MATHS

BOOKS - NAVBODH MATHS (HINGLISH)

DIFFERENTIAL EQUATIONS

Solved Examples

1. find the order and degree of D.E : (1) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = e^x$ (2) $\sqrt{1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$ (3) $e^{\frac{dy}{dx}} + \frac{dy}{dx} = x$

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2. Form the differential equations by elimniating the arbitary constants from the following equations :

(1) $y = c^2 + \frac{c}{x}$ (2) $x^3 + y^3 = 4ax$ (3) $y = Ae^{5x} + Be^{-5x}$

(4) $y = A\cos\alpha x + B\sin\alpha x$

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3. Verify the solution problems: Show that $y = e^{-x} + ax + b$ is solution of the differential equation $e^x \frac{d^y}{dx^2} = 1$

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4. $x^2 + y^2 = r^2$ is a solution of the D.E. $y = x\frac{dy}{dx} + r\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

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5. y sec x = tan x + c is a solution of D.E . $\frac{dy}{dx} + y\tan x = \sec x.$

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6. $y = \log x + c$ is a solution of the differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

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7. $y = Ae^{x} + Be^{-2x}$ is a solution of the D.E. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$

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8. from the differential equation by eliminating the arbitrary constants from the following equations :

$(1) y = e^{x}(A\cos x + B\sin x)$

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9. From the differential equation by eliminating A and B in $Ax^2 + By^2 = 1$

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10. `y = A cos (log x) + B sin (log x)

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11. $y = c_1 e^{3x} + c_2 e^{2x}$

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12. Form the differential equation of the family of circles touching the x-axis at origin.

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13. Fid the differential equation of all the parabolas with latus rectum '$4a$' and whose axes are parallel to x-axis.

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14. Solve the following differential equation :

1. $\frac{dy}{dx} = 1 + x + y + xy$

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15. $y - x\frac{dy}{dx} = 0$

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16. $\sec^2 x \tan y dx + \sec^2 y \tan x dy = dy = 0$

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17. Solve the following differential equation.

$(1)e^x \tan^2 y dx + (e^x - 1)\sec^2 y dy = 0$

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18. $y - x\frac{dy}{dx} = a\left(y^2 + \frac{dy}{dx}\right)$

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19. Find the general solution of each of the following differential equations:

$$\left(x^2 - yx^2\right)dy + \left(y^2 + xy^2\right)dx = 0$$

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20. Solve the following differential equations with the help of the subsitutions shown against then :

$$\frac{dy}{dx} = \cos(x + y), x + y = v$$

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21. $(x - y)^2 \frac{dy}{dx} = a^2, x - y = u$

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22. $\left(x\frac{dy}{dx}-y\right)\sin\left(\frac{y}{x}\right)=x^2e^x, y=vx$

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23. Solve $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$

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24. $\frac{dy}{dx}=(4x+y+1)^2$

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25. $(2x-2y+3)dx-(x-y+1)dy=0, x-y=u$

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26. Solve the following differential equations : (1) $\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}$

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27. $y^2 dx + \left(xy + x^2\right) dy = 0$

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28. Solver the following differential equation :

(1) $x \frac{dy}{dx} + 2y = x^2 . \log x$

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29. Solve the following differential equation: $\cos^2 x \frac{dy}{dx} + y = \tan x$

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30. $y \log y \frac{dx}{dy} + x - \log y = 0$

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31. The slope of the tangent to the curve at any point is equal to y + 2x. Find the equation of the curve passing through the origin .

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32. Find the particular solutions of the following differential equation :

(1) xdx + ydy =0 , when x =3 , y =4

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33.

$\sec^2 y \tan x dy + \sec^2 x \tan y dx = 0, \quad \text{when} \quad x = y = \frac{\pi}{4}$

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34. $\frac{dy}{dx} = e^{2y} \cos x, \quad \text{when} \quad x = \frac{\pi}{6}, y = 0$

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35. Find the particular solutions of the following differential equation :

(1) $y(1+\log x)\frac{dx}{dy} - x\log x = 0, when, x = e, y = e^2$

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36.

$3e^x \tan y dx + (1+e^x)\sec^2 dy = 0, \quad \text{when} x = 0 \text{ and } y = \pi$

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37. cos (x+y) dy =dx , when x =0 and y =0

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38. $x + y\frac{dy}{dx} = \sec\left(x^2 + y^2\right)$ when $x = y = 0$

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39. $\frac{dy}{dx} - y = e^x$ when$x = 0$ and $y = 1$

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40. If the population of a country doubles in 60 years , in how many years will it be triple (treble) under the assumption that the rate of increase is proportional to the number of inhabitants ?

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41. The rate of growth of bacteria is proportional to the number present . If initially, there were 1000 bacteria and the number doubles in 1 hours. Find the number of bacteria after $2\frac{1}{2}$ hours . [take $\sqrt{2} = 1.414$]

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42. The rate of disintegration of a radioactive element at any time t is proportional to its mass at that time. Find the time during which the original mass of 1.5 g m will disintegrate into its mass of 0.5 gm.

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43. A right circular cone has height 9 cm and radius of the base 5 cm. it is inverted and water is poured into it. If an any instant the water level rises at the rate of $\left(\frac{\pi}{A}\right)$ cm /sec, where A is the area of the water surface at the instant , show that the vessel will be full in 75 seconds

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44. A body is heated to $110^{\circ}C$ and placed in air at $10^{\circ}C$. After 1 hour its temperature is $60^{\circ}C$. How much additional time is required for it to cool to $35^{\circ}C$?

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Examples For Practice

1. Find the order and degree of the D.E. :

(1) $\frac{d^2y}{dx^2} + x\frac{dy}{dx0 + y = 2\sin x}$

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2. find order and degree $\frac{d^2y}{dx^2} = \sqrt[3]{1 + \left(\frac{dy}{dx}\right)^2}$

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3. find order and degree of given differential equation :

$x + \frac{dy}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

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4. find order and degree $\left[\frac{d^3y}{dx^3}+x\right]^{\frac{5}{2}}=\frac{d^2y}{dx^2}$

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5. find order and degree $\frac{d^2y}{dx^2}+\frac{dy}{dx}+x=\sqrt{1+\frac{d^3y}{dx^3}}$

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6. Find Degree and Order of $\left(\frac{d^2y}{dx^3}\right)^2+\cos\left(\frac{dy}{dx}\right)=0$

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7. Form the differential equations by eliminating the arbitrary constants from the following equations :

$(1)(x-a)^2+y^2=1$

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8. What is the differential equation of the curve $y=ax^2+bx$?

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9. $y=(c_1+c_2x)e^x$

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10. $y=c_1e^{2x}+c_2e^{-2x}$

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11. If y= a cos 2x + b sin 2x , then

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12. $y^2 = (x + c)^3$

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13. If $y = e^{ax}$ Show that $x\frac{dy}{dx} = y \log y.$

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14. In the following questions verify that the given function is a solution of the given differential equation :

$y = c_1 \sin x + c_2 \cos x, \frac{d^2y}{dx^2} + y = 0$

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15. The solution of the differential equation $\frac{d^2y}{dx^2} = e^{-2x}$, is

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Examples For Practice 2

1. Form the differential equations by eliminating the arbitrary constants from the following equations :

1. $(1)xy = Ae^{x} + Be^{-x} + x^{2}(2)y = e^{-x}(A\cos 2x + B\sin 2x)$

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2. $(1)y = ae^{4x} - be^{-3x} + c(2)xy = ae^{5x} + be^{-5x}$

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3. Form the differential equation for the given solution: $b^{2}x^{2} + a^{2}y^{2} = a^{2}b^{2}$

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4. The differential equation of all circles passing through the origin and having their centres on the x-axis is (1)

$x^2 = y^2 + xy\frac{dy}{dx}$ (2) $x^2 = y^2 + 3xy\frac{dy}{dx}$ (3)

$y^2 = x^2 + 2xy\frac{dy}{dx}$ (4) $y^2 = x^2 - 2xy\frac{dy}{dx}$

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5. the family of circles having their centres on the line y=10 and touching the X-axis.

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6. All parabolas whose axis is the Y-axis.

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7. Solve the following differential equations : (1) $\frac{dy}{dx} = x^2y + y$

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8. $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

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9. Solve: $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

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10. $y^3 - \frac{dy}{dx} = x^2\frac{dy}{dx}$

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11. $\cos x \cos y dy - \sin x \sin y dx = 0$

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12. sec xdy + cosec y dx =0

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13. $\tan y \frac{dy}{dx} = \sin x + \cos x$

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14. $\frac{dy}{dx} = y\left(\frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}\right)$

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15. $2e^{x+2y}dx - 3dy = 0$

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16. $\frac{dy}{dx} = x\sqrt{25 - x^2}$

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17. $\frac{dy}{dx} = 4^{x+y}$

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18. $\log\left(\frac{dy}{dx}\right) = 2x + 3y$

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Examples For Practice 3

1. Solve the following differential equations :

$$y - x\frac{dy}{dx} = 3\left(1 + x^2\frac{dy}{dx}\right)$$

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2. Solution of the differential equation $\frac{dy}{dx}\tan y = \sin(x + y) + \sin(x - y)$ is

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3. $y(2\log y+1)dy=(\sin x+x\cos x)dx$

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4. $y\sqrt{1-x^2}dy+x\sqrt{1-y^2}dx=0$

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5. $e^{-x}\dfrac{dy}{dx}=y\left(1+\tan x+\tan^2 x\right)$

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6. $xy\log ydx + \left(1+x^2\right)dy = 0$

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Examples For Practice 4

1. Using the substitution x + y = u :

(1) $(x+y)dy = a^2dx$

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2. $\sin^{-1}\left(\frac{dy}{dx}\right) = x+y$

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3. The solution of differential edquation $\frac{dy}{dx} + 1 = \cos ec(x + y)$ is

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4. Using the substitution x-y =u : $\cos^{-2}(x - y)\frac{dy}{dx} = 1$

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5. $(x - y)\left(1 - \frac{dy}{dx}\right) = e^x$

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6. using the subsitiution y = vx :

(1) $\left(x\frac{dy}{dx}-y\right)^{e^{\frac{y}{x}}=x^2\cos x}$

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7. $x\sin\left(\frac{y}{x}\right)dy=\left[y\sin\left(\frac{y}{x}\right)-x\right]dx$

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8. Using the substitution shown against them :

$\cos(x-2y)+2\frac{dy}{dx}=0, x-2y=u$

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9. $x + y\frac{dy}{dx} = x^2 + y^2$

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10. $\frac{dy}{dx} = (4x + 3y - 1)^2$

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11. $(x + 2y + 1)dx - (2x + 4y + 3)dy = 0, x + 2y = u$

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Examples For Practice 5

1. Solve the following differential equations : (1) $x^2 \frac{dy}{dx} = x^2 + xy + y^2$

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2. Solve: $x^2 y dx = \left(x^3 + y^3\right) dy = 0$

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3. $\left(x^2 + y^2\right) dx - 2xy dy = 0$

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4. Solve the following differential equations:

$\left(x^2+3xy+y^2\ \hat{}\ \right)dx-x^2dy=0$

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5. $y^2-x^2\frac{dy}{dx}=xy\frac{dy}{dx}$

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6. $xy\frac{dy}{dx}=x^2+2y^2$

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Examples For Practice 6

1. Solve the following differential equations :

(1) $\frac{dy}{dx} + \frac{y}{x} = x^2 - 3$

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2. $\frac{dy}{dx} + 2y \tan x = \sec x$

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3. $ydx - xdy + \log x dx = 0$

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4. $x \sin x \frac{dy}{dx} + (x \cos x + \sin x) = \sin x$

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5. $\left(x+2y^3\right)\frac{dy}{dx}=y$

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6. $ydx+\left(x-y^2\right)dy=0$

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7. Find the equation of a curve passing through the point (0, 2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

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8. Find the equation of a curve passing through the point (0, 1). If the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x coordinate (abscissa) and the product of the x coordinate and y coordinate (ordinate) of t

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9. Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

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Examples For Practice 7

1. Find the particular solutions of the following differential equations : $\frac{dx}{x+2}+\frac{dy}{y+2}=0, \quad \text{when} \quad x=1, y=2$

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2. $xdy+2ydx=0, \quad \text{when} \quad x=2, y=1$

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3. $\frac{dy}{dx}=3^{x+y}, \quad \text{when} \quad x=y=0$

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4. For each of the folowing differential equations, find a particular solution satisfying the given condition:

$$\cos\left(\frac{dy}{dx}\right) = a, \quad \text{where} \quad a \in R \text{ and } y = 2 \quad \text{when} \quad x = 0.$$

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5. $\frac{dy}{dx} + xy = xy^2 \quad \text{when} \quad x = 1, y = 4$

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6. $y - x\frac{dy}{dx} = 0, \quad \text{when} \quad x = 2, y = 3$

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Examples For Practice 8

1. Find particular solutions of the following differential equations : (1)

$$(x - y^2x)dx - (y + x^2y)dy = 0, \quad \text{when} \quad x = 2, y = 0$$

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2. $(x + 1)\dfrac{dy}{dx} - 1 = 2e^{-y}, y = 0, \quad \text{when} \quad x = 1$

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3. $\left(y + x\dfrac{dy}{dx}\right)\sin xy = \cos x, \quad \text{when} \quad x = 0, y = 0$

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4. The differential equations, find a particular solution satisfying the given condition:

$$\frac{dx}{dy} + 2y\tan x = \sin x; y = 0 \text{when } x = \frac{\pi}{3}$$

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5. $e^{\frac{dy}{dx}} = x + 1, y(0) = 3$

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6. $xy\frac{dy}{dx} = x^2 + 2y^2, y(1) = 0$

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7.

$(2x - 2y + 3)dx - (x - y + 1)dy = 0, \quad \text{when} x = 0, y = 1$

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Examples For Practice 9

1. The population of a town increases at a rate proportional to the population at that time . If the population increases from 40 thousands to 60 thousands in 40 years, What will be the population in another 20 years ? = [Given: $\sqrt{\frac{3}{2}} = 1.2247$]

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2. Bacteria increases at the rate proportional to the number of bacteria present. If the original number N doubles in 3 hours, find in how many hours the number of bacteria will by 4N ?

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3. The population grows in at the rate of 8% per year. Find the time taken for the population to become double . (Given : log 2 = 0.6912)

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4. The rate of decay of certain substance is directly proportional to the amount present at that instant . Initially

there are 27 gm of certain substance and three hours later it is found that 8 gm are left. Find the amount left after one more hour.

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5. Water at $100^\circ C$ cools in 10 minutes to $88^\circ C$ in a room temperature of $25^\circ C$ .Find the temperature of water after 20 minutes.

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MCQ

1. The degree and order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}}=7\left(\frac{d^2y}{dx^2}\right)$ respectively are

A. 2,3

B. 3,2

C. 2,2

D. 3,3

Answer: 2,3

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2. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{6}}-\left(\frac{dy}{dx}\right)^{\frac{1}{3}}=0$ are respectively .

A. 3,2

B. 2,3

C. 6,3

D. 3,1

Answer: 3,1

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3. The order of the differential equation of the family of parabolas whose axis is the X-axis is

A. 2

B. 1

C. 3

D. 4

Answer: 2

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4. The differential equation of $y = c^2 + \frac{c}{x}$ is

A. $x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = y$

B. $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

C. $x^3\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} = y$

D. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$

Answer: $x^2\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = y$

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5. The differential equation of the family of curves $y = c_1 e^x + c_2 e^{-x}$ is

A. $\frac{d^2y}{dx^2} + y = 0$

B. $\frac{d^2y}{dx^2} - y = 0$

C. $\frac{d^2y}{dx^2} + 1 = 0$

D. $\frac{d^2y}{dx^2} - 1 = 0$

Answer: $\frac{d^2y}{dx^2} - y = 0$

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6. The integrating factor of linear differential equation $\frac{dy}{dx} + y\sec x = \tan x$ is

A. $\sec x - \tan x$

B. sec x .tan x

C. sec x + tanx

D. sec x. cot x .

Answer: C

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7. The solution of the differential equation $\frac{dy}{dx} = \sec x - y\tan x$ is

A. y sec x = tan x +c

B. y sec x + tan x =c

C. sec x = y tan x + c

D. sec x + y tan x=c

Answer: y sec x = tanx +c

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8. The solution of the DE $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is

A. $\sin^{-1} x - \sin^{-10y=c}$

B. $\sin^{-1} x + \sin^{-1} y = c$

C. $\left(1-x^2\right)\left(1-y^2\right) = c$

D. $2\sin^{-1}y+\sin^{-1}x=c$

Answer: $\sin^{-1}x+\sin^{-1}y=c$

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9. If $\sin x$ is an integrating factor of the differential equation $\frac{dy}{dx}+Py=Q$, then write the value of P.

A. tanx

B. cot x

C. log | sin x|

D. $-\cot x$

Answer: cot x

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10. Integrating factor of linear differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$ is

A. $\frac{1}{x^2}$

B. $\frac{1}{x}$

C. x

D. x^2

Answer: x^2

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11. The solution of $\frac{dy}{dx} = \frac{y + \sqrt{x^2 - y^2}}{x}$ is

A. $\sin\left(\frac{y}{x}\right) = \log|x| + c$

B. $\sin^{-1}\left(\frac{y}{x}\right) = \log|x| + c$

C. $\sin\left(\frac{x}{y}\right) = \log|x| + c$

D. $\sin^{-1}\left(\frac{y}{x}\right) = 2\log|x| + c$

Answer: $\sin^{-1}\left(\frac{y}{x}\right) = \log|x| + c$

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12. The solution of $\frac{dy}{dx} - y = e^x, y(0) = 1$, is

A. $y = (x+1)e^x$

B. $y = (x-1)e^x$

C. $y = (x^2+1)e^x$

D. $y = (1 - x)e^x$

Answer: $y = (x + 1)e^x$

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