MATHS

BOOKS - NAVBODH MATHS (HINGLISH)

APPLICATIONS OF DERIVATIVES

Solved Examples

1. Find the equation of the tangent to the curve $y = 3x^2 - x + 1$ at P(1, 3).

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2. Find the equation of the normal to the curve $\sqrt{x} - \sqrt{y} = 1$ at (9, 4).

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3. Find the coordinates of the point on the curve $y = x - \frac{4}{x}$, where the tangent is parallel to the line $y = 2x$.

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4. If the line $y = 4x - 5$ touches the curve $y^2 = ax^3 + b$ at the point (2, 3), show that $7a + 2b = 0$.

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5. The displacement s of a moving particle at a time t is given by $s = 5 + 20t - 2t^2$. Find its acceleration when the velocity is zero.

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6. The area of a square is increasing the rate of $0.5 \ \ \mathrm{cm}^2/\mathrm{sec}$. Find the rate of increase of its perimeter when the side of square is 10 cm long.

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7. The radius of a soap bubble is increasing at the rate of 0.2 cm/sec. It its radius is 5 cm, find the rate of increase of its volume.

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8. An aeroplane at an altitude of 1 km flying horizontally at 800 km/hr passes directly over an observer. Find the rate at which it is approaching the observer when it is 1250 metres away from him.

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9. A point source of light is hung 30 feet directly above a straight horizontal path on which a man of 6 feet in height is walking. How fast is the man's shadow lengthening and how fast the tip of shadow is moving when he is walking away from the light at the rate of 100 ft/min.

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10. The surface area of a spherical balloon is increasing at the rate of $2 \ \mathrm{cm}^2/\mathrm{sec}$. At what rate is the volume of the ballon is increasing, when the radius of the ballon is 6 cm ?

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11. Find the approximate value of $\sqrt{8.95}$

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12. Find the approximate value of $\sqrt[3]{27.027}$.

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13. Find the approximate values of :

$f(x) = x^3 - 3x + 5 \quad \text{at} \quad x = 1.99.$

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14. Find the approximate values of :

$\cos(89^\circ 30), \quad \text{given} \quad 1^\circ = 0.0175^c.$

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15. Find the approximate values of :

$\tan^{-1}(1.001).$

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16. Find the approximate value of $e^{1.005}$ (given $e = 2.7183$)

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17. Find the approximate value of $\log_{10}(1016)$ given $\log_{10} e = 0.4343$

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18. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 9$ on $[1, 4]$.

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19. Verify LMVT for the following function $f(x) = x + \frac{1}{x}, x \in [1, 3]$.

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20. Test whether the following functions are increasing or decreasing :

$f(x) = x^3 - 6x^2 + 12x - 16, x \in R$

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21. Test whether the following functions are increasing or decreasing :

$f(x) = \cos x, 0 < x < \pi.$

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22. Find the values of x, for which the function $f(x) = x^3 + 12x^2 + 36x + 6$ is increasing.

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23. Find the values of x such that $f(x) = 2x^3 - 15x^2 - 84x - 7$ is a decreasing function.

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24. Examine the function $f(x) = x^3 - 9x^2 + 24x$ for maxima and minima.

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25. Show that $f(x) = \frac{\log x}{x}, x \neq 0$ is maximum at $x = e$.

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26. A rectangle has an area of 50 cm^2. Find its dimensions for least perimeter.

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27. An open box is to be made out of a piece of a square card board of sides 18 cm by cutting off equal squares

from the corners and turning up the sides. Find the maximum volume of the box.

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28. A wire of length l is cut into two parts. One part is bent into a circle and the other into a square. Prove that the sum of the areas of the circle and the square is the least, if the radius of the circle is half of the side of the square.

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29. Find the maximum volume of right circular cylinder, if the sum of its radius and height is 6 units.

A. 24π

B. 28π

C. 32π

D. None of These

Answer: C

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30. A telephone company in the town has 5000 subscribers on its list and collects fixed rent charges of

Rs. 3000 per year from each person. The company proposes to increase the annual rent and it is believed that for every increase of 1 rupee in the rent, one subscriber will be discontinued. Find what increased annual rental will bring the maximum annual income to the company.

A. 900

B. 1000

C. 1200

D. 1500

Answer: B

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31. A box with a square base is to have an open top. The surface area of the box is 192 sq. cm. What should be its dimensions in order that the volume is as large as possible ?

A. 3

B. 4

C. 5

D. 6

Answer: B

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Examples For Practice

1. Find the equations of tangent and normal to the curves at the indicated points on it $y = x^2 + 4x + 1$ at $(-1, -2)$

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2. Find the equations of the tangents to the following curves at the given points :

$x^2 + y^2 + xy = 3$ at $(1, 1.$

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3. Find the equations of the normals to the following curves at the given points :

$2x^2 + 3y^2 - 5 = 0$ at (1, 1)

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4. Find the equations of the normals to the following curves at the given points :

$y = x^2 - 4x + 3$ at (4, 3).

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5. Show that the tangent to the curve $8y = (x - 2)^2$ at the point (-6, 8) is parallel to the tangent to the curve

$y = x + (3/x)$ at the point (1, 4).

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6. Find the points on the curve $y = \sqrt{x - 3}$, where the tangent is perpendicular to the line $6x + 3y - 5 = 0$.

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7. Find the equation of the tangent to the curve $y = 6 - x^2$, where the normal is parallel to the line $x - 4y + 3 = 0$.

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8. If the line $x + y = 0$ touches the curve $2y^2 = ax^2 + b$ at (1, -1), find a and b.

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9. The displacement s of a particle at a time t is given by $s = t^3 - 4t^2 - 5t$. Find its velocity and acceleration at t = 2.

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10. A particle moves according to law $s = t^3 - 6t^2 + 9t + 15$. Find the velocity when t = 0.

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11. The displacement x of a particle at time t is given by $x = 160t - 16t^2$. Show that its velocity at t = 1 amd t = 9 are equal in magnitude but opposite in directions.

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12. The side of a square is increasing at the rate of 0.5 cm/sec. Find the rate of increase of its area, when the side of square is 20 cm long.

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13. The radius of a circular blot of oil is increasing at the rate of 2 cm/min.

Find the rate of change of its area when its radius is 3 cm.

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14. The radius of a circular blot of oil is increasing at the rate of 2 cm/min. The rate of change of its circumference is

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15. A spherical snow ball is melting so that its volume is decreasing at the rate of 8 cc/sec. Find the rate at which its radius is decreasing when it is 2 cm.

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16. A man of 2 metres height walks at a uniform speed of 6 km/hr away from a lamp post of 6 metres high. Find the rate at which the length of his shadow increases.

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17. Aladder of 5 m long rest with one end against a vertical wall of height 3 m and the other end on the

lower ground. If its top slides down at the rate of 10 cm/sec, find the rate at which the foot of the ladder is sliding.

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18. If the volume of spherical ball is increasing at the rate of $4\pi \quad \mathrm{cc}/\mathrm{s}$, then the rate of change of its surface area when the volume is 288 π cc is

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19. Sand is pouring from a pipe at the rate of 12 cm^3/s . The falling sand forms a cone on the ground in such a

way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when t

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20. Water is being poured at the rate of $36 \ \mathrm{m}^3/\mathrm{sec}$ in a cylindrical vessel of base radius 3 metres. Find the rate at which water level is rising.

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21. Find the approximate values of :

$(4.01)^5$

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22. Find the approximate values of :

$\sqrt{144.02}$

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23. Find the approximate values of :

$\sqrt[3]{26.96}$

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24. Find the approximate values of :

$f(x) = x^3 + 5x^2 - 7x + 10 \quad \text{at} \quad x = 1.1.$

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25. Find the approximate values of :

$\cos(60^\circ 30')$, given $1^\circ = 0.0175^c$ and $\sin 60^\circ = 0.8660$

.

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26. The apprximate value of $\sin(31^\circ)$, given that $1^\circ = 0.0175, \cos 30^\circ = 0.8660$ is

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27. Find the approximate values of :

$\tan(44^\circ), \quad \text{given} \quad 1^\circ = 0.0175^c.$

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28. Approximate value of $\tan^{-1}(0.999)$ is

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29. Find the approximate values of :

$\cot^{-1}(1.001)$

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30. Find the approximate values of :

$\sin^{-1}(0.51), \quad \text{given} \quad \sqrt{3} = 1.7321.$

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31. Find the approximate value of $\log_e(101)$ given $\log_e 10 = 2 \cdot 3026$

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32. Find the approximate values of :

$\log_{10}(1002), \quad \text{given that} \quad \log_{10} e = 0.4343.$

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33. Find the approximate value of $:e^{2.1}$ given that $e^2 = 7.389$

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34. Check whether the conditions of Rolle's theorem are satisfied by the following functions or not :

$f(x) = x^2 - 4x + 10, x \in [0, 4]$

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35. Check whether the conditions of Rolle's theorem are satisfied by the following functions or not :

$f(x) = (x-1)(x-2)(x-3), x \in [1, 3]$

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36. Check whether the conditions of Rolle's theorem are satisfied by the following functions or not :

$f(x) = 2\sin x + \sin 2x, x \in [0, \pi]$.

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37. If Rolle's theorem holds for the function

$f(x) = ax^3 + bx^2 + 5x + 1, x \in [1, 2]$ with $c = \frac{3}{2}$,

find the values of a and b.

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38. Verify LMVT for the following functions on the intervals shown against them and find the possible values of c :

$f(x) = x(x+4)^2, x \in [0, 4]$

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39. Verify LMVT for the following functions on the intervals shown against them and find the possible values of c :

$f(x) = \log x, x \in [1, e].$

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40. Find c, if LMVT is applicable for $f(x) = (x-3)(x-6)(x-9), x \in [3,5]$.

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41. Verify Rolle's theorem for each of the following functions :

$f(x) = e^{-x}(\sin x - \cos x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

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42. Test whether the following funtions are increasing or decreasing :

$f(x) = x - \frac{1}{x}, x \in R, x \neq 0.$

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43. Test whether the following funtions are increasing or decreasing :

$f(x) = 2 - 3x + 3x^2 - x^3, x \in R.$

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44. Find the values of x such that $f(x) = x^3 + 12x^2 + 36x + 6$ is an increasing function.

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45. Find the values of x such that $f(x) = 2x^3 - 9x^2 + 12x + 2$ is a decreasing function.

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46. Find the values of x if $f(x) = \frac{x}{x^2 + 1}$ is an increasing function

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47. Find the values of x if $f(x) = \frac{x}{x^2 + 1}$ is a decreasing function.

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48. Examine the following functions for maxima and minima :

$f(x) = 2x^3 - 21x^2 + 36x - 20$

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49. Examine the following functions for maxima and minima :

$f(x) = x^2 + \frac{16}{x^2}$

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50. Examine the following functions for maxima and minima :

$f(x) = 3x^3 - 9x^2 - 27x + 15$

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51. Examine the following functions for maxima and minima :

$f(x) = x^2 e^x$.

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52. Examine the following functions for maxima and minima :

Show that $f(x) = x^x$ is minimum when $x = \frac{1}{e}$.

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53. Examine the following functions for maxima and minima :

Find the maximum value of the function $f(x) = x - e^x$.

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54. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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55. Divide the number 84 into two parts such that the product of one part and the square of other is

maximum.

A. 42, 42

B. 48, 36

C. 56, 28

D. None of These

Answer: C

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56. If $x + y = 8$, then show that the maximum value of x^2y is $\frac{2048}{27}$.

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57. A manufacturer can sell x items at the price of Rs. $(330 - x)$ each. The cost of producing xitems is Rs. $x^2 + 10x - 12$. How many items must be sold so that his profit is maximum?

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58. Find the position of the point P on seg AB of length 8 cm, so that $\mathrm{AP}^2 + \mathrm{BP}^2$ is minimum.

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59. Determine the points on the curve $x^2 = 4y$ which are nearest to the point $(0,\ 5)$.

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60. The combined resistance R of two resistors R_1 and R_2 where $R_1, R_2 > 0$ is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

If $R_1 + R_2 = C$ (constant), show that the maximum reistance R is obtained by chossing $R_1 = R_2$

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61. Two sides of a triangle are given. The angle between them such that the area is maximum, is given by

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Multiple Choice Questions

1. Find the equation of the tangent to the curve $y = 3x^2 - x + 1$ at P(1, 3).

A. $y = 5x + 2$

B. $y = 5x - 2$

C. $y = \frac{1}{5}x + 2$

D. $y = \frac{1}{5}x - 2.$

Answer: B

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2. The equation of tangent to the curve $y = x^2 + 4x + 1$ at (-1, -2) is

A. $2x - y = 0$

B. $2x + y - 5 = 0$

C. $2x - y - 1 = 0$

D. $x + y - 1 = 0.$

Answer: B

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3. Find the equation of the tangent to the curve $y = 3x^2 - x + 1$ at P(1, 3).

A. $5x - y = 2$

B. $x + 5y = 16$

C. $5x - y + 2 = 0$

D. $5x = y.$

Answer: B

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4. Equation of the tangent to the curve $2x^2 + 3y^2 - 5 = 0$ at (1, 1) is

A. $2x - 3y - 5 = 0$

B. $2x + 3y - 5 = 0$

C. $2x + 3y + 5 = 0$

D. $3x + 2y + 5 = 0.$

Answer: B::C

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5. Parabola $y^2 = x + 1$ cuts the Y-axis in A and B. The sum of slopes of tangents to the parabola at A and B is

A. 0

B. 1

C. 2

D. 3

Answer:

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6. A particle moves according to the law $x = t^3 - 6t^2 + 9t + 5$. The displacement of the particle at the time when its acceleration is zero, is

A. 9

B. -7

C. 7

D. 0

Answer:

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7. The radius of a circular blot of oil is increasing at the rate of 2 cm/min. The rate of change of its circumference is

A. 4 cm/sec

B. $4\pi \quad \mathrm{cm}/\mathrm{sec}$

C. $2\pi \quad \mathrm{cm}/\mathrm{sec}$

D. $\pi \quad \mathrm{cm}/\mathrm{sec}.$

Answer: C::D

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8. Find the approximate value of $\cos(29^\circ 30')$ given $1^\circ = 0.0175^c$ and $\cos 30^\circ = 0.8660$

A. 0.0870375

B. 0.87375

C. 0.870375

D. $0.00870375.$

Answer: C

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9. If the function $f(x) = \log x, x \in [1, e]$, satisfies all the conditions of LMVT, then the value of c is

A. 1

B. e

C. $\frac{1}{e}$

D. $e - 1$.

Answer: A

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10. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$is increasing in R.

A. increasing

B. decreasing

C. increasing and decreasing

D. neither increasing nor decreasing.

Answer: A::C

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11. Function $f(x) = x^2 - 3x + 4$ has minimum value at

x = ...

A. 0

B. $\frac{-3}{2}$

C. 1

D. $\frac{3}{2}$.

Answer: B::C

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12. If the function $f(x) = 2x^3 - 9ax^2 + 12x^2x + 1, wherea > 0,$ attains its maximum and minimum at $pandq$, respectively, such that $p^2 = q,$ then a equal to 1 (b) 2 (c) $\frac{1}{2}$ (d) 3

A. $\frac{1}{2}$

B. 1

C. 2

D. 4

Answer: B

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