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MATHS

BOOKS - NAVBODH MATHS (HINGLISH)

CONTINUITY

Solved Examples

1. Discuss the conjinuity of the functions at the points shown against them. If a function is discontinuous, determine whether the discontinuity is removable. In this case, redefine the function, so that it becomes

continuous :

$$f(x) = \left.\begin{array}{ll} \frac{\log(100(0.01+x))}{3x}, & \text{for} x \neq 0 \\ & ,\text{for} x = 0 \end{array}\right\} at = 0. \quad = \frac{100}{3}$$

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2. Discuss the continuity of the functions at the points shown against them . If a function is discontinuous , determine whether the discontinuity is removable . In this case , redefine the function , so that it becomes continuous :

$$\left.\begin{array}{ll} F(x) = \frac{4^x - e^x}{6^x - 1} & , \text{for} \quad x \neq 0 \\ = \log\left(\frac{2}{3}\right) & , \text{for} \quad x = 0 \end{array}\right\} \text{at } x = 0.$$

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3. **Discuss the continuity of the functions at the points shown against them . If a function is discontinuous , determine whether the discontinuity is removable . In this case , redefine the function , so that it becomes continuous :** $\left.\begin{array}{ll} f(x) = x \sin\left(\frac{1}{x}\right) & , \text{for } x \neq 0 \\ = 0 & , \text{for } x = 0 \end{array}\right\}$ **at** $x = 0$

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4. Discuss the continuity of the following functions at the points shown against them :

$$\left.\begin{array}{ll} f(x) = \frac{10^x + 7^x - 14^x - 5^x}{1 - \cos 4x} & , \text{for} x \neq 0 \\ = \frac{10}{7} & \text{for} x = 0 \end{array}\right\} at x = 0.$$

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5. Discuss the continuity of the following functions at the points shown against them :

$$\left.\begin{array}{ll} f(x) = \frac{\log x - \log 8}{x - 8}, & \text{for} x \neq 0 \\ = \frac{10}{7}, & \text{for} x = 0 \end{array}\right\} at x = 8.$$

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6. Discuss the continuity of the following functions at the points shown against them :

$$\left.\begin{array}{ll} f(x) = \frac{1 - \sin x}{\left(\frac{\pi}{2} - x\right)^2}, & \text{for} x \neq \frac{\pi}{2} \\ = 3, & \text{for} x = \frac{\pi}{2} \end{array}\right\} at x = \frac{\pi}{2}.$$

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7. Discuss the continuity of the following functions at the points shown against them :

$$\left.\begin{array}{ll} f(x) = \frac{x}{|x|}, & \text{for} x \ne 0 \\ = 1, & \text{for} x = 0 \end{array}\right\} at x = 0.$$

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8. Diseuss the continuity of the function :

$$\left.\begin{array}{ll} f(x) = \frac{\sin 2x}{\sqrt{1 - \cos 2x}} & , \text{for} 0 < x \le \frac{\pi}{2} \\ = \frac{\cos x}{\pi - 2x} & , \text{for} \frac{\pi}{2} < x < \pi \end{array}\right\} at x = \frac{\pi}{2}.$$

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9. Find the value of k, if the function f given by :

$$f(x) = \left[\tan\left(\frac{\pi}{4} + x\right)^{\frac{1}{x}}\right], \quad \text{for} x \neq 0$$

$$= k, \quad \text{for} x = 0$$

is continous at $x = 0$.

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10. Find the value of k, if the function f given by :

$$f(x) = \frac{1 - \tan x}{1 - \sqrt{2}\sin x}, \quad \text{for} x \neq \frac{\pi}{4}$$

$$= \frac{k}{2}, \quad \text{for} x = \frac{\pi}{4}$$

is continous at $x = \frac{\pi}{4}$.

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11. Find the value of k, if the function f given by :

$$F(X) = \frac{8^x - 2^x}{k^x - 1}, \quad \text{for} x \neq 0$$
$$= 2, \quad \text{for} x = 0$$

is continous at $x = 0$.

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12. If $(x) = \frac{1 - \sqrt{3}\tan x}{\pi - 6x}$, $\text{for} x \neq \frac{\pi}{6}$ is continous at $x = \frac{\pi}{6}$, find $f\left(\frac{\pi}{6}\right)$.

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13. if $f(x) = \frac{e^{x^2} - \cos x}{x^2}$, for $x \neq 0$ is continuous at $x = 0$, then value of f(0) is

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14. Discuss the continous of the following function on its domain, where

$f(x) = x^2 - 4, \text{for} 0 \le x \le 2$

$= 2x + 3, \text{for} 2 < x \le 4$

$= x^2 - 05, \text{for} 4 < x \le 6.$

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15. Show that the function defined b y $f(x) = |\cos x|$ is continous function.

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16. If f(x) $= x^2 + a$, for $x \geq 0$

$= 2\sqrt{x^2 + 1} + b, \text{ for} x <$ and $f\left(\frac{1}{2}\right) = 2,$ is

continuous at x = 0 , find a and b .

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17. A functon f(x) is defined as

$f(x) = x + a, \text{ for} x < 0$

$= x, \text{ for} 0 \leq x < 1,$

$= b - x, \text{ for} x \geq 1$

is continous on its domain. Find $a + b$.

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18. If the function f(x) is continous on the interval $[-2, 2]$, find the values of a and b, where

$$f(x)\frac{\sin ax}{x}-2, \quad \text{for} -2 \leq \ < 0$$
$$= 2x+1, \quad \text{for} 0 \leq x \leq 1$$
$$= 2b\sqrt{x^2+3}-1, \quad \text{for} 1 < x \leq 2.$$

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Examples For Practice

1. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes continuous :

$$\left.\begin{array}{ll} f(x)=\dfrac{x^2-4x}{\sqrt{x^2+9}-5}, & \text{for} x\neq 4 \\ =3, & \text{for} x=4 \end{array}\right\} at x=4.$$

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2. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes continuous :

$$\left.\begin{array}{ll} f(x)\sin x-\cos x, & \text{for} x\neq 0 \\ =-1, & \text{for} x=0 \end{array}\right\} at x=0.$$

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3. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes continuous :

$$\left.\begin{aligned} f(x) &= \frac{e^{5x} - e^{2x}}{\sin 3x}, \quad && \text{for} x \neq 0 \\ &= 1, && \text{for} x = 0 \end{aligned}\right\} at x = 0.$$

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4. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes

continuous :

$$f(x) = (1+\cos 2x)^{4\sec 2x}, \quad \text{for} x \neq \frac{\pi}{4} \\ = e^4, \quad \text{for} x = \frac{\pi}{4} \Bigg\} at x = \frac{\pi}{4}.$$

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5. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes continuous :

$$f(x) = \frac{e^{5x} - e^{2x}}{\sin 3x}, \quad \text{for} x \neq 0 \\ = 1, \quad \text{for} x = 0 \Bigg\} at x = 0.$$

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6. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes continuous :

$$\left.\begin{array}{ll} f(x)=\frac{1-\cos 3x}{x\tan x}, & \text{for} x\neq 0 \\ =9, & \text{for} x=0 \end{array}\right\} at x=0.$$

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7. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes

continuous :

$$f(x) = \frac{\log x - \log 7}{x - 7}, \quad \text{for} x \neq 7 \\ = 7, \quad \text{for} x = 7 \Bigg\} at x = 7.$$

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8. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes continuous :

$$f(x) = \frac{\left(3^{\sin x} - 1\right)^2}{x \log(1 + x)}, \quad \text{for} x \neq 0 \\ = 2\log 3, \quad \text{for} x = 0 \Bigg\} at x = 0.$$

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9. Discuss the continuity of the functions at the points given against them. If a function is discontinuous, determine whether the discontiunity is removable. In this case, redefine the function, so that it becomes continuous :

$$\left.\begin{array}{ll} f(x) = \frac{\log(2+x) - \log(2-x)}{\tan x}, & \text{for} x \neq 0 \\ = 1, & \text{for} x = 0 \end{array}\right\} at x = 0.$$

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10. Find the value of k, if the functions are continuous at the points given against them :

$$\left.\begin{array}{ll} f(x) = \frac{1 - \cos kx}{x \sin x}, & \text{for} x \neq 0 \\ = 9, & \text{for} x = 0 \end{array}\right\} at x = 0.$$

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11. Find the value of k, if the functions are continuous at the points given against them :

$$\left.\begin{array}{ll} f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}, & \text{for} x \neq \frac{\pi}{3} \\ = k, & \text{for} x = \frac{\pi}{3} \end{array}\right\} at x = \frac{\pi}{3}.$$

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12. Find the value of k, if the functions are continuous at the points given against them :

$$\left.\begin{array}{ll} f(x) = \frac{x - \frac{\pi}{4}}{\sin x - \cos x}, & \text{for} x \neq \frac{\pi}{4} \\ = k, & \text{for} x = \frac{\pi}{4} \end{array}\right\} at x = \frac{\pi}{4}.$$

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13. Find the value of k, if the functions are continuous at the points given against them :

$$\left.\begin{array}{ll} f(x) = x^2 + 1, & \text{for} x \geq 0 \\ = 2\sqrt{x^2 + 1} + k, & \text{for} x < 0 \end{array}\right\} at x = 0.$$

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14. Find the value of k, if the functions are continuous at the points given against them :

$$\left.\begin{array}{ll} f(x) = \frac{e^{kx} - 1}{\sin x}, & \text{for} x \neq 0 \\ = 4, & \text{for} x = 0 \end{array}\right\} at x = 0.$$

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15. If f (x) is continuous at $x = \pi$, where $f(x) = \frac{\sqrt{2 + \cos x} - 1}{(\pi - 2)^2}$, for $x \neq \pi$, find $f(\pi)$.

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16. If the function $f(x) = \frac{(4^{\sin x} - 1)^2}{x \log(1 + 2x)}$, for $x \neq 0$, is continous at $x = 0$, find $f(0)$.

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17. If f(x) is continous at $x = 0$, where $f(x) \frac{(e^{3x} - 1)\sin x}{x \log(1 + x)}$, for $x \neq 0$, find f(0).

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18. Discuss the continuity of the functions on te intervals shown below them or against them :

$$f(x)=\frac{x^2+x-12}{x^2-3x+2}, on[0,4].$$

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19. Discuss the continuity of the functions on te intervals shown below them or against them :

$$\begin{aligned} f(x) &= 3x+5, && \text{for} 0\le x<3 \\ &= 2x+8, && \text{for} 3\le x<5 \\ &= x+13, && \text{for} 5\le x\le 10 \end{aligned}$$

on its domain.

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20. Discuss the continuity of the functions on te intervals shown below them or against them :

$$f(x) = \frac{2x+5}{x+1}, \quad \text{for} 0 \leq x < 2$$
$$= 4x - 5, \quad \text{for} 2 \leq x \leq 4$$
$$= \frac{x^2+2}{x-5}, \quad \text{for} 4 < x \leq 6, x \neq 5$$

on its domain.

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21. If f (x) is continous on $[0, 8]$ defined as

$$f(x) = x^2 + ax, \text{for} 0 \leq x < 2$$

$$= 3x + 2, \text{for} 2 \leq x \leq 4$$

$= 2ax + 5b, \text{for} 4 < x \leq 8,$

find a and b.

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22. If $f(x)\frac{x^2 - 9}{x - 3} + \alpha, \text{for} x > 3$

$= 5, \text{for} x = 3$

$= 2x^2 + 3x + \beta, \text{for} x < 3$

is continous at $x = 3$, find α and β.

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23. If (f) is continuous on its domian $[-2, 2]$, where

$f(x) = \frac{\sin x}{x} + a, \text{for} - 2 \leq x < 0$

$= 3x + 5, \text{ for} 0 \leq x \leq 1$ `=sqrt(x^(2)+8)-b, "for" 1 lt x

le2, find the values of a and b.

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24. Find α and β, so the function f (x) defined by

$$f(x) = -2\sin x, \text{ for } -\pi \leq x \leq -\frac{\pi}{2}$$

$$= \alpha \sin x + \beta \text{for} -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$= \cos x, \text{ for} \frac{\pi}{2} \leq x \leq \pi,$$

is continuous on $[-\pi, \pi]$.

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25. Find the values of a and b so that the function

$$f(x) = \begin{cases} x + a\sqrt{2}\sin x, & 0 \le x \le \pi/4 \\ 2x\cot x + b, & \pi/4 \le x \le \pi/2 \\ a\cot 2x - b\sin x, & \pi/2 < x \le \pi \end{cases}$$

is continuous for $0 \le x \le \pi$.

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26. If f (x) is continuous on $0 - 4, 2]$, defined as

$$f(x) = 6b - 3ax, \text{ for } -4 \le x < -2$$

$$= 4x + 1, \quad \text{for } -2 \le x \le 2,$$

find the value of $a + b$.

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Multiple Choice Questions

1. Select the write the most appropriate answer from the given alternatives in each of the following :

$$f(x)\frac{\sin 5x}{3x}, \quad x \neq 0$$
$$= k, \qquad x = 0.$$

If f is continuous at $x = 0$, then the value of k is

A. $\frac{3}{5}$

B. $\frac{1}{2}$

C. $\frac{5}{3}$

D. 0.

Answer:

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2. Select the write the most appropriate answer from the given alternatives in each of the following :

If f (x) continuous at $x = 0,$ where

$$f(x)\frac{1-\cos kx}{x^2}, \quad \text{for} x \neq 0$$
$$= \frac{1}{2}, \quad \text{for} x = 0.$$

then the value of k is

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. ± 2

D. $\pm 1.$

Answer:

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3. Select the write the most appropriate answer from the given alternatives in each of the following :

If the function,

$$f(x) = k + x, \quad \text{for} x < 1$$
$$= 4x + 3, \qquad \text{for} x \geq 1$$

is continous at $x = 1$, then $k = \ldots\ldots\ldots$

A. 7

B. 8

C. 6

D. -6

Answer:

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4. Select the write the most appropriate answer from the given alternatives in each of the following :

If f (x) is continuous at $x = 0$, where

$$f(x) = \frac{3^x - 3^{-x}}{\sin x}, \quad \text{for} x \neq 0$$
$$= k, \quad \text{for} x = 0.$$

then the value of k is

A. 0

B. log 3

C. $(\log 3)^2$

D. $\log 9$.

Answer:

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5. Select the write the most appropriate answer from the given alternatives in each of the following :

If the function

$$f(x)\frac{\sin 3x}{7x}+a, \quad \text{for} x>0$$
$$=x+3-b, \quad \text{for} x<0$$

is continous at $x=0$, then $a+b$ is equal to

A. $\frac{2}{7}$

B. $\frac{7}{18}$

C. $\frac{18}{7}$

D. $-\frac{18}{7}$.

Answer:

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6. Select the write the most appropriate answer from the given alternatives in each of the following :

The function $f(x)=\frac{|x|}{x^2+2x}, x\neq 0 \text{ and } f(0)=0$ is not continuous at $x=0$ because

A. $\lim_{x\to 0} f(x)\neq f(0)$

B. $\lim_{x\to 0} f(x)$ does not exist

C. $\lim_{x\to 0} f(x)\frac{1}{2}$

D. $\lim_{x \to 0} f(x) = -\frac{1}{2}$.

Answer:

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