# MATHS

# BOOKS - NAVBODH MATHS (HINGLISH)

## PAIR OF STRAIGHT LINES

**Solved Examples**

**1.** Find the separate equation of the lines represented by the following equations :

(1) $3x^2 - 10xy - 8y^2 = 0$

(2) $x^2 + 2xy - y^2 = 0.$

**2.** Find the combined equation of the lines passing through the origin and having inclinations $\dfrac{\pi}{3}$ and $\dfrac{5\pi}{3}$.
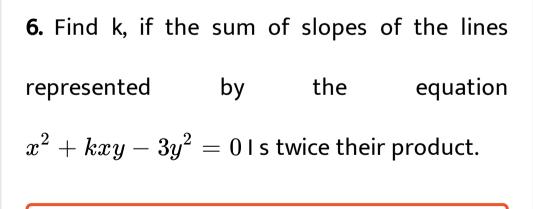
**3.** Find the joint equation of the lines passing through the origin and perpendicular to the lines $x + 2y = 19$ and $3x + y = 18$.

**4.** Find k, if one of the lines given by $6x^2 + kxy + y^2 = 0$ is $2x + y = 0$.

**5.** Find k, if the slope of one of the lines given by $kx^2 + 4xy - y^2 = 0$ exceeds the slope of other by 8.

**6.** Find k, if the sum of slopes of the lines represented by the equation $x^2 + kxy - 3y^2 = 0$ Is twice their product.

**7.** Find the measure of the acute angle between the lines represented by $\left(a^2 - 3b^2\right)x^2 + 8abxy + \left(b^2 - 3a^2\right)y^2 = 0.$

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**8.** If the acute angle between the lines $ax^2 + 2hxy + by^2 = 0$ is $60°$, then show that $(a + 3b)(3a + b) = 4h^2.$

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**9.** The slopes of the lines represented by $x^2 + 2hxy + 2y^2 = 0$ are in the ratio 1 : 2, then find 'h'.

**10.** Find the joint equation of the pair of lines through the origin which are perpendicular to the lines given by $5x^2 + 2xy - 3y^2 = 0$.

**11.** Show that the lines $x^2 - 4xy + y^2 = 0$ and $x + y = 10$ contain the sides of an equilateral triangle.

**12.** $\Delta OAB$ is formed by the lines $x^2 - 4xy + y^2 = 0$ and the line AB. The equation of line AB is $2x + 3y - 1 = 0$.Find the equation of the median of the triangle drawn from the origin.

**13.** If the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines $2x^2 - 5xy + 3y^2 = 0$, then show that $100(h^2 - ab) = (a + b)^2$.

**14.** Find the joint equaiton of the pair of the lines through the origin each of which is

making an angle of $30°$ with the line $3x + 2y - 11 = 0$.

15. Find the joint equation of the lines passing through the origin, each of which making angle of measure $15°$ with the line $x - y = 0$.
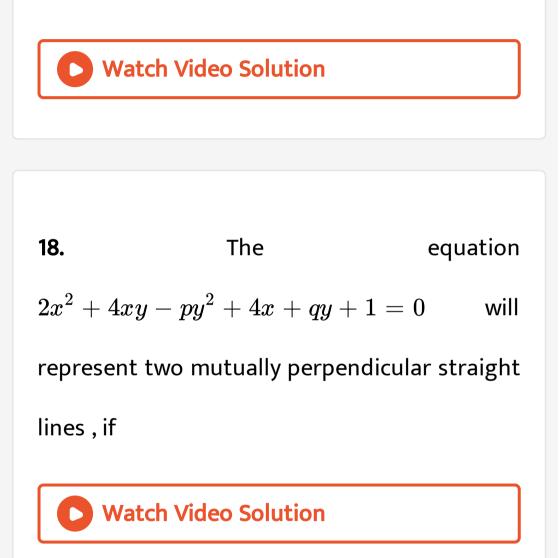
**16.** Show that the equation

$$x^2 - 6xy + 5y^2 + 10x - 14y + 9 = 0$$

represents a pair of lines. Find the acute angle between them. Also find the point of intersection of the lines.
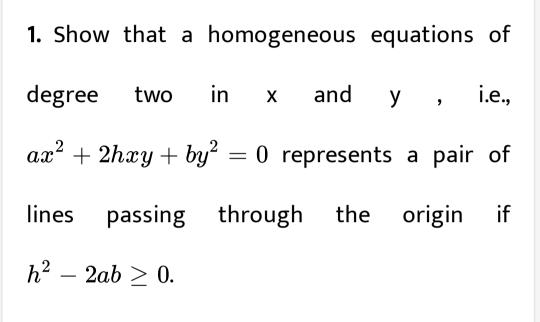
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**17.** Find the value of k, if the equation

$$2x^2 + 4xy - 2y^2 + 4x + 8y + k = 0$$

represents a pair of lines, Further find whether these lines are parallel or intersecting.

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**18.** The equation

$$2x^2 + 4xy - py^2 + 4x + qy + 1 = 0 \qquad \text{will}$$

represent two mutually perpendicular straight lines , if

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**1.** Show that a homogeneous equations of degree two in x and y , i.e., $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - 2ab \geq 0$.

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**2.** If $\theta$ is the measure of acute angle between the pair of line repseented by

$ax^2 + 2hxy + by^2 = 0$ , then prove that

$$\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|, \, a + b \neq 0$$

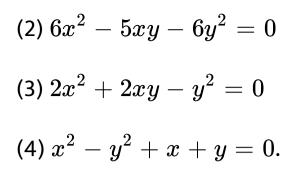Hence find the acture angle between the lines

$$x^2 - 4xy + y^2 = 0$$

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**Examples For Practice**

**1.** Find the separate equations of the lines

represented by :

(1) $3x^2 - 7xy + 4y^2 = 0$

(2) $6x^2 - 5xy - 6y^2 = 0$

(3) $2x^2 + 2xy - y^2 = 0$

(4) $x^2 - y^2 + x + y = 0.$

**2.** Find the combined equation of te lines :
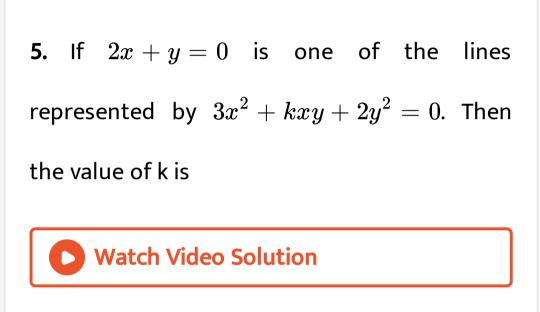
(1) Through the origin having inclinations $60°$ and $120°$ with the X-axis.

(2) bisecting the angle between the coordinate axes.

**3.** Find the combined equation of the lines passing through the origin such that

(1) one is parallel to $x + 2y = 5$ and other is perpendicular to $2x - y + 3 = 0$.

(2) which are perendicular to the lines $3x + 2y - 1 = 0$ and $x - 3y + 2 = 0$

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**4.** If slopes of lines represented by $kx^2 + 5xy + y^2 = 0$ differ by 1, then k=

**5.** If $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$. Then the value of k is

**6.** Find the value of k, if the slope of one of the lines given by $4x^2 + kxy + y^2 = 0$ is four times the other.

7. If $m_1$ and $m_2$ are slopes of lines represented by equation $3x^2 + 2xy - y^2 = 0$, then find the value of $(m_1)^2 + (m_2)^2$.

8. If the lines $px^2 - qxy - y^2 = 0$ make the angles $\alpha$ and $\beta$ with X-axis, then find the value of $\tan(\alpha + \beta)$.

**9.** Find the measure of the acute angle between the lines represented by $x^2 + 4xy + y^2 = 0$.

**10.** If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is four times the other, prove that $16h^2 = 25ab$.

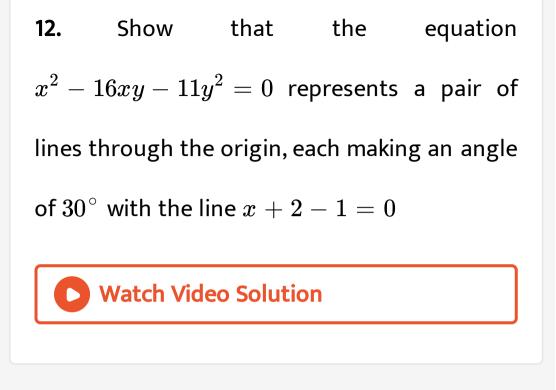**11.** Find the combined equation of the lines through the origin :

(1) each making an ange of $45°$ with the line $3x + y = 2.$

(2) each making an angle of $\pi/6$ with the line $3x + y - 6 = 0$.

(3) which form an equilateral triangle with the line $3x + 4y = 8.$

**12.** Show that the equation $x^2 - 16xy - 11y^2 = 0$ represents a pair of lines through the origin, each making an angle of $30°$ with the line $x + 2 - 1 = 0$

**13.** Find the combined equation of the pair of lines through the origin and perpendicular to the lines represented by :

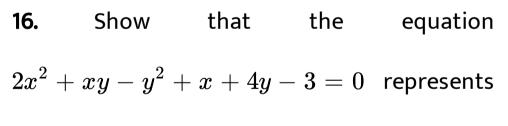(1) $5x^2 - 8xy + 3y^2 = 0$

(2) $x^2 + 4xy - 5y^2 = 0$

(3) $ax^2 + 2hxy + by^2 = 0.$

**14.** IF the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is square of the slope of the other line, show that $a^2b + ab^2 + 8h^3 = 6abh.$

**15.** Find the joint equation of the pair of lines through the origin and making an equilateral triangle with the line x = 3.

**16.** Show that the equation $2x^2 + xy - y^2 + x + 4y - 3 = 0$ represents a pair of lines.

**17.** Show that the equation

$$9x^2 - 6xy + y^2 + 18x - 6y + 8 = 0$$

represents a pair of lines. Find the acute angle between them.

**18.** Find k, if each of the following equation represents a pair of lines : (1)

$$3x^2 + 10xy + 3y^2 + 16y + k = 0 \qquad (2)$$

$$kxy + 10x + 6y + 4 = 0.$$

**19.** Find p and q, if the following equation represent a pair of lines perpendicular to each other :

(1) $px^2 - 8xy + 3y^2 + 14x + 2y + q = 5$
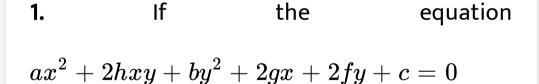
(2) $12x^2 + 7xy - py^2 + 18x + qy + 6 = 0$.

**20.** Find p and q, if the equation

$px^2 - 6xy + y^2 + 18x - qy + 8 = 0$

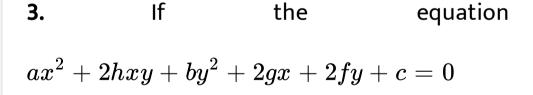represents a pair of parallel lines.

## Combined Equation Of Any Two Lines

**1.** If the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel lines, then

**2.** the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents an ellipse , if

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**3.** If the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represents a pair of parallel lines, then

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**1.** Joint equation of lines, trisecting angles is second and fourth quadrant is

A. $\sqrt{3}(x^2 + y^2) - 4xy = 0$

B. $\sqrt{3}(x^2 - y^2) + 4xy = 0$

C. $\sqrt{3}(x^2 + y^2) + 4xy = 0$

D. $\sqrt{x^2 - y^2} - 4xy = 0$

**Answer: C**

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**2.** The joint equation of the pair of lines passing through (2, 3) and parallel to the coordinate axes is

A. $xy - 3x - 2y + 6 = 0$

B. $xy + 3x + 2y + 6 = 0$

C. $xy = 0$

D. $xy - 3x - 2y - 6 = 0$

**Answer: A**

**3.** If the slope of one of the lines represented by $ax^2 - 6xy + y^2 = 0$ is square of the other, then
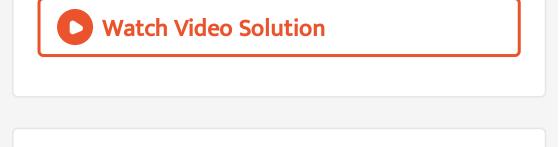
A. a = 1

B. a = 2

C. a = 4

D. a = 8

**Answer: D**

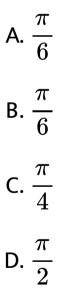**4.** If $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$. Then the value of k is

A. $\dfrac{1}{2}$

B. $\dfrac{11}{2}$

C. $\dfrac{5}{2}$

D. $-\dfrac{11}{2}$

**Answer: B**

**5.** The measure of the angle between the lines

$$\left(\sin^2\theta - 1\right)x^2 - 2xy\cos^2\theta + y^2\cos^2\theta = 0 \text{ is}$$

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{\pi}{4}$

D. $\dfrac{\pi}{2}$

**Answer: D**

**6.** If the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product, thent the vlaue of 'k' is

A. 2

B. 1

C. $-1$

D. $-2$

**Answer: D**

7. The slopes of the lines given by $12x^2 + bxy - y^2 = 0$ differ by 7. Then the value of b is
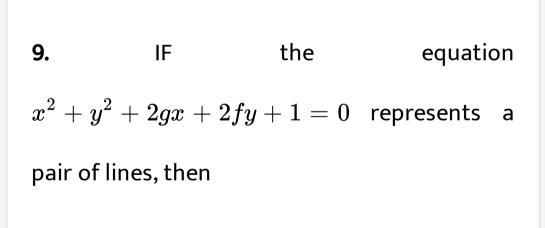
A. 2

B. $\pm 2$

C. $\pm 1$

D. 1

**Answer: C**

**8.** IF the angle between the pair of straight lines represented by the equation $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$ is $\tan^{-1}\left(\dfrac{1}{3}\right)$, where $\lambda$ is non-negative real number, then value of $\lambda$ is

A. 0

B. $\dfrac{1}{2}$

C. 2

D. 4

**Answer: C**

**9.** IF the equation

$$x^2 + y^2 + 2gx + 2fy + 1 = 0 \quad \text{represents a}$$

pair of lines, then

A. $g^2 - f^2 = 0$

B. $f^2 - g^2 = 1$

C. $g^2 + f^2 = \dfrac{1}{2}$

D. $f^2 - g^2 = 1.$

**Answer: B**

**10.** If the equation $hxy + gx + fy + c = 0$

represents a pair of straight lines, then

A. $fg = ch$

B. $gh = fc$

C. $fh = gc$

D. $fh = -gc$

**Answer: A**