MATHS

BOOKS - NAVBODH MATHS (HINGLISH)

LINE

Solved Example

1. Find the cartesian equation of the line passing through the points $A(3, 4, -7)$ and $B(6, -1, 1)$

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2. Find the vector equation of a line which passes through the point with position vector $4\hat{i} - \hat{j} + 2\hat{k}$ and is in the direction of $-2\hat{i} + \hat{j} + \hat{k}$.

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3. The cartesian equation of a line is $3x + 1 = 6y - 2 = 1 - z$.

Find the vector equation of the line.

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4. Find the direction cosines of the line $\frac{x+2}{2} = \frac{2y-5}{3}, z = -1$

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5. Find the angle between the pair of lines $\bar{r} = \left(3\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and $\bar{r} = \left(5\hat{i} - 2\hat{k}\right) + \mu\left(3\hat{i} + 2\hat{j}\right.$

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6. The equation of line passing through (3,-1,2) and perpendicular to the lines

$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$

and $\vec{r} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ is

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7. Find the vector equation and cartesian equation of a line passing through the points A(3 ,4, -7) and B(6, -1, 1)

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8. Find the equation of the line passing through the point (3, 1, 2) and perpendicualr to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$

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9. Show that the lines $\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1}$ and $\frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4}$ intersect

each other and find the coordinates of the points of intersection .

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10. Show that the lines $\bar{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$ and $\bar{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$ intersect. Find their point of intersection .

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11. Find the coordinates of the foot of perpendicular drawn from the point $2\hat{i} - \hat{j} + \hat{5}k$ to the line $\bar{r} = \left(11\hat{i} + 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$. Also , find the length of perpendicular .

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12. Find the length of the perpendicular from (3,2,1) to the line $\frac{x-7}{-2} = \frac{y-7}{2} = \frac{z-6}{3}$.

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13. Find the shortest distance between the lines $\bar{r} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\bar{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(\hat{i} + 4\hat{j} - 5\hat{k}\right)$.

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14. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

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15. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k.

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16. Find the two points on the line $\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+5}{2}$ on either side of $(2,\ -3,\ -5)$ which are at a distance of 3 units from it.

A. $\frac{x-2}{1}=\frac{z+4}{1}, y=3$

B. $x=2, y=3+\lambda, z=-4$

C. $x=-2, y=-3+\lambda, z=4$

D. $x=-2, y=\lambda+3, z=4$

Answer:

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Theory

1. parallel to the vector $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$, where λ is a scalar.

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Examples For Practice

1. Find the equartion of the line in vector form passing through the point (4 , -2,5) and parrallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$.

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2. Find the vector equation of the line passing through (4 , 2, 1) and (2, -1, ,3) .

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3. If the points A(5,5 λ) , B (- 1, 3 , 2) and C (-4 , 2 , -2) are collinear , find the value of λ .

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4. Find the vector equation of a line passing through the point with position vector $2\hat{i} + \hat{j} - \hat{k}$ and parallel to the line joining the points - $\hat{i} + \hat{j} + 4\hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$.

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5. The cartesian equation of a line is $\frac{x-6}{2} = \frac{y+4}{7} = \frac{z-5}{3}$, find its vector equation .

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6. The cartesian equation of a line is 3x - 1 = 6y + 2 = 1 - z . Find the vector equation of the line .

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7. Find the angle between the pair of lines

$$\frac{x-1}{4}=\frac{y-3}{1}=\frac{z}{8} \text{ and } \frac{x-2}{2}=\frac{y+1}{2}=\frac{z-4}{1}.$$

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8. if the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-5}{1}=\frac{z-6}{-5}$ are at night angle , then find the value of k .

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9. A line passing through the points A (-2 , -1 , 5) and B (1, 3 , -1) ,find the equation of the line in parametric form. Also, write the equation in non - parametric form.

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10. Find the vector equation of the line passing through the point with position vector $2\hat{i} - \hat{j} + \hat{k}$ and parallel to the line joining the points with position vectors $-\hat{i} + 4\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 2\hat{k}$. Also, find the cartesian equation of the line.

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11. Find the vector equation of the line passing through the point $2\hat{i} + \hat{j} - 3\hat{k}$ and perpendicular to the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$

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12. Find the equation of the line passing throught the point $(2, 1, 3)$ and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

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13. Find the vector equation of the line passing throught the point $(-1, -1, 2)$ and parallel to the line $2x - 2 = 3y + 1 = 6z - 2$.

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14. Find the coordinates of the point of intersection for the following lines :

$$\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-1}{-1} \text{ and } \frac{x-4}{2} = \frac{y-2}{-1} = \frac{z-4}{2}$$

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15. Find the foot of the perpendicular from the point $(0, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Also, find the length of the perpendicular.

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16. Find the co-ordinates of the foot of perpendicular and length of perpendicular drawn from point $\left(\hat{i}+6\hat{j}+3\hat{k}\right)$ to the line $\vec{r}=\hat{j}+2\hat{k}+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$.

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17. Find the shortest distance between the following pair of line: $\vec{r}=\hat{i}+2\hat{j}+\hat{k}+lamd\left(\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu\left(2\hat{i}+\hat{j}+$

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18. Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}and\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$.

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19. Determine whether the following pair of lines intersect: $\vec{r} = \hat{i} - \hat{j} + \lambda\left(2\hat{i} + \hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

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20. Find the coordinates of the points on the line $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z+3}{6}$, which are at a distance of 3 units from the point (-1 , 2 , -3).

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Multiple Choice Question

1. Equation of the line passing through the point (2,3, -4) and perpendicular to XZ-plane is

A. $\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 13\hat{k}\right)$

B. $\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left(8\hat{i} + 5\hat{j} + \hat{k}\right)$

C. $\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} - k\right)$

D. $\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} - 13\hat{k}\right)$

Answer: (b) x = 2 , y = 3 + lamda , z = -4

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2. The vector form of the equation of the line passing through the points (3, 4, -7) and (5, 1 , 6) is

A. $\bar{r} = \left(-i - j + 2\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + \hat{k}\right)$

B. $\bar{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

C. $\bar{r} = \left(-\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$

D. $\bar{r} = \left(-\hat{i} + -\hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$

Answer: (a) $\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left(2\hat{i} - 3\hat{j} + 13\hat{k}\right)$

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3. The vector equation of the line passing through the point (-1, -1 ,2) and parallel to the line 2x - 2= 3y +1 = 6z -2 is

A. $\frac{11}{70}$

B. $\frac{70}{11}$

C. $-\frac{10}{11}$

D. $\frac{10}{11}$.

Answer: (d) $\bar{r} = \left(-\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + \hat{k}\right)$

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4. If the lines $\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angle, then the value of λ is

A. $\bar{r} = \left(2\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

B. $\bar{r} = \left(-\frac{1}{3}\hat{i} + \frac{1}{3}\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

C. $\bar{r} = \left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

D. $\bar{r} = \left(-\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

Answer: (b) (70)/(11)

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5. The vector form of the equation of the line 6x - 2 = 3y + 1 = 2z - 2 is

A. $\frac{\cos^{-1}(19)}{21}$

B. $\frac{\cos^{-1}(1)}{21}$

C. $\frac{\pi}{2}$

D. $\frac{\cos^{-1}(9)}{21}$.

Answer: $\bar{r} = \left(\frac{1}{3}\hat{i} - \frac{1}{3}\hat{j} + \hat{k}\right)\Big) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

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6. The angle between the lines $\bar{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right) \text{and} \bar{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\right.$ is

A. $\frac{\left|\left(\overline{a_2} - \overline{a_1}\right).\left(\overline{b_1} \times \bar{b}_2\right)\right|}{\left|\bar{b}_1 \times \bar{b}_2\right|}$

B. $\frac{\left|\left(\overline{a_2} - \overline{b_1}\right).\left(\overline{a_2} \times \bar{b}_2\right)\right|}{\left|\bar{b}_1 \times \bar{b}_2\right|}$

C. $\frac{\left|\left(\overline{a_2} - \overline{b_2}\right).\left(\overline{a_1} \times \bar{b}_1\right)\right|}{\left|\bar{b}_1 \times \bar{b}_2\right|}$

D. $\frac{\left|\left(\overline{a_1} - \overline{b_2}\right).\left(\overline{b_1} \times \bar{a}_2\right)\right|}{\left|\bar{b}_1 \times \bar{b}_2\right|}$

Answer: $\frac{\cos^{-1}(19)}{21}$

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7. The shortest distance between the skew lines $\bar{r} = \overline{a_1} + \lambda\overline{b_1} \text{and} \bar{r} = \overline{a_2} + \mu\overline{b_2}$ is

A. -2

B. -5

C. 5

D. 2

Answer: $\frac{\left|(\overline{a_2}-\overline{a_1}).\left(\overline{b_1}\times\overline{b_2}\right)\right|}{\left|\overline{b_1}\times\overline{b_2}\right|}$

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8. If the straight lines $\frac{x-1}{k}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x-2}{3}=\frac{y-3}{k}=\frac{z-1}{2}$ intersect at a point, then the integer k is equal to (1) -5 (2) 5 (3) 2 (4) -2

A. -1

B. 1

C. 8

D. -8

Answer: -5

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9. If the point A (λ , 5 , -2) lies on the line $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$, then the value of λ is

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Solved Examples

1. Maximize $z = 6x + 4y$, subject to $x \leq 2, x + y \leq 3, -2x + y \leq 1, x \geq 0, y \geq 0$. Also , find maximum value of z.

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2. Maximize $z = 10x + 25y$, subject to $x \leq 3, y \leq 3, x + y \leq 5, x \geq 0, y \geq 0$.

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3. Minimize $z = 6x + 2y$, subject to $5x + 9y \leq 90, x + y \geq 4, y \leq 8, x \geq 0, y \geq 0.$

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4. Minimize $z = x + 2y$, subject to $x + 2y \geq 50$, $2x - y \leq 0, 2x + y \leq 100, x, y \geq 0.$

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5. Minimize $z = 6x + 4y$, subject to $3x + 2y \geq 12, x + y \geq 5, 0 \leq x \leq 4, 0 \leq y \leq 4.$

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6. Maximize $z = x_1 + x_2,$ subject to $x_1 + x_2 \leq 10, 3x_2 - 2x_1 \leq 15 x_1 \leq 6, x_1, x_2 \geq 0.$

Show that the maximum value of z occurs at more than two points. What is your conclusion ?

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7. A company manufactures bicycles and tricycles, each of which must be processed through two machines A and B. Machine A has maximum of 120 hours available and machine B has a maximum of 180 hours available. Manufacturing a bicycle requires 6 hours on machine A and 3 hours on machine B. Manufacturing a tricycle requires 4 hours on machine A and 10 hours on machine B.

If profits are ₹ 180 for a bicycle and ₹ 220 for a tricycle, determine the number of bicycles and tricycles that should be manufactured in order to maximize the profit.

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8. A diet of sick person contains at least 48 units of vitamin A and 64 uints of vitamin B. Two foods F_1 and F_2 are available . Food F_1 costs Rs. 6 per unit and food F_2 costs Rs. 10 per unit. One unit of food F_1 contains 6 units of vitamin A and 7 units of vitamin B. One unit of of food F_2 contain 8 units of vitamin A and 12 units of vitamin B. Formulate the LPP, for the minimum cost for the diet that consists of mixture of these two foods and also meeting the minimal nutrition requirements

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Examples For Practice

1. Maximize $z = 4x + 5y$, subject to

$2x + y \geq 7, 2x + 3y \leq 15, x \leq 3, x \geq 0, y \geq 0$

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2. Maximize $z = 3x_1 - x_2$, subject to

$2x_1 + x_2 \geq 2, x_1 + 3x_2 \leq 2, x_2 \leq 2, x_1 \geq 0, x_2 \geq 0$

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3. Maximize $z = 4x + 6y$,subject to

$3x + 2y \leq 12, x + y \geq 4, x \geq 0, y \geq 0$

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4. Minimize $z = 7x + y$, subject to

$5x + y \geq 5, x + y \geq 3, x \geq 0, y \geq 0.$

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5. Minimize $z = 20x + 10y$, subject to

$x + 2y \leq 40, 3x + y \geq 30, 4x + 3y \geq 60, x \geq 0, y \geq 0.$

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6. Maximize $z = 6x + 4y$, subject to

$2x + 3y \leq 30, 3x + 2y \leq 24, x + y \geq 3, x \geq 0, y \geq 0.$

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7. A carpenter has 45, 40 and 25 running feet of teak wood, plywood and rosewood respectively. A table requires 2, 1 and 1 running feet and a chair requires 1, 2, 1 running feet of teak wood, plywood and rosewood respectively. If a table would sell for ₹ 4800 per unit and a chair for ₹ 1600 per unit, how many tables and chairs should the carpener make and sell in order to obtain maximum income out of his stock of wood ?

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8. A company produces soft drinks that has a contract which requires that a minimum of 80 units of the chemical A and 60 units of the chemical B

go into each bottle of the drink. The chemicals are available in prepared mix packets from two different suppliers. Supplier S had a packet of mix of 4 units of A and 2 units of B that costs Rs.10. The supplier T has a packet of mix of 1 unit of A and 1 unit of B costs Rs.4. How many packets of mixed from S and T should the company purchase to honour the contract requirement and yet minimize cost? Make a LPP and solve graphically.

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