# MATHS

## BOOKS - GURUKUL BOOKS & PACKAGING MATHS (HINGLISH)

## OCTOBER 2014

### Section I

1. If $\left[\vec{a}\ \vec{b}\ \vec{c}\right] \neq 0$ and $\vec{P} = \dfrac{\vec{b} \times \vec{c}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}, \vec{q} = \dfrac{\vec{c} \times \vec{a}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}, \vec{r} = \dfrac{\vec{a} \times \vec{b}}{\left[\vec{a}\ \vec{b}\ \vec{c}\right]}$, then $\vec{a}\,.\,\vec{p} + \vec{b}\,.\,\vec{q} + \vec{c}\,.\,\vec{r}$ is equal to ............

A. 0

B. 1

C. 2

D. 3

**Answer: D**

Watch Video Solution

**2.** Select and write the most appropriate answer from the given alternatives in each of the following :

The inverse of the matrix $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ is

A. $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

B. $\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

D. $-\frac{1}{2}\begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

**Answer: A**

Watch Video Solution

**3.** Direction cosines of the line passing through the points $A(-4, 2, 3)$ and $B(1, 3, -2)$ are

A. $\pm\frac{1}{\sqrt{51}}, \ \pm\frac{5}{\sqrt{51}}, \ \pm\frac{1}{\sqrt{51}}$

B. $\pm\frac{5}{\sqrt{51}}, \ \pm\frac{1}{\sqrt{51}}, \ \pm\frac{-5}{\sqrt{51}}$

C. $\pm 5, \ \pm 1, \ \pm 5$

D. $\pm\sqrt{51}m\sqrt{51}, \ \pm\sqrt{51}$

**Answer: B**

Watch Video Solution

**4.** Write the truth values of the following statements :

(a) $\sqrt{5}$ is an irrational number but $3+\sqrt{5}$ is a complex number

Watch Video Solution

**5.** Write the truth values of the following statements :

(b) $\exists n \in N$ such that $n + 5 > 10$

Watch Video Solution

**6.** If $\bar{c} = 3\bar{a} - 2\bar{b}$, then prove that $[\bar{a}\bar{b}\bar{c}]0$

View Text Solution

**7.** Find the vector equation of the plane which is at a distance of 5 units from the origin and which is normal to the vector $2\hat{i} + \hat{j} + 2\hat{k}$.

Watch Video Solution

**8.** The Cartesian equationof a line are $3x + 1 = 6y = 2 = 1 - z$. Find the direction ratios and write down its equation in vector form.

Watch Video Solution

**9.** Using truth table, prove the following logical equivalence

$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

Watch Video Solution

**10.** Find the joint equation of the pair of the lines through the origin each of which is making an angle of $30^\circ$ with the line $3x + 2y - 11 = 0$.

Watch Video Solution

**11.** Prove that:$2\frac{\sin^{-1} 3}{5} = \frac{\tan^{-1}(24)}{7}$

Watch Video Solution

**12.** Solve the following equations by the method of reduction

$2x - y + z = 1, x + 2y + 3x = 8, 3x + y - 4z = 1.$

Watch Video Solution

**13.** Prove that volume of a parallelopiped with coterminal edges as $\bar{a}, \bar{b}, \bar{c}$ is $[\bar{a}, \bar{b}, \bar{c}]$. Hence find the volume of the parallelopiped with coterminal edges $\hat{\text{i}} + \hat{j}, \hat{j} + \hat{k}$ and $\hat{k} + \hat{\text{i}}$.

Watch Video Solution

**14.** Solve the following LPP by using graphical method.

Maximize : $Z = 6x + 4y,$

Subject to $x \leq 2, x + y \leq 2, \ -2x + y \leq 1, x \geq 0, y \geq 0.$ Also find maximum value of Z.

Watch Video Solution

15. In $\triangle ABC$ , with the usual notations , prove that

$$2\left\{a\sin^2.\frac{C}{2}+c\sin^2.\frac{A}{2}\right\}=a+c-b$$

Watch Video Solution

16. Assuming p and q as given, write the verbal statement for the following symoblic statements :

p : It is a day time. " " q : It is warm.

(i) $p \wedge$ ~$q$ (ii) ~$p \rightarrow q$ (iii) $q \rightarrow p$

Watch Video Solution

17. If the lines

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4} and \frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$$

intersect, then find the value of $k$.

Watch Video Solution

**18.** Parametric form of the equation of the plane is $\bar{r} = \left(2\hat{i} + \hat{k}\right) + \lambda\hat{i} + \mu\left(\hat{i} + 2\hat{i} - 3\hat{k}\right)\lambda \text{ and } \mu$ are parameters. Find normal to the plane and hence equation of the plane in normal form. Write its Cartesion form.

Watch Video Solution

**19.** If the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0,$ then show that $100\left(h^2 - ab\right) = (a + b)^2$.

Watch Video Solution

**20.** $\sin x \tan x - 1 = \tan x - \sin x$

Watch Video Solution

**21.** The differential equation of the family of curves $y = c_1 e^x + c_2 e^{-x}$ is

A. $\frac{d^2y}{dx^2} + y = 0$

B. $\frac{d^2y}{dx^2} - y = 0$

C. $\frac{d^2y}{dx^2} + 1 = 0$

D. $\frac{d^2y}{dx^2} - 1 = 0$

**Answer: B**

Watch Video Solution

## Section Ii

**1.** If X is the a random variable with probability mass function $P(x) = kx$, for x = 1,2,3

= 0 , otherwise

then k = ........

A. $\frac{1}{5}$

B. $\frac{1}{4}$

C. $\frac{1}{6}$

D. $\frac{2}{3}$

**Answer: C**

Watch Video Solution

**2.** If $\sec\left(\frac{x+y}{x-y}\right)=a^2, \text{ then } \frac{d^2y}{dx^2}=\ldots\ldots$

A. y

B. x

C. $\frac{y}{x}$

D. 0

**Answer: C**

Watch Video Solution

**3.** If $y = \sin^{-1}(3x) + \sec^{-1}\left(\frac{1}{3x}\right)$, find $\frac{dy}{dx}$.

Watch Video Solution

**4.** Evaluate : $\int x \log x dx$

Watch Video Solution

**5.** if $\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$ then find the value of $k$

Watch Video Solution

**6.** The probability that a certain kind of component will survive a check test is 0.5. Find the probability that exactly two of the next four components tested will survive.

Watch Video Solution

**7.** Find the area of the region bounded by the curve $y = \sin x$, the lines $x = -\frac{\pi}{2}, x = \frac{\pi}{2}$ and X- axis.

Watch Video Solution

**8.** Examine the continuity of the following function at given point :

$$f(x) = \frac{\log x - \log 8}{x - 8}, \text{for} \quad x \neq 8$$

$8$ ,for $x = 8$

at , $x = 8$

View Text Solution

**9.** If $x = \phi(t)$ is a differentiable function of 't', then prove that $\int f(x)dx = \int f[\phi(t)]\phi'(t)dt$.

View Text Solution

**10.**

$3e^x \tan y dx + (1 + e^x)\sec^2 dy = 0, \quad \text{when} x = 0 \text{ and } y = \pi$

Watch Video Solution

**11.** A point source of light is hung 30 feet directly above a straight horizontal path on which a man of 6 feet in height is walking. How fast is the man's shadow lengthening and how fast the tip of shadow is moving when he is walking away from the light at the rate of 100 ft/min.

Watch Video Solution

**12.** Evaluate the following integrals:

$$\int \frac{\log x}{(1+\log x)^2} dx$$

Watch Video Solution

**13.** If $x = f(t), y = g(t)$ are differentiable functions of parameter 't' then prove that y is a differentiable function of 'x' and

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \frac{dx}{dt} \neq 0$$

Hence find $\frac{dy}{dx}$ if $x = a\cos t, y = a\sin t$.

Watch Video Solution

**14.** Show that the function defined by $f(x) = |\cos x|$is a continuous function.

Watch Video Solution

**15.** Solve the differential equation $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$.

Watch Video Solution

**16.** Given X ~ B(n,p) If n= 20,, E(x)= 10 , Find p, Car(X) and S.D (X)

Watch Video Solution

**17.** A bakerman sells 5 types of cakes. Profit due to the sale of each type of cake is respectively Rs3, Rs 2.5, Rs 2, Rs 1.5, Rs 1. The demands for these cakes are

$10\%$, $5\%$, $25\%$, $45\%$ and $15\%$ respectively.. What is he expected profit per cake?

Watch Video Solution

**18.** Varify Lagrange's mean value theorem for the function $f(x) = x + \frac{1}{x}, x \in [1, 3]$

Watch Video Solution

**19.** $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. Hence evaluate : $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx$.

Watch Video Solution