow{AN} = K\overrightarrow{AC}$ and $\overrightarrow{AM} = \frac{\overrightarrow{AB}}{3}$ If $\overrightarrow{BN}$ and $\overrightarrow{CM}$ are orthogonalthen the value of K is equal to

A. $\frac{1}{5}$

B. $\frac{1}{4}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer: A

Watch Video Solution

33. In a quadrilateral ABCD, AC is the bisector of the (AB, AD) which is $\frac{2\pi}{3}$, $15|AC| = 3|AB| = 5|AD|$, then $\cos(BA, CD)$ is equal to

A. $\frac{-\sqrt{14}}{7\sqrt{2}}$

B. $-\frac{\sqrt{21}}{7\sqrt{3}}$

C. $\frac{2}{\sqrt{7}}$

D. $\frac{2\sqrt{7}}{14}$

Answer: C

Watch Video Solution

34. If the distance from the point P(1, 1, 1) to the line passing through the points $Q(0, 6, 8)$ and $R(-1, 4, 7)$ is expressed in the form $\sqrt{\frac{p}{q}}$, where p and q are co-prime, then the value of $\frac{(q+p)(p+q-1)}{2}$ is equal to

A. 4950

B. 5050

C. 5150

D. None of these

Answer: A

Watch Video Solution

35. Given the vectors $\vec{u} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{v} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{w} = \hat{v} - \hat{k}$ If the volume of the parallelopiped having $-c\vec{u}, \vec{v}$ and $c\vec{w}$ as concurrent edges, is 8 then 'c' canbe equal to

A. ± 2

B. 4

C. 8

D. cannot be determine

Answer: A

Watch Video Solution

36. Vector $\vec{c}$ is perpendicular to vectors $\vec{a} = (2, -3, 1) and \vec{b} = (1, -2, 3)$ and satisfies the condition $\vec{\cdot}\left(\hat{i} + 2\hat{j} - 7\hat{k}\right) = 10.$ Then vector $\vec{c}$ is equal to $(7, 5, 1)$ b. $-7, -5, -1$ c. $1, 1, -1$ d. none of these

A. $(7, 5, 1)$

B. $(-7, , -5, -1)$

C. $(1, 1, -1)$

D. None of these

Answer: B

Watch Video Solution

37. Let $\vec{a} = \hat{j} + \hat{j}, \vec{b} = \hat{j} + \hat{k}$ and $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. If the vectors, $\hat{i} - 2\hat{j} + \hat{k}, 3\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c}$ are coplanar then $\frac{\alpha}{\beta}$ is

A. 1

B. 2

C. 3

D. -3

Answer: D

Watch Video Solution

38. A rigid body rotates about an axis through the origin with an angular velocity $10\sqrt{3}$rad/s. If ω points in the direction of $\hat{i}+\hat{j}+\hat{k}$, then the equation to the locus of the points having tangential speed 20m/s.

A. $x^2 + y^2 + z^2 - xy - yz - xz - 1 = 0$

B. $x^2 + y^2 + z^2 - 2xy - 2yz - 2xz - 1 = 0$

C. $x^2 + y^2 + z^2 - xy - yz - xz - 2 = 0$

D. $x^2 + y^2 + z^2 - 2xy - 2yz - 2xz - 2 = 0$

Answer: C

Watch Video Solution

39. A rigid body rotates with constant angular velocity $omaga$ about the line whose vector equation is, $r = \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$. The speed of the particle

at the instant it passes through the point with position vector $\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$ is equal to

A. $\omega\sqrt{2}$

B. 2ω

C. $\dfrac{\omega}{\sqrt{2}}$

D. None of these

Answer: A

Watch Video Solution

40. Consider ΔABC with $A = (\vec{a}); B = (\vec{b})$ and $C = (\vec{c})$. If $\vec{b}.(\vec{a} + \vec{c}) = \vec{b}.\vec{b} + \vec{a}.\vec{c}; \left|\vec{b} - \vec{a}\right| = 3; \left|\vec{c} - \vec{b}\right| = 4$ then the angle between the medians $\vec{AM}$ and $\vec{BD}$ is

A. $\pi - \cos^{-1}\left(\dfrac{1}{5\sqrt{13}}\right)$

B. $\pi - \cos^{-1}\left(\dfrac{1}{13\sqrt{5}}\right)$

C. $\cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$

D. $\cos^{-1}\left(\frac{1}{13\sqrt{5}}\right)$

Answer: A

Watch Video Solution

41. Given unit vectors m, n and p such that angle between m and n. Angle between p and $(m \times n) = \frac{\pi}{6}$, then [n p m] is equal to

A. $\frac{\sqrt{3}}{4}$

B. $\frac{3}{4}$

C. $\frac{1}{4}$

D. None of these

Answer: A

Watch Video Solution

42. If $\vec{a}$ and $\vec{b}$ are two unit vectors, then the vector $\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)$ is parallel to the vector

A. $a+b$

B. b - a

C. $2a$ - b

D. `a+2b

Answer: B

Watch Video Solution

43. If $\vec{a}$ and $\vec{b}$ are othogonal unit vectors, then for a vector $\vec{r}$ non - coplanar with $\vec{a}$ and $\vec{b}$ vector $\vec{r}\times\vec{a}$ is equal to

A. $\left[r\hat{a}\hat{b}\right]\left(\hat{a}+\hat{b}\right]$

B. $\left[r\hat{a}\hat{b}\right]\hat{a}+\left(r\cdot\hat{a}\right)\left(\hat{a}\times\hat{b}\right)$

C. $\left[r\hat{a}\hat{b}\right]\hat{b} + \left(r \cdot \hat{b}\right)\left(\hat{a} \times \hat{b}\right)$

D. $\left[r\hat{a}\hat{b}\right]\hat{b} + \left(r \cdot \hat{a}\right)\left(\hat{a} \times \hat{b}\right)$

Answer: C

Watch Video Solution

44. If vector $\vec{i} + 2\vec{j} + 2\vec{k}$ is rotated through an angle of 90°, so as to cross the positivedirection of y-axis, then the vector in the new position is

A. $-\frac{2}{\sqrt{5}}\hat{i} + \sqrt{5}\hat{j} - \frac{4}{\sqrt{5}}\hat{k}$

B. $-\frac{2}{\sqrt{5}}\hat{i} - \sqrt{5}\hat{j} + \frac{4}{\sqrt{5}}\hat{k}$

C. $4\hat{i} - \hat{j} - \hat{k}$

D. None of these

Answer: A

Watch Video Solution

45. 10 different vectors are lying on a plane out of which four are parallel with respect to each other. Probability that three vectors chosen from them will satisfy the equation $\lambda_1 a + \lambda_2 b + \lambda_3 c = 0$, where λ_1, λ_2 and $\lambda_3 \neq\ = 0$ is

A. $\dfrac{.^6C_2 \times .^4C_1}{.^{10}C_3}$

B. $\dfrac{\left(.^6C_3 \times .^4C_1\right) + ,^6C_3}{.^{10}C_3}$

C. $\dfrac{\left(.^6C_3 + \ \times .^4C_1\right) + ,^4C_3}{.^{10}C_3}$

D. $\dfrac{\left(.^6C_3 + .^4C_1\right) + ,^6C_2 \times .^4C_1}{.^{10}C_3}$

Answer: D

Watch Video Solution

46. If $\hat{a}$ is a unit vector and projection of x along $\hat{a}$ is 2 units and $\left(\hat{a} \times x\right) + b = x,$ then x is equal to

A. $\frac{1}{2}\left(\hat{a} - b + \left(\hat{a} \times b\right)\right)$

B. $\frac{1}{2}\left(2\hat{a} + b + \left(\hat{a} \times b\right)\right)$

C. $\left(\hat{a} + \left(\hat{a} \times b\right)\right)$

D. None of these

Answer: B

Watch Video Solution

47. If a, b and c are any three non-zero vectors, then the component of $a \times (b \times c)$ perpendicular to b is

A. $a \times (b \times c) + \frac{(a \times b) \cdot (c \times a)}{|b|^2}b$

B. $a \times (b \times c) + \frac{(a \times c) \cdot (a \times b)}{|b|^2}b$

C. $a \times (b \times c) + \frac{(a \times b) \cdot (b \times a)}{|b|^2}b$

D. $a \times (b \times c) + \frac{(a \times b) \cdot (b \times c)}{|b|^2}b$

Answer: D

Watch Video Solution

48. The position vector of a point P is $r = x\hat{i} + y\hat{j} + \hat{k}z$, where $x, y, z \in N$ and $a = \hat{i} + 2\hat{j} + \hat{k}. If r \cdot a = 20$ and the number of possible of P is 9λ, then the value of λ is

A. 81

B. 49

C. 100

D. 36

Answer: A

Watch Video Solution

49. . Let a, b > 0 and $\vec{\alpha} = \frac{\hat{i}}{a} + 4\frac{\hat{j}}{b} + b\hat{k}$ and $\beta = b\hat{i} + a\hat{j} + \frac{\hat{k}}{b}$ then the maximum value of $\frac{30}{5 + \alpha . \beta}$

A. 1

B. 2

C. 4

D. 8

Answer: A

Watch Video Solution

50. If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors forming a linearly independent system, then $\forall \theta \in R$

$\vec{p} = \vec{a}\cos\theta + \vec{b}\sin\theta + \vec{c}(\cos 2\theta)$

$\vec{q} = \vec{a}\cos\left(\frac{2\pi}{3} + \theta\right) + \vec{b}\sin\left(\frac{2\pi}{3} + \theta\right) + \vec{c}(\cos 2)\left(\frac{2\pi}{3} + \theta\right)$

and $\vec{r} = \vec{a}\cos\left(\theta - \frac{2\pi}{3}\right) + \vec{b}\sin\left(\theta - \frac{2\pi}{3}\right) + \vec{c}\cos 2\left(\theta - \frac{2\pi}{3}\right)$

then $\left[\vec{p}\vec{q}\vec{r}\right]$

A. $[abc]\sin\theta$

B. [a b c]$\cos 2\theta$

C. [a b c]$\cos 3\theta$

D.

Answer: D

Watch Video Solution

51. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' make a right angle withe the side AB then the cosine of the angle α is given by

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: B

Watch Video Solution

52. If in a $\triangle ABC, BC = \frac{e}{|e|} - \frac{f}{|f|}$ and $AC = \frac{2e}{|e|} : |e| \neq |f|,$ then the value of $\cos 2A + \cos 2B + \cos 2C$ must be

A. -1

B. 0

C. 2

D. $\frac{-3}{2}$

Answer: A

Watch Video Solution

53. Let the unit vectors a and b be perpendicular and the unit vector c be inclined at an angle θ to both a and b. If $c = \alpha a + \beta b + \gamma(a \times b)$, then

A. $\alpha = \beta = -\cos\theta, y^2 = \cos 2\theta$

B. $\alpha = \beta = \cos\theta, y^2 = \cos 2\theta$

C. $\alpha = \beta = \cos\theta, y^2 = -\cos 2\theta$

D. $\alpha = \beta = -\cos\theta, y^2 = -\cos 2\theta$

Answer: C

Watch Video Solution

54. In triangle ABC the mid point of the sides AB, BC and AC respectively (l, 0, 0), (0, m, 0) and (0, 0, n). Then, $\dfrac{AB^2 + BC^2 + CA^2}{l^2 + m^+ n^2}$ is equal to

A. 2

B. 4

C. 8

D. 16

Answer: C

Watch Video Solution

55. The angle between the lines whose directionn cosines are given by $2l - m + 2n = 0, lm + mn + nl = 0$ is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

Watch Video Solution

56. A line makes an angle θ both with x-axis and y-axis. A possible range of θ is

A. $\left[0, \frac{\pi}{4}\right]$

B. $\left[0, \frac{\pi}{2}\right]$

C. $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

D. $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$.

Answer: C

Watch Video Solution

57. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the three vectors having magnitudes, 1,5 and 3, respectively, such that the angle between $\vec{a}$ and $\vec{b}$ is θ and $\vec{a} \times \left(\vec{a} \times \vec{b}\right) = \vec{c}$. Then $\tan\theta$ is equal to

A. 0

B. $\frac{2}{3}$

C. $\frac{3}{5}$

D. $\frac{3}{4}$

Answer: D

Watch Video Solution

58. Find the perpendicular distance of a corner of a unit cube from a diagonal not passing through it.

A. $\sqrt{\frac{3}{2}}$

B. $\sqrt{\frac{2}{3}}$

C. $\sqrt{\frac{3}{4}}$

D. $\sqrt{\frac{4}{3}}$

Answer: B

Watch Video Solution

Watch Video Solution

59. If p,q are two-collinear vectors such that $(b - c)p \times q + (c - a)p + (a - b)q = 0$ where a, b, c are lengths of sides of a triangle, then the triangle is

A. right angled

B. obtuse

C. equilateral

D. right angled isosceles triangle

Answer: C

Watch Video Solution

60. Let $a = \hat{i} + \hat{j} + \hat{k}, b = -\hat{i} + \hat{j} + \hat{k}, c = \hat{i} - \hat{j} + \hat{k}$ and $d = \hat{i} + \hat{j} - \hat{k}$. Then, the line of intersection of planes one determined by a, b and other determined by c, d is perpendicular to

A. X-axis

B. Y-axis

C. Both X and Y axes

D. Both y and z-axes

Answer: D

View Text Solution

61. A parallelepiped is formed by planes drawn parallel to coordinate axes through the points A=(1,2,3) and B=(9,8,5). The volume of that parallelepiped is equal to (in cubic units)

A. 192

B. 48

C. 32

D. 96

Answer: D

Watch Video Solution

62. Let $\bar{a}, \bar{b}, \bar{c}$ be three non-coplanar vectors and $\bar{d}$ be a non-zero vector, which is perpendicularto $\bar{a} + \bar{b} + \bar{c}$. Now, if $\bar{d} = (\sin x)\left(\bar{a} \times \bar{b}\right) + (\cos y)\left(\bar{b} \times \bar{c}\right) + 2\left(\bar{c} \times \bar{a}\right)$ then minimum value of $x^2 + y^2$ is equal to

A. π^2

B. $\frac{\pi^2}{2}$

C. $\frac{\pi^2}{4}$

D. $\frac{5\pi^2}{4}$

Answer: D

Watch Video Solution

63. If $\alpha(a \times b) + \beta(b \times c) + \gamma(c \times a) = 0$, then

A. a, b, c are coplanar if all of $\alpha, \beta, \gamma \neq 0$

B. a, b, c are non-coplanar if any one $\alpha, \beta\gamma = 0$

C. a, b, c are non-coplanar for any α, β, γ.

D. None of these

Answer: A

Watch Video Solution

64. Let area of faces $\triangle OAB = \lambda_1$, $\triangle OAC = \lambda_2$, $\triangle OBC = \lambda_3$, $\triangle ABC = \lambda_4$ and h_1, h_2, h_3, h_4 be perpendicular height from 0 to face $\triangle ABC$, A to the face $\triangle OBC$, B to the face $\triangle OAC$, C to the face $\triangle OAB$, then the face $\frac{1}{3}\lambda_1 h_4 \cdot \frac{1}{3}\lambda_2 h_3 + \frac{1}{3}\lambda_3 h_2 + \frac{1}{3}\lambda_4 h_1$

A. $\frac{2}{3}$|[AB AC OA]|

B. $\frac{1}{3}|[AB\ AC\ OA]|$

C. $\frac{2}{3}|[OA\ OB\ OC]|$

D.

Answer: A

Watch Video Solution

65. Given four non zero vectors $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$. The vectors $\bar{a}, \bar{b}$ and $\bar{c}$ are coplanar but not collinear pair by pairand vector $\bar{d}$ is not coplanar with vectors $\bar{a}, \bar{b}$ and $\bar{c}$ and $\widehat{\bar{a}\bar{b}} = \widehat{\bar{b}\bar{c}} = \frac{\pi}{3}, \left(\bar{d}\bar{b}\right) = \beta$,If $\left(\bar{d}\bar{c}\right) = \cos^{-1}(m\cos\beta + n\cos\alpha)$ then $m - n$ is :

A. $\cos^{-1}(\cos\beta - \cos\alpha)$

B. $\sin^{-1}(\cos\beta - \cos\alpha)$

C. $\sin^{-1}(\sin\beta - \sin\alpha)$

D. $\cos^{-1}(\tan\beta - \tan\alpha)$

Answer: A

Watch Video Solution

66. The shortest distance between a diagonal of a unit cube and the edge skew to it, is

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{3}}$

D. $\frac{1}{\sqrt{6}}$

Answer: A

Watch Video Solution

67. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k} and \vec{W} = \hat{i} + 3\hat{k}$ If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is -1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

A. -1

B. $\sqrt{35}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: B

Watch Video Solution

68. The length of the edge of the regular tetradedron ABCD is 'a'. Points E and F are taken on the edges AD and BD respectively such that 'E' divides DA and 'F' divides BD in the ratio of 2:1 each. Then, area of $\triangle CEF$ is

A. $\dfrac{5a}{12\sqrt{3}}$ sq. units

B. $\dfrac{a}{12\sqrt{3}}$ sq. units

C. $\dfrac{a^2}{12\sqrt{3}}$ sq. unit

D. $\dfrac{5a^2}{12\sqrt{3}}$ sq. units

Answer: D

View Text Solution

69. If the two adjacent sides of two rectangles are represented by vectors $\vec{p} = 5\vec{a} - 3\vec{b}; \vec{q} = -\vec{a} - 2\vec{b} and \vec{r} = -4\vec{a} - \vec{b}; \vec{s} = -\vec{a} + \vec{b},$ respectively, then the angel between the vector $\vec{x} = \frac{1}{3}(\vec{p} + \vec{r} + \vec{s}) and \vec{y} = \frac{1}{5}(\vec{r} + \vec{s})$ is $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ b. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ c. $\pi\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$ d. cannot be evaluate

A. $\pi - \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

B. $\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

C. $-\cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$

D. $\pi - \cos^{-1}\left(\frac{19}{\sqrt{43}}\right)$

Answer: B

Watch Video Solution

70. Let $\vec{a}, \vec{b}, \vec{c}$ are three vectors along the adjacent edges ofa tetrahedron, if $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right| = 2$ and $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 2$ then volume of tetrahedron is (A) $\frac{1}{\sqrt{2}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $2\frac{\sqrt{2}}{3}$

A. $\frac{1}{\sqrt{2}}$

B. $\frac{2}{\sqrt{3}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2\sqrt{2}}{3}$

Answer: D

Watch Video Solution

71. The angle θ between two non-zero vectors a and b satisfies the relation $\cos\theta = \left(a \times \hat{i}\right) \cdot \left(b \times \hat{i}\right) + \left(a \times \hat{j}\right) \cdot \left(b \times \hat{j}\right) + \left(a \times \hat{k}\right) \cdot \left(b \times \hat{k}\right)$, then the least value of $|a| + |b|$ is equal to

A. $\frac{1}{2}$

B. 2

C. $\sqrt{2}$

D. 4

Answer: C

View Text Solution

72. If the angle between the vectors $\vec{a} = \hat{i} + (\cos x)\hat{j} + \hat{k}$ and

$\vec{b} = \left(\sin^2 x - \sin x\right)\hat{i} - (\cos x)\hat{j} + (3 - 4\sin x)\hat{k}$

is obutse and x in $\left(0, \frac{\pi}{2}\right)$, then the exhaustive set of values of 'x' is equal to-

A. $x \in \left(0, \frac{\pi}{6}\right)$

B. $x \in \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

C. $x \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$

D. $x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

Answer: B

Watch Video Solution

73. If position vectors of the points A, B and C are a, b and c respectively and the points D and E divides line segment AC and AB in the ratio 2:1 and 1:3, respectively. Then, the points of intersection of BD and EC divides EC in the ratio

A. $2:1$

B. $1:3$

C. $1:2$

D. $3:2$

Answer: D

Watch Video Solution

Exercise (More Than One Correct Option Type Questions)

1. If vectors a and b are non-collinear, then $\frac{a}{|a|}+\frac{b}{|b|}$ is

A. a unit vector

B. in the plane of a and b

C. equally inclined to a and b

D. perpendicular to $a \times b$

Answer: B::C::D

Watch Video Solution

2. If $a \times (b \times c) = (a \times b) \times c$, then

A. $(c \times a) \times b = 0$

B. $c \times (a \times b) = 0$

C. $b \times (c \times a) = 0$

D. $b \times (c \times a) = 0$

Answer: A::C::D

Watch Video Solution

3. Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - \left(\vec{a}.\vec{b}\right)\vec{b}$ and $\vec{=} \vec{a} \times \vec{b}$, then $\left|\vec{v}\right|$ is

A. $|u|$

B. $|u| + |u \cdot a|$

C. $|u| + |u \cdot b|$

D. $|u| + u \cdot (a + b)$

Answer: A::C

Watch Video Solution

4. The scalars l and m such that $la + mb = c$, where a, b and c are given vectors, are equal to

A. $l = \dfrac{(c \times b) \cdot (a \times b)}{(a \times b)^2}$

B. $l = \dfrac{(c \times sa) \cdot (b \times a)}{(b \times a)^2}$

C. $m = \dfrac{(c \times sa) \cdot (b \times a)}{(b \times a)^2}$

D. $n = \dfrac{(c \times sa) \cdot (b \times a)}{(b \times a)^2}$

Answer: A::C

Watch Video Solution

5. Let $\vec{r}$ be a unit vector satisfying $\vec{r} \times \vec{a} = \vec{b}$, where $\left|\vec{a}\right| = \sqrt{3}$ and $\left|\vec{b}\right| = \sqrt{2}$. Then $\vec{r}$-

A. $\hat{r} = \dfrac{2}{3}(a + a \times b)$

B. $\hat{r} = \dfrac{1}{3}(a + a \times b)$

C. $\hat{r} = \dfrac{2}{3}(a - a \times b)$

D. $\hat{r} = \frac{1}{3}(-a + a \times b)$

Answer: B::D

Watch Video Solution

6. $a_1, a_2, a_3, \in R - \{0\}$ and $a_1 + a_2\cos 2x + a_3\sin^2 x = 0 f$ or $all x \in R,$ then vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} and \vec{b} = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} and \vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$ are parallel to each other vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length$\sqrt{6}$ units, then one of the ordered triple $(a_1, a_2, a_3) = (1, -1, -2)$ are perpendicular to each other if $2a_1 + 3a_2 + 6a_3 = 26, then \left|a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right| is 2\sqrt{6}$

A. vectors $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b = 4\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to each other

B. vectors $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b = -\hat{i} + \hat{j} + \hat{k}$ are perpendicular to each other

C. if vectors $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is of length $\sqrt{6}$ units, then one of the ordered triplet $\left(a_1, a_2, a_3\right) = (1, \ -1, \ -2)$

D. if vectors $2a_1 + 3a_2 + 6a_3$, then $\left|a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right|$ is $2\sqrt{6}$.

Answer: A::B::C::D

Watch Video Solution

7. If $\vec{a}$ and $\vec{b}$ are two vectors and angle between them is θ, then

A. $|a \times b|^2 + (a \cdot b)^2 = |a|^2|b|^2$

B. $|a \times b| = (a \cdot b), \quad \text{if} \quad \theta = \dfrac{\pi}{4}$

C. $a \times b = (a \cdot b)\hat{n}$, (where $\hat{n}$ is a normal unit vector), if $\theta = \dfrac{\pi}{4}$

D. $|a \times b| \cdot (a + b) = 0$

Answer: A::B::C::D

Watch Video Solution

8. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $\left|\vec{a} - \vec{b}\right| < 1$ and $0 \leq \theta \leq \pi$ then theta lies in the intervasl. (A) [0,pi/6] $(B)\left(5\frac{\pi}{6}, \pi\right]$ (C) [pi/2,5pi/6](D)[pi/6,pi/2]`

A. $\left[0, \frac{\pi}{6}\right]$

B. $\left(\frac{5\pi}{6}, \pi\right]$

C. $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

D. $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right]$

Answer: A::B

Watch Video Solution

9. If $\vec{a} \times \left(\vec{b} \times \vec{c}\right) + \left(\vec{a} \cdot \vec{b}\right)\vec{b} = (4 - 2\beta - \sin\alpha)\vec{b} + \left(\beta^2 - 1\right)\vec{c}$ and $\left(\vec{c} \cdot \vec{c}\right)\vec{a} = \vec{c}, \vec{b}, \vec{c}$ being non-collinear then

A. $x = 1$

B. $x = -1$

C. $y = (4n+1)\frac{\pi}{2}, n \in I$

D. $y = (2n+1)\frac{\pi}{2}, n \in I$

Answer: A::C

Watch Video Solution

10. If in triangle $ABC, \vec{AB} = \frac{\vec{u}}{|\vec{u}|} - - \frac{\vec{v}}{|\vec{v}|} and \vec{AC} = \frac{2\vec{u}}{|\vec{u}|}, where |\vec{u}| \neq |\vec{v}|,$ then $1 + \cos 2A + \cos 2B + \cos 2C = 0$ b.$\sin A = \cos C$ c. projection of AC on BC is equal to BC d. projection of AB on BC is equal to AB

A. $1 + \cos 2A + \cos 2B + \cos 3C = 0$

B. $\sin A = \cos C$

C. projection of AC on BC is equal to BC

D. projection of AB on BC is equal to AB

Answer: A::B::C

Watch Video Solution

11. If a, b and c be the three non-zero vectors satisfying the condition $a \times b = c$ and $b \times c = a$, then which of the following always hold(s) good?

A. a, b and c are orthogonal in pairs

B. [a b c]=|b|

C. [a b c] $= \left|c^2\right|$

D. $|b| = |c|$

Answer: A::C

Watch Video Solution

12. Given the following informations about the non-zero vectors A, B and C

$(i)(A \times B) \times A = 0 : (ii)B \cdot B = 4$

$(iii) A \cdot B = -6 : (iv) B \cdot C = 6$

which one of the following holds good?

A. $A \times B = 0$

B. $A \cdot (B \times C) = 0$

C. $A \cdot A = 8$

D. $A \cdot C = -1$

Answer: A::B

Watch Video Solution

13. Let a, b and c are non-zero vectors such that they are not orthogonal pairwise and such that $V_1 = a \times (b \times c)$ and $V_2 = (a \times b) \times c$, are collinear then which of the following holds goods?

A. a and b are orthogonal

B. a and c are collinear

C. b and c are orthogonal

D. $b = \lambda(a \times c)$ when λ is a scalar

Answer: B::D

Watch Video Solution

14. Given three vectors $U = 2\hat{i} + 3\hat{j} - 6\hat{k}, V = 6\hat{i} + 2\hat{j} + 2\hat{k}$ and $W = 3\hat{i} - 6\hat{j} - 2\hat{k}$ which of the following hold good for the vectors U, V and W ?

A. U, V and W are linearly dependent

B. $(U \times V) \times W = 0$

C. U, V and W form a triplet of mutually perpendicular vectors

D. $U \times (V \times W) = 0$

Answer: B::C::D

Watch Video Solution

15. Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} = \hat{k} and \vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b} and \vec{c}$, whose projection on $\vec{a}$ is of magnitude $\sqrt{2/3}$, is $2\hat{i} + 3\hat{j} - 3\hat{k}$ b. $2\hat{i} - 3\hat{j} + 3\hat{k}$ c. $-2\hat{i} - \hat{j} + 5\hat{k}$ d. $2\hat{i} + \hat{j} + 5\hat{k}$

A. $2\hat{i} + 3\hat{j} - 3\hat{k}$

B. $2\hat{i} + 3\hat{j} + 3\hat{k}$

C. $-2\hat{i} - \hat{j} + 5\hat{k}$

D. $2\hat{i} + \hat{j} + 5\hat{k}$

Answer: A::C

Watch Video Solution

16. Three vectors $\vec{a}, \vec{b}, \vec{c}$ are such that $\vec{a} \times \vec{b} = 3(\vec{a} \times \vec{c})$Also $|\vec{a}| = |\vec{b}| = 1, |\vec{c}| = \frac{1}{3}$ If the angle between $\vec{b}$ and $\vec{c}$ is 60° then

A. $b = 3c + a$

B. $b = 3c - a$

C. $a = 6c + 2b$

D. $a = 6c - 2b$

Answer: A::B

Watch Video Solution

17. Let a, b and c be non-zero vectors and $|a| = 1$ and r is a non-zero vector such that $\rtimes a = b$ and $r \cdot c = 1$, then

A. $a \perp b$

B. $r \perp b$

C. $r \cdot a = \dfrac{1 - [abc]}{a \cdot b}$

D. [r a b]=0

Answer: A::B::C

View Text Solution

18. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpendicular to each other and $\vec{c} = \lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3\left(\vec{a} \times \vec{b}\right)$ then the following is (are) true (A) $\lambda_1 = \vec{a}.\vec{c}$ (B) $\lambda_2 = \left|\vec{b} \times \vec{c}\right|$ (C) $\lambda_3 = \left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right|$ (D) $\lambda_1 + \lambda_2 + \lambda_3 = \left(\vec{a} + \vec{b} + \vec{a} \times \vec{b}\right).\vec{c}$

A. $\lambda_1 = a \cdot c$

B. $\lambda_2 = |a \times b|$

C. $\lambda_3 = |(a \times b) \times c|$

D. $\lambda_1 + \lambda_2 + \lambda_3 = (a + b + a \times b) \cdot c$

Answer: A::D

Watch Video Solution

19. Given three non-coplanar vectors OA=a, OB=b, OC=c. Let S be the centre of the sphere passing through the points O, A, B, C if OS=x, then

A. x must be linear combination of a, b, c

B. x must be linear combination of $b \times c, c \times a$ and $a \times b$

C. $x = \dfrac{a^2(b \times c) + b^2(c \times a) + c^2(a \times b)}{2[abc]}$, $a = |a|, b = |b|. C = |c|$

D. $x = a + b + c$

Answer: A::B::C

View Text Solution

20. If $a = \hat{i} + \hat{j} + \hat{k}$ and $b = \hat{i} - \hat{j}$, then the vectors $\left(a \cdot \hat{i}\right)\hat{i} + \left(a \cdot \hat{j}\right)\hat{j} + \left(a \cdot \hat{k}\right)\hat{k}$, $\left(b \cdot \hat{i}\right)\hat{i} + \left(b \cdot \hat{j}\right)\hat{j} + \left(b \cdot \hat{k}\right)\hat{k}$ and $\hat{i} + \hat{j} - 2\hat{k}$

A. are mutually perpendicular

B. are coplanar

C. form a parallepiped of volume 3 units

D. form a parallelopiped of volume 6 units

Answer: A::D

Watch Video Solution

21. If $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}, \vec{b} = y\hat{i} + z\hat{j} + x\hat{k} and \vec{c} = z\hat{i} + x\hat{j} + y\hat{k}, then \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ is parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$ orthogonal to $\hat{i} + \hat{j} + \hat{k}$ orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$ orthogonal to $x\hat{i} + y\hat{j} + z\hat{k}$

A. parallel to $(y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k}$

B. orthogonal to $\hat{i} + \hat{j} + \hat{k}$

C. orthogonal to $(y + z)\hat{i} + (z + x)\hat{j} + (x + y)\hat{k}$

D. parallel to $\hat{i} + \hat{j} + \hat{k}$

Answer: A::B::C

Watch Video Solution

22. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, then which of the following statement(s) is/are true?

A. $a \times (b \times c), b \times (c \times a), c \times (a \times b)$ form a right handed system

B. $c, (a \times b) \times , a \times b$ form a right handed system

C. $a \cdot b + b \cdot c + c \cdot a < 0, \quad \text{if} \quad a + b + c = 0$

D. $\dfrac{(a \times b) \cdot (b \times c)}{(b \times c) \cdot (a \times c)} = -1, \quad \text{if} \quad a + b + c = 0$

Answer: B::C::D

Watch Video Solution

23. Let the unit vectors a and b be perpendicular and the unit vector c be inclined at an angle θ to both a and b. If $c = \alpha a + \beta b + \gamma(a \times b)$, then

A. $l = m$

B. $n^2 = 1 - 2l^2$

C. $n^2 = -\cos 2\alpha$

D. $m^2 = \dfrac{1 + \cos 2\alpha}{2}$

Answer: A::B::C::D

Watch Video Solution

24. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors, then which of the following statement(s) is/are true?

A. $a \times (b \times c), b \times (c \times a), c \times (a \times b)$ form a right handed system

B. $c, (a \times b) \times , a \times b$ form a right handed system

C. $a \cdot b + b \cdot c + c \cdot a < 0, \quad \text{if} \quad a + b + c = 0$

D. $\dfrac{(a \times b) \cdot (b \times c)}{(b \times c) \cdot (a \times c)} = -1, \quad \text{if} \quad a + b + c = 0$

Answer: C::D

Watch Video Solution

25. Let $\vec{a}$ and $\vec{b}$ be two non- zero perpendicular vectors. A vector $\vec{r}$ satisfying the equation $\vec{r} \times \vec{b} = \vec{a}$ can be

A. $b - \dfrac{a \times b}{|b|^2}$

B. $2b - \dfrac{a \times b}{|b|^2}$

C. $|a|b - \dfrac{a \times b}{|b|^2}$

D. $|b|b - \dfrac{a \times b}{|b|^2}$

Answer: A::B::C::D

Watch Video Solution

26. If $\vec{a}$ and $\vec{b}$ are any two unit vectors, then find the greatest postive integer in the range of $\dfrac{3\left|\vec{a} + \vec{b}\right|}{2} + 2\left|\vec{a} - \vec{b}\right|$

A. 2

B. 3

C. 4

D. 5

Answer: B::C::D

Watch Video Solution

27. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{r}$ is non-zero vector such that

$p\vec{r} + \left(\vec{r}\overset{\cdot}{}\vec{a}\right)\vec{b} = \vec{c}$, then $\vec{r} = \dfrac{\vec{c}}{p} - \dfrac{\left(\vec{a}\overset{\cdot}{}\vec{c}\right)\vec{b}}{p^2}$ (b) $\dfrac{\vec{a}}{p} - \dfrac{\left(\vec{\cdot}\,\vec{b}\right)\vec{a}}{p^2}$ $\dfrac{\vec{a}}{p} - \dfrac{\left(\vec{a}\overset{\cdot}{}\vec{b}\right)\vec{c}}{p^2}$ (d)

$\dfrac{\vec{c}}{p^2} - \dfrac{\left(\vec{a}\overset{\cdot}{}\vec{c}\right)\vec{b}}{p}$

A. [r a c]=0

B. $p^2 r = pa - (c \cdot a)b$

C. $p^2 r = pb - (a \cdot b)c$

D. $p^2 r = pc - (b \cdot c)a$

Answer: A::D

Watch Video Solution

28. In a four-dimensional space where unit vectors along the axes are $\hat{i}, \hat{j}, \hat{k} and \hat{l}, and \vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non-zero vectors such that no vector can be expressed as a linear combination of others and $(\lambda - 1)\left(\vec{a}_1 - \vec{a}_2\right) + \mu\left(\vec{a}_2 + \vec{a}_3\right) + \gamma\left(\vec{a}_3 + \vec{a}_4 - 2\vec{a}_2\right) + \vec{a}_3 + \delta\vec{a}_4 = 0$, then a. $\lambda = 1$ b. $\mu = -2/3$ c. $\gamma = 2/3$ d. $\delta = 1/3$

A. $\lambda = 1$

B. $\mu = \frac{-2}{3}$

C. $\lambda = \frac{2}{3}$

D. $\delta = \frac{1}{3}$

Answer: A::B::D

Watch Video Solution

29. A vector(d) is equally inclined to three vectors $a = \hat{i} - \hat{j} + \hat{k}, b = 2\hat{i} + \hat{j}$ and $c = 3\hat{j} - 2\hat{k}$. Let x, y, z be three vectors in the plane a, b:b, c:c, a respectively, then

A. $x \cdot d = 14$

B. $y \cdot d = 3$

C. $z \cdot d = 0$

D. $r \cdot d = 0$, where $r = \lambda x + \mu y + \delta z$

Answer: C::D

Watch Video Solution

30. If a, b, c are non-zero, non-collinear vectors such that a vectors such that a vector $p = ab\cos(2\pi - (a, c))c$ and $aq = ac\cos(\pi - (a, c))$ then b+q is

A. parallel to a

B. perpendicular to a

C. coplanar with b and c

D. coplanar with a and c

Answer: B::C

View Text Solution

31. Given three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-zero and non-coplanar vectors. Then which of the following are coplanar.

A. $a + b, b + c, c + a$

B. $a - b, b + c, c + a$

C. $a + b, b - c, c + a$

D. $a + b, b + c, c - a$

Answer: B::C::D

Watch Video Solution

32. If $r = \hat{i} + \hat{j} + \lambda\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$ and $r \cdot \left(\hat{i} + 2\hat{j} - \hat{k} = 3\right.$ are equations of a line and a plane respectively, then which of the following is incorrect?

A. line is perpendicular to the plane

B. line lies in the plane

C. line is parallel to the plane but not lie in the plane

D. line cuts the plane obliquely

Answer: C::D

Watch Video Solution

33. If vectors $\vec{a}$ and $\vec{b}$ are two adjacent sides of parallelograsm then the vector representing the altitude of the parallelogram which is perpendicular to $\vec{a}$ is (A) $\vec{b} + \frac{\vec{b} \times \vec{a}}{|\vec{a}|^2}$ (B) $\frac{\vec{a}.\vec{b}}{\vec{b}|^2}$ (C) $\vec{b}$ - $\frac{\vec{b}.\vec{a}}{(|\vec{a}|)^2}\Big)$ (D) $\frac{\vec{a} \times \left(\vec{b} \times \vec{a}\right)}{\vec{b}|^{20}}$

A. $b + \frac{b \times a}{|a|^2}$

B. $\frac{a \cdot b}{|b|^2} b$

C. b - $\frac{a \cdot b}{|b|^2} b$

D. $\frac{a \times (b \times a)}{|a|^2}$

Answer: C::D

Watch Video Solution

34. Let a, b, c be three vectors such that each of them are non-collinear, a+b and b+c are collinear with c and a respectively and a+b+c=k. Then (|k|, |k|) lies on

A. $y^2 = 4ax$

B. $x^2 + y^2 - ax - by = 0$

C. $x^2 - y^2 = 1$

D. $|x| + |y| = 1$

Answer: A::B

Watch Video Solution

35. If a, b and c are non-collinear unit vectors also b, c are non-collinear and $2a \times (b \times c) = b + c$, then

A. angle between a and c is $60\ ^\circ$

B. angle between b and c is $30\ ^\circ$

C. angle between a and b is $120\ ^\circ$

D. b is perpendicular to c

Answer: A::C

Watch Video Solution

36. If $a = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) : b = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right) : c = c_1\hat{i} + c_2\hat{j} + c_2\hat{k}$ and

matrix $A = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \\ c_1 & c_2 & c_3 \end{bmatrix}$ and $AT^T = I$, then c

A. $\dfrac{3\hat{i} + 6\hat{j} + 2\hat{k}}{7}$

B. $\dfrac{3\hat{i} - 6\hat{j} + 2\hat{k}}{7}$

C. $\dfrac{-3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$

D. $-\dfrac{3\hat{i} + 6\hat{j} + 2\hat{k}}{7}$

Answer: B::C

Watch Video Solution

Exercise (Statement I And Ii Type Questions)

1. Statement 1: A component of vector $\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ in the direction perpendicular totehdirectin of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k} is \hat{i} - \hat{j}$. Statement 2: A component of vector in the direction of $\vec{a} = \hat{i} + \hat{j} + \hat{k} is 2\hat{i} + 2\hat{j} + 2\hat{k}$.

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: C

Watch Video Solution

2. Statement-I $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are three mutually perpendicular unit vector, then $a_1\hat{i} + b_1\hat{j} + c_1\hat{k}, a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ and $a_3\hat{i} + b_3\hat{j} + 3\hat{k}$ may be mutually perpendicular unit vectors.

Statement-II Value of determinant and its transpose are the same .

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

Watch Video Solution

3. Consider three vectors $\vec{a}, \vec{b} and \vec{\cdot}$ Statement 1

$\vec{a} \times \vec{b} = \left(\left(\hat{i} \times \vec{a}\right)\dot{\vec{b}}\right)\hat{i} + \left(\left(\hat{j} \times \vec{a}\right)\dot{\vec{b}}\right)\hat{j} + \left(\left(\hat{k} \times \vec{a}\right)\dot{\vec{b}}\right)\hat{k}$ Statement 2:

$\vec{c} = \left(\hat{i}\dot{\vec{c}}\right)\hat{i} + \left(\hat{j}\dot{\vec{c}}\right)\hat{j} + \left(\hat{k}\dot{\vec{c}}\right)\hat{k}$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

Watch Video Solution

4. Statement 1: Distance of point D(1,0,-1) from the plane of points A(1,-2,0) , B (3, 1,2) and C(-1,1,-1) is $\frac{8}{\sqrt{229}}$

Statement 2: volume of tetrahedron formed by the points A,B, C and D is $\frac{\sqrt{229}}{2}$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: D

5. Statement 1: If $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}, \vec{B} = \hat{i} + \hat{j} - 2\hat{k} and \vec{C} = \hat{i} + 2\hat{j} + \hat{k}$, then $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right)\dot{\vec{C}}\right| = 243.$ Statement 2: $\left|\vec{A} \times \left(\vec{A} \times \left(\vec{A} \times \vec{B}\right)\right)\dot{\vec{C}}\right| = \left|\vec{A}\right|^2 \left|\left[\vec{A}\vec{B}\vec{C}\right]\right|.$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: D

6. Statement-I The number of vectors of unit length and perpendicular to both the vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is zero.

Statement-II a and b are two non-zero and non-parallel vectors it is true that $a \times b$ is perpendicular to the plane containing a and b

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: D

Watch Video Solution

7. Statement-I $\left(S_1\right)$: If $A\left(x_1, y_1\right), B\left(x_2, y_2\right), C\left(x_3, y_3\right)$ are non-collinear points. Then, every point (x, y) in the plane of $\triangle ABC$, can be expressed in the form $\left(\dfrac{kx_1 + lx_2 + mx_3}{k + l + m}, \dfrac{ky_1 + ly_2 + my_3}{k + l + m}\right)$

Statement-II $\left(S_2\right)$ The condition for coplanarity of four A(a), B(b), C(c), D(d) is that there exists scalars l, m, n, p not all zeros such that $la + mb + nc + pd = 0$ where $l + m + n + p = 0$.

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

Watch Video Solution

8. If a, b are non-zero vectors such that $|a + b| = |a - 2b|$, then

Statement-I Least value of $a \cdot b + \dfrac{4}{|b|^2 + 2}$ is $2\sqrt{2} - 1$.

Statement-II The expression $a \cdot b + \dfrac{4}{|b|^2 + 2}$ is least when magnitude of b is $\sqrt{2\tan\left(\dfrac{\pi}{8}\right)}$.

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

Watch Video Solution

9. Statement-I If $a = 3\hat{i} - 3\hat{j} + \hat{k}, b = -\hat{i} + 2\hat{j} + \hat{k}$ and $c = \hat{i} + \hat{j} + \hat{k}$ and $d = 2\hat{i} - \hat{j}$, then there exist real numbers α, β, γ such that $a = \alpha b + \beta c + \gamma d$

Statement-II a, b, c, d are four vectors in a 3-dimensional space. If b, c, d are non-coplanar, then there exist real numbers α, β, γ such that $a = \alpha b + \beta c + \gamma d.$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: B

View Text Solution

10. Statement 1: Let $\vec{a}, \vec{b}, \vec{c} and \vec{d}$ be the position vectors of four points $A, B, CandD$ and $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = 0$. Then points $A, B, C, andD$ are coplanar. Statement 2: Three non-zero, linearly dependent coinitial vector $\left(\vec{P}Q, \vec{P}Rand\vec{P}S\right)$ are coplanar. Then $\vec{P}Q = \lambda\vec{P}R + \mu\vec{P}S, where\lambda and\mu$ are scalars.

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

Watch Video Solution

11. If $a = \hat{i} + \hat{j} - \hat{k}, b = 2\hat{i} + \hat{j} - 3\hat{k}$ and r is a vector satisfying $2r + \rtimes a = b$.

Statement-I r can be expressed in terms of a, b and $a \times b$.

Statement-II $r = \frac{1}{7}\left(7\hat{i} + 5\hat{j} - 9\hat{k} + a \times b\right)$.

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: A

Watch Video Solution

12. Let $\hat{a}$ and $\hat{b}$ be unit vectors at an angle $\frac{\pi}{3}$ with each other. If $\left(\hat{a} \times \left(\hat{b} \times \hat{c}\right)\right) \cdot \left(\hat{a} \times \hat{c}\right) = 5$ then

Statement-I $\left[\hat{a}\hat{b}\hat{c}\right] = 10$

Statement-II [x y z]=0, if x=y or y=z or z=x

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: B

Watch Video Solution

Exercise (Passage Based Questions)

1. Consider three vectors $\vec{p} = \hat{i} + \hat{j} + \hat{k}, \vec{q} = 2\hat{i} + 4\hat{j} - \hat{k}$ and $\vec{r} = \hat{i} + \hat{j} + 3\hat{k}$ and let $\vec{s}$ be a unit vector, then $\vec{p}, \vec{q}$ and $\vec{r}$ are

A. linealy dependent

B. can form the sides of a possible triangle

C. such that the vectors (q-r) is orthogonal to p

D. such that each one of these can be expressed as a linear combination of the other two

Answer: C

Watch Video Solution

2. Consider three vectors $p = \hat{i} + \hat{j} + \hat{k}, q = 2\hat{i} + 4\hat{j} - \hat{k}$ and $r = \hat{i} + \hat{j} + 3\hat{k}$ and let s be a unit vector, then

Q. If $(p \times q) \times r = up + vq + wr$, then (u+v+w) is equal to

A. 8

B. 2

C. -2

D. 4

Answer: B

Watch Video Solution

3. Consider three vectors $p = \hat{i} + \hat{j} + \hat{k}, q = 2\hat{i} + 4\hat{j} - \hat{k}$ and $r = \hat{i} + \hat{j} + 3\hat{k}$ and let s be a unit vector, then Q. The magnitude of the vector $(p \cdot s)(q \times r) + (q \cdot s)(r \times p) + (r \cdot s)(p \times q)$ is

A. 4

B. 8

C. 18

D. 2

Answer: A

Watch Video Solution

4. Consider the three vectors p, q, r such that $p = \hat{i} + \hat{j} + \hat{k}$ and $q = \hat{i} - \hat{j} + \hat{k}, p \times r = q + cp$ and $p \cdot r = 2$

Q.The value of [p q r] is

A. $\dfrac{5\sqrt{2}c}{|r|}$

B. $-\dfrac{8}{3}$

C. 0

D. greater than 0

Answer: B

Watch Video Solution

5. Consider the three vectors p, q, and r such that $\vec{p} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{q} = \vec{i} - \vec{j} + \vec{k}; p \times r = q + cp$ and $p.r = 2$

A. $c\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

B. a unit vector

C. independent, as [p q r]

D. $-\dfrac{\hat{i} - 2\hat{j} + \hat{k}}{2}$

Answer: D

Watch Video Solution

6. Consider the three vectors p, q, r such that $p = \hat{i} + \hat{j} + \hat{k}$ and $q = \hat{i} - \hat{j} + \hat{k}, p \times r = q + cp$ and $p \cdot r = 2$

Q. If y is a vector satisfying $(1 + c)y = p \times (q \times r)$, then the vectors x, y and r

A. are collinear

B. are coplanar

C. represent the coterminus edges of a tetrahedron whose volume is c cu. Units

D. represent the coterminus edges of a parallelopiped whose volume is c cu. Units

Answer: C

View Text Solution

7. Let P, Q are two points on the curve $y = \log_{\frac{1}{2}}(x - 0.5) + \log_2\sqrt{4x^2 4x + 1}$ and P is also on the $x^2 + y^2 = 10$, Q lies inside the given circle such that its abscissa is an integer.

A. $(1, 2)$

B. $(2, 4)$

C. $(3, 1)$

D. $(3, 5)$

Answer: C

Watch Video Solution

8. Let P and Q are two points on the curve $y = \log_{\frac{1}{2}}(x - 0.5) + \log_2\sqrt{4x^2 - 4x + 1}$ and P is also on the circle $x^2 + y^2 = 10$. Q lies inside the given circle such that its abscissa is an integer.

Q. $OP \cdot OQ$, O being the origin is

A. 4 or 7

B. 4 or 2

C. 2 or 3

D. 7 or 8

Answer: A

Watch Video Solution

9. Let P, Q are two points on the curve $y = \log_{\frac{1}{2}}(x - 0.5) + \log_2\sqrt{4x^2 4x + 1}$ and P is also on the $x^2 + y^2 = 10$, Q lies inside the given circle such that its abscissa is an integer.

A. 1

B. 4

C. 0

D.

Answer: D

Watch Video Solution

10. If a, b, c are three given non-coplanar vectors and any arbitratry vector

r is in space, where $\Delta_1 = \begin{vmatrix} r\cdot a & b\cdot a & c\cdot a \\ r\cdot b & b\cdot b & c\cdot b \\ r\cdot c & b\cdot c & c\cdot c \end{vmatrix} : \Delta_2 = \begin{vmatrix} a\cdot a & r\cdot a & c\cdot a \\ a\cdot b & r\cdot b & c\cdot b \\ a\cdot c & r\cdot c & c\cdot c \end{vmatrix}$

$\Delta_3 = \begin{vmatrix} a\cdot a & b\cdot a & r\cdot a \\ a\cdot b & b\cdot b & r\cdot b \\ a\cdot c & b\cdot c & r\cdot c \end{vmatrix}, \Delta_4 = \begin{vmatrix} a\cdot a & b\cdot a & c\cdot a \\ a\cdot b & b\cdot b & c\cdot b \\ a\cdot c & c\cdot c & c\cdot c \end{vmatrix}$

Q. The vector r is expressible in the form

A. $r = \frac{\Delta_1}{2\Delta}a + \frac{\Delta_2}{2\Delta}b + \frac{\Delta_3}{2\Delta}c$

B. $r=\frac{2\Delta_1}{\Delta}a+\frac{2\Delta_2}{\Delta}b+\frac{2\Delta_3}{\Delta}c$

C. $r=\frac{\Delta}{\Delta_1}a+\frac{\Delta}{\Delta_2}b+\frac{\Delta}{\Delta_3}c$

D. $r=\frac{\Delta_1}{\Delta}a+\frac{\Delta_2}{\Delta}b+\frac{\Delta_3}{\Delta}c$

Answer: D

Watch Video Solution

11. If a, b, c are three given non-coplanar vectors and any arbitratry vector

r is in space, where $\Delta_1=\begin{vmatrix} r\cdot a & b\cdot a & c\cdot a\\ r\cdot b & b\cdot b & c\cdot b\\ r\cdot c & b\cdot c & c\cdot c\end{vmatrix}:\Delta_2=\begin{vmatrix} a\cdot a & r\cdot a & c\cdot a\\ a\cdot b & r\cdot b & c\cdot b\\ a\cdot c & r\cdot c & c\cdot c\end{vmatrix}$

$\Delta_3=\begin{vmatrix} a\cdot a & b\cdot a & r\cdot a\\ a\cdot b & b\cdot b & r\cdot b\\ a\cdot c & b\cdot c & r\cdot c\end{vmatrix},\Delta_4=\begin{vmatrix} a\cdot a & b\cdot a & c\cdot a\\ a\cdot b & b\cdot b & c\cdot b\\ a\cdot c & c\cdot c & c\cdot c\end{vmatrix}$

Q. The vector r is expressible as

A. $r=\frac{[rbc]}{2[abc]}a+\frac{[rbc]}{2[abc]}b+\frac{[rbc]}{2[abc]}c$

B. $r=\frac{2[rbc]}{[abc]}a+\frac{2[rbc]}{[abc]}b+\frac{2[rbc]}{[abc]}c$

C. $r = \frac{1}{[abc]}([rbc]a + [rca]b + [rab]c)$

D. None of these

Answer: D

Watch Video Solution

12. If a, b, c are three given non-coplanar vectors and any arbitratry vector

r is in space, where $\Delta_1 = \begin{vmatrix} r\cdot a & b\cdot a & c\cdot a \\ r\cdot b & b\cdot b & c\cdot b \\ r\cdot c & b\cdot c & c\cdot c \end{vmatrix} : \Delta_2 = \begin{vmatrix} a\cdot a & r\cdot a & c\cdot a \\ a\cdot b & r\cdot b & c\cdot b \\ a\cdot c & r\cdot c & c\cdot c \end{vmatrix}$

$\Delta_3 = \begin{vmatrix} a\cdot a & b\cdot a & r\cdot a \\ a\cdot b & b\cdot b & r\cdot b \\ a\cdot c & b\cdot c & r\cdot c \end{vmatrix}, \Delta_4 = \begin{vmatrix} a\cdot a & b\cdot a & c\cdot a \\ a\cdot b & b\cdot b & c\cdot b \\ a\cdot c & c\cdot c & c\cdot c \end{vmatrix}$

Q. The vector r is expressible as

A. $a = \frac{1}{[abc]}[(a\cdot a)(b\times c) + (b\cdot b)(c\times a) + c\cdot c(a\times b)]$

B. $a = \frac{1}{[abc]}[(a\cdot a)(b\times c) + (b\cdot a)(c\times a) + (a\cdot a)(a\times b)]$

C. $a = [(a\cdot a)(b\times c) + (a\cdot b)(c\times a) + (c\cdot a)(a\times b)]$

D. None of these

Answer: C

Watch Video Solution

13. If a, b, c are three given non-coplanar vectors and any arbitratry vector r is in space, where $\Delta_1 = \begin{vmatrix} r\cdot a & b\cdot a & c\cdot a \\ r\cdot b & b\cdot b & c\cdot b \\ r\cdot c & b\cdot c & c\cdot c \end{vmatrix} : \Delta_2 = \begin{vmatrix} a\cdot a & r\cdot a & c\cdot a \\ a\cdot b & r\cdot b & c\cdot b \\ a\cdot c & r\cdot c & c\cdot c \end{vmatrix}$

$\Delta_3 = \begin{vmatrix} a\cdot a & b\cdot a & r\cdot a \\ a\cdot b & b\cdot b & r\cdot b \\ a\cdot c & b\cdot c & r\cdot c \end{vmatrix}, \Delta_4 = \begin{vmatrix} a\cdot a & b\cdot a & c\cdot a \\ a\cdot b & b\cdot b & c\cdot b \\ a\cdot c & c\cdot c & c\cdot c \end{vmatrix}$

Q. The vector r is expressible as

A. $(p \times q)[a \times bb \times cc \times a]$

B. $2(p \times q)[a \times bb \times cc \times a]$

C. $4(p \times q)[a \times bb \times cc \times a]$

D. $(p \times q)\sqrt{[a \times bb \times cc \times a]}$

Answer: B

Watch Video Solution

Watch Video Solution

14. Let $g(x) = \int_0^x \left(3t^2 + 2t + 9\right)dt$ and $f(x)$ be a decreasing function $\forall x \geq 0$ such that $AB = f(x)\hat{i} + g(x)\hat{j}$ and $AC = g(x)\hat{i} + f(x)\hat{j}$ are the two smallest sides of a triangle ABC whose circumcentre lies outside the triangle $\forall c > 0$. Q. Which of the following is true (for $x \geq o$)

A. $f(x) > 0, g(x) < 0$

B. $f(x) < 0, g(x) < 0$

C. $f(x) > 0, g(x) > 0$

D. $f(x) < 0, g(x) > 0$

Answer: D

Watch Video Solution

15. Let $g(x) = \int_0^x \left(3t^2 + 2t + 9\right)dt$ and $f(x)$ be a decreasing function $\forall x \geq 0$ such that $AB = f(x)\hat{i} + g(x)\hat{j}$ and $AC = g(x)\hat{i} + f(x)\hat{j}$ are the two smallest sides of a triangle ABC whose circumcentre lies outside the triangle

$\forall c > \odot$ Q. $\lim_{t \to 0} \lim_{x \to \infty} \left(\cos\left(\frac{\pi}{4}\left(1 - t^2\right)\right)^{f(x)g(x)} \right)$

A. 0

B. 1

C. e

D. does not exist

Answer: A

Watch Video Solution

16. Let x, y, z be the vector, such that $|x| = |y| = |z| = \sqrt{2}$ and x, y, z make angles of 60° with each other also, $x \times (y \times z) = a$ and $y \times (z \times x) = bx \times y = c$, then Q. The value of x is

A. $(a + b) \times x - (a + b)$

B. $(a + b) - (a + b) \times c$

C. $\frac{1}{2}\{(a+b)\times c - (a+b)\}$

D. None of these

Answer: C

Watch Video Solution

17. Let x,y,z be the vector, such that $|x| = |y| = |z| = \sqrt{2}$ and x,y,z make angles of 60° with each other also.

The value of y is:

A. $\frac{1}{2}[(a+b)+(a+b)\times c]$

B. $2[(a+b)+(a+b)\times c]$

C. $4[(a+b)+(a+b)\times c]$

D. None of these

Answer: A

Watch Video Solution

18. Let x, y, z be the vector, such that $|x| = |y| = |z| = \sqrt{2}$ and x, y, z make angles of $60\,^{\circ}$ with each other also, $x \times (y \times z) = a$ and $y \times (z \times x) = bx \times y = c,$. The value of z is

A. $\frac{1}{2}[(b - a) \times c + (a + b)]$

B. $\frac{1}{2}[(b - a) + c \times (a + b)]$

C. $[(b - a) \times c + (a + b)]$

D. None of these

Answer: B

Watch Video Solution

19. a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non-zero scalars satisfying $a \times b + b \times c = pa + qb + rc.$ Now, answer the following questions. Q. Volume of parallelopiped with edges a, b and c is equal to

A. $p + (q + r)\cos\theta$

B. $(p + q + r)\cos\theta$

C. $2p$ - $(q + r)\cos\theta$

D. None of these

Answer: A

Watch Video Solution

20. a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non-zero scalars satisfying $a \times b + b \times c = pa + qb + rc$. Now, answer the following questions. Q. $\frac{q}{p} + 2\cos\theta$ is equal to

A. 1

B. 2[a b c]

C. 0

D. None of these

Answer: C

Watch Video Solution

21. a, b, c are non-zero unit vectors inclined pairwise with the same angle θ, p, q, r are non-zero scalars satisfying $a \times b + b \times c = pa + qb + rc$. Now, answer the following questions. Q. $|(q + p)\cos\theta + r|$ is equal to

A. $(1 + \cos\theta)\left(\sqrt{1 - 2\cos\theta}\right)$

B. $2\frac{\sin(\theta)}{2}\sqrt{(1 + 2\cos\theta)}$

C. $(1 - \sin\theta)\sqrt{1 + 2\cos\theta}\Big)$

D. None of these

Answer: B

Watch Video Solution

Exercise (Single Integer Answer Type Questions)

1. Let $\hat{u}, \hat{v}$ and $\hat{w}$ are three unit vectors, the angle between $\hat{u}$ and $\hat{v}$ is twice that of the angle between $\hat{u}$ and $\hat{w}$ and $\hat{v}$ and $\hat{w}$, then $\left[\hat{u}\hat{v}\hat{w}\right]$ is equal to

Watch Video Solution

2. If a, b and c are three vectors such that [a b c]=1, then find the value of [a+b b+c c+a]+[$a \times b\ b \times c\ c \times a$]+[$a \times (b \times c)\ b \times (c \times a)\ c \times (a \times b)$]

Watch Video Solution

3. If $\hat{a}, \hat{b}$ and $\hat{c}$ are the three unit vector and α, β and γ are scalars such that $\hat{c} = \alpha\hat{a} + \beta\hat{b} + \gamma\left(\hat{a} \times \hat{b}\right)$. If is given that $\hat{a} \cdot \hat{b} = o$ and $\hat{c}$ makes equal angle with both $\hat{a}$ and $\hat{b}$, then evaluate $\alpha^2 + \beta^2 + \gamma^2$.

Watch Video Solution

4. The three vectors $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}$ taken two at a time form three planes, The three unit vectors drawn perpendicular to these planes form a parallelopiped of volume:

Watch Video Solution

5. Let $\hat{c}$ be a unit vector coplanar with $a = \hat{i} - \hat{j} + 2\hat{k}$ and $b = 2\hat{i} - \hat{j} + \hat{k}$ such that $\hat{c}$ is perpendicular to a . If P be the projection of $\hat{c}$ along, where $p = \frac{\sqrt{11}}{k}$ then find k.

Watch Video Solution

6. Let a, b and c are three vectors hacing magnitude 1, 2 and 3 respectively satisfying the relation [a b c]=6. If $\hat{d}$ is a unit vector coplanar with b and c such that $b \cdot \hat{d} = 1$, then evaluate $|(a \times c) \cdot d|^2 + \left|(a \times c) \times \hat{d}\right|^2$.

Watch Video Solution

7. Let $A\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right), B\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$ and $C\left(\lambda\hat{i} + 5\hat{j} + \mu\hat{k}\right)$ are vertices of a triangle and its median through A is equally inclined to the positive directions of the axes, the value of $2\lambda - \mu$ is equal to

Watch Video Solution

8. If V is the volume of the parallelopiped having three coterminus edges as a,b and c, then the volume of the parallelopiped having the edges as $\alpha = (a.a)a + (a.b)b + (a.c)c; \beta = (a.b)a + (b.b)b + (b.c)b; \gamma = (a.c)a + (b.c)b +$, is

Watch Video Solution

9. If $\vec{a}, \vec{b}$ are vectors perpendicular to each other and $|\vec{a}| = 2, |\vec{b}| = 3, \vec{c} \times \vec{a} = \vec{b}$, then the least value of $2|\vec{c} - \vec{a}|$ is

Watch Video Solution

10. M and N are mid-point of the diagnols AC and BD respectivley of quadrilateral ABCD, then$\overline{AB} + \overline{AD} + \overline{CB} + \overline{CD}$=

Watch Video Solution

11. If $a \times b = c, b \times c = a, c \times a = b$. If vectors a, b and c are forming a right handed system, then the volume of tetrahedron formed by vectors $3a - 2b + 2c, \ - a - 2c$ and $2a - 3b + 4c$ is

Watch Video Solution

12. Let $\vec{a}$ and $\vec{c}$ be unit vectors inclined at $\pi/3$ with each other. If $\left(\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right). \left(\vec{a} \times \vec{c}\right) = 5$, then $\left[\vec{a}\vec{b}\vec{c}\right]$ is equal to

Watch Video Solution

13. Volume of parallelopiped formed by vectors $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}$ and $\vec{c} \times \vec{a}$ is 36 sq.units, then the volume of the parallelopiped formed by the vectors

$\vec{a}, \vec{b}$ and $\vec{c}$ is.

Watch Video Solution

14. If α and β are two perpendicular unit vectors such that $x = \hat{\beta} - (\alpha \times x)$, then the value of $4|x|^2$ is.

Watch Video Solution

15. The volume of the tetrahedron whose vertices are the points with position vectors $\hat{i} + \hat{j} + \hat{k}$, $-\hat{i} - 3\hat{j} + 7\hat{k}$, $\hat{i} + 2\hat{j} - 7\hat{k}$ and $3\hat{i} - 4\hat{j} + \lambda\hat{k}$ is 22, then the digit at unit place of λ is.

Watch Video Solution

16. The volume of a tetrahedron formed by the coterminous edges $\vec{a}, \vec{b}, and \vec{c}$ is 3. Then the volume of the parallelepiped formed by the coterminous edges $\vec{a} + \vec{b}, \vec{b} + \vec{c} and \vec{c} + \vec{a}$ is 6 b. 18 c. 36 d. 9

Watch Video Solution

Exercise (Subjective Type Questions)

1. For any two vectors $\vec{a}$ and $\vec{b}$ we always have $\left|\vec{a}\cdot\vec{b}\right| \leq |\vec{a}||\vec{b}|$(Cauchy-Schwartz inequality).

Watch Video Solution

2. P and Q are two points on the curve $y = 2^{x+2}$ in the rectangular cartesian coordinate system such that $\overline{OP}.\bar{C} = -1$ and $\overline{OQ}.\bar{C} = 2.$ where $\bar{c}$ is the unit vector along the positive direction of the x-axis. Then $\overline{OQ} - 4\overline{OP} =$ (A) $3i + 8j$ (B) $4i + 6j$ (C) $2(3i + 4j)$ (D) $(4i + 3j)$

Watch Video Solution

3. O is the origin and A is a fixed point on the circle of radius 'a' with centre O.The vector $\vec{OA}$ is denoted by $\vec{a}$. A variable point P lie on the tangent at A and $\vec{OP} - \vec{r}$. Show that $\vec{a}\vec{r} = a^2$. Hence if $P(x, y)$ and $A(x_1, y_1)$, deduce the equation of tangent at A to this circle.

Watch Video Solution

4. If a is real constant $A, BandC$ are variable angles and $\sqrt{a^2 - 4\tan A} + a\tan B\sqrt{a^2 + 4\tan c} = 6a,$ then the least vale of $\tan^2 A + \tan^2 b + \tan^2 Cis\ 6$ b. 10 c. 12 d. 3

Watch Video Solution

5. Given , the edges A, B and C of triangle ABC. Find $\cos\angle BAM$, where M is mid-point of BC.

Watch Video Solution

6. Distance of point $A(1, 4, -2)$ is the distance from BC, where B and C The coordinates are respectively $(2, 1, -2)$ and $(0, -5, 1)$, respectively

Watch Video Solution

7. Given, the angles A, B and C of $\triangle ABC$. Let M be the mid-point of segment AB and let D be the foot of the bisector of $\angle C$. Find the ratio of $\frac{AreaOf \triangle CDM}{Areaof \triangle ABC}$ and also $\cos\phi = \cos\angle DCM$.

View Text Solution

8. In ABC, a point P is taken on AB such that $AP/BP = 1/3$ and point Q is taken on BC such that $CQ/BQ = 3/1$. If R is the point of intersection of the lines $AQandCP$, ising vedctor method, find the are of ABC if the area of BRC is 1 unit

Watch Video Solution

9. If one diagonal of a quadrilateral bisects the other, then it also bisects the quadrilateral.

Watch Video Solution

10. Two forces $F_1 = \{2, 3\}$ and $F_2 = \{4, 1\}$ are specified relative to a general cartesian form. Their points of application are respectivel, A=(1, 1) and B=(2, 4). Find the coordinates of the resultant and the equation of the straight line l containing it.

View Text Solution

11. A non zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$and the plane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ can be

Watch Video Solution

12. Vector $\vec{OA} = \hat{i} + 2\hat{j} + 2\hat{k}$ turns through a right angle passing through the positive x-axis on the way. Show that the vector in its new position is $\frac{4\hat{i} - \hat{j} - \hat{k}}{\sqrt{2}}$.

Watch Video Solution

13. Let $\vec{u} and \vec{v}$ be unit vectors such that $\vec{u} \times \vec{v} + \vec{u} = \vec{w}$ and $\vec{w} \times \vec{u} = \vec{v}$. Find the value of $\left[\vec{u}\vec{v}\vec{w}\right]$.

Watch Video Solution

14. A, B and C are three vectors given by $2\hat{i} + \hat{k}, \hat{i} + \hat{j} + \hat{k}$ and $4\hat{i} - 3\hat{j} + 7\hat{k}$. Then, find R, which satisfies the relation $R \times B = C \times B$ and $R \cdot A = 0$.

Watch Video Solution

15. If $x \cdot a = 0, x \cdot b = 1,$ [x a b]=1 and $a \cdot b \ne 0$, then find x in terms of a and b.

Watch Video Solution

16. Let $\hat{x}, \hat{y}$ and $\hat{z}$ be unit vectors such that $\hat{x} + \hat{y} + \hat{z} = a. \hat{x} \times (\hat{y} \times \hat{z}) = b, (\hat{x} \times \hat{y}) \times \hat{z} = c, a \cdot \hat{x} = \frac{3}{2}, a \cdot \hat{y} = \frac{7}{4}$ and $|a| = 2$. Find x, y and z in terms of a, b and c.

Watch Video Solution

17. Let p, q, r be three mutually perpendicular vectors of the same magnitude. If a vector R satisfies th equation p x ((X-q) x p) q x ((x-r)xq)+rx ((x-p) x r) Then x is given by :

Watch Video Solution

18. Given vectors $\bar{C}B = \bar{a}, \bar{C}A = \bar{b}$ and $\bar{C}O = \bar{x}$ where O is the centre of circle circumscribed about ΔABC, then find vector $\bar{x}$

Watch Video Solution

Exercise (Questions Asked In Previous 13 Years Exam)

1. Let O be the origin and let PQR be an arbitrary triangle. The point S is such that $\overline{OP} \cdot \overline{OQ} + \overline{OR} \cdot \overline{OS} = \overline{OR} \cdot \overline{OP} + \overline{OQ} \cdot \overline{OS} = \overline{OQ} \cdot \overline{OR} + \overline{OP} \cdot \overline{OS}$

Then the triangle PQR has S as its

A. centroid

B. orthogonal

C. incentre

D. circumcentre

Answer: B

Watch Video Solution

2. Let O be the origin and OX, OY, OZ be three unit vectors in the directions of the sides, QP, RP, QR respectively of a $\triangle PQR$.

Q. If the triangle PQR varies, then the minimum value of $\cos(P+Q)+\cos(Q+R)+\cos(R+P)$ is

A. $\frac{-3}{2}$

B. $\frac{3}{2}$

C. $\frac{5}{3}$

D. $\frac{-5}{3}$

Answer: A

Watch Video Solution

3. Let O be the origin, and OX, OY, OZ be three unit vectors in the direction of the sides QR , RP , PQ , respectively of a triangle PQR.

$|OX \times OY| =\ s \in (P + R)$ (b) $\sin 2R$ $(c)\sin(Q + R)$ (d) $\sin(P + Q)$

A. $\sin(P + Q)$

B. $\sin(P + R)$

C. $\sin(Q + R)$

D. $\sin 2R$

Answer: A

Watch Video Solution

4. Let a, b and c be three unit vectors such that $a \times (b \times c) = \frac{\sqrt{3}}{2}(b + c)$. If b is not parallel to c , then the angle between a and b is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer: D

Watch Video Solution

5. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then the value of $\sin\theta$ is:

A. $\frac{2\sqrt{2}}{3}$

B. $\frac{-\sqrt{2}}{3}$

C. $\frac{2}{3}$

D. $-\frac{2\sqrt{3}}{3}$

Answer: (a)

Watch Video Solution

6. If a, b and c are unit vectors satisfying $|a - b|^2 + |b - c|^2 + |c - a|^2 = 9$, then $|2a + 5b + 5x|$ is

Watch Video Solution

7. The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ are perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is are

A. $\hat{j} - \hat{k}$

B. $-\hat{i} + \hat{j}$

C. $\hat{i} - \hat{j}$

D. $-\hat{j} + \hat{k}$

Answer: A

Watch Video Solution

8. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be threevectors. A vector in the plane of $\vec{a}$ and $\vec{b}$, whose projectionon $\vec{c}$ is $\frac{1}{\sqrt{3}}$, is given by

A. $\hat{i} - 3\hat{j} + 3\hat{k}$

B. $-3\hat{i} - 3\hat{j} - \hat{k}$

C. $3\hat{i} - \hat{j} + 3\hat{k}$

D. $\hat{i} + 3\hat{j} - 3\hat{k}$

Answer: C

Watch Video Solution

9. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' make a right angle withe the side AB then the cosine of the angle α is given by

A. $\frac{8}{9}$

B. $\frac{\sqrt{17}}{9}$

C. $\frac{1}{9}$

D. $\frac{4\sqrt{5}}{9}$

Answer: B

Watch Video Solution

10. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}, 4\hat{i}, 3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$, respectively. The quadrilateral PQRS must be

A. parallelogram, which is neither a rhombus nor a rectangle

B. square

C. rectangle, but not a square

D. rhombus, but not a square

Answer: (a)

Watch Video Solution

11. If a and b are vectors in space given by $a = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $b = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $(2a + b) \cdot [(a \times b) \times (a - 2b)]$ is

Watch Video Solution

12. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are unit vectors such that $\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{c} \times \vec{d}\right) = 1$ and $\vec{a} . \vec{c} = \frac{1}{2}$, then

A. a, b, c are non-coplanar

B. a, b, d are non-coplanar

C. b, d are non-parallel

D. a, d are parallel and b, c are parallel

Answer: C

Watch Video Solution

13. The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors $\hat{a}, \hat{b}, \hat{c}$ such that $\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{c} = \hat{c} \cdot \hat{a} = \frac{1}{2}$. Then, the volume of the parallelopiped is

A. $\frac{1}{\sqrt{2}}$ cu units

B. $\frac{1}{2\sqrt{2}}$ cu units

C. $\frac{\sqrt{3}}{2}$ cu units

D. $\frac{1}{\sqrt{3}}$ cu units

Answer: A

Watch Video Solution

14. Lelt two non collinear unit vectors $\hat{a}$ and $\hat{b}$ form and acute angle. A point P moves so that at any time t the position vector $\overrightarrow{OP}$ (where O is the origin) is given by $\hat{a}\cos t + \hat{b}\sin t$. When P is farthest from origin O, let M be the length of $\overrightarrow{OP}$ and $\hat{u}$ be the unit vector along $\overrightarrow{OP}$ Then (A) $\hat{u} = \frac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|}$ and $M = \left(1 + \hat{a}.\hat{b}\right)^{\frac{1}{2}}$ (B) $\hat{u} = \frac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|}$ and $M = \left(1 + \hat{a}.\hat{b}\right)^{\frac{1}{2}}$ (C) $\hat{u} = \frac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|}$ and $M = \left(1 + 2\hat{a}.\hat{b}\right)^{\frac{1}{2}}$ (D) $\hat{u} = \frac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|}$ and $M = \left(1 + 2\hat{a}.\hat{b}\right)^{\frac{1}{2}}$

A. $\hat{u} = \frac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|}$ and $M = \left(1 + \hat{a} \cdot \hat{b}\right)^{\frac{1}{2}}$

B. $\hat{u} = \frac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|}$ and $M = \left(1 + \hat{a} \cdot \hat{b}\right)^{\frac{1}{2}}$

C. $\hat{u} = \frac{\hat{a} + \hat{b}}{\left|\hat{a} + \hat{b}\right|}$ and $M = \left(1 + 2\hat{a} \cdot \hat{b}\right)^{\frac{1}{2}}$

D. $\hat{u} = \frac{\hat{a} - \hat{b}}{\left|\hat{a} - \hat{b}\right|}$ and $M = \left(1 + 2\hat{a} \cdot \hat{b}\right)^{\frac{1}{2}}$

Answer: A

Watch Video Solution

15. Let the vectors PQ, QR, RS, ST, TU and UP represent the sides of a regular hexagon.

Statement-I $PQ \times (RS + ST) \neq 0$, becouse

Statement-II $PQ \times RS = 0$ and $PQ \times ST \neq 0$

A. Both Statement-I and Statement-II are correct and Statement-II is the correct explanation of Statement-I

B. Both Statement-I and Statement-II are correct but Statement-II is not the correct explanation of Statement-I

C. Statement-I is correct but Statement-II is incorrect

D. Statement-II is correct but Statement-I is incorrect

Answer: C

Watch Video Solution

16. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}, \hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is

A. 0

B. 1

C. $\pm\sqrt{2}$

D. 3

Answer: C

Watch Video Solution

17. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$. Which of the following is correct?

A. $a \times b = b \times c = c \times a = 0$

B. $a \times b = b \times c = c \times a \neq 0$

C. $a \times b = b \times c = a \times c = 0$

D. $a \times b, b \times c, c \times a$ are mutually perpendicular

Answer: B

Watch Video Solution

18. Let A be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and that P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector A and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is

A. $\frac{\pi}{2}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{3\pi}{4}$

Answer: B::D

Watch Video Solution

19. Let $a = \hat{i} + 2\hat{j} + \hat{k}, b = \hat{i} - \hat{j} + \hat{k}, c = \hat{i} + \hat{j} - \hat{k}$. A vector coplanar to a and b has a projection along c of magnitude $\frac{1}{\sqrt{3}}$, then the vector is

A. $4\hat{i} - \hat{j} + 4\hat{k}$

B. $4\hat{i} + \hat{j} - 4\hat{k}$

C. $2\hat{i} + \hat{j} + \hat{k}$

D. None of these

Answer: A

Watch Video Solution

20. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three non-zero, non coplanar vector $\vec{b}_1 = \vec{b} - \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{c}_1 = \vec{c} - \frac{\vec{\cdot}\,\vec{a}}{|\vec{a}|^2}\vec{a} + \frac{\vec{b}\cdot\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $, c_2 = \vec{c} - \frac{\vec{\cdot}\,\vec{a}}{|\vec{a}|^2}\vec{a} - \frac{\vec{b}\cdot\vec{c}}{|\vec{b}_1|^2}$,

$b_1, \vec{c}_3 = \vec{c} - \frac{\vec{} \cdot \vec{a}}{|\vec{c}|^2}\vec{a} + \frac{\vec{b}\vec{c}}{|\vec{c}|^2}\vec{b}_1$, $\vec{c}_4 = \vec{c} - \frac{\vec{} \cdot \vec{a}}{|\vec{c}|^2}\vec{a} = \frac{\vec{b}\vec{c}}{|\vec{b}|^2}\vec{b}_1$ then the set of orthogonal vectors is $\left(\vec{a}, \vec{b}_1, \vec{c}_3\right)$ b. $\left(\vec{a}, \vec{b}_1, \vec{c}_2\right)$ c. $\left(\vec{a}, \vec{b}_1, \vec{c}_1\right)$ d. $\left(\vec{a}, \vec{b}_2, \vec{c}_2\right)$

A. $\{a, b_1, c_1\}$

B. $\{a, b_1, c_2\}$

C. $\{a, b_2, c_3\}$

D. $\{a, b_2, c_4\}$

Answer: B

Watch Video Solution

21. The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with the vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is (A) $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$ (B) $\frac{2\hat{i} - 3\hat{j}}{\sqrt{3}}$ (C) $3\hat{j} - \hat{k}\frac{)}{\sqrt{10}}$ (D) $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$

A. $\dfrac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$

B. $\dfrac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$

C. $\dfrac{3\hat{j} - \hat{k}}{\sqrt{10}}$

D. $\dfrac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$.

Answer: C

Watch Video Solution

22. The value of a so that the volume of parallelepiped formed by $\hat{i} + a\hat{j} + \hat{k}, \hat{j} + a\hat{k} and a\hat{i} + \hat{k}$ is minimum is -3 b. 3 c. $1/\sqrt{3}$ d. $\sqrt{3}$

A. -3

B. 3

C. $\dfrac{1}{\sqrt{3}}$

D. $\sqrt{3}$

Answer: C

Watch Video Solution

.

23. If $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right), \vec{a}\vec{b} = 1 and \vec{a} \times \vec{b} = \hat{j} - \hat{k}, then \hat{b}$ is $\hat{i} - \hat{j} + \hat{k}$ b. $2\hat{j} - \hat{k}$ c. $\hat{i}$ d. $2\hat{i}$

A. $\hat{i} - \hat{j} + \hat{k}$

B. $2\hat{j} - \hat{k}$

C. $\hat{i}$

D. $2\hat{i}$

Answer: C

Watch Video Solution

24. Let $\vec{V} = 2\hat{i} + \hat{j} - \hat{k} and \vec{W} = \hat{i} + 3\hat{k}$ If $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[UVW]$ is -1 b. $\sqrt{10} + \sqrt{6}$ c. $\sqrt{59}$ d. $\sqrt{60}$

A. -1

B. $\sqrt{10} + \sqrt{6}$

C. $\sqrt{59}$

D. $\sqrt{60}$

Answer: C

Watch Video Solution

25. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is

A. 45°

B. 60°

C. $\cos^{-1}\left(\frac{1}{3}\right)$

D. $\cos^{-1}\left(\frac{2}{7}\right)$

Answer: B

Watch Video Solution

26. Let $a = 2\hat{i} - 2\hat{k}, b = \hat{i} + \hat{j}$ and c be a vectors such that $|c - a| = 3, |(a \times b) \times c| = 3$ and the angle between c and $a \times b$ is 30°. Then a. c is equal to

A. $\frac{25}{8}$

B. 2

C. 5

D. $\frac{1}{8}$

Answer: B

Watch Video Solution

27. If $[a \times bb \times cc \times a] = \lambda[abc]^2$, then λ is euqual to

A. 0

B. 1

C. 2

D. 3

Answer: C

Watch Video Solution

28. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other then the angle between $\hat{a}$ and $\hat{b}$ is (A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

Watch Video Solution

29. Let ABCD be a parallelogram such that $\vec{AB} = \vec{q}, \vec{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is

A. $r = 3p + \frac{3(q \cdot p)}{p \cdot p} p$

B. $r = -p + \frac{(q \cdot p)}{p \cdot p} p$

C. $r = p - \frac{(q \cdot p)}{p \cdot p} p$

D. $r = -3p + \frac{3(q \cdot p)}{p \cdot p} p$

Answer: B

Watch Video Solution

30. $\frac{1}{\sqrt{10}}\left(3\hat{j}+\hat{k}\right)$ and $\vec{b}=\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$, then the value of $\left(2\vec{a}-\vec{b}\right).\left[\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right]$ is:

A. -3

B. 5

C. 3

D. -5

Answer: D

Watch Video Solution

31. The vectors a and b are not perpendicular and c and d are two vectors satisfying $b\times c=b\times d$ and $a.d=0$. The vectors d is equal to

A. $c+\left(\frac{a\cdot c}{a\cdot b}\right)b$

B. $b+\left(\frac{b\cdot c}{a\cdot b}\right)c$

C. $c - \left(\frac{a \cdot c}{a \cdot b}\right)b$

D. $b - \left(\frac{b \cdot c}{a \cdot b}\right)c$

Answer: C

Watch Video Solution

32. If the vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}, \hat{i} + \hat{j} + c\hat{k}$, where a, b, c are coplanar, then $a + b + c - abc =$

A. -2

B. 2

C. 0

D. -1

Answer: A

Watch Video Solution

33. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector b satisfying $\vec{a}x\vec{b} + \vec{c} = 0$ and $\vec{a}.\vec{b} = 3$, is

A. $-\hat{i} + \hat{j} - 2\hat{k}$

B. $2\hat{i} - \hat{j} + 2\hat{k}$

C. $\hat{i} - \hat{j} - 2\hat{k}$

D. $\hat{i} + \hat{j} - 2\hat{k}$

Answer: D

Watch Video Solution

34. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}.\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{=} \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ)

A. $(-3, 2)$

B. $(2, -3)$

C. $(-2, 3)$

D. $(3, -2)$

Answer: A

Watch Video Solution

35. If $\vec{u}, \vec{v}, \vec{w}$ are non -coplanar vectors and $p, q,$ are real numbers then the equality

$\left[3\vec{u}p\vec{v}p\vec{w}\right] - \left[p\vec{v}\vec{w}q\vec{u}\right] - \left[2\vec{w} - q\vec{v}q\vec{u}\right] = 0$ holds for

A. exactly two values of (p, q)

B. more than two but not all values of (p, q)

C. all values of (p,q)

D. exactly one value of (p, q)

Answer: D

Watch Video Solution

36. The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{\text{i}} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of $\alpha \text{and} \beta$? (1) $\alpha = 2, \beta = 2$ (2) $\alpha = 1, \beta = 2$ (3) $\alpha = 2, \beta = 1$ (4) $\alpha = 1, \beta = 1$

A. $\alpha = 1, \beta = 1$

B. $\alpha = 2, \beta = 2$

C. $\alpha = 1, \beta = 2$

D. $\alpha = 2, \beta = 1$

Answer: D

Watch Video Solution

37. If $\vec{u}$ and $\vec{v}$ are unit vectors and θ is the acute angle between them, then $2u\vec{u} \times 3\vec{v}$ is a unit vector for

A. exactly two values of θ

B. more than two but not all values of θ

C. no value of θ

D. exactly one value of θ

Answer: D

Watch Video Solution

38. Let $\bar{a} = \hat{i} + \hat{j} + \hat{k}, b = \hat{i} - \hat{j} + 2\hat{k}$ and $\bar{c} = x\hat{i} + (x - 2)\hat{j} - \hat{k}$. If the vector c lies in the plane of a and b , then x equals (1) 0 (2) 1 (3) -4 (4) -2

A. 0

B. 1

C. -4

D. -2

Answer: D

Watch Video Solution

39. If $\left(\vec{a}\times\vec{b}\right)\times\vec{c}=\vec{a}\times\left(\vec{b}\times\vec{c}\right)$, $Where\vec{a},\vec{b}$ and $\vec{c}$ and any three vectors such that $\vec{a}.\vec{b}=0, \vec{b}.\vec{c}=0,$ then $\vec{a}$ and $\vec{c}$ are

A. inclined at an angle of $\frac{\pi}{6}$ between them

B. perpendicular

C. parallel

D. inclined at an angle $\frac{\pi}{3}$ between them

Answer: C

Watch Video Solution

40. The values of a for which the points $A, B,$ and C with position vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k},$ and $a\hat{i}-3\hat{j}+\hat{k}$,respectively, are the vertices of a right-angled triangle with $C=\frac{\pi}{2}$ are

A. -2 and -1

B. -2 and 1

C. 2 and -1

D. 2 and 1

Answer: D

Watch Video Solution

41. The distance between the line $r = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda\left(\hat{i} - \hat{j} + 4\hat{k}\right)$ and the plane $r \cdot \left(\hat{i} + 5\hat{j} + \hat{k}\right) = 5,$ is

A. $\frac{10}{3}$

B. $\frac{3}{10}$

C. $\frac{10}{3\sqrt{3}}$

D. $\frac{10}{9}$

Answer: C

Watch Video Solution

42. If $\vec{a}$ is any vector, then $\left(\vec{a} \times \vec{i}\right)^2 + \left(\vec{a} \times \vec{j}\right)^2 + \left(\vec{a} \times \vec{k}\right)^2$ is equal to

A. $4a^2$

B. $2a^2$

C. a^2

D. $3a^2$

Answer: B

Watch Video Solution

43. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number then $[\lambda\left(\vec{a} + \vec{b}\right)\ \lambda^2\vec{b}\ \lambda\vec{c}] = [\vec{a}\ \ \vec{b} + \vec{c}\ \ \vec{b}]$ for: a. exactly two values of λ b. exactly three values of λ c. no value of λ d. exactly one values of λ

A. exactly two values of λ

B. exactly three values λ

C. no value of λ

D. exactly one value of λ

Answer: C

Watch Video Solution

44. If $\vec{a} = ht(i) - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$

$\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}.$

then $\vec{a}.\left(\vec{b} \times \vec{c}\right)$ depends on

A. neither x nor y

B. both x and y

C. only x

D. only y

Answer: A

Watch Video Solution

45. Let $\vec{u}$, $\vec{v} and \vec{w}$ be such that $|\vec{u}| = 1$, $|\vec{v}| = 2 and |\vec{w}| = 3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and vectors $\vec{v} and \vec{w}$ are perpendicular to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals 2 b. $\sqrt{7}$ c. $\sqrt{14}$ d. 14

A. 2

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 14

Answer: C

Watch Video Solution

46. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then the value of $\sin\theta$ is:

A. $\frac{1}{3}$

B. $\frac{\sqrt{2}}{3}$

C. $\frac{2}{3}$

D. $\frac{2\sqrt{2}}{3}$

Answer: D

Watch Video Solution

47. A particle acted by constant forces $4\hat{i} + \hat{j}$ - $3\hat{k}and3\hat{i} + \hat{9}$ - $\hat{k}$ is displaced from point $\hat{i} + 2\hat{j} + 3\hat{k}$ to point $5\hat{i} + 4\hat{j} + \dot{\hat{k}}$ find the total work done by the forces in units.

A. 40 units

B. 30units

C. 25 units

D. 15 units

Answer: A

Watch Video Solution

48. If $\bar{u}, \bar{v}, \bar{w}$ are three non coplanar vectors then $(\bar{u} + \bar{v} - \bar{w}).\ \{(\bar{u} - \bar{v}) \times (\bar{v} - \bar{w})\} =$

A. 0

B. $u \cdot v \times w$

C. $u \cdot w \times v$

D. $3u \cdot v \times w$

Answer: B

Watch Video Solution

49. a, b, c are three vectors, such that $a + b + c = 0 |a| = 1, |b| = 2, |c| = 3,$ then $a \cdot b + b \cdot c + c \cdot a$ is equal to

A. 0

B. -7

C. 7

D. 1

Answer: B

Watch Video Solution

50. A tetrahedron has vertices $O(0, 0, 0), A(1, 2, 1), B(2, 1, 3), andC(-1, 1, 2)$, then angle between face $OABandABC$ will be a. $\cos^{-1}\left(\frac{17}{31}\right)$ b. 30^0 c. 90^0 d. $\cos^{-1}\left(\frac{19}{35}\right)$

A. $\cos^{-1}\left(\frac{19}{35}\right)$

B. $\cos^{-1}\left(\frac{17}{31}\right)$

C. 30°

D. 90°

Answer: A

Watch Video Solution

51. Let $\hat{u} = \hat{i} + \hat{j}, \hat{v} = \hat{i} - \hat{j} and \hat{w} = \hat{i} + 2\hat{j} + 3\hat{k}^{\cdot}$ If $\hat{n}$ is a unit vector such that $\hat{u}\dot{\hat{n}} = 0 and \hat{v}\dot{\hat{n}} = 0,$ then find the value of $\left|\hat{w}\dot{\hat{n}}\right|^{\cdot}$

A. 0

B. 1

C. 2

D. 3

Answer: D

Watch Video Solution

52. Given, two vectors are $\hat{i} - \hat{j}$ and $\hat{i} + 2\hat{j}$, the unit vector coplanar with the two vectors and perpendicular to first is

A. $\frac{1}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

B. $\frac{1}{\sqrt{5}}\left(2\hat{i} + \hat{j}\right)$

C. $\pm \frac{1}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

D. None of these

Answer: (a)

Watch Video Solution