MATHS

NCERT - NCERT MATHEMATICS (ENGLISH)

SEQUENCES AND SERIES

Miscellaneous Exercise

1. If the sum of three numbers in A.P., is 24 and their product is 440, find the numbers.

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2. The sum of some terms of G. P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.

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3. A man deposited Rs 10000 in a bank at the rate of 5% simple interest annually. Find the amount in 15^{th} year since he deposited the amount and also calculate the total amount after 20 years.

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4. A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

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5. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on third day and so on.

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6. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spend on the postage when 8th set of letter is mailed.

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7. Shamshad Ali buys a scooter for Rs. 2200. He pays Rs. 4000 cash and agrees to pay the balance in annual instalments of Rs. 1000 plus 10% interest on the unpaid amount. How much the scooter will cost him?

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8. Find the sum of the first n terms of the series : $3 + 7 + 13 + 21 + 31 + \ldots .$

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9. Find the 20^{th} term of the series $2 \times 4 + 4 \times 6 + 6 \times 8 + \vdots + n$ terms.

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10. Find the sum of the following series up to n terms:(i) $5 + 55 + 555 + \ldots$. (ii) . $6 + .66 + .666 + \ldots$.

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11. If a, b, c are in A.P., b, c, d are in G.P. and $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$ are in A.P. prove that a, c, e are in G.P.

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12. A farmer buys a used tractor for Rs. 12000. He pays Rs. 6000 cash and agrees to pay the remaining balance in annual instalments of Rs. 500 plus 12% interest on the unpaid amount. How much the tractor cost him?

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13. Show that $\frac{1 \times 2^2 + 2 \times 3^2 + \vdots + n \times (n+1)^2}{1^2 \times 2 + 2^2 \times 3 + \vdots + n^2 \times (n+1)} = \frac{3n+5}{3n+1}$.

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14. Find the sum of the series $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} +$ up to n terms.

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15. If S_1, S_2, S_3 are the sum of first n natural numbers, their squares and their cubes, respectively, show that $9S_2^2 = S_3(1 + 8S_1)$.

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16. A G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find its common ratio.

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17. If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P. prove that a, b, c are in A.P.

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18. If a, b, c, d are in G.P., prove that $(a^n + b^n), (b^n + c^n), (c^n + a^n)$ are in G.P.

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19. Let S e the sum, P the product, adn R the sum of reciprocals of n terms in a G.P. Prove that $P^2R^n = S^n$.

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20. The p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c, respectively. Show that $(q - r)a + (r - p)b + (p - q)c = 0$.

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21. The sum of the first four terms of an A.P. is 56. The sum of the last four terms is 112. If its first term is 11, then find the number of terms.

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22. If $\frac{a+bx}{a-bx}=\frac{b+cx}{b-cx}=\frac{c+dx}{c-dx}(x\neq 0)$, then show that a, b, c and d are in G.P.

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23. The sum of three numbers m GP is 56. If we subtract 1.7,21 from these numbers in that order, we obtain an arithmetic progression. Find the numbers.

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24. If $a\ and\ b$ are the roots of $x^2-3x+p=0\ and\ c,\ d$ are the roots $x^2-12x+q=0$ where a, b, c, d form a G.P. Prove that $(q+p):(q-p)=17:15.$

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25. The ratio of the A.M. and G.M. of two positive numbers a and b, is m : n. Show that a : b = $\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$.

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26. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ terms of an A.P. is equal to twice the m^{th} term.

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27. Let the sum of n, 2n, 3n terms of an A.P. be S_1, S_2 and S_3, respectively, show that $S_3 = 3(S_2 - S_1)$.

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28. Find the sum of integers from 1 to 100 that are divisible by 2 or 5.

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29. Find the sum of all numbers between 200 and 400 which are divisible by 7.

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30. If f is a function satisfying $f(x+y)=f(x)f(y)$for all $x, y \in X$such that $f(1)=3$and $\sum_{x=1}^{n} f(x)=120$, find the value of n.

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31. Find the sum of all two digit numbers which when divided by 4, yields 1 as remainder.

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32. The first term of a G.P. is 1. The sum of the third term and fifth term is 90. Find the common ratio of G.P.

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Solved Examples

1. Insert 6 numbers between 3 and 24 such that the resulting sequence is an A. P.

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2. Find the 10^{th} and n^{th} terms of the G.P. 5, 25, 125, . . .

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3. Write the first three terms in each of the following sequences defined by the following:(i) $a_n = 2n + 5$ (ii) $a_n = \frac{n-3}{4}$

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4. What is the 20^{th}term of the sequence defined by $a_n = (n-1)(2-n)(3+n)$?

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5. Let the sequence $a_n$be defined as follows : $a_1 = 1, a_n = a_{n-1} + 2$for $n \geq 2$. Find first five terms and write corresponding series.

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6. In an A.P. if m^{th}term is n and the n^{th}term is m, where $m \neq n$, find the pth term.

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7. The income of a person is Rs. 3, 00,000, m the first year and he receives an increase of Rs. 10.000 to his income per year for the next 19 years. Find the total amount, he received m 20 years.

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8. Find the sum of first n terms of the following series: $5 + 11 + 19 + 29 + 41 + \cdot$

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9. If A.M. and GM. of two positive numbers a and b are 10 and 8, respectively find the numbers.

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10. How many terms of the G.P. 3, $\frac{3}{2}, \frac{3}{4}, \ldots$ are needed to give the sum $\frac{3069}{512}$?

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11. Find the sum of first n terms and the sum of first 5 terms of the geometric series $1+\frac{2}{3}+\frac{4}{9}+\ldots$

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12. In a GP the 3rd term is 24 and the 6th term is 192. Find the 10th term.

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13. Which term of the G.P., 2, 8, 32, . . . up to n terms in 131072?

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14. Insert three numbers between 1 and 256 so that the resulting sequence is a G.P.

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15. A person has 2 parents, 4 grandparents, 8 great grand parents, and so on. Find the number of his ancestors during the ten generations preceding his one.

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16. Find the sum of the sequence 7, 77, 777, 7777, . . . to n terms.

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17. The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is 1. Find the common ratio and the terms.

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18. If p, q, r are in G.P. and the equations, $px^2 + 2qx + r = 0$and $dx^2 + 2ex + f = 0$have a common root, then show that $\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$are in A.P.

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19. If a, b, c are in G.P. and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$, prove that x, y, z are in A.P.

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20. If a, b, c, d and p are different real numbers such that $\left(a^2 + b^2 + c^2\right)p^2 - 2(ab + bc + cd)p + \left(b^2 + c^2 + d^2\right) \leq 0$, then show that a, b, c and d are in G.P.

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21. Find the sum of n terms of the series whose nth term is: $n(n+3)$

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22. If p^{th}, q^{th}, $r^{th} and\ s^{th}$ terms of an A.P. are in G.P., then show that $(p-q),\ (q-r),\ (r-s)$ are also in G.P.

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23. If the sum of n terms of an A.P. is $nP + \frac{1}{2}n(n-1)Q$, where P and Q are constants, find the common difference.

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24. The sum of n terms of two arithmetic progressions are in the ratio $(3n+8):(7n+15)$. Find the ratio of their 12th terms.

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Exercise 9 2

1. Sum of the first p, q and r terms of an A.P are a, b and c, respectively.Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

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2. Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

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3. A mail starts repaying a loan as first instalment of Rs. 100. If he increases the instalment by Rs 5 every month, what amount he will pay in the 30^{th}instalment?

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4. The difference between any two consecutive interior angles of a polygon is $5o$. If the smallest angle is $120o$, find the number of the sides of the polygon.

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5. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an A. P. and the ratio of 7^{th} and $(m-1)^{th}$ numbers is 5 : 9. Find the value of m.

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6. If the sum of first p terms of an A.P. is equal to the sum of the first q terms, then find the sum of the first (p + q) terms.

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7. If the sum of n terms of an A.P. is $3n^2 + 5$and its m^{th}term is 164, find the value of m.

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8. The ratio of the sum of m and n terms of an A.P. is $m^2 : n^2$. Show that the ratio mth and nth term is (2m-1) : (2n-1).

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9. If $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$ is the A.M. between a and b, then find the value of n.

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10. If the sum of n terms of an A.P. is $\left(pn + qn^2\right)$, where p and q are constants, find the common difference.

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11. The sums of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18^{th} terms.

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12. If the sum of a certain number of terms of the A.P. 25, 22, 19.... is 116. Find the last term.

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13. Find the sum to n terms of the A.P., whose k^{th} term is $5k + 1$.

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14. How many terms of the A.P. $6, -\frac{11}{2}, -5, \ldots$ are needed to give the sum 25?

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15. In an A,P if the pth term is $\frac{1}{q}$ and q^{th} terms is $\frac{1}{p}$. Prove that the sum of first pq term is $\frac{1}{2}$ (pq+1) where, $(p \neq q)$

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16. Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

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17. In an A.P., the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20th term is 112.

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18. Find the sum of odd integers from 1 to 2001.

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Exercise 9 3

1. If the first and the nth terms of a GP are a and b respectively and if P is the product of the first n terms, then P^2 is equal to

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2. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9, and the second term is greater than the 4^{th} by 18.

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3. Show that the products of the corresponding terms of the sequences a, $ar, ar^2, \ldots ar^{n-1}$ and $A, AR, AR^2, \ldots AR^{n-1}$, form a G.P, and find the

common ratio.

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4. Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

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5. Insert two number between 3 and 81 so that the resulting sequence is G.P.

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6. If a, b, c and d are in G.P. show that $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=(ab+bc+cd)^2$.

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7. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour?

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8. What will Rs. 500 amounts to in 10 years after its deposit in a bank which pays annual interest are 10% compounded annually?

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9. If A.M. and GM. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

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10. If A and G be A.M. and GM., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

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11. The sum of two numbers is 6 times their geometric means, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

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12. If the p^{th},$q^{th}$and r^{th}terms of a GP are a, b and c, respectively. Prove that $a^{q-r}b^{r-p}c^{p-q}=1$.

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13. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$. 9873740001

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14. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

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15. If the 4^{th}, 10^{th} and 16^{th} terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P.

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16. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the GP.

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17. Given a G.P. with a = 729 and 7^{th} term 64, determine S_7.

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18. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

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19. How many terms of G.P. $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

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20. Find the sum to indicated number of terms in each of the geometric progressions : $x^3, x^5, x^7, \ldots n$ terms $(\quad \text{if} \quad x \neq \pm 1)$.

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21. Evaluate $\sum_{k=1}^{11}\left(2+3^{k}\right)$

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22. Find the sum to n terms of the sequence, 8, 88, 888, 8888 . . . .

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23. Find the sum of the products of the corresponding terms of the sequences $2, 4, 8, 16, 32$ *and* $128, 32, 8, 2, \frac{1}{2}$.

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24. Find the 20^{th} and n^{th} terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

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25. The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

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26. Find the 12^{th} term of a G.P. whose 8th term is 192 and the common ratio is 2.

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27. Which term of the following sequences:(a) $2, 2\sqrt{2}, 4, \ldots$ is 128? (b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ is 729?(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ is $\frac{1}{19683}$?

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28. The 4^{th} term of a G.P. is square of its second term, and the first term is 3. Determine its 7^{th} term.

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29. Find the sum to indicated number of terms in each of the geometric progressions : 0.15, 0.015, 0.0015, . . . , 20 terms.

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30. For what value of x, the number $-\frac{2}{7}, x, -\frac{2}{7}$ are in G.P.?

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31. Find the sum to indicated number of terms in each of the geometric progressions : $1, -a, a^2 - a^3, \dot{\dot{n}}$ terms $(\text{if } a \neq -1)$

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32. Find the sum to indicated number of terms in each of the geometric progressions : $\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots$ n terms.

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Exercise 9 1

1. Write the first five terms of the sequence whose n^{th}terms are : $a_n = n\frac{n^2+5}{4}$

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2. Find the indicated terms of the sequence whose n^{th}terms are : $a_n = \frac{n(n-2)}{n+3}; a_{20}$

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3. Write the first five terms of the sequence and obtain the corresponding series : $a_1 = 3, a_n = 3a_{n-1} + 2$for all $n > 1$

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4. Write the first five terms of the sequence and obtain the corresponding series : $a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$

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5. Write the first five terms of the sequence and obtain the corresponding series : $a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2.$

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6. The Fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2}, n > 2$. Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5.

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7. Find the indicated terms of the sequence whose n^{th}terms are :

$a_n = (-1)^{n-1} n^3; a_9$

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8. Find the indicated terms of the sequence whose n^{th}terms are :

$a_n = \frac{n^2}{2^n}; a_7$

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9. Find the indicated terms of the sequence whose n^{th}terms are :

$a_n = 4n - 3; a_{17}, a_{24}$

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10. Write the first five terms of the sequence whose n^{th}terms are : $a_n = (-1)^{n-1}5^{n+1}$

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11. Write the first five terms of the sequence whose n^{th}terms are : $a_n = \frac{2n-3}{6}$

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12. Write the first five terms of the sequence whose n^{th}terms are : $a_n = 2^n$

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13. Write the first five terms of the sequence whose n^{th}terms are : $a_n = \frac{n}{n+1}$

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14. Write the first five terms of the sequence whose n^{th}terms are : $a_n = n(n+2)$

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Exercise 9 4

1. Find the sum to n terms of the series, whose n^{th}terms is given by : $(2n-1)^2$

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2. Find the sum to n terms of the series, whose n^{th}terms is given by : $n(n+1)(n+4)$

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3. Find the sum to n terms of the series in Questions 8 to 10 whose nth terms is given by :

$n^2 + 2^n$

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4. Find the sum to n terms of the series :

$1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \ldots\ldots$

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5. Find the sum to n terms of the series :

$1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \vdots$

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6. Find the sum to n terms of the series :$3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \vdots$

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7. Find the sum to n terms of the series : $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\vdots$

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8. Find the sum to n terms of the series : $5^2+6^2+7^2+\vdots+20^2$

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9. Find the sum to n terms of the series 3.8 +6.11 +9.14 + ...

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10. Find the sum to n terms of the series : $1^2+\left(1^2+2^2\right)+\left(1^2+2^2+3^2\right)+\vdots$

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Exercise 9 5

1. Find the sum to infinity of the following Geometric Progression: $6, 1.2, 0.24, \ldots$

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2. Find the sum to infinity of the following Geometric Progression: $5, \frac{20}{7}, \frac{80}{49}, \ldots$

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3. Find the sum to infinity of the following Geometric Progression: $1, \frac{1}{3}, \frac{1}{9}, \ldots$

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4. If

$x = 2 + a + a^2 + \infty, where |a| < 1 and y = 1 + b + b^2 + \infty, where |b| < 1$

prove that: $1 + ab + a^2b^2 + \infty = \frac{xy}{x + y - 1}$

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5. Prove that: $3^{\frac{1}{2}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{8}} \times \ldots = 3$

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6. Find the sum to infinity of the following Geometric Progression: $\frac{-3}{4}, \frac{3}{16}, \frac{-3}{64}, \ldots$

A. $-\frac{3}{5}$

B. $-\frac{2}{5}$

C. $\frac{3}{5}$

D. $\frac{2}{5}$

Answer: A

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