MATHS

BOOKS - ARIHANT MATHS (HINGLISH)

SETS, RELATIONS AND FUNCTIONS

Examples

1. Write the set of the letter of the word 'ALLAGABAD'. Also find the number of subsets of this set.

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2. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. show that $B = C$

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3. Let A and B be any two sets. If for some set X, $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$, prove that A = B.

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4. If A and B are any two sets, prove that

P(A) = P(B) implies A = B.

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5. If A and B be two sets containing 6 and 3 elements respectively, what can be the minimum number of elements in $A \cup B$? Also, find the maximum number of elements in $A \cup B$.

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6. Suppose $A_1, A_2 \ldots . A_{30}$ are thirty sets each having 5 elements and $B_1 B_2 \ldots . B_n$ are n sets each having 3 elements ,Let $\bigcup_{i=1}^{30} A_1 = \bigcup_{j=1}^{n} B_j = s$ and each element of S belongs to exactly 10 of the A_1 and exactly 9 of the value of n.

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7. In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only? How many can speak Bengali only? How many can speak both Hindi and Bengali?

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8. in a survey of 500 TV views , it was found that 285 watch cricket , 195 watch football and 115 watch tannis . also , 45 watch both cricket and football, 70 watch both cricket and tennis and 50 watch football and

tennis . if 50 do not watch any game on tv . then the no. of views watch all three games is ?

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9. A class has 175 students. The following table shows the number of students studying one or more of the following subjects in this case.

Subjects	Number of students
Mathematics	100
Physics	70
Chemistry	46
Mathematics and Physics	30
Mathematics and Chemistry	28
Physics and Chemistry	23
Mathematics, Physics and Chemistry	18

How many students are enrolled in Mathematics alone, Physics alone and Chemistry alone? Are there students who have not offered any one of these subjects?

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10. In a pollution study of 1500 Indian rivers the following data were reported. 520 were pulleted by sulphur compounds, 335 were polluted by phosphates, 425 were polluted by crude oil, 100 were polluted by both crude oil and sulphur compounds, 180 were polluted by both sulphur compounds and phosphates, 150 were polluted by both phosphates and crude oil and 28 were polluted by sulphur compounds, phosphates and crude oil. How many of the rivers were polluted by atleast one of the three impurities?

How many of the rivers were polluted by exactly one of the three impurities?

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11. If A = {1, 2, 3} and B = {4, 5}, find $A \times B$, $B \times A$ and show that $A \times B \neq B \times A$.

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12. If A and B be two sets and $A \times B = \{(3, 3), (3, 4), (5, 2), (5, 4)\}$, find A and B.

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13. If A and B are two sets given in such a way that $A \times B$ consists of 6 elements and if three elements of $A \times B$ are (1,5), (2,3) and (3,5), what are the remaining elements?

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14. Let A = {1, 2, 3} and R = {(a,b): $a, b \in A$, a divides b and b divides a}. Show that R is an identity relation on A.

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15. Let A = {3,5}, B = {7,1}.

Let R = {(a,b): $a \in A, b \in B, a - b$ is even}.

Show that R is an universal relation from A to B.

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16. Prove that the relation R defined on the set N of natural numbers by xRy $\Leftrightarrow 2x^2 - 3xy + y^2 = 0$ is not symmetric but it is reflexive.

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17. Let R be a relation on the set N of natural numbers defined by $n\ R\ m$ iff n divides m. Then, R is (a) Reflexive and symmetric (b) Transitive and symmetric (c) Equivalence (d) Reflexive, transitive but not symmetric

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18. Statement-1: The relation R on the set $N \times N$ defined by (a, b) R (c, d) $\Leftrightarrow$ a+d = b+c for all a, b, c, d $\in$ N is an equivalence relation.

Statement-2: The intersection of two equivalence relations on a set A is an equivalence relation.

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19. A relation R on the set of complex numbers is defined by $z_1 R z_2$ if and oly if $\frac{z_1 - z_2}{z_1 + z_2}$ is real Show that R is an equivalence relation.

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20. Let R be a relation such that $R = \{(1, 4), (3, 7), (4, 5), (4, 6), (7, 6)\}$, find

(i) $R^{-1} O R^{-1}$ and (ii) $\left(R^{-1} O R\right)^{-1}$

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21. Let f : N $\to N: f(x) = 2x$ for all $x \in N$

Show that f is one -one and into.

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22. Let the function $f: R \to R$ be defined by $f(x) = \cos x, \forall x \in R.$

Show that f is neither one-one nor onto.

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23. Let $f: R \to R$ be defined by $f(x) = \cos(5x + 2)$. Is f invertible?

Justify your answer.

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24. If $f: R \to R, g: R \to R$ defined as $f(x) = \sin x$ and $g(x) = x^2$, then find the value of $(gof)(x)$ and $(fog)(x)$and also prove that $gof \neq fog.$

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25. If $f: R \to R$ and $g: R \to R$ be two mapping such that f(x) = sin x and g(x) = x^2, then find the values of (fog) $\frac{\sqrt{\pi}}{2}$ and $(\text{gof})\left(\frac{\pi}{3}\right)$.

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26. If the mapping f and g are given by

f = {(1, 2), (3, 5), (4, 1)}

and g = {(2, 3), (5, 1), (1, 3)},

write down pairs in the mapping fog and gof.

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27. Let I = $\{0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\}$ and $R = \{(a, b): (a - b)/4 = k, k \in l\}$ is an equivalence relation, find

equivalence class[0].

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28. Find congruent solutions of $155 \equiv 7 \pmod 4$.

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29. Find all congruent solutions of $8x \equiv 6 \pmod{14}$.

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30. Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The value of m and n is

A. 7, 6

B. 6, 3

C. 5, 1

D. 8, 7

Answer: B

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31. If $aN = \{ax : x \in N\}$ and $bN \cap cN = dN$, where $b, c \in N$ are relatively prime, then show that $d = bc$.

A. d = bc

B. c = bd

C. b = cd

D. None of these

Answer: A

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32. In a town of 10,000 families it was found that 40% family buy newspaper A, 20% buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2%families buy all the three newspapers, then find the number of families which buy A only

A. 3100

B. 3300

C. 2900

D. 1400

Answer: B

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33. Let R be the relation on the set R of all real numbers defined by a Rb Iff $|a-b| \leq 1$. Then R is

A. reflexive and symmetric

B. symmetric only

C. transitive only

D. anti-symmetric only

Answer: A

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34. The relation R defined in $A = \{1, 2, 3\}$ by aRb if $\left|a^2 - b^2\right| \leq 5$. Which of the following is faise

A. R = {(1,1),(2,2),(3,3),(2,1),(1,2),(2,3),(3,2)}

B. $R^{-1} = R$

C. Domain of R = {1, 2, 3}

D. Range of R = {5}

Answer: D

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35. If $f(x) = \frac{1}{(1-x)}, g(x) = f\{f(x)\}$ and $h(x) = f[f\{f(x)\}]$. Then the value of f(x).g(x).h(x) is

A. 6

B. -1

C. 1

D. 2

Answer: B

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36. If I is the set of integers and if the relation R is defined over I by aRb, iff a - b is an even integer, $a, b \in l$, the relation R is

A. reflexive

B. anti-symmetric

C. symmetric

D. equivalence

Answer: A::C::D

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37. If $f(x) = \frac{a - x}{a + x}$, the domain of $f^{-1}(x)$ contains

A. $(-\infty, \infty)$

B. $(-\infty, -1)$

C. $(-1, \infty)$

D. $(0, \infty)$

Answer: B::C::D

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38. If $f(x) = \frac{\sin([x]\pi)}{x^2 + x + 1}$, where $[.]$ denotes the greatest integer function, then $fisoneone$ $fis\neg one - oneandnon - constant$ $fisaconstantfunction\ noneofthese$

A. f is one-one

B. f is not one-one and non-constant

C. f is constant function

D. f is zero function

Answer: C::D

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39. If $A = \{x : |x| < 2\}, B = \{x : |x - 5| \leq 2\},$

$C = \{x : |x| > x\}$ and $D = \{x : |x| < x\}$

The number of integral values in $A \cup B$ is

A. 4

B. 6

C. 8

D. 10

Answer: C

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40. If $A = \{x : |x| < 2\}, B = \{x : |x - 5| \leq 2\},$

$C = \{x : |x| > x\}$ and $D = \{x : |x| < x\}$

The number of integral values in $A \cup C$ is

A. 1

B. 2

C. 3

D. 0

Answer: A

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41. If $A = \{x : |x| < 2\}, B = \{x : |x - 5| \leq 2\},$

$C = \{x : |x| > x\}$ and $D = \{x : |x| < x\}$

The number of integral values in $A \cap D$ is

A. 2

B. 4

C. 6

D. 0

Answer: D

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42. If $A = \{x : x^2 - 2x + 2 > 0\}$ and $B = \{x : x^2 - 4x + 3 \leq 0\}$

$A \cap B$ equals

A. $[1, \infty]$

B. [1,3]

C. $(-\infty, 3]$

D. $(-\infty, 1) \cup (3, \infty)$

Answer: B

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43. If $A = \{x : x^2 - 2x + 2 > 0\}$ and $B = \{x : x^2 - 4x + 3 \leq 0\}$

A - B equals

A. $(-\infty, \infty)$

B. (1,3)

C. $(3, \infty)$

D. $(-\infty, 1) \cup (3, \infty)$

Answer: D

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44. If $A = \{x : x^2 - 2x + 2 > 0\}$ and $B = \{x : x^2 - 4x + 3 \leq 0\}$

$A \cup B$ equals

A. $(-\infty, 1)$

B. $(3, \infty)$

C. $(-\infty, \infty)$

D. (1,3)

Answer: C

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45. If $f : R^+ \to A$, where $A = \{x : -5 < x < \infty\}$ is defined by f(x) = x^2 - 5 and if

$f^{-1}(13) = \left\{-\lambda\sqrt{(\lambda - 1)}, \lambda\sqrt{(\lambda - 1)}\right\}$, the value of λ is

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46. If $A = \{2, 3\}, B = \{4, 5\}$ and $C = \{5, 6\}$, then $n\{(A \times B) \cup (B \times C)\}$ is

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47.

	Column I		Column II
(A)	$R = \{(x, y) : x < y\ ;\ x, y \in N\}$	(p)	Reflexive
(B)	$S = \{(x, y) : x + y = 10\ ;\ x, y \in N\}$	(q)	Symmetric
(C)	$T = \{(x, y) : x = y$ or $x - y = 1\ ;\ x, y \in N\}$	(r)	Transitive
(D)	$U = \{(x, y) : x^y = y^x;\ x, y \in N\}$	(s)	Equivalence

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48. Statement-1 If $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then B = C.

Statement-2 $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

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49. Statement-1 If U is universal set and B = U - A, then n(B) = n(U) - n(A).

Statement-2 For any three arbitrary sets A, B and C, if C = A - B, then n(C) = n(A) - n(B).

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50. If $A = A \cup B$, prove that $B = A \cap B$.

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51. Find the smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$.

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52. If P, Q and R are the subsets of a set A, then prove that $R \times (P^c \cup Q^c)^c = (R \times P) \cap (R \times Q)$.

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53. Check the following relations R and S for reflexivity, symmetry and transitivity: aRb iff b is divisible by a, $a,\ b \in N$ (ii) $l_1\ S\, l_2$ iff $l_1 \perp l_2$, where l_1 and l_2 are straight lines in a plane.

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54. Check the following relations R and ρ for reflexive, symmetry and transitivity.

$\alpha\rho\beta$ iff α is perpendicular to β, where α and β are straight lines in a plane.

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55. Let f:[0, 1] $\rightarrow$ [0, 1] be defined by $f(x) = \dfrac{1-x}{1+x}, 0 \leq x \leq 1$ and $g: [0,1] \rightarrow [0,1]$ be defined by $g(x) = 4x(1-x), 0 \leq x \leq 1$

Determine the functions fog and gof.

Note that [0,1] stands for the set of all real members x that satisfy the condition $0 \leq x \leq 1$.

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56. If A, B are two sets, prove that $A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$.

Hence or otherwise prove that

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

where, n(A) denotes the number of elements in A.

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57. If $A = \left\{\theta : 2\cos^2\theta + \sin\theta \leq 2\right\}$, and $B = \left\{\theta : \frac{\pi}{2} \leq \theta \leq 3\frac{\pi}{2}\right\}$, then the region for $(A \cap B)$ is

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58. An investigator interviewed 100 students to determine their preferences for the three drinks, milk (M), coffee (C) and tea (T). He reported the following: 10 students has all three drinks M, C, T, 20 had M and C, 30 had C and T, 25 had M and T, 12 had M only, 5 had C only and 8 had T only. Using a Venn diagram, find how many did not take any of the three drinks?

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59. In a certain city, only two newspapers A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% reads A and B. It is also known that 30% of those who rad A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements while 50% of those who read both A and B, look into advertisement. What percent of the population read on advertisement?

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60. An analysis of 100 personal injury claims made upon a motor insurance company revealed that loss or injury in respect of an eye, an arm, a leg occurred in 30, 50 and 70 cases, respectively. Claims involving this loss or injury to two of these members numbered 44. How many claims involved loss or injury to all the three, we must assume that one or another of three members was mentioned in each of the 100 claims?

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61. Let N denote the set of all natural numbers and R be the relation on NxN defined by $(a, b)R(c, d)ad(b + c) = bc(a + d)$. Check whether R is an equivalence relation on NxN.

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62. The sets S and E are defined as given below:

$S = \{(x, y) : |x - 3| < 1 \text{ and } |y - 3| < 1\}$ and

$E = \{(x, y) : 4x^2 + 9y^2 - 32x - 54y + 109 \leq 0\}$.

Show that $S \subset E$.

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Exercise For Session 1

1. If $X = \{4^n - 3n - 1 : n \in N\}$ and $y = \{9(n-1) : n \in N\}$, then $X \cup Y$ equals

A. X

B. Y

C. N

D. None of these

Answer: B

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2. If $N_a = \{an : n \in N\}$, then $N_5 \cap N_7$ equals

A. N

B. N_5

C. N_7

D. N_{35}

Answer: D

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3. If A and B are two sets, then $A \cap (A \cup B)$ equals

A. A

B. B

C. ϕ

D. None of these

Answer: C

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4. Let U be the universal set and $A \cup B \cup C = \cup$ then $\{(A - B) \cup (B - C) \cup (C - A)\}'$ is equal to

A. $A \cup B \cup C$

B. $A \cap B \cap C$

C. $A \cup (B \cap C)$

D. $A \cap (B \cup C)$

Answer: B

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5. If A and B are two sets ,then $(A - B) \cup (B - A) \cup (A \cap B)$ equals

A. $A \cup B$

B. $A \cap B$

C. A

D. B'

Answer: A

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6. If A = {x : x is a multiple of 4} and B = {x : x is a multiple of 6}, then A $\subset$ B consists of all multiple of

A. 4

B. 8

C. 12

D. 16

Answer: C

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7. A set contains 2n+1 elements. The number of subsets of this set containing more than n elements :

A. 2^{n-1}

B. 2^n

C. 2^{n+1}

D. 2^{2n}

Answer: D

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8. Power set of the set $A = \{\phi, \{\phi\}\}$ is

A. A

B. $\{\phi, \{\phi\}, A\}$

C. $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

D. None of these

Answer: C

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9. Given n(U) = 20, n(A) = 12, n(B) = 9, $n(A \cap B) = 4$, where U is the universal set, A and B are subsets of U, then $n((A \cup B)')$ equals

A. 3

B. 9

C. 11

D. 17

Answer: A

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10. A survey shows that 63% of the Indians like cheese, whereas 76% like apples. If x % of the Indians like both cheese and apples, then x can be

A. 40

B. 65

C. 39

D. None of these

Answer: C

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11. In a class of 55 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics, 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. Find the number of students who have taken exactly one subject.

A. 6

B. 7

C. 9

D. 22

Answer: D

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Exercise For Session 2

1. If A = {2, 3, 5}, B = {2, 5, 6}, then $(A - B) \times (A \cap B)$ is

A. $\{(3, 2), (3, 3), (3, 5)\}$

B. $\{(3, 2), (3, 5), (3, 6)\}$

C. $\{(3, 2), (3, 5)\}$

D. None of these

Answer: C

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2. If $n(A) = 4, n(B) = 3, n(A \times B \times C) = 24$, then n(C) equals

A. 1

B. 2

C. 17

D. 288

Answer: B

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3. The relation R defined on the set of natural numbers as {(a,b) : a differs from b by 3} is given by

A. $\{(1, 4), (2, 5), (3, 6), \ldots\}$

B. $\{(4, 1), (5, 2), (6, 3), \ldots\}$

C. $\{(1, 3), (2, 6), (3, 9), \ldots\}$

D. None of these

Answer: B

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4. Let A be the set of the children in a family. The relation Â‘x is a brother of y' relation on A is

A. reflexive

B. anti-symmetric

C. transitive

D. equivalence

Answer: C

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5. Let n(A) = n, then the number of all relations on A, is

A. 2^n

B. $2^{n!}$

C. 2^{n^2}

D. None of these

Answer: C

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6. If S = {1, 2, 3,...,20}, K = {a, b, c, d}, G = {b, d, e, f}. The number of elements of $(S \times K) \cup (S \times G)$ is

A. 40

B. 100

C. 120

D. 140

Answer: C

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7. The relation R is defined on the set of natural numbers as {(a,b): a = 2b}, the R^{-1} is given by

A. $\{(2, 1)(4, 2)(6, 3), \ldots\}$

B. $\{(1, 2)(2, 4)(3, 6), \ldots\}$

C. R^{-1} is not defined

D. None of these

Answer: B

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8. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on a set A={1, 2, 3} is

A. reflexive but not symmetric

B. reflexive but not transitive

C. symmetric and transitive

D. Neither symmetric nor transitive

Answer: A

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9. The number of equivalence relations that can be defined on set {a, b, c}, is

A. 5

B. $3!$

C. 2^3

D. 3^3

Answer: A

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10. If R be a relation $<$ from $A = \{1, 2, 3, 4) \to B = (1, 3, 5)$ that is $(a, b) \in R \Leftrightarrow a < b$, then RoR^{-1} is

A. $\{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\}$

B. $\{(3, 1), (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)\}$

C. $\{(3, 3), (3, 5), (5, 3), (5, 5)\}$

D. $\{(3, 3), (3, 4), (4, 5)\}$

Answer: C

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Exercise For Session 3

1. The values of $bandc$ for which the identity of $f(x + 1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d, are$

$b = 2, c = 1$ (b) $b = 4, c = -1$ $b = -1, c = 4$ (d) $b = -1, c = 1$

A. b = 2, c = 1

B. b = 4, c = - 1

C. b = - 1, c = 4

D. b = - 1, c = 1

Answer: B

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2. If $f(x) = \frac{x-1}{x+1}$, then $f(f(ax))$ in terms of $f(x)$ is equal to $\frac{f(x)-1}{a(f(x)-1)}$ (b) $\frac{f(x)+1}{a(f(x)-1)}$ $\frac{f(x)-1}{a(f(x)+1)}$ (d) $\frac{f(x)+1}{a(f(x)+1)}$

A. $\frac{f(x)+a}{1+af(x)}$

B. $\frac{(a-1)f(x)+a+1}{(a+1)f(x)+a-1}$

C. $\frac{(a+1)f(x)+a-1}{(a-1)f(x)+a+1}$

D. None of these

Answer: C

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3. Let f be a function satisfying $f(x+y)=f(x)+f(y)$ for all $x, y \in R$. If $f(1)=k$ then $f(n), n \in N$ is equal to

A. k^n

B. nk

C. k

D. None of these

Answer: B

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4. Is $g=\{(1,1),(2,3),(3,5,),(4,7)\}$ a function? If this is described by the formula, $g(x)=\alpha x+\beta$, then what values should be assigned to

$\alpha and \beta$?

A. $\alpha = 1, \beta = 1$

B. $\alpha = 2, \beta = -1$

C. $\alpha = 1, \beta = -2$

D. $\alpha = -2, \beta = -1$

Answer: B

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5. The value of parameter α, for which the function $f(x) = 1 + \alpha x, \alpha \neq 0$ is the inverse of itself

A. -2

B. -1

C. 1

D. 2

Answer: B

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6. If $f(x) = (a - x^n)^{1/n}$, where a > 0 and $n \in N$, then fof (x) is equal to

A. a

B. x

C. x^n

D. a^n

Answer: B

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7. If $f(x) = \left(ax^2 + b\right)^3$, then find the function g such that $f(g(x)) = g(f(x))$.

A. $g(x) = \left(\frac{b - x^{1/3}}{a}\right)^{1/2}$

B. $g(x) = \frac{1}{(ax^2 + b)^3}$

C. $g(x) = (ax^2 + b)^{1/3}$

D. $g(x) = \left(\frac{x^{1/3} - b}{a}\right)^{1/2}$

Answer: D

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8. Which of the following functions from I to itself are bijections?

A. $f(x) = x^3$

B. $f(x) = x + 2$

C. $f(x) = 2x + 1$

D. $f(x) = x^2 + x$

Answer: B

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9. Let f : R - {n} $\rightarrow$ R be a function defined by $f(x) = \frac{x-m}{x-n}$, where $m \neq n$. Then,

A. f is one-one onto

B. f is one-one into

C. f is many-one onto

D. is many-one into

Answer: B

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10. If $f(x+2y, x-2y) = xy,$ then $f(x, y)$ equals

A. $\frac{x^2-y^2}{8}$

B. $\frac{x^2-y^2}{4}$

C. $\frac{x^2+y^2}{4}$

D. $\frac{x^2-y^2}{2}$

Answer: A

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Exercise Single Option Correct Type Questions

1. If A and B are two sets, then $A \cap (A \cup B)$ equals

A. A

B. B

C. ϕ

D. None of these

Answer: A

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2. If R is a relation from a set A to a set B and S is a relation from B to a set C, then the relation SoR

A. is from A to C

B. is from C to A

C. does not exist

D. None of these

Answer: A

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3. Let R = {(1, 3), (2, 2), (3, 2)} and S = {(2, 1), (3, 2), (2, 3)} be two relations on set A = {(1, 2, 3)}. Then, SoR is equal

A. {(2, 3), (3, 2), (2, 1)}

B. {(1, 3), (2, 2), (3, 2), (2, 1), (2, 3)}

C. {(3, 2), (1, 3)}

D. {(2, 3), (3, 2)}

Answer: A

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4. If X and Y are two sets, then $X \cap (Y \cap X)'$ equals

A. X

B. Y

C. ϕ

D. None of these

Answer: D

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5. For real numbers x and y, we write x* y, if $x - y + \sqrt{2}$ is an irrational number. Then, the relation * is an equivalence relation.

A. reflexive

B. symmetric

C. transitive

D. None of these

Answer: A

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6. Let f(x) = $(x+1)^2 - 1, (x \geq -1)$. Then, the set S = {x : f(x) = $f^{-1}(x)$} is

A. $\left\{0, -1, \frac{-3+i\sqrt{3}}{2}, \frac{-3-i\sqrt{3}}{2}\right\}, i = \sqrt{-1}$

B. {0, 1, -1}

C. {0, -1}

D. empty

Answer: C

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7. The number of elements of the power set of a set containing n elements is

A. 2^{n-1}

B. 2^n

C. $2^n - 1$

D. 2^{n+1}

Answer: B

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8. Which one of the following is not true?

A. $A - B \subseteq A$

B. $B' - A' \subseteq A$

C. $A \subseteq A - B$

D. $A \cap B' \subseteq A$

Answer: C

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9. If A = {1, 2, 3} and B = {3, 8}, then $(A \cup B) \times (A \cap B)$is

A. {(3, 1), (3, 2), (3, 3), (3, 8)}

B. {(1, 3), (2, 3), (3, 3), (8, 3)}

C. {(1, 2), (2, 2), (3, 3), (8, 8)}

D. {(8, 3), (8, 2), (8, 1), (8, 8)}

Answer: B

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10. Let A = {p, q, r}. Which of the following is an equivalence relation on A?

A. $R_1 = \{(p, q), (q, r), (p, r), (p, p)\}$

B. $R_2 = \{(r, q), (r, p), (r, r), (q, q)\}$

C. $R_3 = \{(p, p), (q, q), (r, r), (p, q)\}$

D. None of the above

Answer: D

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11. Let A = {x : x is a multiple of 3} and B = {x : x is a multiple of 5), then $A \cap B$ is given by

A. {3, 6, 9}

B. {5, 10, 15, 20, ...}

C. {15, 30, 45, ...}

D. None of these

Answer: C

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12. Let A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}, the $A \cup (B \cap C)$ is

A. {3}

B. {1, 2, 3, 4}

C. {1, 2, 5, 6}

D. {1, 2, 3, 4, 5, 6}

Answer: B

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13. Let A = {x, y, z), B = {u, v, w} and f : A $\rightarrow$ B be defined by f(x) = u, f(y) = v, f(z) = w. Then, f is

A. surjective but not injective

B. injective but not surjective

C. bijective

D. None of the above

Answer: C

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14. If A = {2, 4) and B = {3, 4, 5), then $(A \cap B) \times (A \cup B)$ is

A. {(2, 2), (3, 4), (4, 2), (5, 4)}

B. {(2, 3), (4, 3), (4, 5)}

C. {(2, 4), (3, 4), (4, 4), (4, 5)}

D. {(4, 2), (4, 3), (4, 4), (4, 5)}

Answer: D

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15. In the set X = {a, b, c, d}, which of the following functions in X?

A. $R_1 = \{(b, a), (a, b), (c, d), (a, c)\}$

B. $R_2 = \{(a, d), (d, c), (b, b), (c, c)\}$

C. $R_3 = \{(a, b), (b, c), (c, d), (b, d)\}$

D. $R_4 = \{(a, a), (b, b), (c, c), (a, d)\}$

Answer: B

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16. The composite mapping fog of the maps $f: R \to R, f(x) = \sin x$ and $g: R \to R, g(x) = x^2$, is

A. $x^2 \sin x$

B. $(\sin x)^2$

C. $\sin x^2$

D. $\sin x / x^2$

Answer: C

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17. Which of the following is the empty set

A. {x : x is a real number and $x^2 - 1 = 0$}

B. {x : x is a real number and $x^2 + 1 = 0$}

C. {x : x is a real number and $x^2 - 9 = 0$}

D. {x : x is a real number and $x^2 = x + 2$}

Answer: B

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18. In order that a relation R defined on a non-empty set A is an equivalence relation, it is sufficient, if R

A. is reflexive

B. is symmetric

C. is transitive

D. possesses all the above three properties

Answer: D

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19. If $A = \{p, q, r, s\}$ and $B = \{1, 2, 3\}$, find which of the following is not a function from A to B?

A. $R_1 = \{(p, 1), (q, 2), (r, 1), (s, 2)\}$

B. $R_2 = \{(p, 1), (q, 2), (r, 1), (s, 1)\}$

C. $R_3 = \{(p, 1), (q, 2), (r, 2), (r, 2)\}$

D. $R_4 = \{(p, 2), (q, 3), (r, 2), (s, 2)\}$

Answer: C

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20. For $n, m \varepsilon N$, $n \mid m$ means that n is a factor of m then relation $\mid$ is

A. reflexive and symmetric

B. transitive and symmetric

C. reflexive, transitive and symmetric

D. reflexive, transitive and not symmetric

Answer: D

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21. The solution of $8x = 6(\mod 14)$ is

A. [8],[6]

B. [8],[14]

C. [6],[13]

D. [8],[14],[16]

Answer: C

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22. Let A be a set containing 10 distinct elements,then the total number of distinct functions from A to A is

A. $10!$

B. 10^{10}

C. 2^{10}

D. $2^{10} - 1$

Answer: B

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23. Let A and B be two non empty subsets of set X such that A is not a subset of B, then:

A. A is a subset of the complement of B

B. B is a subset of A

C. A and B are disjoint

D. A and the complement of B are non-disjoint

Answer: D

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24. f and h are function from A $\rightarrow$ B, where A = {a, b, c, d} and B = {s, t, u} defined as follows

f(a) = t, f (b) = s, f(c) = s

f(d) = u, h(a) = s, h(b) = t

h(c) = s, h(a) = u, h(d) = u

Which one of the following statement is true?

A. f and h are functions

B. f is a function and h is not a function

C. f and h are not functions

D. None of the above

Answer: B

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25. Let I be the set of integer and f : I $\rightarrow$ I be defined as f(x) = $x^2, x \in I$, the function is

A. bijection

B. injection

C. surjection

D. None of these

Answer: D

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26. Which of the four statements given below is different from other?

A. $f: A \to B$

B. $f: x \to f(x)$

C. f is a mapping of A into B

D. f is a function of A into B

Answer: B

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27. Let $A = \{1, 2, \ldots, n\}$ and $B = \{a, b\}$. Then number of subjections from A into B is nP2 (b) $2^n - 2$ (c) $2^n - 1$ (d) nC2

A. $.^n P_2$

B. $2^n - 2$

C. $2^n - 1$

D. None of these

Answer: B

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28. Let $f: R \to R$ be defined by f(x)=3x-4. Then, f^{-1}(x) is

A. $\frac{1}{3}(x + 4)$

B. $\frac{1}{3}x - 4$

C. $3x + 4$

D. not defined

Answer: A

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29. $f: R \to R$ is a function defined by f(x)=$10x - 7$, if $g = f^{-1}$ then g(x)=

A. $\frac{1}{10x - 7}$

B. $\frac{1}{10x + 7}$

C. $\frac{x + 7}{10}$

D. $\frac{x - 7}{10}$

Answer: C

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30. Let R be a relation defined by R = {(a, b) : $a \geq b$}, where a and b are real numbers, then R is

A. reflexive, symmetric and transitive

B. reflexive, transitive but not symmetric

C. symmetric, transitive but not reflexive

D. neither transitive, nor reflexive, not symmetric

Answer: B

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31. If the sets A and B are defined are defined as $A = \{(x, y) : y = e^x, x \in R\}, B = \{(x, y) : y = x, x \in R\}$then

A. $B \subset A$

B. $A \subset B$

C. $A \cap B = \phi$

D. $A \cup B$

Answer: B

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32. If f : A $\rightarrow$ B is a bijective function, then $f^{-1}of$ is equal to

A. fof^{-1}

B. f

C. f^{-1}

D. I_A (the identity map of the set A)

Answer: D

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33. If $f(y) = \frac{y}{\sqrt{1-y^2}}, g(y) = \frac{y}{\sqrt{1+y^2}}$, then (fog) y is equal to

A. $\frac{y}{\sqrt{1-y^2}}$

B. $\frac{y}{\sqrt{1+y^2}}$

C. y

D. $\frac{(1-y^2)}{\sqrt{1-y^2}}$

Answer: C

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34. $f: R \to R$ is defined as $f(x) = 2x + |x|$ then $f(3x) - f(-x) - 4x =$

A. f(x)

B. $-f(x)$

C. $f(-x)$

D. 2f(x)

Answer: D

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35. Let R and S be two non-void relations on a set A. Which of the following statements is false?

A. R and S are transitive $\Rightarrow R \cup S$ is transitive

B. R and S are transitive implies $R \cap S$ is symmetric

C. R and S are symmetric implies $R \cup S$ is symmetric

D. R and S are reflexive implies $R \cap S$ is reflexive

Answer: A

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36. Let $f: R \to R, g: R \to R$ be two functions given by $f(x) = 2x - 3, g(x) = x^3 + 5$. Then $(fog)^{-1}$ is equal to

A. $\left(\frac{x+7}{2}\right)^{1/3}$

B. $\left(x-\frac{7}{2}\right)^{1/3}$

C. $\left(\frac{x-2}{7}\right)^{1/3}$

D. $\left(\frac{x-7}{2}\right)^{1/3}$

Answer: D

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37. If f(x)=ax+b and g(x)=cx+d, then f(g(x))=g(f(x)) is equivalent to

A. f(a) = g(c)

B. f(b) = g(b)

C. f(d) = g(b)

D. f(c) = g(a)

Answer: C

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38. If $f: R \overrightarrow{R} \overrightarrow{R}$ are two given functions, then prove that $2m \in$ if $(x) - g(x), 0 = f(x) - |g(x) - f(x)|$

A. f(x) + g(x) - |g(x) - f(x)|

B. f(x) + g(x) + |g(x) - f(x)|

C. f(x) - g(x) + |g(x) - f(x)|

D. f(x) - g(x) - |g(x) - f(x)|

Answer: D

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39. Let $f: R \overrightarrow{R} and g: R \overrightarrow{R}$ be two given functions such that f is injective and g is surjective. Then which of the following is injective? gof (b) fog (c) gog (d) none of these

A. gof

B. fog

C. gog

D. fof

Answer: D

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Exercise More Than One Correct Option Type Questions

1. Let L be the set of all straight lines in the Euclidean plane. Two lines l_1 and l_2 are said to be related by the relation R iff l_1 is parallel to l_2. Then, the relation R is not

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: A::B::C::D

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2. Let $X = \{1, 2, 3, 4\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is relations from X to Y

A. $R_1 = \{(x, y) : y = 2 + x, x \in X, y \in Y\}$

B. $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$

C. $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$

D. $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

Answer: A::B::C

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3. Let the function $f; R - \{-b\} \to R - \{1\}$ be defined by $f(x) = \dfrac{x + a}{x + b}, a \neq b,$ then

A. f is one-one but not onto

B. f is onto but not one-one

C. f is both one-one and onto

D. $f^{-1}(2) = a - 2b$

Answer: C::D

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Exercise Passage Based Questions

1. Let f and g be real valued functions defined as

$$f(x) = \begin{cases} 7x^2 + x - 8, & x \leq 1 \\ 4x + 5, & 1 < x \leq 7 \\ 8x + 3, & x > 7 \end{cases} \quad g(x) = \begin{cases} |x|, & x < -3 \\ 0, & -3 \leq x < 2 \\ x^2 + 4, & x \geq 2 \end{cases}$$

The value of (gof) (0) + (fog) (-3) is

A. -8

B. 0

C. 8

D. 16

Answer: B

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2. Let f and g be real valued functions defined as

$$f(x) = \begin{cases} 7x^2 + x - 8, & x \leq 1 \\ 4x + 5, & 1 < x \leq 7 \\ 8x + 3, & x > 7 \end{cases} \quad g(x) = \begin{cases} |x|, & x < -3 \\ 0, & -3 \leq x < 2 \\ x^2 + 4, & x \geq 2 \end{cases}$$

The value of 2(fog) (7) - (gof) (6) is

A. 9

B. 11

C. 13

D. 15

Answer: A

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3. Let f and g be real valued functions defined as

$$f(x)=\begin{cases}7x^2+x-8, & x\leq 1\\ 4x+5, & 1<x\leq 7\\ 8x+3, & x>7\end{cases} \quad g(x)=\begin{cases}|x|, & x<-3\\ 0, & -3\leq x<2\\ x^2+4, & x\geq 2\end{cases}$$

The value of 4(gof) (2) - (fog) (9) is

A. 0

B. 2

C. 5

D. 9

Answer: D

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4. R_1 on Z defined by $(a,b)\in R_1$ iff $|a-b|\leq 7$, R_2 on Q defined by $(a,b)\in R_2$ iff $ab=4$ and R_3 on R defined by

$(a, b) \in R_3$ iff $a^2 - 4ab + 3ab^2 = 0$

Relation R_2 is

A. reflexive and symmetric

B. symmetric and transitive

C. reflexive and transitive

D. equivalence

Answer: A

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5. R_1 on Z defined by $(a, b) \in R_1$ iff $|a - b| \leq 7$, R_2 on Q defined by $(a, b) \in R_2$ iff $ab = 4$ and R_3 on R defined by $(a, b) \in R_3$ iff $a^2 - 4ab + 3ab^2 = 0$

Relation R_2 is

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: B

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6. R_1 on Z defined by $(a, b) \in R_1$ iff $|a - b| \leq 7$, R_2 on Q defined by $(a, b) \in R_2$ iff $ab = 4$ and R_3 on R defined by $(a, b) \in R_3$ iff $a^2 - 4ab + 3b^2 = 0$ Relation R_3 is

A. reflexive

B. symmetric

C. transitive

D. equivalence

Answer: A

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Exercise Single Integer Answer Type Questions

1. In a group of 45 students, 22 can speak Hindi only and 12 can speak English only. If $(2\lambda + 1)$ student can speak both Hindi and English, the value of λ is

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2. If $A = \left\{x \mid \cos x > -\frac{1}{2} \text{ and } 0 \leq x \leq \pi\right\}$ and $B = \left\{x \mid \sin x > \frac{1}{2} \text{ and }\right.$, the value of $(\lambda + \mu)$ is

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3. If $S = R, A = \{x : -3 \leq x < 7\}$ and $B = \{x : 0 < x < 10\}$, the number of positive integers in $A \Delta B$ is

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4. Two finite sets have m and n elements. The total number of subsets of the first set is 48 more than the total number of subsets of the second set. The value of m - n is

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5. If two sets A and B are having 99 elements in common, the number of elements common to each of the sets $A \times B$ and $B \times A$ are $121\lambda^2$, the value of λ is

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Exercise Matching Type Questions

1. The functions defined have domain R

Column I		Column II	
(A)	$7x + 1$	(p)	onto $[-1, 1]$ but not one-one $[0, \pi]$
(B)	$\cos x$	(q)	one-one on $[0, \pi]$ but not onto R
(C)	$\sin x$	(r)	one-one and onto R
(D)	$1 + \ln x$	(s)	one-one on $(0, \infty)$

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2. The domain of the function f(x) is denoted by D_f

Column I		Column II	
(A)	$f(x) = \sqrt{(3 - x)} + \sin^{-1}\left(\frac{3 - 2x}{5}\right)$, then D_f is	(p)	$\bigcup_{k \in I} [2k\pi, (2k + 1)\pi]$
(B)	$f(x) = \log_{10}(1 - \log_{10}(x^2 - 5x + 16))$, then D_f is	(q)	$[-4, -\pi] \cup [0, \pi]$
(C)	$f(x) = \cos^{-1}\left(\frac{2}{2 + \sin x}\right)$, then D_f is	(r)	$(2, 3)$
(D)	$f(x) = \sqrt{(\sin x)} + \sqrt{(16 - x^2)}$, then D_f is	(s)	$[-1, 3]$

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Exercise Statement I And Ii Type Questions

1. Statement-1 If a set A has n elements, then the number of binary relations on $A = n^{n^2}$.

Statement-2 Number of possible relations from A to $A = 2^{n^2}$.

A. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

B. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true, Statement-2 is false

D. Statement-1 is false, Statement-2 is true

Answer: B

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2. Statement-1 If A = {x |g(x) = 0} and B = {x| f(x) = 0}, then $A \cap B$ be a root of $\{f(x)\}^2 + \{g(x)\}^2 = 0$

Statement-2 $x \in A \cap B \Rightarrow x \in A \text{ or } x \in B$.

A. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

B. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true, Statement-2 is false

D. Statement-1 is false, Statement-2 is true

Answer: C

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3. Statement-1 $P(A) \cap P(B) = P(A \cap B)$, where P(A) is power set of set A.

Statement-2 $P(A) \cup P(B) = P(A \cup B)$.

A. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

B. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true, Statement-2 is false

D. Statement-1 is false, Statement-2 is true

Answer: C

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4. Statement-1 If Sets A and B have three and six elements respectively, then the minimum number of elements in $A \cup B$ is 6.

Statement-2 $A \cap B = 3$.

A. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

B. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

C. Statement-1 is true, Statement-2 is false

D. Statement-1 is false, Statement-2 is true

Answer: A

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Exercise Subjective Type Questions

1. If A = {x : x is a natural number}, B = {x : x is an even natural number}, C = {x : x is an odd natural number} and D = {x : x is a prime number}, Find :

(i) $A \cap B$

(ii) $A \cap C$

(iii) $A \cap D$

(iv) $B \cap C$

(v) $B \cap D$

(vi) $C \cap D$.

A. $A \cap B$

B. $A \cap C$

C. $B \cap D$

D. $C \cap D$

Answer: A::B::C::D

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2. Let U be the set of all people and M = {Males},

S = {College students},

T = {Teenagers}, W = {People having height more than five feet}.

Express each of the following in the notation of set theory.

(i) College student having heights more than five feet

(ii) People who are not teenagers and have their height less five feet

(iii) All people who are neither males nor teenagers nor college students.

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3. The set X consists of all points within and on the unit circle $x^2 + y^2 = 1$, whereas the set Y consists of all points on and inside the rectangular boundary $x = 0, x = 1$, y = - 1 and y = 1. Determine $X \cup Y$ and $X \cap Y$. Illustrate your answer by diagrams.

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4. In a group of children, 35 play football out of which 20 play football only, 22 play hockey, 25 play cricket out of which 11 play cricket only. Out of these 7 play cricket and football but not hockey, 3 play football and hockey but not cricket and 12 play football and cricket both.

How many play all the three games ? How many play cricket and hockey but not football, how many play hockey only? What is the total number of children in the group?

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5. Of the members of three athletic teams in a certain school, 21 are in the basketball team, 26 in hockey team and 29 in the football team. 14 play hockey and basket ball, 15 play hockey and football, 12 play football and basketball and 8 play all the three games bow many members are there in all?

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6. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry , 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

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7. In a survey of population of 450 people, it is found that 205 can speak English, 210 can speak Hindi and 120 people can speak Tamil. If 100 people

can speak both Hindi and English, 80 people can speak both English and Tamil, 35 people can speak Hindi and Tamil and 20 people can speak all the three languages, find the number of people who can speak English but not a Hindi or Tamil. Find also the number of people who can speak neither English nor Hindi nor Tamil.

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8. A group of 123 workers went to a canteen for cold drinks, ice-cream and tea, 42 workers took ice-cream, 36 tea and 30 cold drinks. 15 workers purchased ice-cream and tea, 10 ice-cream and cold drinks, and 4 cold drinks and tea but not ice-cream, 11 took ice-cream and tea but not cold drinks. Determine how many workers did not purchase anything?

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9. Let n be a fixed positive integer. Define a relation R on Z as follows: $(a, b)Ra - b$ is divisible by n. Show that R is an equivalence relation on Z.

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10. Prove that a relation R defined on $N \times N$ where $(a, b)R(c, d) \Leftrightarrow ad = bc$ is an equivalence relation.

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11. The following relations are defined on the set of real numbers.

(i) $aRb \Leftrightarrow |a - b| > 0$

(ii) $aRb \Leftrightarrow |a| = |b|$

(iii) $aRb \Leftrightarrow |a| \geq |b|$

(iv) $aRb \Leftrightarrow 1 + ab > 0$

(v) $aRb \Leftrightarrow |a| \leq b$

Find whether these relations are reflexive, symmetric or transitive.

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12. Let $A = \{x : -1 \leq x \leq 1\}$ = B for each of the following functions from A to B. Find whether it is surjective, injective or bijective

(i) f(x) = $\frac{x}{2}$

(ii) g(x) = |x|

(iii) h(x) = x|x|

(iv) $k(x) = x^2$

(v) I(x) = $\sin \pi x$

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13. If the functions f and g defined from the set of real number R to R such that $f(x) = e^x$ and g(x) = 3x - 2, then find functions fog and gof. Also, find the domain of the functions $(fog)^{-1}$ and $(gof)^{-1}$.

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14. If $f(x) = \frac{x^2 - x}{x^2 + 2x}$, then find the domain and range of f. Show that f is one-one. Also, find the function $\frac{d\left(f^{-1}(x)\right)}{dx}$ and its domain.

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15. If the functions f, g and h are defined from the set of real numbers R to R such that

$f(x) = x^2 - 1, g(x) = \sqrt{\left(x^2 + 1\right)},$

$h(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ \text{x}, & \text{if } x \geq 0 \end{cases}$

Then find the composite function hofog and determine whether the function fog is invertible and h is the identity function.

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Exercise Questions Asked In Previous 13 Years Exam

1. Let $R = \{(3,3),(6,6),(9,9),(6,12),(3,9),(3,12),(3,6)\}$ is a relation on set $A = \{3,6,9,12\}$ then R is

A. an equivalence relation

B. reflexive and symmetric only

C. reflexive and transitive only

D. reflexive only

Answer: C

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2. Let w denote the words in the english dictionary. Define the relation R by: R = $\{(x,y) \in W \times W \mid$ words x and y have at least one letter in common$\}$. Then R is: (1) reflexive, symmetric and not transitive (2) reflexive, symmetric and transitive (3) reflexive, not symmetric and transitive (4) not reflexive, symmetric and transitive

A. not reflexive, symmetric and transitive

B. reflexive, symmetric and not transitive

C. reflexive, symmetric and transitive

D. reflexive, not symmetric and transitive

Answer: B

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3. Let R be the real line. Consider the following subsets of the plane $R \times R$. $S = \{(x, y) : y = x + 1 and 0 < x < 2\}, T = \{(x, y) : x - y$ is an integer }. Which one of the following is true? (1) neither S nor T is an equivalence relation on R (2) both S and T are equivalence relations on R (3) S is an equivalence relation on R but T is not (4) T is an equivalence relation on R but S is not

A. Both S and T are equivalence relations on R

B. S is an equivalence relation on R but T is not

C. T is an equivalence relation on R but S is not

D. Neither S nor T is an equivalence relations on R

Answer: C

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4. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then (1) $A = B$ (2) $A = C$ (3) $B = C$ (4) $A \cap B = \varphi$

A. $A \cap B = \phi$

B. A = B

C. A = C

D. B = C

Answer: D

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5. Let $S = \{1, , 2, 34\}$. The total number of unordered pairs of disjoint subsets of S is equal a.25 b. 34 c. 42 d. 41

A. 25

B. 34

C. 42

D. 41

Answer: D

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6. Consider the following relations: R = {(x, y) | x, y are real numbers and x = wy for some rational number w}; $S = \left\{ \left(\frac{m}{n}, \frac{p}{q} \right) \mathrm{m} , \mathrm{n} , \mathrm{pandqa r ei n t e g e r ss u c ht h a tn} , q \neq 0 \mathrm{andq m} =$. Then (1) neither R nor S is an equivalence relation (2) S is an equivalence relation but R is not an equivalence relation (3) R and S both are

equivalence relations (4) R is an equivalence relation but S is not an equivalence relation

A. neither R nor S is an equivalence relation

B. S is an equivalence relation but R is not an equivalence relation

C. R and S both are equivalence relations

D. R is an equivalence relation but S is not an equivalence relation

Answer: B

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7. Let $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$ and $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$ be two ses. Then,

A. $P \subset Q$ and $A - P \neq \phi$

B. $Q \not\subset P$

C. $P \not\subset Q$

D. P = Q

Answer: D

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8. find the value of the

A. $\pm\sqrt{n\pi}, n \in \{0, 1, 2, \ldots\}$

B. $\pm\sqrt{n\pi}, n \in \{1, 2, 3, \ldots\}$

C. $\frac{\pi}{2} + 2n\pi, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

D. $2n\pi, n \in \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Answer: A

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9. Let R be the set of real numbers. Statement-1 : $A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R. Statement-2 : $B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number a$\}$ is an equivalence relation on R. Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1. Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1. Statement-1 is true, Statement-2 is false. Statement-1 is false, Statement-2 is true.

A. Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for Statement-1

B. Statement-1 is true, Statement-2 is false

C. Statement-1 is false, Statement-2 is true

D. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1

Answer: A

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10. If A and B two sets containing 2 elements and 4 elements, respectively. Then, the number of subsets of $A \times B$ having 3 or more elements, is

A. 220

B. 219

C. 211

D. 256

Answer: B

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11. If $X = \{4^n - 3n - 1 : n \in N\} and Y = \{9(n-1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to (1) N (2) Y - X (3) X (4) Y

A. X

B. Y

C. N

D. Y - X

Answer: B

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12. Let A and B be too sets containing four and two elements respectively then the number of subsets of set $A \times B$ having atleast 3 elements is

A. 275

B. 510

C. 219

D. 256

Answer: C

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