MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

SEQUENCES AND SERIES

Examples

1. If $f: N \rightarrow R$, where $f(n)=a_n=\frac{n}{(2n+1)^2}$ write the sequence in ordered pair from.

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2. The Fibonacci sequence is defined by $1=a_1=a_2(\text{ and } a)_n=a_{n-1}+a_{n-2}, n>2$. Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5.

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3. If the sum of n terms of a series is $2n^2 + 5n$ for all values of n, find its 7th term.

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4. (i) Write $\sum_{r=1}^{n} \left(r^2 + 2\right)$ in expanded form.

(ii) Write the series $\frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{4}{6} + \ldots + \frac{n}{n+2}$ in sigma form.

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5. (i) 1,3,5,7,...

(ii) $\pi, \pi + e^{\pi}, \pi + 2e^{\pi}, \ldots.$

(iii) $a, a - b, a - 2b, a - 3b, \ldots.$

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6. Show that the sequence t_n defined by $t_n = 5n + 4$ is AP, also find its common difference.

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7. Show that the the sequence defined by $T_n = 3n^2 + 2$ is not an AP.

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8. Which term of the sequence $20, \ 19\frac{1}{4}, \ 18\frac{1}{2}, \ 17\frac{3}{4}, \cdot$ is the first negative term?

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9. If mth term of an AP is 1/n and its nth term is 1/m , then show that its (mn)th term is 1

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10. If $|x-1|, 3$ and $|x-3|$ are first three terms of an increasing AP, then find the 6th term of on AP .

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11. In the sequence 1, 2, 2, 3, 3, 3, 4, 4,4,4,......, where n consecutive terms have the value n, the 150 term is

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12. If a_1, a_2, a_3, a_4 and a_5 are in AP with common difference $\neq 0,$ find the value of $\sum_{i=1}^{5} a_i$ when $a_3 = 2.$

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13. The ratio of the sum of m and n terms of an A.P. is $m^2 : n^2$. The ratio of the mth and nth terms is

A. $(2m+1):(2n-1)$,

B. $m:n$

C. $(2m-1):(2n-1)$

D. None of these

Answer: C

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14. The sums of n terms of two arithmetic progresssions are in the ratio $(7n+1):(4n+17)$. Find the ratio of their nth terms and also common differences.

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15. The sums of n terms of two AP's are in the ratio $(3n-13):(5n+21)$. Find the ratio of their 24th terms.

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16. How many terms of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + ...$ must be taken to make 300?

Explain the double answer.

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17. Find the arithmetic progression consisting of 10 terms , if sum of the terms occupying the even places is equal to 15 and the sum of those occupying the odd places is equal to $\frac{25}{2}$

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18. If the set of natural numbers is partitioned into subsets $S_1 = \{1\}, S_2 = \{2, 3\}, S_3 = \{4, 5, 6\}$ and so on then find the sum of the terms in S_{50}.

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19. Find the sum of first 24 terms of on AP $t_1, t_2, t_3, \ldots.,$ if it is known that $t_1 + t_5 + t_{10} + t_{15} + t_{20} + t_{24} = 225.$

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20. If $(1 + 3 + 5 + \quad + p) + (1 + 3 + 5 + \quad + q) = (1 + 3 + 5 + \quad + r)$ where each set of parentheses contains the sum of consecutive odd integers as shown, the smallest possible value of $p + q + r(wherep > 6)$ is 12 b. 21 c. 45 d. 54

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21. If S_1,S_2,........S_p are the sum of n terms of an A.P.,whose first terms are 1,2,3,...... and common differences are 1,3,5,7.....Show that S_1+S_2+.......+S_p= $\frac{np}{2}[np + 1]$

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22. Let α and β be roots of the equation $X^2 - 2x + A = 0$ and let γ and δ be the roots of the equation $X^2 - 18x + B = 0$. If $\alpha < \beta < \gamma < \delta$ are in arithmetic progression then find the valus of A and B.

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23. The digits of a positive integer, having three digits, are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number.

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24. If three positive real numbers a,b,c are in AP such that abc=4, then the minimum value of b is

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25. If a, b, c, d are distinct integers in A. P. Such that $d = a^2 + b^2 + c^2$, then a + b + c + d is

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26. $1, 2, 4, 8, 16, \ldots.$

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27. $9, 3, 1, \frac{1}{3}, \frac{1}{9}, \ldots.$

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28. $-2, -6, -18, \ldots.$

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29. $-8, \ -4, \ -2, \ -1, \ -\frac{1}{2}, \dots$

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30. $5, \ -10, 20, \dots.$

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31. $5, 5, 5, 5, \dots.$

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32. $1, 1+I, 2i, \ -2+2i, \dots.i = \sqrt{-1}$

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33. Show that the sequence t_n defined by $t_n = \frac{2^{2n-1}}{3}$ for all values of $n \in N$ is a GP. Also, find its common ratio.

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34. Show that the sequence t_n defined by $t_n = 2 \cdot 3^n + 1$ is not a GP.

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35. If first term of a GP is a, third term is b and $(n+1)th$ term is c. The $(2n+1)th$ term of a GP is

A. $c\sqrt{\frac{b}{a}}$

B. $\frac{bc}{a}$

C. abc

D. $\frac{c^2}{a}$

Answer:

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36. In a GP if the $(m+n)th$ term is p and $(m-n)th$ term is q then mth term is

A. $p\left(\frac{q}{p}\right)^{\frac{m}{2n}}$

B. $\sqrt{pq}$

C. $\sqrt{\frac{p}{q}}$

D. None of these

Answer:

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37. If $\sin\theta, \sqrt{2}(\sin\theta+1), 6\sin\theta+6$ are in GP, than the fifth term is

A. 81

B. $81\sqrt{2}$

C. 162

D. $162\sqrt{2}$

Answer: c

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38. The 1025th term in the sequence are $1, 22, 4444, 88888888, \ldots$ is 2^N find the value of N

A. 9

B. 10

C. 11

D. 12

Answer: 10

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39. If a, b, c are real numbers such that $3(a^2 + b^2 + c^2 + 1) = 2(a + b + c + ab + bc + ca)$, than a, b, c are in

A. AP only

B. GP only

C. GP and AP

D. None of these

Answer: C

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40. Find the value of 0.3258.

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41. Find the sum upto n terms of the series

$a + aa + aaa + aaaa + \ldots\ldots, \forall a \in N$ and $1 \leq a \leq 9.$

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42. Find the sum upto n terms of the series

$0.b + 0.bb + 0.bbb + 0.bbbb + \ldots\ldots, \forall$ b $\in N$ and $1 \leq b \leq 9.$

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43. If N, the set of natural numbers is partitionaed into groups

$S_1 = \{1\}, S_2 = \{2, 3\}, S_3 = \{4, 5, 6, 7\}, S_4 = \{8, 9, 10, 11, 12, 13, 14, 15\}, .$

find the sum of the numbers n C_{50}.

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44. If $S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{n-1}}$ and $2 - S_n < \frac{1}{100}$, then the

least value of n must be :

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45. If $X = 1 + a + a^2 + a^3 + ... + \infty$ and $y = 1 + b + b^2 + b^3 + ... + \infty$ show that $1 + ab + a^2b^2 + a^3b^3 + ... + \infty = \frac{xy}{x + y - 1}$, where $0 < a < 1$ and $0 <$

.

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46. If $1 + ab + a^2b^2 + a^3b^3 + ... + \infty = \frac{xy}{x + y - 1}$ are the sum of infinire geometric series whose first terms are $1, 2, 3, \ldots.., p$ and whose common ratios are $S_1, S_2, S_3, \ldots., S_p \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{1}{p+1}$ respectively, prove that $S_1 + S_2 + S_3 + \ldots. + S_p = \frac{p(p+3)}{2}$.

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47. If x_1, x_2 be the roots of the equation $x^2 - 3x + A = 0$ and x_3, x_4 be those of the equation $x^2 - 12x + B = 0$ and x_1, x_2, x_3, x_4 be an increasing GP. find find A and B.

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48. Suppose a, b, c are in AP and a^2, b^2, c^2 are in GP, If $a > b > c$ and $a + b + c = \frac{3}{2}$, than find the values of a and c.

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49. If the continued product of three numbers in GP is 216 and the sum of their products in pairs is 156, then find the sum of three numbers.

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50. Find a three digit numberwhose consecutive digits form a GP. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now, if we increase the second digit of the required number by 2, then the resulting digits will form an AP.

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51. A square is drawn by joining mid pint of the sides of a square. Another square is drawn inside the second square in the same way and the process is continued in definitely. If the side of the first square is 16 cm, then what is the sum of the areas of all the squares ?

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52. One side of an equilateral triangle is 24 cm. The midpoints of its sides are joined to form another triangle whose midpoints are in turn joined to form still another triangle this process continues indefinitely. The sum of the perimeters of all the triangles is

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53. Let $S_1, S_2,$ be squares such that for each $n \geq 1,$ the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of $S_1 is 10 cm,$ then for which of the following value of n is the area of S_n less than 1 sq. cm? a. 5 b. 7 c. 9 d. 10

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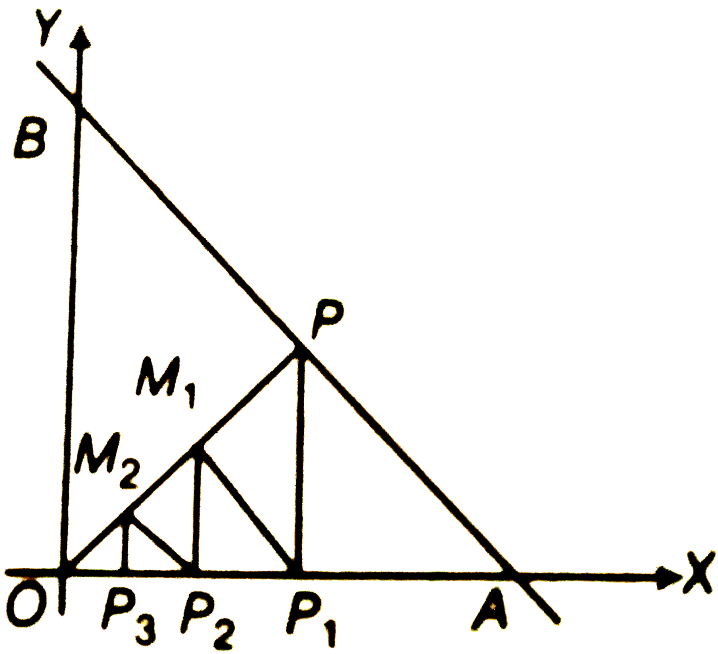
54. The line $x + y = 1$ meets X-axis at A and Y-axis at B,P is the mid-point of AB, P_1 is the foot of perpendicular from P to OA, M_1 is that of P_1 from OP, P_2 is that of M_1 from OA, M_2 is that of P_2 from OP, P_3 is that of M_2 from OA and so on. If P_n denotes theb nth foot of the perpendicular on OA, then find OP_n



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55. Dipesh writes letters to four of his friends. He asks each of them to copy the letter and mail to four different persons with the request that they continue the chain similarly. Assuming that the chain is not broken and that it costs 25 paise to mail one letter, find the total money spent on postage till the 8th set of letters is mailed.

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56. An insect starts from a point and travels in a straight path 1 mm in the first second and half of the distance covered in the previous second in the succeeding second. In how much time would it reach a point 3 mm away from its starting point.

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57. The pollution in a normal atmosphere is less than 0.01%. Due to leakage of a gas from a factory, the pollution is increased to 20%. If every day 80% of pollution is neutralised, in how many days the atmosphere will be normal?

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58. If a, b, c are in HP, then $\frac{a-b}{b-c}$ is equal to

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59. Find the first term of a HP whpse secpmd ter, os $\frac{5}{4}$ and the third term is $\frac{1}{2}$.

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60. If $\frac{1}{a}+\frac{1}{a-b}+\frac{1}{c}+\frac{1}{c-b}=0$ and $a+c-b\neq 0$, then prove that a, b, c are in H.P.

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61. If $a_1, a_2, a_3, \ldots\ldots, a_n$ are in HP, than prove that $a_1a_2+a_2a_3+a_3a_4+\ldots\ldots+a_{n-1}a_n=(n-1)a_1a_n$

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62. The sum of three numbers in HP is 37 and the sum of their reciprocals is $\frac{1}{4}$. Find the numbers.

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63. If pth, qth and rth terms of a HP be respectivelya, b and c, has prove that $(q-r)bc+(r-p)ca+(p-q)ab=0.$

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64. If $a, b, c,$ are in AP, a^2, b^2, c^2 are in HP, then prove that either $a=b=c$ or $a, b, -\frac{c}{2}$ from a GP (2003, 4M)

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65. If a, b, c are in HP,b,c,d are in GP and c, d, e are in AP, than show that $e=\frac{ab^2}{(2a-b)^2}$.

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66. Given a,b,c are in A.P.,b,c,d are in G.P and c,d,e are in H.P .If a=2 and e=18 , then the sum of all possible value of c is _______.

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67. If three positive numbers a, b and c are in AP, GP and HP as well, than find their values.

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68. If a, b, c are in AP and p is the AM between a and b and q is the AM between b and c, then show that b is the AM between p and q.

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69. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between $aandb$.

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70. There are n AM's between 3 and 54.Such that the 8th mean and $(n-2)$th mean is 3 ratio 5. Find n.

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71. If 11 AM's are inserted between 28 and 10, than find the three middle terms in the series.

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72. If a, b, c are in AP, than show that $a^2(b+c)+b^2(c+a)+c^2(a+b)=\frac{2}{9}(a+b+c)^3$.

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73. If a be one A.M and G_1 and G_2 be then geometric means between b and c then $G_1^3 + G_2^3 =$

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74. If one geometric mean G and two arithmetic means p,q be inserted between two given numbers, then prove that, $G^2 = (2p - q)(2q - p)$.

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75. Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between $aandb$.

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76. Insert five geimetrec means between $\frac{1}{3}$ and 9 and verify that their product is the fifth power of the geometric mean between $\frac{1}{3}$ and 9.

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77. AM between two numbers whose sum is 100 is ti the GM as 5:4`, find the numbers.

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78. If $a_1, a_2, a_3, \ldots. a_n$ are positive real numbers whose product is a fixed number c, then the minimum value of $a_1 + a_2 + \ldots. + a_{n-1} + 2a_n$ is

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79. If H be the harmonic mean between x and y, then show that $\frac{H+x}{H-x} + \frac{H+y}{H-y} = 2$

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80. IF $a_1, a_2, a_3, \ldots .a_{10}$ be in AP and $h_1, h_2, h_3, \ldots .h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then find value of $a_4 h_7$.

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81. Find n, so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n} (a \neq b)$ be HM beween a and b.

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82. Insert 6 harmonic means between 3 and $\frac{6}{23}$

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83. If $A^x = G^y = H^z$, where A, G, H are AM,GM and HM between two given quantities, then prove that x, y, z are in HP.

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84. The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation $2A + G^2 = 27$. Find two numbers.

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85. If the geometric mea is $\frac{1}{n}$ times the harmonic mean between two numbers, then show that the ratio of the two numbers is $1 + \sqrt{1 - n^2} : 1 - \sqrt{1 - n^2}$.

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86. Statement -1: If a,b,c are distinct real numbers in H.P, then $a^n + c^n > 2b^n \text{ for all } n \in N.$

Statement -2: $AM > GM > HM$

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87. If a, b, c, d be four distinct positive quantities in AP, then

(a) $bc > ad$

(b) $c^{-1}d^{-1} + a^{-1}b^{-1} > 2(b^{-1}d^{-1} + a^{-1}c^{-1} - a^{-1}d^{-1})$

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88. If a, b, c, d be four distinct positive quantities in GP,then

(a) $a + d > b + c$

(b) $c^{-1}d^{-1} + a^{-1}b^{-1} > 2(b^{-1}d^{-1} + a^{-1}c^{-1} - a^{-1}d^{-1})$

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89. If a, b, c, d be four disinct positive quantities in HP, then

(a) $a + d > b + c$

(b)$ad > bc$

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90. Find the sum of n terms of the series $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\cdot$

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91. The sum to infinity of the series

$1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots,$ is

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92. If the sum to infinity of the series $1+4x+7x^2+10x^3+\ldots\ldots\ldots\ldots$ is $\frac{35}{16}$ then $x=$ (A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{7}$ (D) $\frac{1}{7}$

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93. Find the sum of the series

$1+2^2x+3^2x^2+4^2x^3+\ldots.$ upto $\infty |x|<1.$

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94. Find the sum of the series $1^2 + 3^2 + 5^2 + \rightarrow n$ terms.

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95. Find the sum to n terms of the series $1 \cdot 2^2 + 2 \cdot 3^2 + 3 \cdot 4^2 + \dots$

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96. Find the sum of n terms of the series whose nth terms is

(i) $n(n-1)(n+1)$.

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97. Find the sum of n terms of the series whose nth terms is

(ii) $n^2 + 3^n$.

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98. Find the sum of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+$ up to n terms.

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99. Show that $S_n=\frac{n(2n^2+9n+13)}{24}$.

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100. Find the sum of the series $1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+\ldots.\ \text{upto n terms}$.

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101. Find sum to n terms of the series $1+(2+3)+(4+5+6)+\ldots..$

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102. Find the sum of the series $1 \cdot n + 2 \cdot (n-1) + 3 \cdot (n-2) + 4 \cdot (n-3) + \dots (n-1).2 + n.1$ also, find the coefficient of x^{n-1} in th cxpansion of $(1 + 2x + 3x^2 + \dots nx^{n-1})^2$.

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103. Find the nth term and sum to n tems of the following series: 1+5+12+22+.............

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104. Find the sum of the series $1 + 3 + 7 + 15 + 31 + \dots\dots$ n terms.

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105. Find the nth term of the series $= 1 + 4 + 10 + 20 + 35 + \ldots$

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106. Find the nth term of the series $1 + 5 + 18 + 58 + 179 + \ldots.$

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107. Sum of the following series to n term: $2 + 4 + 7 + 11 + 16 +$

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108. Find the sum of the following series to n terms $5 + 7 + 13 + 31 + 85 +$

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109. Find the nth term of the series $1 + 2 + 5 + 12 + 25 + 46 + \ldots..$

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110. Find the nth term of the series $2 + 5 + 12 + 31 + 86 + \ldots.$

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111. Find the nth term and sum to n terms of the series $12 + 40 + 90 + 168 + 280 + 432 + \ldots..$

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112. Find the sum upto n terms of the series $1 \cdot 4 \cdot 7 \cdot 10 \cdot 13 \cdot 16 + \ldots..$

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113. Find the sum to n terms of the series $\frac{1}{1\cdot 3\cdot 5\cdot 7\cdot 9}+\frac{1}{3\cdot 5\cdot 7\cdot 9\cdot 11}+\frac{1}{5\cdot 7\cdot 9\cdot 11\cdot 13}+\ldots\ldots.$ Also, find the sum to infinty terms.

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114. If $\sum_{r=1}^{n} T_r=\frac{n(n+1)(n+2)(n+3)}{12}$ where T_r denotes the rth term of the series. Find $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{T_r}$.

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115. If $yz+zx+xy=12, where x, y, z$ are positive values, find the greatest value of $xyz\cdot$

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116. Find the greatest value of x^3y^4 if $2x+3y=7$ and $x\geq 0, y\geq 0$.

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117. Find the least value of $3x + 4y$ for positive values of x and y, dubject to the condition $x^2y^3 = 6$.

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118. The minimum value of $P = bcx + cay + abz$, when $xyz = abc$, is

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119. If a, b, c are positive real numbers such that $a + b + c = 1$, then prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}$

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120. If a + b - 1, a $>$ 0,b $>$ 0, prove that $\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2 \geq \frac{25}{2}$

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121. If $b-c, 2b-\lambda, b-a$ are in HP, then $a-\frac{\lambda}{2}, b-\frac{\lambda}{2}, c-\frac{\lambda}{2}$ are is

A. AP

B. GP

C. HP

D. None of these

Answer: B

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122. Let $a_1, a_2, a_3, ...a_{10}$ are in GP with $a_{51} = 25$ and $\sum_{i=1}^{101} a_i = 125$ than the value of $\sum_{i=1}^{101}\left(\frac{1}{a_i}\right)$ equals.

A. 5

B. $\frac{1}{5}$

C. $\frac{1}{25}$

D. $\frac{1}{125}$

Answer: B

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123. If $x = 111....(20 digits), y = 333....(10 digits)$ and $z = 222.....2(10 digits), th$

equals.

A. $\frac{1}{2}$

B. 1

C. 2

D. 4

Answer: B

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124. Consider the sequence $1, 2, 2, 3, 3, 3, \ldots\ldots,$ where n occurs n times that occuts as 2011th terms is

A. 61

B. 62

C. 63

D. 64

Answer: C

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125. Let $S=\sum_{r=1}^{117}\frac{1}{2\left[\sqrt{r}\right]+1}$, when $[\cdot]$ denites the greatest integer function and if $S=\frac{p}{q}$, when p and q are co-primes, the value of $p+q$ is

A. 20

B. 76

C. 19

D. 69

Answer: B

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126. If a,b,c are non-zero real numbers, then the minimum value of the expression $\frac{\left(a^8+4a^4+1\right)\left(b^4+3b^2+1\right)\left(c^2+2c+2\right)}{a^4b^2}$ equals

A. 12

B. 24

C. 30

D. 60

Answer: C

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127. If the sum of m consecutive odd integers is m^4 , then the first integer is

A. $m^3 + m + 1$

B. $m^3 + m - 1$

C. $m^3 - m - 1$

D. $m^3 - m + 1$

Answer: D

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128. $\sum_{r=1}^{\infty} \frac{(4r+5)5^{-r}}{r(5r+5)}$

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{1}{25}$

D. $\frac{2}{25}$

Answer: A

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129. Let λ be the greatest integer for which $5p^2 - 16, 2p\lambda, \lambda^2$ are jdistinct consecutive terms of an AP, where $p \in R$. If the common difference of the Ap is $\left(\frac{m}{n}\right), n \in N$ and m ,n are relative prime, the value of $m + n$ is

A. 133

B. 138

C. 143

D. 148

Answer: C

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130. If $2\lambda, \lambda$ and $[\lambda^2 - 14], \lambda \in R - \{0\}$ and $[\cdot]$ denotes the gratest integer function are the first three terms of a GP in order, then the 51th term of the sequence, $1, 3\lambda, 6\lambda, 10\lambda, \ldots.,$ is

A. 5104

B. 5304

C. 5504

D. 5704

Answer: B

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131. The first three terms of a sequence are $3, 1, -1$. The next terms is

A. 2

B. -3

C. $-\frac{5}{27}$

D. $-\frac{5}{9}$

Answer: B

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132. There are two numbers a and b whose product is 192 and the quotient of AM by HM of their greatest common divisor and least common multiple is `(169)/(48). The smaller of a and b is

A. 2

B. 4

C. 6

D. 12

Answer: B::D

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133. Consider a series $\frac{1}{2}+\frac{1}{2^2}+\frac{2}{2^3}+\frac{3}{2^4}+\frac{5}{2^5}+\dots\dots\dots\dots+\frac{\lambda n}{2^n}$. If S_n denotes its sum to n tems, then S_n cannot be

A. 2

B. 3

C. 4

D. 5

Answer: A::B::C::D

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134. If $S_r = \sqrt{r + \sqrt{r + \sqrt{r + \sqrt{......\infty}}}}, r > 0$ then which the following is\are correct.

A. $S_r, S_6, S_{12}, S_{20},$ are in AP

B. S_4, S_9, S_{16} are irrational

C. $(2S_4 - 1)^2, (2S_5 - 1)^2, (2S_6 - 1)^2$ are in AP

D. S_2, S_{12}, S_{56} are in GP

Answer: A::B::C::D

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135. If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P and a,b -2c, are in G.P where a,b,c are non-zero then

A. $a^3 + b^3 + c^3 = 3abc$

B. $-2a, b, -2c$ are in AP

C. $-2a, b, -2c$ are inGP

D. $a^2, b^2, 4c^2$ are in GP

Answer: A::B::D

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136. The nature of the $S_n = 3n^2 + 5n$ series is

A. AP

B. GP

C. HP

D. AGP

Answer: A

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137. For the $S_n = 3n^2 + 5n$ sequence, the number 5456 is the ____th term

A. 153

B. 932

C. 707

D. 909

Answer: 909

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138. Sum of the squares of the first 3 terms of the given series is

A. 1100

B. 660

C. 799

D. 1000

Answer: B

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139. The number of terms common to the two A.P 's 3,7,11,......, 407 2,9,16,....709 is __________

A. 14

B. 21

C. 28

D. 35

Answer: A

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140. The 10th common term between the series 3+7+11+ . . . And 1+6+11+ . . . ., is

A. 189

B. 191

C. 211

D. 213

Answer: B

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141. The largest term common to the sequences $1, 11, 21, 31, \rightarrow 100$ terms and $31, 36, 41, 46, \rightarrow 100$ terms is 381 b. 471 c. 281 d. none of these

A. 281

B. 381

C. 471

D. 521

Answer: D

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142. If $x > 0, y > 0, z > 0$ and $x + y + z = 1$ then the minimum value of $\frac{x}{2-x} + \frac{y}{2-y} + \frac{z}{2-z}$ is

A. 0.2

B. 0.4

C. 0.6

D. 0.8

Answer: C

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143. If $\sum_{i=1}^{n} a_i^2 = \lambda, \forall a_i \geq 0$ and if greatest and least values of $\left(\sum_{i=1}^{n} a_i\right)^2$ are λ_1 and λ_2 respectively, then $(\lambda_1 - \lambda_2)$ is

A. $n\lambda$

B. $(n-1)\lambda$

C. $(n+2)\lambda$

D. $(n+1)\lambda$

Answer: B

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144. If sum of the mth powers of first n odd numbers is λ, $Aam > 1$, then

A. $\lambda < n^m$

B. $\lambda > n^m$

C. $\lambda < n^{m+1}$

D. $\lambda > n^{m+1}$

Answer: D

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145. A squence of positive terms $A_1, A_2, A_3, \ldots., A_n$ satisfirs the relation $A_{n+1} = \frac{3(1 + A_n)}{(3 + A_n)}$. Least integeral value of A_1 for which the sequence is decreasing can be

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146. When the ninth term of an AP is divided by its second term we get 5 as the quotient, when the thirteenth term ia devided ny sixth term the quotient is 2 and the remainderis 5, then the seonnd term is

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147. Match the following Column I to Column II

	Column I		Column II
(A)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_6 + a_{10} + a_{21} + a_{25} + a_{30} = 120$, then $\sum_{i=1}^{30} a_i$ is	(p)	400
(B)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_5 + a_9 + a_{13} + a_{17} + a_{21} + a_{25} = 112$, then $\sum_{i=1}^{25} a_i$ is	(q)	600
(C)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_4 + a_7 + a_{10} + a_{13} + a_{16} = 375$, then $\sum_{i=1}^{16} a_i$ is	(r)	800
		(s)	1000

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148. Match the following Column I to Column II

	Column I		Column II
(A)	If $a > 0, b > 0, c > 0$ and the minimum value of $a(b^2 + c^2) + b(c^2 + a^2) + c(a^2 + b^2)$ is λabc, then λ is	(p)	2
(B)	If a, b, c are positive, $a + b + c = 1$ and the minimum value of $\left(\frac{1}{a} - 1\right)\left(\frac{1}{b} - 1\right)\left(\frac{1}{c} - 1\right)$ is λ, then λ is	(q)	4
(C)	If $a > 0, b > 0, c > 0, s = a + b + c$ and the minimum value of $\frac{2s}{s-a} + \frac{2s}{s-b} + \frac{2s}{s-c}$ is $(\lambda - 1)$, then λ is	(r)	6
(D)	If $a > 0, b > 0, c > 0, a, b, c$ are in GP and the the minimum value of $\left(\frac{a}{b}\right)^{\lambda} + \left(\frac{c}{b}\right)^{\lambda}$ is 2, then λ is	(s)	8
		(t)	10

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149. Statement 1 The sum of first n terms of the series $1^2 - 2^2 + 3^2 - 4^2 \_ 5^2 - \ldots\ldots$ can be $= \pm \frac{n(n+1)}{2}$. Statement 2 Sum of first n narural numbers is $\frac{n(n+1)}{2}$

A. Statement 1 is true, Statement 2 is true, Statement 2 is a corrct explanation for Statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is false, Statement 2 is true.

Answer: A

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150. Statement 1 If a,b,c are three positive numbers in GP, then $\left(\frac{a+b+c}{3}\right)\left(\frac{3abc}{ab+bc+ca}\right)=(abc)^{\frac{2}{3}}$.

Statement 2 $(AM)(HM)=(GM)^2$ is true for positive numbers.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a corrct explanation for Statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is false, Statement 2 is true.

Answer: C

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151. Consider an AP with a as the first term and d is the common difference such that S_n denotes the sum to n terms and a_n denotes the nth term of the AP. Given that for some m,$n \in N$, $\frac{S_m}{S_n} = \frac{m^2}{n^2}(\neq n)$.

Statement 1 $d = 2a$ because

Statement 2 $\frac{a_m}{a_n} = \frac{2m+1}{2n+1}$.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a corrct explanation for Statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is false, Statement 2 is true.

Answer: C

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152. Statement 1 $1, 2, 4, 8, \ldots..$ is a GP,$4, 8, 16, 32, \ldots.$ is a GP and $1 + 4, 2 + 8, 4 + 16, 8 + 32, \ldots.$ is also a GP. Statement 2 Let general term of a GP with common ratio r be T_{k+1} and general term of another GP with common ratio r be T'_{k+1}, then the series whode general term $T''_{k+1} = T_{k+1} + T'_{k+1}$ is also a GP woth common ratio r.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a corrct explanation for Statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is false, Statement 2 is true.

Answer: A

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153. In a set of four numbers, the first three are in GP and the last three are in AP with a common difference of 6. If the number is same as the fourth, then find the four numbers.

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154. The natural number a for which $\sum_{k=1,n} f(a+k) = 16(2^n - 1)$ where the function f satisfies the relation $f(x+y) = f(x).\ f(y)$ for all natural numbers x,y and further $f(1) = 2$ is:- A) 2 B) 3 C) 1 D) none of these

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155. If n is a root of $x^2(1-ac)-x\left(a^2+c^2\right)-(1+ac)=0$ and if n harmonic means are inserted between a and c, find the difference between the first and the last means.

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156. A number consists of three digits which are in GP the sum of the right hand digits exceeds twice the middle digits by 1 and the sum of the left hand and middle digits is two thirs of the sum of the middle and right hand digits. Find the number.

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157. $S=\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j}1$

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158. Three numbers are in GP whose sum is 70. If the extremes be each multiplied by 4 and the mean by 5, then they will be in AP. Find the numbers.

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159. If the sum of m terms of an A.P. is equal to the sum of either the next n terms or the next p terms, then prove that $(m+n)\left(\frac{1}{m}-\frac{1}{p}\right)=(m+p)\left(\frac{1}{m}-\frac{1}{n}\right)$.

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160. Find the sum of the product of every pair of the first n of natural number

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161. If $l_n = \int_0^{\frac{\pi}{4}} \tan^n x dx$ show that $\frac{1}{l_2 + l_4}, \frac{1}{l_3 + l_5}, \frac{1}{l_4 + l_6}, \frac{1}{l_5 + l_7}, \dots$ from an AP. Find its common difference.

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162. If the sum of the terms of an infinitely decreasing GP is equal to the greatest value of the fuction $f(x) = x^3 + 3x - 9$ on the iterval $[-5, 3]$ and the difference between the first and second terms is $f'(0)$, then show that the common ratio of the progression is $\frac{2}{3}$.

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163. Solve the following equaions for x and y $\log_{10} x + \frac{1}{2}\log_{10} x + \frac{1}{4}\log_{10} x + \dots = y$ and $\frac{1 + 3 + 5 + \dots + (2y - 1}{4 + 7 + 10 + \dots + (3y - 1}$

.

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164. If $0 < x < \frac{\pi}{2}$ exp $\left[\left(\sin^2 x + \sin^4 x + \sin^6 x + ' \ldots . + \infty\right)\log_e 2\right]$ satisfies the quadratic equation $x^2 - 9x + 8 = 0$, find the value of $\frac{\sin x - \cos x}{\sin x + \cos x}$.

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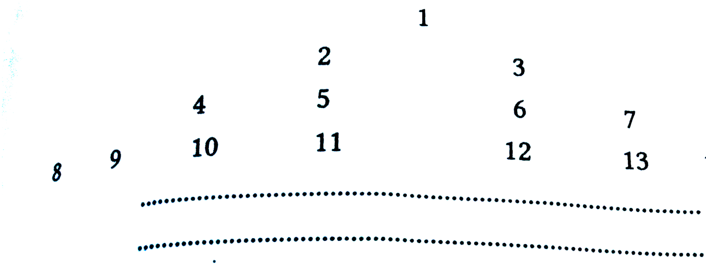
165. The natural numbers arearranged innthe form given below



The rth group containing 2^{r-1} numbers. Prove that sum of the numbers in the nth group is $2^{n-2}\left[2^n + 2^{n+1} - 1\right]$.

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166. If a,b,c are in HP, then prove that $\frac{a+b}{2a-b} + \frac{c+b}{2c-b} > 4$.

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167. Find the sum to n terms of the series: $\frac{1}{1+1^2+1^4}+\frac{1}{1+2^2+2^4}+\frac{1}{1+3^2+3^4}+$

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168. The value of xyz is 55 or $\frac{343}{55}$ according as the series a, x, y, z, b is an AP or HP. Find the alues of a and b given that they are positive integers.

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169. Find the sum of n terms of the series $1^3+3.2^2+3^3+3.4^2+5^3+3.6^2+\ldots\ldots$ when (i)n is odd (ii)n is even

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170. Find out the largest term of the sequence $\frac{1}{503}, \frac{4}{524}, \frac{9}{581}, \frac{16}{692}, \ldots.$

.

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171. IF $f(r) = 1 + \frac{1}{2} + \frac{1}{3} + \ldots. + \frac{1}{r}$ and $f(0) = 0,$ find $\sum_{r=1}^{n} (2r+1) f(r).$

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172. If the equation $x^4 - 4x^3 + ax^2 + bx + 1 = 0$ has four positive roots, fond the values of a and b.

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173. Evaluate $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n \cdot 3^m + m \cdot 3^n)}.$

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174. $\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{3^i 3^i 3^k}$

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175. Let $S_n, n = 1, 2, 3, \ldots$ be the sum of infinite geometric series, whose first term is n and the common ratio is $\frac{1}{n+1}$. Evaluate $\lim_{n \to \infty} \frac{S_1 S_n + S_2 S_{n-1} + S_3 S_{n-2} + \ldots + S_n S_1}{S_1^2 + S_2^2 + \ldots\ldots + S_n^2}$.

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176. The nth term of a series is given by $t_n = \frac{n^5 + n^3}{n^4 + n^2 + 1}$ and if sum of its n terms can be expressed as $S_n = a_n^2 + a + \frac{1}{b_n^2 + b}$ where a_n and b_n are the nth terms of some arithmetic progressions and a, b are some constants, prove that $\frac{b_n}{a_n}$ is a costant.

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Exercise For Session 1

1. First term of a sequence is 1 and the $(n+1)th$ term is obtained by adding $(n+1)$ to the nth term for all natural numbers n, the 6th term of the sequence is

A. 7

B. 13

C. 21

D. 27

Answer: C

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2. The first three terms of a sequence are $3, 3, 6$ and each term after the sum of two terms preceding it, then the $8th$ term of the sequence

A. 15

B. 24

C. 39

D. 63

Answer: D

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3. If $a_n = \sin\left(\frac{n\pi}{6}\right)$ then the value of $\sum a_n^2$

A. 2

B. 3

C. 4

D. 7

Answer: B

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4. If for a sequence $\{a_n\}$, $S_n = 2n^2 + 9n$, where S_n is the sum of n terms, the value of a_{20} is

A. 65

B. 75

C. 87

D. 97

Answer: C

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5. If $a_1 = 2$ and $a_n = 2a_{n-1} + 5$ for $n > 1$, the value of $\sum_{r=2}^{5} a_r$ is

A. 130

B. 160

C. 190

D. 220

Answer: C

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Exercise For Session 2

1. If nth term of the series $25 + 29 + 33 + 37......$ and $3 + 4 + 6 + 9 + 13 + ...$ are equal, then n equal

A. 11

B. 12

C. 13

D. 14

Answer: B

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2. The rth term of the series $2\left(\frac{1}{2}\right)+1\left(\frac{7}{13}\right)+1\left(\frac{1}{9}\right)+\frac{20}{23}+\ldots.$ is

A. $\frac{20}{5r+3}$

B. $\frac{20}{5r-3}$

C. $20(5r+3)$

D. $\frac{20}{5r^2+3}$

Answer: A

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3. In a certain AP, 5 times the 5th term is equal to 8 times the 8th term, its 13th term is

A. 0

B. -1

C. -12

D. -13

Answer: A

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4. If the 9th term of an AP is zero, then prove that its 29th term is twice its 19th term.

A. $1:2$

B. $2:1$

C. $1:3$

D. $3:1$

Answer: B

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5. If pth,qth and rth terms of an A.P. are a, b, c respectively, then show that

(i) a(q-r)+b(r-p)+c(p-q)=0

A. 1

B. -1

C. 0

D. $\frac{1}{2}$

Answer: C

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6. The $6th$ term of an AP is equal to 2, the value of the common difference of the AP which makes the product $a_7a_4a_5$ least is given by

A. $\frac{8}{5}$

B. $\frac{5}{4}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: C

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7. The sum of first $2n$ terms of an AP is α. and the sum of next n terms is β, its common difference is

A. $\frac{\alpha - 2\beta}{3n^2}$

B. $\frac{2\beta - \alpha}{3n^2}$

C. $\frac{\alpha - 2\beta}{3n}$

D. $\frac{2\beta - \alpha}{3n}$

Answer: B

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8. The sum of three numbers in AP is -3 and their product is 8, then sum of squares of the numbers is

A. 9

B. 10

C. 12

D. 21

Answer:

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9. Let S_n denote the sum of first n terms of an AP and $3S_n = S_{2n}$ What is $S_{3n} : S_n$ equal to? What is $S_{3n} : S_{2n}$ equal to?

A. 9

B. 6

C. 16

D. 12

Answer:

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10. The sum of the products of 2n numbers $\pm 1, \pm 2, \pm 3, \ldots, n$ taking two at time is

A. -65

B. 165

C. -55

D. 95

Answer:

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11. If $a_1, a_2, a_3, \ldots., a_n$ are in AP, where $a_i > 0$ for all I, the value of $\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots.. + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$ is

A. $\frac{1}{\sqrt{a_1}+\sqrt{a_n}}$

B. $\frac{1}{\sqrt{a_1}-\sqrt{a_n}}$

C. $\frac{n}{\sqrt{a_1}-\sqrt{a_n}}$

D. $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

Answer:

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Exercise For Session 3

1. The fourth ;seventh and last terms of a GP are 10; 80 and 2560 respectively . Find the first term and the no. of terms in the GP.

A. $\frac{4}{5}, 12$

B. $\frac{4}{5}, 10$

C. $\frac{5}{4}, 12$

D. $\frac{5}{4}, 10$

Answer: B

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2. If the first and the nth terms of a GP are a and b respectively and if P is the product of the first n terms, then P^2 is equal to

A. ab

B. $(ab)^{\frac{n}{2}}$

C. $(ab)^n$

D. None of these

Answer: C

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3. If $a_1, a_2, a_3 (a_1 > 0)$ are three successive terms of a GP with common ratio r, the value of r for which $a_3 > 4a_2 - 3a_1$ holds is given by

A. $1 < r < 3$

B. $-3 < r < -1$

C. $r < 1$ or $r > 3$

D. None of these

Answer: B

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4. If $x, 2x+2, 3x+3$ are the first three terms of a GP, then what is its fourth term?

A. 27

B. -27

C. 13.5

D. -13.5

Answer: C

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5. In a sequence of 21 terms the first 11 terms are in A.P. with common difference 2. and the lastterms are in G.P. with common ratio 2. If the middle tem of the A.P. is equal to themiddle term of the G.P., then the middle term of the entire sequence is

A. $-\frac{10}{31}$

B. $\frac{10}{31}$

C. $-\frac{32}{31}$

D. $\frac{32}{31}$

Answer: D

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6. Three distinct numbers x,y,z form a GP in that order and the numbers $7x+5y, 7y+5z, 7z+5x$ form an AP in that order. The common ratio of GP is

A. -4

B. -2

C. 10

D. 18

Answer:

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7. The sum to n terms of the series $11+103+1005+\ldots.$ is

A. $\frac{1}{9}(10^n-1)+n^2$

B. $\frac{1}{9}(10^n - 1) + 2n$

C. $\frac{10}{9}(10^n - 1) + n^2$

D. $\frac{10}{9}(10^n - 1) + 2n$

Answer:

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8. In a n increasing G.P. , the sum of the first and the last term is 66, the product of the second and the last but one is 128 and the sum of the terms is 126. How many terms are there in the progression?

A. 6

B. 8

C. 10

D. 12

Answer:

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9. If S_1, S_2, S_3be respectively the sum of n,2n and 3n terms of a GP, then $\frac{S_1(S_3 - S_2)}{(S_2 - S_1)^2}$ is equal to

A. 1

B. 2

C. 3

D. 4

Answer:

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10. If $|a| < 1|b| < 1$ and $|x| < 1$ then the solution of $\sin^{-1}\left(\frac{2a}{1+a^2}\right) - s\frac{\cos^{-1}(1-b^2)}{1+b^2} = \frac{\tan^{-1}(2x)}{1-x^2}$ is

A. $\frac{1}{(1-a)(1-b)}$

B. $\frac{1}{(1-a)(1-ab)}$

C. $\frac{1}{(1-b)(1-ab)}$

D. $\frac{1}{(1-a)(1-b)(1-ab)}$

Answer:

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11. If the sides of a triangle are in G.P., and its largest angle is twice the smallest, then the common ratio r satisfies the inequality `0

A. $0 < r < \sqrt{2}$

B. $1 < r < \sqrt{2}$

C. $1 < r < 2$

D. $r > \sqrt{2}$

Answer:

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12. If $ax^3 + bx^2 + cx + d$ is divisible by $ax^2 + c$, then a, b, c, d are in (a) AP (b) GP (c) HP

A. AP

B. GP

C. HP

D. None of these

Answer:

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13. If $(r)_n$, denotes the number $rrr...(ndigits)$, where $r = 1, 2, 3, ..., 9$ and $a = (6)_n, b = (8)_n, c = (4)_{2n}$, then

A. $a^2 + b + c = 0$

B. $a^2 + b - c = 0$

C. $a^2 + b2c = 0$

D. $a^2 + b - 9c = 0$

Answer:

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14. $0.4\overline{27}$ represents the rational number

A. $\frac{47}{99}$

B. $\frac{47}{110}$

C. $\frac{47}{999}$

D. $\frac{49}{99}$

Answer:

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15. If the product of three numbers in GP be 216 and their sum is 19, then the numbers are

A. $4, 6, 9$

B. $4, 7, 8$

C. $3, 7, 9$

D. None of these

Answer:

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Exercise For Session 4

1. If a,b,c are in AP and b,c,d be in HP, then

A. $ab = cd$

B. $ad = bc$

C. $ac = bd$

D. $abcd = 1$

Answer: C

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2. If a,b,c are in AP, then $\frac{a}{bc}, \frac{1}{c}, \frac{1}{b}$ are in

A. AP

B. GP

C. HP

D. None of these

Answer: C

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3. If a, b, c are in AP and a, b, d are in GP, show that a, $(a-b)$ and $(d-c)$ are in GP.

A. AP

B. GP

C. HP

D. None of these

Answer: C

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4. If $x, 1, z$ are in AP and $x, 2, z$ are in GP, then $x, 4, z$ will be in

A. AP

B. GP

C. HP

D. None of these

Answer: D

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5. If a,b,c are in GP,$a - b, c - a, b - c$ are in HP, then $a + 4b + c$ is equal to

A. 0

B. 1

C. -1

D. None of these

Answer: A

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6. if $(m+1)th, (n+1)th \text{ and } (r+1)th$ term of an AP are in GP.and m, n and r in HP. . find the ratio of first term of A.P to its common difference

A. $-\frac{2}{n}$

B. $\frac{2}{n}$

C. $-\frac{n}{2}$

D. $\frac{n}{2}$

Answer: A

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7. If a,b,c are in AP and a^2, b^2, c^2 are in HP, then

A. $a = b = c$

B. $2b = 3a + c$

C. $b^2 = \sqrt{\frac{ac}{8}}$

D. None of these

Answer:

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8. If a, b, c are in HP., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in

A. AP

B. GP

C. HP

D. None of these

Answer:

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9. If $\frac{x+y}{2}, y, \frac{y+z}{2}$ are in HP, then x,y, z are in

A. AP

B. GP

C. HP

D. None of these

Answer:

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10. if $\frac{a+b}{1-ab}, b, \frac{b+c}{1-bc}$ are in AP then $a, \frac{1}{b}, c$ are in

A. AP

B. GP

C. HP

D. None of these

Answer:

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Exercise For Session 5

1. If the AM of two positive numbers a and b $(a > b)$ is twice of their GM, then $a : b$ is

A. $2 + \sqrt{3} : 2 - \sqrt{3}$

B. $7 + 4\sqrt{3} : 7 - 4\sqrt{3}$

C. $2 : 7 + 4\sqrt{3}$

D. $2 : \sqrt{3}$

Answer: C

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2. Let α and β be two positive real numbers. Suppose A_1, A_2 are two arithmetic means; G_1, G_2 are tow geometrie means and $H_1 H_2$ are two harmonic means between α and β, then

A. $A_1 H_2$

B. $A_2 H_1$

C. G_1G_2

D. None of these

Answer: A

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3. The GM between -9 and -16, is

A. 12

B. -12

C. -13

D. None of these

Answer: A

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4. Let $n \in N, n > 25.$ Let A, G, H deonote te arithmetic mean, geometric man, and harmonic mean of 25 and n. The least value of n for which $A, G, H \in \{25, 26, n\}$ is a. 49 b. 81 c.169 d. 225

A. 49

B. 81

C. 169

D. 225

Answer: C

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5. If nine arithmetic means and nine harmonic means are inserted between 2 and 3 alternatively, then prove that $A + 6/H = 5$ (where A is any of the A.M.'s and H the corresponding H.M.).

A. 8

B. 9

C. 10

D. None of these

Answer: B

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6. If $H_1 . H_2 .... , H_n$ are n harmonic means between a and b$(\ne a)$, then the value of $\dfrac{H_1 + a}{H_1 - a} + \dfrac{H_n + b}{H_n - b}$=

A. n

B. $n + 1$

C. 2n

D. $2n - 2$

Answer: B

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7. The AM of teo given positive numbers is 2. If the larger number is increased by 1, the GM of the numbers becomes equal to the AM to the given numbers. Then, the HM of the given numbers is

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{1}{2}$

D. 2

Answer: B

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8. if $a, a_1, a_2, a_3, \ldots\ldots\ldots, a_{2n}, b$ are in $A.P.$ and $a, g_1, g_2, \ldots\ldots\ldots\ldots g_{2n}, b$ are in $G.P.$ and h is $H.M.$ of a, b then $\frac{a_1 + a_{2n}}{g_1 \cdot g_{2n}} + \frac{a_2 + a_{2n-1}}{g_2 \cdot g_{2n-1}} + \ldots\ldots\ldots\ldots + \frac{a_n + a_{n+1}}{g_n \cdot g_{n+1}}$ is equal

A. $\frac{2n}{h}$

B. 2nh

C. nh

D. $\frac{n}{h}$

Answer: B

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Exercise For Session 6

1. The sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\dots$ is equal to

A. $2^n - n - 1$

B. $1 - 2^{-n}$

C. $n + 26(-n) - 1$

D. $26(n) - 1$

Answer: B

Watch Video Solution

2. Prove that: $2^{\frac{1}{4}}.4^{\frac{1}{8}}, 8^{\frac{1}{16}}.\ 16^{\frac{1}{32}}........\infty = 2.$

A. 1

B. $\frac{3}{2}$

C. 2

D. $\frac{5}{2}$

Answer: D

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3. $1 + 3 + 7 + 15 + 31 + \ldots + \text{ to n terms}$

A. $2^{n+1} - n$

B. $2^{n+1} - n - 2$

C. $2^n - n - 2$

D. None of these

Answer: B

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4. 99^{th} term of the series $2 + 7 + 14 + 23...$

A. 9998

B. 9999

C. 10000

D. 100000

Answer: C

Watch Video Solution

5. Find the sum of the series

$1\cdot 2\cdot 3+2\cdot 3\cdot 4+3\cdot 4\cdot 5+\ldots.$ upto n terms .

A. $n(n+1)(n+2)$

B. $(n+1)(n+2)(n+3)$

C. $\frac{1}{4}n(n+1)(n+2)(n+3)$

D. $\frac{1}{4}(n+1)(n+2)(n+3)$

Answer: A

Watch Video Solution

6. $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots.+\frac{1}{n(n+1)}$ equals

A. $\frac{1}{n(n+1)}$

B. $\frac{n}{n+1}$

C. $\frac{2n}{n+1}$

D. $\frac{2}{n(n+1)}$

Answer: B

Watch Video Solution

7. Sum of the n terms of the series $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^3}+\ldots\ldots.$ is

A. $\frac{2n}{n+1}$

B. $\frac{4n}{n+1}$

C. $\frac{6n}{n+1}$

D. $\frac{9n}{n+1}$

Answer: A

Watch Video Solution

8. If $t_n=\frac{1}{4}(n+2)(n+3)$ for $n=1, 2, 3, \ldots.$ then $\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}+\ldots.+\frac{1}{t_{2003}}=$

A. $\frac{4006}{3006}$

B. $\frac{4003}{3007}$

C. $\frac{4006}{3008}$

D. $\frac{4006}{3009}$

Answer: C

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9. The value of $\frac{1}{(1+a)(2+a)}+\frac{1}{(2+a)(3+a)}+\frac{1}{(3+a)(4+a)}.......$ upto ∞ is (where, a is constant)

A. $\frac{1}{1+a}$

B. $\frac{2}{1+a}$

C. ∞

D. None of these

Answer: B

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10. If f is a function satisfying $f(x+y)=f(x)\times f(y)$ for all $x, y \in N$ such that $f(1)=3$ and $\sum_{x=1}^{n} f(x)=120$, find the value of n .

A. 4

B. 5

C. 6

D. None of these

Answer: C

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Exercise For Session 7

1. The minimum value of $4^x + 4^{2-x}, x \in R$ is

A. 0

B. 2

C. 4

D. 8

Answer: A

Watch Video Solution

2. If $0 < \theta < \pi$, then the minmum value of $\sin^3 \theta + \cos ec^3 \theta + 2$ is

A. 0

B. 2

C. 4

D. 8

Answer: C

Watch Video Solution

3. If a,b,c and d are four real numbers of the same sign, then the value of $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}$ lies in the interval

A. $[2, \infty)$

B. $[3, \infty)$

C. $(4, \infty)$

D. $[4, \infty)$

Answer: B

Watch Video Solution

4. If $0 < x < \frac{\pi}{2}$, then the minimum value of $2(\sin x + \cos x + \cos ec2x)^3$ is

A. 27

B. 13.5

C. 6.75

D. 0

Answer: D

Watch Video Solution

5. If $a+b+c=3$ and $a>0, b>0, c>0$ then the greatest value of $a^2b^2c^2$ is

A. $\dfrac{3^4 \cdot 2^{10}}{7^7}$

B. $\dfrac{3^{10} \cdot 2^4}{7^7}$

C. $\dfrac{3^2 \cdot 2^{12}}{7^7}$

D. $\dfrac{3^{12} \cdot 2^2}{7^7}$

Answer: C

Watch Video Solution

6. If $x+y+z=a$ and the minimum value of $\frac{a}{x}+\frac{a}{y}+\frac{a}{z}$ is 81, then the value of . λ is

A. $\frac{1}{2}$

B. 1

C. $\frac{1}{4}$

D. 2

Answer: C

Watch Video Solution

7. If a, b, c are three positive real numbers such that abc^2 has the greatest value $\frac{1}{64}$, then

A. $a=b=\frac{1}{2}, c=\frac{1}{4}$

B. $a = b = c = \frac{1}{3}$

C. $a = b = \frac{1}{4}, c = \frac{1}{2}$

D. $a = b = c = \frac{1}{4}$

Answer: A

Watch Video Solution

Exercise Single Option Correct Type Questions

1. If the number x,y,z are in H.P. , then $\frac{\sqrt{yz}}{\sqrt{y}+\sqrt{z}}, \frac{\sqrt{xz}}{\sqrt{x}+\sqrt{z}}, \frac{\sqrt{xy}}{\sqrt{x}+\sqrt{y}}$ are in

A. AP

B. GP

C. HP

D. None of these

Answer: A

Watch Video Solution

2. If $a_1, a_2, \ldots..,$ are in HP and $f_k = \sum_{r=1}^{n} a_r - a_k,$ then $2^{\alpha_1}, 2^{\alpha_2}, 2^{\alpha_3} 2^{\alpha_4}, \ldots..$ are in $\left\{ \text{where } \alpha_1 = \frac{a_1}{f_1}, \alpha_2 = \frac{a_2}{f_2}, \alpha_3 = \frac{a_3}{f_3}, \ldots.. \right\}.$

A. AP

B. GP

C. HP

D. None of these

Answer: D

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3. ABC is a right angled triangle in which $\angle B = 90^\circ$ and BC=a. If n points $L_1, L_2, \ldots\ldots, L_n$ on AB are such that AB is divided in $n+1$ equal parts and $L_1M_1, L_2M_2, \ldots\ldots, L_nM_n$ are line segments parallel to BC and $M_1, M_2, M_3, \ldots\ldots, M_n$ are on AC, the sum of the lenghts of $L_1M_1, L_2M_2, \ldots\ldots, L_nM_n$ is

A. $\dfrac{n(n+1)}{(2)}$

B. $\dfrac{a(n-1)}{2}$

C. $\dfrac{an}{2}$

D. Impossible to find from the given data

Answer: C

Watch Video Solution

4. Let S_n denotes the sum of the terms of n series $(1 \leq n \leq 9)$ $1 + 22 + 333 + \ldots.999999999$, is

A. $S_n - S_{n-1} = \frac{1}{9}(10^n - n^2 + n)$

B. $S_n = \frac{1}{9}(10^n - n^2 + 2n - 2)$

C. $9(S_n - S_{n-1}) = n(10^n - 1)$

D. None of these

Answer: C

Watch Video Solution

5. If a, b, c are in GP, then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

A. AP

B. GP

C. HP

D. None of these

Answer: A

Watch Video Solution

6. The sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\ldots.$ is equal to

A. 2^n-n-1

B. $1-2^{-n}$

C. $n+2^{-n}-1$

D. 2^n-1

Answer: C

Watch Video Solution

7. If in a triangle $PQR; \sin P, \sin Q, \sin R$ are in A.P; then

A. the altitudes are in AP

B. the altitudes are in HP

C. the medians are in GP

D. the medians are in AP

Answer: B

Watch Video Solution

8. Let $a_1, a_2, , a_{10}$ be in A.P. and h_1, h_2, h_{10} be in H.P. If $a_1 = h_1 = 2 and a_{10} = h_{10} = 3, then a_4 h_7$ is 2 b. 3 c. 5 d. 6

A. 2

B. 3

C. 5

D. 6

Answer: D

Watch Video Solution

9. If $I_n = \int_0^{\pi} \frac{1 - \sin 2nx}{1 - \cos 2x} dx$ then $I_1, I_2, I_3, \ldots$... are in

A. AP

B. GP

C. HP

D. None of these

Answer: A

Watch Video Solution

10. Show that If $a(b-c)x^2 + b(c-a)xy + c(a-b)y^2 = 0$ is a perfect square, then the quantities a, b, c are in harmonic progresiion

A. AP

B. GP

C. HP

D. None of these

Answer: C

Watch Video Solution

11. The sum to infinity of the series

$1+2\left(1-\frac{1}{n}\right)+3\left(1-\frac{1}{n}\right)^2+\ldots\ldots\ldots$, is

A. n^2

B. $n(n+1)$

C. $n\left(1+\frac{1}{n}\right)^2$

D. None of these

Answer: A

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12. If $\log_3 2, \log_3(2^x-5)$ and $\log_3\left(2^x-\frac{7}{2}\right)$ are in $A.P$, determine the value of x.

A. 2

B. 3

C. 4

D. $2, 3$

Answer: B

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13. If x,y,z be three positive prime numbers. The progression in which $\sqrt{x}, \sqrt{y}, \sqrt{z}$ can be three terms (not necessarily consecutive) is

A. AP

B. GP

C. HP

D. None of these

Answer: D

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14. If n is an odd integer greater than or equal to 1, then the value of

$n^3 - (n-1)^3 + (n-1)^3 - (n-1)^3 + ... + (-1)^{n-1}1^3$

A. $\frac{(n+1)^2(2n-1)}{4}$

B. $\frac{(n-1)^2(2n-1)}{4}$

C. $\frac{(n+1)^2(2n+1)}{4}$

D. None of these

Answer: A

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15. If the sides of a angled triangle are in A.P then the sines of the acute angles are

A. $\frac{3}{5}, \frac{4}{5}$

B. $\sqrt{3}, \frac{1}{3}$

C. $\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}+1}{2}}$

D. $\frac{\sqrt{3}}{2}, \frac{1}{2}$

Answer: A

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16. The $6th$ term of an AP is equal to 2, the value of the common difference of the AP which makes the product $a_7 a_4 a_5$ least is given by

A. $\frac{8}{5}$

B. $\frac{5}{4}$

C. $\frac{2}{3}$

D. None of these

Answer: C

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17. If the arithmetic progression whose common difference is nonzero the sum of first $3n$ terms is equal to the sum of next n terms. Then, find the ratio of the sum of the $2n$ terms to the sum of next $2n$ terms.

A. $\frac{1}{5}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. None of these

Answer: A

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18. The coefficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3)...(x-n)$ is

A. $\frac{n(n^2+2)(3n+1)}{24}$

B. $\frac{n(n^2-1)(3n+2)}{24}$

C. $\frac{n(n^2+1)(3n+4)}{24}$

D. None of these

Answer: B

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19. Consider the pattern shown below:

$$\begin{array}{llllll} \text{Row} & 1 & 1 & & & \\ \text{Row} & 2 & 3 & 5 & & \\ \text{Row} & 3 & 7 & 9 & 11 & \\ \text{Row} & 4 & 13 & 15 & 17 & 19 \end{array} \quad etc.$$

The number at the end of row 60 is

A. 3659

B. 3519

C. 3681

D. 3731

Answer: A

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20. Let a_n be the nth term of an AP, if $\sum_{r=1}^{100} a_{2r} = \alpha \text{ and } \sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the AP is

A. $\alpha - \beta$

B. $\beta - \alpha$

C. $\frac{\alpha - \beta}{2}$

D. None of these

Answer: D

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21. If a_1, a_2, a_3, a_4, a_5 are in HP, then $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$ is eqiual to

A. $2a_1a_5$

B. $3a_1a_5$

C. $4a_1a_5$

D. $6a_1a_5$

Answer: C

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22. If a,b,c and d are four positive real numbers such that abcd=1 , what is the minimum value of $(1+a)(1+b)(1+c)(1+d)$.

A. 1

B. 4

C. 16

D. 64

Answer: C

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23. If a,b,c are in AP and $(a+2b-c)(2b+c-a)(c+a-b)=\lambda abc$, then λ is

A. 1

B. 2

C. 4

D. None of these

Answer: C

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24. If $a_1, a_2, a_3.....$ are in GP with first term a and common rario r, then $\frac{a_1a_2}{a_1^2-a_2^2}+\frac{a_2a_3}{a_2^2-a_3^2}+\frac{a_3a_4}{a_3^2-a_4^2}+.....+\frac{a_{n-1}a_n}{a_{n-1}^2-a_n^2}$ is equal to

A. $\frac{nr}{1-r^2}$

B. $\frac{(n-1)r}{1-r^2}$

C. $\frac{nr}{1-r}$

D. $\frac{(n-1)r}{1-r}$

Answer: B

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25. If the sum of the first ten terms of an $A.P$ is four times the sum of its first five terms, the ratio of the first term to the common difference is:

A. $\frac{1}{2}$

B. 2

C. $\frac{1}{4}$

D. 4

Answer: A

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26. If $\cos(x-y)$, , $\cos x$ and $\cos(x+y)$ are in H.P., are in H.P., then $\cos x \cdot \sec\left(\frac{y}{2}\right)$=

A. $\pm\sqrt{2}$

B. $\frac{1}{\sqrt{2}}$

C. $-\frac{1}{\sqrt{2}}$

D. None of these

Answer: A

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27. If 11 AM's are inserted between 28 and 10, the number of integra AM's is

A. 5

B. 6

C. 7

D. 8

Answer: A

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28. If $x > 1, y > 1, z > 1$ are in GP, then $\frac{1}{1 + 1nx}, \frac{1}{1 + 1ny}, \frac{1}{1 + 1nz}$ are in (1998, 2M) AP (b) HP (c) GP (d) none of these

A. AP

B. GP

C. HP

D. None of these

Answer: C

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29. The minimum value of $\frac{(a^2+3a+1)(b^2+3b+1)(c^2 3c+1)}{abc}$ The minimum value of , where $a, b, c \in R$ is

A. $\frac{11^3}{2^3}$

B. 125

C. 25

D. 27

Answer: B

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30. Let $a_1, a_2, \ldots.$ be in AP and $q_1, q_2, \ldots.$ be in GP. If $a_1 = q_1 = 2$ and $a_{10} = q_{10} = 3$, then

A. $a_7 q_{19}$ is not an integer

B. $a_{19} q_7$ is an integer

C. $a_7 q_{19} = a_{19} = q_{10}$

D. None of these

Answer: C

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Exercise More Than One Correct Option Type Questions

1. Let $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{2^n - 1}$. Then

A. $a(100) < 100$

B. $a(100) > 100$

C. $a(200) > 100$

D. $a(200) < 100$

Answer: A::C

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2. If first and $(2n-1)^{th}$ terms of A.P., G.P. and H.P. are equal and their nth terms are a,b,c respectively, then

A. $a=b=c$

B. $a \geq b \geq c$

C. $a+c=b$

D. $ac-b^2=0$

Answer: B::D

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3. let $0 < \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\phi$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$

A. $xyz = xz + y$

B. $xyz = xy + z$

C. $xyz = x + y + z$

D. $xyz = yz + x$

Answer: B::C

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4. If a,b,c are in A.P. and a^2, b^2, c^2 are in H.P. then which of the following could and true (A) $-\frac{a}{2}, b, care \in G.P.$ (B) $a = b = c$ (C) a^3, b^3, c^3 are in G.P. (D) none of these

A. $-\frac{a}{2}, b, c$ are in GP

B. $a = b = c$

C. a^2, b^2, c^2 are in GP

D. None of these

Answer: A::B

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5. The next term of the G.P. $x, x^2 + 2, and x^3 + 10$ is $\frac{729}{16}$ b. 6 c. 0 d. 54

A. 0

B. 6

C. $\frac{729}{16}$

D. 54

Answer: C::D

Watch Video Solution

6. Consecutive odd integers whose sum is $25^2 - 11^2$ are

A. $n = 14$

B. $n = 16$

C. first odd number is 23

D. last odd number is 49

Answer: A::C::D

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7. The G.M. of two positive numbers is 6. Their arithmetic mean A and harmonic mean H satisfy the equation $90A + 5H = 918$, then A may be equal to (A) $\frac{5}{2}$ (B) 10 (C) 5 (D) $\frac{1}{5}$

A. $\frac{1}{5}$

B. 5

C. $\frac{5}{2}$

D. 10

Answer: A::D

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8. If the sum to n terms of the series $\frac{1}{1\cdot3\cdot5\cdot7}+\frac{1}{3\cdot5\cdot7\cdot9}+\frac{1}{5\cdot7\cdot9\cdot11}+\dots\dots$ is $\frac{1}{90}-\frac{\lambda}{f(n)}$, then find $f(0)$, $f(1)$ and $f(\lambda)$

A. $f(0)=15$

B. $f(1)=105$

C. $f(\lambda)=\frac{640}{27}$

D. $\lambda=\frac{1}{3}$

Answer: A::B::C

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9. For the series, $S=1+\frac{1}{(1+3)}(1+2)^2+\frac{1}{(1+3+5)}(1+2+3)^2+\frac{1}{(1+3+5+7)}($ +... 7th term is 16 7th term is 18 Sum of first 10 terms is $\frac{505}{4}$ Sum of first 10 terms is $\frac{45}{4}$

A. 7th term is 16

B. 7th term is 18

C. sum of first 10 terms is $\frac{505}{4}$

D. sum of first 10 terms is $\frac{405}{4}$

Answer: A::C

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10. Let $E=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+$ Then, $E<3$ b. $E>3/2$ c. $E>2$ d. $E<2$

A. $E<3$

B. $E>\frac{3}{2}$

C. $E<2$

D. $E>2$

Answer: B::C

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11. Let $S_n (n \leq 1)$ be a sequence of sets defined by $S_1\{0\}, S_2 = \left\{\frac{3}{2}, \frac{5}{2}\right\}, S_3 = \left\{\frac{15}{4}, \frac{19}{4}, \frac{23}{4}, \frac{27}{4}\right\}$, ... then

A. third element in S_{20} is $\frac{439}{20}$

B. third element in S_{20} is $\frac{431}{20}$

C. sum of the element in S_{20} is 589

D. sum of the element in S_{20} is 609

Answer: A::C

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12. Which of the following sequences are unbounded?

A. $\left(1 + \frac{1}{n}\right)^n$

B. $\left(\frac{2n+1}{n+2}\right)$

C. $\left(1+\frac{1}{n}\right)^{n^2}$

D. $\tan n$

Answer: C::D

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13. Let a sequence $\{a_n\}$ be defined by $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \ldots\ldots \frac{1}{3n}$, then

A. $a_2 = \frac{11}{12}$

B. $a_2 = \frac{19}{20}$

C. $a_{n+1} - a_n = \frac{(9n+5)}{(3n+1)(3n+2)(3n+3)}$

D. $a_{n+1} - a_n = \frac{-2}{3(n+1)}$

Answer: B::C

Watch Video Solution

14. Let

$$S_n(x) = \left(x^{n-1} + \frac{1}{x^{n-1}}\right) + 2\left(x^{n-2} + \frac{1}{x^{n-2}}\right) + \ldots\ldots + (n-1)\left(x + \frac{1}{x}\right)$$

, then

A. $S_1(x) = 1$

B. $S_1(x) = x + \frac{1}{x}$

C. $S_{100}(x) = \frac{1}{x^{99}}\left(\frac{x^{100}-1}{x-1}\right)^2$

D. $S_{100}(x) = \frac{1}{x^{100}}\left(\frac{x^{100}-1}{x-1}\right)^2$

Answer: A::C

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15. All the terms of an AP are natural numbers and the sum of the first 20 terms is greater than 1072 and lss than 1162.If the sixth term is 32, then

A. first term is 7

B. first term is 12

C. common difference is 4

D. common difference is 5

Answer: A::D

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Exercise Passage Based Questions

1. S_n be the sum of n terms of the series $\frac{8}{5}+\frac{16}{65}+\frac{24}{325}+\dots\dots$

The value of $\lim_{n \to \infty} S_n$ is

A. 0

B. $\frac{1}{2}$

C. 2

D. 4

Answer: C

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2. S_n be the sum of n terms of the series $\frac{8}{5}+\frac{16}{65}+\frac{24}{325}+\ldots\ldots$

The seveth term of the series is

A. $\frac{56}{2505}$

B. $\frac{56}{6505}$

C. $\frac{56}{5185}$

D. $\frac{107}{9605}$

Answer: D

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3. S_n be the sum of n terms of the series $\frac{8}{5}+\frac{16}{65}+\frac{24}{325}+\ldots\ldots$

The value of S_8, is

A. $\frac{288}{145}$

B. $\frac{1088}{545}$

C. $\frac{81}{41}$

D. $\frac{107}{245}$

Answer: A

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4. Two consecutive numbers from $1, 2, 3, \ldots\ldots n$ are removed. The arithmetic mean of the remaining numbers is $\left.\frac{105}{4}\right)$.

The value of n lies in

A. $(41, 51)$

B. $(52, 62)$

C. $(63, 73)$

D. $(74, 84)$

Answer: A

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5. Two consecutive numbers from 1,2,3 ...., n are removed .The arithmetic mean of the remaining numbers is 105/4

The sum of all numbers

A. are less than 10

B. lies between 10 to 30

C. lies between 30 to 70

D. greater than 70

Answer: A

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6. Two consecutive numbers from 1,2,3 ...., n are removed .The arithmetic mean of the remaining numbers is 105/4

The sum of all numbers

A. less than 1000

B. lies between 1200 to 1500

C. greater than 1500

D. None of these

Answer: B

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7. There are two sets A and B each of which consists of three numbers in AP whose sum is 15 and where D and d are the common differences such that $D = 1 + d, d > 0$. If p=7(q-p)`, where p and q are the product of the nymbers respectively in the two series.

The value of p is

A. 105

B. 140

C. 175

D. 210

Answer: A

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8. There are two sets A and B each of which consists of three numbers in AP whose sum is 15 and where D and d are the common differences such that $D = 1 + d, d > 0$. If $p = 7(q - p)$, where p and q are the product of the nymbers respectively in the two series.

The value of q is

A. 200

B. 160

C. 120

D. 80

Answer: C

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9. There are two sets A and B each of which consists of three numbers in AP whose sum is 15 and where D and d are the common differences such that $D = 1 + d, d > 0$. If $p = 7(q - p)$, where p and q are the product of the nymbers respectively in the two series.

The value of $7D + 8d$ is

A. 37

B. 22

C. 67

D. 52

Answer: B

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10. There are two sets A and B each of which consists of three numbers in GP whose product is 64 and R and r are the common ratios sich that

$R = r + 2$. If $\frac{p}{q} = \frac{3}{2}$, where p and q are sum of numbers taken two at a time respectively in the two sets.

The value of p is

A. 66

B. 72

C. 78

D. 84

Answer: D

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11. There are two sets A and B each of which consists of three numbers in GP whose product is 64 and R and r are the common ratios sich that $R = r + 2$. If $\frac{p}{q} = \frac{3}{2}$, where p and q are sum of numbers taken two at a time respectively in the two sets.

The value of q is

A. 54

B. 56

C. 58

D. 60

Answer: B

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12. There are two sets A and B each of which consists of three numbers in GP whose product is 64 and R and r are the common ratios sich that $R = r + 2$. If $\frac{p}{q} = \frac{3}{2}$, where p and q are sum of numbers taken two at a time respectively in the two sets.

The value of $r^R + R^r$ is

A. 5392

B. 368

C. 32

D. 4

Answer: C

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13. The numbers $1, 3, 6, 10, 15, 21, 28. \ldots \ldots$ are called triangular numbers. Let t_n denotes the bth triangular number such that $t_n = t_{n-1} + n, \forall n \geq 2.$

The value of t_{50} is

A. 1075

B. 1175

C. 1275

D. 1375

Answer: C

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14. The numbers $1, 3, 6, 10, 15, 21, 28. \ldots \ldots$ are called triangular numbers. Let t_n denotes the n^{th} triangular number such that $t_n = t_{n-1} + n, \ \forall n \geq 2$. The number of positive integers lying between t_{100} and t_{101} are

A. 99

B. 100

C. 101

D. 102

Answer: B

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15. The numbers $1, 3, 6, 10, 15, 21, 28. \ldots \ldots$ are called triangular numbers. Let t_n denotes the bth triangular number such that $t_n = t_{n-1} + n, \ \forall n \geq 2$.

If $(m + 1)$ is the nth triangular number, then $(n - m)$ is

A. $1+\sqrt{(m^2+2m)}$

B. $1+\sqrt{(m^2+2)}$

C. $1+\sqrt{(m^2+m)}$

D. None of these

Answer: D

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16. Let $A_1, A_2, A_3, ...A_m$ be arithmetic means between -3 and 828 and $G_1, G_2, G_3, ...G_n$ be geometric means between 1 and 2187. Produmt of geometrimc means is 3^{35} and sum of arithmetic means is 14025.

The valjue of n is

A. 45

B. 30

C. 25

D. 10

Answer: D

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17. Let $A_1, A_2, A_3, ...A_m$ be arithmetic means between -3 and 828 and $G_1, G_2, G_3, ...G_n$ be geometric means between 1 and 2187. Produmt of geometrimc means is 3^{35} and sum of arithmetic means is 14025.

The value of m is

A. 17

B. 34

C. 51

D. 68

Answer: B

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18. Let $A_1, A_2, A_3, ...A_m$ be arithmetic means between -3 and 828 and $G_1, G_2, G_3, ...G_n$ be geometric means between 1 and 2187. Produmt of geometrimc means is 3^{35} and sum of arithmetic means is 14025.

The value of m is

A. 2044

B. 1022

C. 511

D. None of these

Answer: D

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19. Suppose α, β are roots of $ax^2 + bx + c = 0$ and γ, δ are roots of $Ax^2 + Bx + C = 0.$

If $\alpha, \beta, \gamma, \delta$ are in AP, then common difference of AP is

A. $\frac{1}{4}\left(\frac{b}{a}-\frac{B}{A}\right)$

B. $\frac{1}{3}\left(\frac{b}{a}-\frac{B}{A}\right)$

C. $\frac{1}{2}\left(\frac{c}{a}-\frac{B}{A}\right)$

D. $\frac{1}{3}\left(\frac{c}{a}-\frac{C}{A}\right)$

Answer: A

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20. Suppose α, β are roots of $ax^2+bx+c=0$ and γ, δ are roots of $Ax^2+Bx+C=0$.

If a,b,c are in GP as well as $\alpha, \beta, \gamma, \delta$ are in GP, then A,B,C are in

A. AP only

B. GP only

C. AP and GP

D. None of these

Answer: B

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21. Suppose α, β are roots of $ax^2 + bx + c = 0$ and γ, δ are roots of $Ax^2 + Bx + C = 0.$

If $\alpha, \beta, \gamma, \delta$ are in GP, then common ratio of GP is

A. $\sqrt{\left(\frac{bA}{aB}\right)}$

B. $\sqrt{\left(\frac{aB}{bA}\right)}$

C. $\sqrt{\left(\frac{bC}{cB}\right)}$

D. $\sqrt{\left(\frac{cB}{bC}\right)}$

Answer: B

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22. Suppose p is the first of $n(n > 1)$ arithmetic means between two positive numbers a and b and q the first of n harmonic means between the same two numbers.

The value of p is

A. $\dfrac{na+b}{n+1}$

B. $\dfrac{nb+a}{n+1}$

C. $\dfrac{na-b}{n+1}$

D. $\dfrac{nb-a}{n+1}$

Answer: A

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23. Suppose p is the first of $n(n > 1)$ arithmetic means between two positive numbers a and b and q the first of n harmonic means between the same two numbers.

The value of q is

A. $\frac{(n-1)ab}{nb+a}$

B. $\frac{(n+1)ab}{nb+a}$

C. $\frac{(n-1)ab}{na+b}$

D. $\frac{(n-1)ab}{na+b}$

Answer: B

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24. If p is the first of the n arithmetic means between two numbers and q be the first on n harmonic means between the same numbers. Then, show that q does not lie between p and $\left(\frac{n+1}{n-1}\right)^2 p$.

A. q lies between p and $\left(\frac{n+1}{n-1}\right)^2 p$

B. q lies between p and $\left(\frac{n+1}{n-1}\right)p$

C. q does not lie between p and $\left(\frac{n+1}{n-1}\right)^2 p$

D. q does not lie between p and $\left(\frac{n+1}{n-1}\right)p$

Answer: C

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Exercise Single Integer Answer Type Questions

1. Let a,b,c,d be positive real numbers with $a < b < c < d$. Given that a,b,c,d are the first four terms of an AP and a,b,d are in GP. The value of $\frac{ad}{bc}$ is $\frac{p}{q}$, where p and q are prime numbers, then the value of q is

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2. If the coefficient of x in the expansion of $\prod_{r=1}^{110}(1+rx)$ is $\lambda(1+110)\left(1+10+10^2\right)$, then the value of λ is

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3. A 3 digit palindrome is a 3 digit number (not starting with zero) which reads the same backwards as forwards For example, 242. The sim of all even 3 digit palindromes is $2^{n_1} \cdot 3^{n_2} \cdot 5^{n_3} \cdot 7^{n_4} \cdot 11^{n_5} \cdot$ value of $n_1 + n_2 + n_3 + n_4 + n_5$ is

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4. If n is a positive integer satisfying the equation $2 + (6 \cdot 2^2 - 4 \cdot 2) + (6 \cdot 3^2 - 4 \cdot 3) + \ldots\ldots + (6 \cdot n^2 - 4 \cdot n) = 140$ then the value of n is

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5. Let $S(x) = 1 + x - x^2 - x^3 + x^4 + x^5 - x^6 - x^7 + \ldots\ldots + \infty$, where $0 < x < 1$. If $S(x) = \dfrac{\sqrt{2}+1}{2}$, then the value of $(x+1)^2$ is

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6. The sequence $a_1, a_2, a_3, \ldots\ldots.,$ is a geometric sequence with common ratio r. The sequence $b_1, b_2, b_3, \ldots\ldots.,$ is also a geometric sequence. If $b_1 = 1, b_2 = \sqrt[4]{7} - \sqrt[4]{28} + 1, a_1 = \sqrt[4]{28}$ and $\sum_{n=1}^{\infty} \frac{1}{a_n} = \sum_{n=1}^{\infty} \frac{1}{b_n}$, then the value of $(1 + r^2 + r^4)$ is

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7. Let (a_1, b_1) and (a_2, b_2) are the pair of real numbers such that 10,a,b,ab constitute an arithmetic progression. Then, the value of $\left(\frac{2a_1a_2 + b_1b_2}{10}\right)$ is

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8. If one root of $Ax^3 + Bx^2 + Cx + D = 0, D \neq 0$ is the arithmetic mean of the other two roots, then the relation $2B^2 + \lambda ABC + \mu A^2D = 0$ holds good. Then, the value of $2\lambda + \mu$ is

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9. If $|x| > 1$,then sum of the series $\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{2^2}{1+x^4} + \frac{2^3}{1+x^8} + \dots\dots \text{upto n terms } \infty$ is $\frac{1}{x-\lambda}$,then the value of λ is

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10. Three non-zero real numbers form a AP and the squares of these numbers taken in same order form a GP. If the possible common ratios are $(2 \pm \sqrt{k})$ where $k \in N$, then the value of $\frac{k}{8} - \frac{8}{k}\Big)$ is (where [] denites the greatest integer function).

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Exercise Matching Type Questions

1. Match the following Column I and Column II

Column I		Column II	
(A)	a, b, c, d are in AP, then	(p)	$a+d>b+c$
(B)	a, b, c, d are in GP, then	(q)	$ad>bc$
(C)	a, b, c, d are in HP, then	(r)	$\frac{1}{a}+\frac{1}{d}>\frac{1}{b}+\frac{1}{c}$
		(s)	$ad<bc$

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2. Match the following Column I and Column II

Column I		Column II	
(A)	For an AP $a_1, a_2, a_3, \ldots, a_n, \ldots$; $a_1 = \frac{5}{2}$; $a_{10} = 16$. If $a_1 + a_2 + \ldots + a_n = 110$, then '$n$' equals	(p)	9
(B)	The interior angles of a convex non-equiangular polygon of 9 sides are in AP. The least positive integer that limits the upper value of the common difference between the measures of the angles in degrees is	(q)	10
(C)	For an increasing GP, $a_1, a_2, a_3, \ldots, a_n, \ldots$; $a_6 = 4a_4$; $a_9 - a_7 = 192$, if $a_4 + a_5 + a_6 + \ldots + a_n = 1016$, then n equals	(r)	11
		(s)	12

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3. Match the following Column I and Column II

Column I		Column II	
(B)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 195$, $\alpha = a_2 + a_7 + a_{18} + a_{23}$ and $\beta = 2(a_3 + a_{22}) - (a_8 + a_{17})$, then	(q)	$\alpha + 2\beta = 260$
(C)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_7 + a_{10} + a_{21} + a_{24} + a_{30} = 225$, $\alpha = a_2 + a_7 + a_{24} + a_{29}$ and $\beta = 2(a_{10} + a_{21}) - (a_3 + a_{28})$, then	(r)	$\alpha + 2\beta = 220$
		(s)	$\alpha - \beta = 5\lambda, \lambda \in I$
		(t)	$\alpha + \beta = 15\mu, \mu \in I$

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Matching Type Questions

	Column I		Column II
(A)	If $a_1, a_2, a_3, \ldots$ are in AP and $a_1 + a_4 + a_7 + a_{14} + a_{17} + a_{20} = 165$, $\alpha = a_2 + a_6 + a_{15} + a_{19}$ and $\beta = 2(a_9 + a_{12}) - (a_3 + a_{18})$, then	(p)	$\alpha = 2\beta$

1.

	Column I		Column II
(A)	If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where a, b, c are non-zero numbers, then a, b, c are in	(p)	AP
(B)	If $17a^2 + 13b^2 + 5c^2 = (3ab + 15bc + 5ca)$, where a, b, c are non-zero numbers, then a, b, c are in	(q)	GP
(C)	If $a^2 + 9b^2 + 25c^2 = abc\left(\frac{15}{a} + \frac{5}{b} + \frac{3}{c}\right)$, where a, b, c are non-zero numbers, then a, b, c are in	(r)	HP
(D)	If $(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + ca) + (a^2 + b^2 + c^2) \le 0$, where a, b, c, p are non-zero numbers, then a, b, c are in		

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Exercise Statement I And Ii Type Questions

1. Statement 1 $4, 8, 16,$ are in GP and 12,16,24 are in HP.

Statement 2 If middle term is added in three consecutive terms of a GP, resultant will be in HP.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: A

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2. Satement 1 If the nth termn of a series is $2n^3 + 3n^2 - 4,$ then the second order differences must be an AP.

Statement 2 If nth term of a series is a polynomial of degree m, then mth order differences of series are constant.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: A

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3. Statement 1 The sum of the products of numbers $\pm a_1, \pm a_2, \pm a_3, \ldots\ldots \pm a_n$ taken two at a time is $-\sum_{i=1}^{n} a_i^2$.

Statement 2 The sum of products of numbers $a_1, a_2, a_3, \ldots\ldots a_n$ taken two at a time is denoted by $\sum_{1 \leq i < j \leq n} \sum a_i a_j$.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: B

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4. Statement 1 $a+b+c=18(a, b, c>0)$, then the maximum value of abc is 216.

Statement 2 Maximum value occurs when $a=b=c$.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: A

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5. If $4a^2 + 9b^2 + 16c^2 = 2(3ab + 6bc + 4ca)$, where a,b,c are non-zero real numbers, then a,b,c are in GP.

Statement 2 If $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2 = 0$, then $a_1 = a_2 = a_3, \forall a_1, a_2, a_3 \in R$.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: D

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6. Statement 1 If a and b be two positive numbers, where $a > b$ and $4 \times GM = 5 \times HM$ for the numbers. Then, $a = 4b$.

Statement 2 $(AM)(HM) = (GM)^2$ is true for positive numbers.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: C

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7. Statement 1 The difference between the sum of the first 100 even natural numbers and the sum of the first 100 odd natural numbers is 100. Statement 2 The difference between the sum opf the first n even natural numbers and sum of the first n odd natural numbers is n.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: A

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Exercise Subjective Type Questions

1. The pth, $(2p)th$ and $(4p)th$ terms of an AP, are in GP, then find the common ratio of GP.

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2. Find the sum of n terms of the series $(a+b)+(a^2+ab+b^2)+(a^3+a^2b+ab^2+b^3)+\ldots\ldots$ where $a \neq 1, b \neq 1$ and $a \neq b$.

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3. The sequence of odd natural numbers is divided into groups $1, 3, 5, 7, 9, 11, \ldots$ and so on. Show that the sum of the numbers in nth group is n^3.

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4. Let a,b,c are respectively the sums of the first n terms, the next n terms and the next n terms of a GP. Show that a,b,c are in GP.

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5. If the first four terms of an arithmetic sequence are $a, 2a, b$ and $(a-6-b)$ for some numbers a and b, find the sum of the first 100 terms of the sequence.

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6. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\infty=\frac{\pi^2}{6}$ then value of $1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\infty=$

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7. If the arithmetic mean of $a_1, a_2, a_3, \ldots\ldots.. a_n$ is a and $b_1, b_2, b_3, \ldots\ldots.. b_n$ have the arithmetic mean b and $a_i + b_i = 1$ for $i = 1, 2, 3, \ldots\ldots.. n,$ prove that $\sum_{i=1}^{n} (a_i - a)^2 + \sum_{i=1}^{n} a_i b_i = nab.$

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8. If $a_1, a_2, a_3, \ldots\ldots..$ is an arithmetic progression with common difference 1 and $a_1 + a_2 + a_3 + \ldots + a_{98} = 137$, then find the value of $a_2 + a_4 + a_6 + \ldots + a_{98}.$

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9. If $t_1 = 1, t_r - t_{r-1} = 2^{r-1}, r \geq 2$, find $\sum_{r=1}^{n} t_r.$

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10. Prove that $I_1, I_2, I_3 \ldots$ form an AP, if

(i) $I_n = \int_0^{\pi} \frac{\sin 2nx}{\sin x} dx$

(ii) $I_n = \int_0^{\pi} \left(\frac{\sin nx}{\sin x}\right)^2 dx.$

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11. Consider the sequence $S = 7 + 13 + 21 + 31 + \ldots\ldots + T_n$, find the value of T_{70}.

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12. Find value of $\left(x + \frac{1}{x}\right)^3 + \left(x^2 + \frac{1}{x^2}\right)^3 + \ldots\ldots\ldots + \left(x^n + \frac{1}{x^n}\right)^3.$

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13. If a_m be the mth term of an AP, show that $a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots. + a_{2n-1}^2 - a_{2n}^2 = \frac{n}{(2n-1)}\left(a_1^2 - a_{2n}^2\right).$

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14. If three unequel numbers are in HP and their squares are in AP, show that they are in the ratio $1+\sqrt{3}: -2: 1-\sqrt{3}$ or $1-\sqrt{3}: -2: 1+\sqrt{3}$.

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15. If $a_1, a_2, a_3, \ldots\ldots.., a_n$ are in AP with $a_1 = 0$, prove that $\frac{a_3}{a_2} + \frac{a_4}{a_3} + \ldots\ldots + \frac{a_n}{a_{n-1}} - a_2\left(\frac{1}{a_2} + \frac{1}{a_3} + \ldots\ldots. + \frac{1}{a_{n-2}}\right) = \frac{a_{n-1}}{a_2} + \frac{}{a}$.

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16. Balls are arranged in rows to form an equilateral triangle. The first row consists of ome ball, the second row of two balls and so on. If 669 more balls are added,then all the balls can be arranged in the shape of a square and each of the sides, then contains 8 balls less than each side of the triangle. Determine the initial number of balls.

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17. If $\theta_1, \theta_2, \theta_3, \ldots\ldots, \theta_n$ are in AP whose common difference is d, then show that

$\sin d\{\sec\theta_1\sec\theta_2 + \sec\theta_2\sec\theta_3 + \ldots\ldots + \sec\theta_{n-1}\sec\theta_n\} = \tan\theta_n - \tan\theta$

.

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18. Show that,

$\left(1+5^{-1}\right)\left(1+5^{-2}\right)\left(1+5^{-4}\right)\left(1+5^{-8}\right)\ldots\ldots\left(1+5^{-2n}\right) = \frac{5}{4}\left(1-5^{-2(n+}\right.$

.

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19. Evaluate $S = \sum_{n=0}^{n} \frac{2^n}{\left(a^{2^n}+1\right)}$ (where $a > 1$).

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20. Find the sum to infinite terms of the series

$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\ldots\ldots\ldots+\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right)+\ldots\ldots$

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21. Find the sum to n terms, whose nth term is

$\tan[\alpha+(n-1)\beta]\tan(\alpha+n\beta)$.

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22. If $\sum_{r=1}^{n} T_r=\frac{n}{8}(n+1)(n+2)(n+3), \quad \text{find} \quad \sum_{r=1}^{n}\frac{1}{T_r}$.

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23. If s_1, s_2, s_3 denote the sum of n terms of 3 arithmetic series whose first terms are unity and their common difference are in H.P., Prove that

$n=\frac{2s_3s_1-s_1s_2-s_2s_3}{s_1-2s_2+s_3}$

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24. The friends whose ages from a G.P. divide a certain sum of money in proportion to their ages. If they do that three years later, when the youngest is halfg the age of the oldest, then he will receive 105 rupees more that the he gets now and the middle friends will get 15 reupees more that he gets now, then ages of the friends are

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Exercise Questions Asked In Previous 13 Years Exam

1. Let a, b, c be in A.P. and $|a| < 1, |b| < 1|c| < 1.$ If $x = 1 + a + a^2 + \ldots. \text{to } \infty, y = 1 + b + b^2 + \ldots. \text{to } \infty$ and $, z = 1 +$

, then x, y, z are in

A. AP

B. GP

C. HP

D. None of these

Answer: C

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2. If $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots(- 1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n = 1 - a_n$, then find the minimum natural number n, such that $b_n > a_n$

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3. If $a_1, a_2, a_3,$ be terms of an A.P. if $\frac{a_1 + a_2 + \ + a_p}{a_1 + a_2 + \ + a_q} = \frac{p^2}{q^2}, p \neq q, then \frac{a_6}{a_{21}}$ equals 41/11 b. 7/2 c. 2/7 d. 11/41

A. $\frac{41}{11}$

B. $\frac{7}{2}$

C. $\frac{2}{7}$

D. $\frac{11}{41}$

Answer: D

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4. If $a_1, a_2, a_3, \ldots . a_n$ are in H.P. and $a_1a_2 + a_2a_3 + a_3a_4 + \ldots\ldots a_{n-1}a_n = ka_1a_n$, then k is equal to

A. $n(a_1 - a_n)$

B. $(n-1)(a_1 - a_n)$

C. na_1a_n

D. $(n-1)a_1a_n$

Answer: D

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5. Let V_r denote the sum of the first r terms of an arithmetic progression (AP) whose first term is r and the common difference is $(2r-1)$. Let T_r=V_(r+1)-V_r-2 and Q_r =T_(r+1)-T_r for r=1,2T_r` is always (A) an odd number (B) an even number (C) a prime number (D) a composite num,ber

A. $\frac{1}{12}n(n+1)(3n^2-n+1)$

B. $\frac{1}{12}n(n+1)(3n^2+n+2)$

C. $\frac{1}{2}n(2n^2-n+1)$

D. $\frac{1}{3}(2n^3-2n+3)$

Answer: B

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6. Let V_r denote the sum of the first r terms of an arithmetic progression (AP) whose first term is r and the common difference is $(2r-1)$. Let T_r=V_(r+1)-V_r-2 and Q_r =T_(r+1)-T_r for r=1,2T_r` is always (A) an odd number (B) an even number (C) a prime number (D) a composite num,ber

A. an odd number

B. an even number

C. a prime number

D. a composite number

Answer: D

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7. Let V_r denote the sum of the first r terms of an arithmetic progression (AP) whose first term is r and the common difference is $(2r-1)$. Let T_r=V_(r+1)-V_r-2 and Q_r =T_(r+1)-T_r for r=1,2 $Which o \neq of the follow \in g is ac$ or $rect statement? (A)$Q_1, Q_2, Q_3..............,$are \in A.P. with common d \Leftrightarrow erence 5(B)$Q_1, Q_2, Q_3..............,$are \in A.P. with common d \Leftrightarrow erence 6(C)$Q_1, Q_2, Q_3..............,$are \in A.P. with common d \Leftrightarrow erence 11(D)$Q_1=Q_2=Q_3`

A. $Q_1, Q_2, Q_3, \ldots..$ are in AP with common difference 5

B. $Q_1, Q_2, Q_3, ...$ are in AP with common differemce 6

C. $Q_1, Q_2, Q_3, ...$ are in AP with common difference 11

D. $Q_1 = Q_2 = Q_3 = ...$

Answer: B

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8. Let A_1, G_1, H_1denote the arithmetic, geometric and harmonic means respectively, of two distinct positive numbers. For $n > 2$, let A_{n-1}, G_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as $A_n, G_N, H_N,$ respectively.

A. $G_1 > G_2 > G_3 > ....$

B. $G_1 < G_2 < G_3 < ....$

C. $G_1 = G_2 = G_3 = ...$

D. $G_1 < G_3 < G_5 < ....$ and G_(2)gtG_(4)gtG_(6)gt.......

Answer: C

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9. Let A_1, G_1, H_1denote the arithmetic, geometric and harmonic means respectively, of two distinct positive numbers. For $n > 2$, let A_{n-1}, G_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as $A_n, G_N, H_N,$ respectively.

A. $A_1 > A_2 > A_3 > \ldots\ldots.$

B. $A_1 < A_2 < A_3 < \ldots\ldots.$

C. $A_1 > A_3 > A_5 > \ldots\ldots. \text{ and } A_2 < A_4 < A_6 < \ldots\ldots.$

D. $A_1 < A_3 < A_5 < \ldots\ldots. \text{ and } A_2 > A_4 > A_6 > \ldots\ldots.$

Answer: A

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10. Let A_1, G_1, H_1denote the arithmetic, geometric and harmonic means respectively, of two distinct positive numbers. For $n > 2$, let A_{n-1}, G_{n-1} and H_{n-1} has arithmetic, geometric and harmonic means as $A_n, G_N, H_N,$ respectively.

A. $H_1 > H_2 > H_3 > \ldots\ldots.$

B. $H_1 < H_2 < H_3 < \ldots\ldots.$

C. $H_1 > H_3 > H_5 > \ldots\ldots. \text{ and } H_2 < H_4 < H_6 < \ldots\ldots.$

D. $H_1 < H_3 < H_5 < \ldots\ldots. \text{ and } H_2 > H_4 > H_6 > \ldots\ldots.$

Answer: B

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11. in a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals-

A. $\frac{1}{2}(1-\sqrt{5})$

B. $\frac{1}{2}\sqrt{5}$

C. $\sqrt{5}$

D. $\frac{1}{2}(\sqrt{5}-1)$

Answer: D

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12. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1 + , a_b = b_1 + a_2, b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

Statement -1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.

Statement -2: The numbers b_1, b_2, b_3, b_4 are in H.P.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: C

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13. The first two terms of a geometric progression add up to 12. The sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is (1) 4 (2) 12 (3) 12 (4) 4

A. -12

B. 12

C. 4

D. -4

Answer: A

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14. If the sum of first n terms of an AP is cn^2, then the sum of squares of these n terms is

A. $\frac{n(4n^2-1)c^2}{6}$

B. $\frac{n(4n^2+1)c^2}{3}$

C. $\frac{n(4n^2-1)c^2}{3}$

D. $\frac{n(4n^2+1)c^2}{6}$

Answer: C

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15. $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\frac{14}{3^4}+\ldots\infty=$

A. 6

B. 2

C. 3

D. 4

Answer: C

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16. Let $S_k, k = 1, 2, , 100,$ denotes thesum of the infinite geometric series whose first term s $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$, then the value of $\frac{100^2}{100!} + \sum_{k=2}^{100} \left(k^2 - 3k + 1\right) S_k$ is ______.

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17. Le $a_1, a_2, a_3, , a_{11}$ be real numbers satisfying $a_2 = 15, 27 - 2a_2 > 0 and a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, , 11.$ If $\frac{a12 + a22 + \ldots + a112}{11} = 90,$ then the value of $\frac{a1 + a2 + \; + a11}{11}$ is equals to ______.

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18. A person is to cout 4500 currency notes. Let a_n denotes the number of notes he counts in the nth minute. If $a_1 = a_2 = \ldots\ldots. = a_{10} = 150$ and $a_{10}, a_{11}, \ldots\ldots,$ are in AP with common difference -2, then the time taken by him to count all notes is

A. 34 min

B. 125 min

C. 135 min

D. 24 min

Answer: A

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19. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is

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20. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of swrvice will be Rs. 11040 after

A. 19 months

B. 20 months

C. 21 months

D. 18 months

Answer: C

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21. Let a_n be the nth term of an AP, if $\sum_{r=1}^{100} a_{2r} = \alpha \text{ and } \sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the AP is

A. $\frac{\alpha - \beta}{200}$

B. $\alpha - \beta$

C. $\frac{\alpha - \beta}{100}$

D. $\beta - \alpha$

Answer: C

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22. If $a_1, a_2, a_3, \ldots\ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

A. 22

B. 23

C. 24

D. 25

Answer: D

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23. Statement 1 The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \ldots\ldots + (361 + 380 + 4$

is 8000.

Statement 2 $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$ for any natural number n.

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1

C. Statement 1 is true, Statement 2 is false

D. Statement 1 is false, Statement 2 is true

Answer: A

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24. If 100 times the 100th term of an AP with non-zero common difference equals the 50 times its 50th term, then the 150th term of this AP is

A. 150 times its 50th term

B. 150

C. zero

D. -150

Answer: C

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25. If x,y,z are in AP and $\tan^{-1} x, \tan^{-1} y, \tan^{-1} z$ are also in AP, then

A. $2x = 3y = 6z$

B. $6x = 3y = 2z$

C. $6x = 4y = 3z$

D. $x = y = z$

Answer: D

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26. The sum of first 20 terms of the sequence $0.7, 0.77, 0.777, \ldots\ldots$ is

A. $\frac{7}{9}\left(99 - 10^{-20}\right)$

B. $\frac{7}{81}\left(179 + 10^{-20}\right)$

C. $\frac{7}{9}\left(99 + 10^{-20}\right)$

D. $\frac{7}{81}\left(179 - 10^{-20}\right)$

Answer: B

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27. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} \cdot k^2$, then S_n can take value

A. 1056

B. 1088

C. 1120

D. 1332

Answer: A::D

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28. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k-20$ is equal to

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29. If $(10)^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + \dots\dots + (10)(11)^9 = k(10)^9$, then k is equal to

A. 100

B. 110

C. $\frac{121}{10}$

D. $\frac{441}{100}$

Answer: A

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30. Three positive numbers form an increasing GP. If the middle terms in this GP is doubled, the new numbers are in AP. Then, the common ratio of the GP is

A. $2-\sqrt{3}$

B. $2+\sqrt{3}$

C. $\sqrt{2}+\sqrt{3}$

D. $3+\sqrt{2}$

Answer: B

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31. Let a,b,c be positive integers such that $\frac{b}{a}$ is an integer. If a,b,c are in geometric progression and the arithmetic mean of a,b,c is $b+2$, the value of $\frac{a^2+a-14}{a+1}$ is

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32. The sum of first 9 terms of the series $\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+........$ is

A. 192

B. 71

C. 96

D. 142

Answer: C

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33. If m is the AM of two distinct real numbers l and n $(l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals

A. $4l^2m^2n^2$

B. $4l^2mn$

C. $4lm^2n$

D. $4lmn^2$

Answer: C

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34. Soppose that all the terms of an arithmetic progression (AP) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is $6 : 11$ and the seventh term lies between 130 and 140, then the common difference of this AP is

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35. If the $2_{nd}, 5_{th}$ and 9_{th} term of A.P. are in G.P. then find the common ratio of G.P.

A. 1

B. $\frac{7}{4}$

C. $\frac{8}{5}$

D. $\frac{4}{3}$

Answer: D

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36. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots\ldots,$ is $\frac{16}{5}$ m, then m is equal to: (1) 102 (2) 101 (3) 100 (4) 99

A. 100

B. 99

C. 102

D. 101

Answer: D

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37. Let $b_i > 1$ for $i = 1, 2, \ldots\ldots, 101.$ Suppose $\log_e b_1, \log_e b_2, \log_e b_3, \ldots\ldots\ldots \log_e b_{101}$ are in Arithmetic Progression (AP) with the common difference $\log_e 2$. Suppose $a_1, a_2, a_3, \ldots\ldots\ldots a_{101}$ are in AP. Such that, $a_1 = b_1$ and $a_{51} = b_{51}.$ If $t = b_1 + b_2 + \ldots\ldots\ldots + b_{51}$ and $s = a_1 + a_2 + \ldots\ldots\ldots + a_{51}$, then

A. $s > t$ and $a_{101} > b_{101}$

B. $s > t$ and $a_{101} < b_{101}$

C. $s < t$ and $a_{101} > b_{101}$

D. $s < t$ and $a_{101} < b_{101}$

Answer: B

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38. For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)\cdot$ Then : a, b and$c$ are in $A\dot{P}\cdot$ (2) a, b and c are in $G\dot{P}\cdot$ b, c and a are in $G\dot{P}\cdot$ (4) b, c and a are in $A\dot{P}\cdot$

A. a,b and c are in GP

B. b,c and a are in GP

C. b,c and a are in AP

D. a,b and c are in AP

Answer: C

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