# MATHS

# BOOKS - NAVBODH MATHS (HINGLISH)

# PROBABILITY DISTRIBUTION

## Solved Examples

**1.** Verify whether each of the following functions can be regarded as p.mf for the given value of X :

(1)

| $X=x$ | $-2$ | $-1$ | $1$ | $2$ |
|---|---|---|---|---|
| $P(X=x)$ | $0.5$ | $-0.1$ | $0.6$ | $0$ |

(1)

(2) $P(X) = \begin{cases} \frac{x-1}{3} & x = 1,2,3 \\ 0 & \text{Otherwise} \end{cases}$

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**2.** Three balance coins are tossed simultaneoulsy. If X denotes the number of heads, find probaility distribution of X.

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**3.** Two cards are drawn form a pack of 52 cards . If X = number of red cards darwn, find probability mass

function of X.

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**4.** Find k ,such that the function

$$P(x) = \left\{ \left( k\binom{4}{x}, , x = 0,1,2,3,4, k > 0 \right), (0, \text{otherwise.}) \right.$$

is a probability mass function (p.m.f.)

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**5.** A random variable X has the follwing probability distribution:

| $X=x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P[X=x]$ | $k$ | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

Find k.

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**6.** A random variable X has the follwing probability distribution:

| $X=x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P[X=x]$ | $k$ | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

Find $P(0 < X < 4)$

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**7.** A random variable X has the follwing probability distribution:

| $X=x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P[X=x]$ | $k$ | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

Obtain cumulative distribution function (c.d.f)of x.

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**8.** The probability distribution of a random variable X is as below:

| $X=x$ | $-1.5$ | $-0.5$ | 0.5 | 1.5 | 2.5 |
|---|---|---|---|---|---|
| $P[X=x]$ | 0.05 | 0.2 | 0.15 | 0.25 | 0.35 |

(1) Construct c.d.f F(x) of X.

(2) obtain $P(X \leq 0.5), F(-0.5), F(2), P(X \leq 4)$

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**9.** Following is the distribution function F(x) of a X.

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $F(x)$ | 0.2 | 0.37 | 0.48 | 0.62 | 0.85 | 1 |

Find the probability distribution of X.

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**10.** Following is the distribution function F(x) of a X.

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $F(x)$ | 0.2 | 0.37 | 0.48 | 0.62 | 0.85 | 1 |

Find $P(X \leq 3), P(2 \leq X \leq 5)$

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**11.** Following is the distribution function F(x) of a X.

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $F(x)$ | 0.2 | 0.37 | 0.48 | 0.62 | 0.85 | 1 |

Find $P(X \leq 5 \mid X \geq 3)$

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**12.** A fair coin is tossed 3 times. Let X be the number of heads obtained. Find E(X) and V(X).

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**13.** A backerman sells 5 types of cakes .Profit due ot sale of each type of cake is respectively 3,2.5,2,1.5 and 1. The demands for these cakes are 10%,5%,20%,50 and 15% respectivley. What is the expected profit per cake ?

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**14.** Let X have p.m.f $P(x) = kx^2 \quad x = 1,2,3,4$
$= 0 \quad \text{otherwise}$

Find mean and varince of X.

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**15.** Find k, if the following is p.d.f or r.v.f X.

$$f(x) = \begin{cases} kx^2(1-x) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

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**16.** For the following p.d.f of X, find $P(X < 1)$ and $P(|X| \leq 1)$ :

$$f(x) = \begin{cases} \frac{x+2}{18} & -2 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

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**17.** The time (in minutes) for a lab assistant to prepare the equipment for a certain experiment is a random variable taking values between 25 and 35 minutes with p.d.f $f(x) = \begin{cases} \frac{1}{10} & 25 \leq x \leq 35 \\ 0 & \text{otherwise} \end{cases}$ What is the probability that preparation time exceeds 33 minutes ? . Also find the c.d.f of X.

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**18.** Let X = time (in minutes ) that lapses between the bell and the end of the lectures in cases of a collge professor. Suppose X has p.d.f

$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$

Find the value of k.

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**19.** Let X = time (in minutes ) that lapses between the bell and the end of the lectures in cases of a collge professor. Suppose X has p.d.f

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that lecture ends within 1 minute of the bell ringing ?

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**20.** Let the random variable X is defined as time (in minutes) that elapses between the bell and end of the lecture in case of collagen professor whrer pdf is defined

as $f(x) = \begin{cases} kx^2, 0 \leq x < 2 \\ 0, \quad \text{elsewhere} \end{cases}$

find the probability that lecture continue for atleast 90s beyond the bell

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## Example For Practice

**1.** Verify whether the following functions can be regarded as the p.m.f for the given value of X :

(1) $P(X = x) = \begin{cases} \frac{x^2}{5} & x = 0,1,2 \\ 0 & \text{otherwise} \end{cases}$

(2) $P(X = x) \begin{cases} \frac{x-2}{5} & x = 1,2,3,4 \\ 0 & \text{otherwise} \end{cases}$

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**2.** Find the probability distribution of the number of sixes in three tosses of a die.

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**3.** It is know that a box of 8 batteries contains 3 defective pieces and a person randomly selects 2 batteries form this box. Find the probability distrubtion of the number of defective batteries.

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**4.** Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability

distribution of the number of aces.

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**5.** The probability distrubtion of a discrete random variable X is as below :

| $X=x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X=x)$ | $k$ | $2k$ | $3k$ | $4k$ | $5k$ |

Find $(PX \leq 4)$

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**6.** Obtain the expected value and variance of X for the following probability distribution:

| $X=x$ | $-2$ | $-1$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $P(X=x)$ | 0.2 | 0.3 | 0.1 | 0.15 | 0.25 |

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**7.** A r.v X has the following probability distribution:

| $X=x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $P(X=x)$ | 0.1 | $k$ | 0.2 | $2k$ | 0.3 | $k$ |

Find the value of k and calculate mean and varieance of X.

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**8.** The probability mass funciton (p.m.f) of X is given below :

| $X=x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X=x)$ | $\frac{1}{5}$ | $\frac{2}{5}$ | $\frac{2}{5}$ |

Find $E(X^2)$

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**9.** A player tosses 2 fair coins. He wins 5 if 2 heads apper , 2if 1 head appers and 1 if no head appers. Find his expected winning amount and variance of winning amount .

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**10.** Find k so that the function f(x) defined by

$f(x) = ke^{-3x}, x > 0$

= 0 , otherwise.

is a probability density function.

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**11.** Find k, if the function f defined by $f(x) = kx, 0 < x < 2$ =0 , otherwise is the p.d.f of a random variable X.

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**12.** The p.d.f of a random variable X is given by

$f(x) = 3\left(1 - 2x^2\right), 0 < x < 1$

= 0 , otherwise

Find $P\left(\frac{1}{4} < X < \frac{1}{3}\right)$

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**13.** Let X = amount of time for which a book is taken out of a collage library by a randomly selected student and suppose X has p.d.f.

$$f(x) = \begin{cases} 0.5x & 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Calculate (1) $P(X \leq 1)$ (2) $P(0.5 \leq X \leq 1.5)$

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**14.** Find k if the function f(x) is defined by

$f(x) = kx(1-x)$, for $0 < x < 1$

=0 , otherwise, is the probability density function (p.d.f.) of a random varible (r.v) X. Also find P $\left(X < \frac{1}{2}\right)$

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**15.** The following is the p.d.f of continuous random variable X. $f(x) = \frac{x}{8}, 0 < x < 4$

=0 , otherwise.

Find the expression for c.d.f of X

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**16.** The following is the p.d.f of continuous random variable X. $f(x) = \frac{x}{8}, 0 < x < 4$

=0 , otherwise.

Also , find its value at x = 0.5 , 1.7 and 5

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**17.** Given the p.d.f of a continous r.v.X was $f(x) = \frac{x^2}{3}, -1 < x < 2$ =0, Otherwise Determine the c.d.f of X and hence find .

$P(X < 1), P(X \leq -2), P(X > 0), P(1 \leq X \leq 2)$

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**18.** The following is the p.d.f (Probability Density Function ) of a continous random variable X :

$f(x) = \frac{x}{32}, 0 < x < 8$

= 0 , otherwise

Find the expression for c.d.f (Cumulative Distribution Function ) of X

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**19.** The following is the p.d.f (Probability Density Function ) of a continous random variable X :

$f(x) = \frac{x}{32}, 0 < x < 8$

= 0 , otherwise

Also, find its value at x = 0.5 and 9.

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**20.** Show that the function f(x) defined by $.f(x) = \frac{1}{7}$, for $1 \leq x \leq 8$ =0, otherwise, is a probability density function for a random variable. Hence find $P(3 < X < 10)$

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## Multiple Choice Questions

**1.** c.d.f F(x) of a continous random variable X is defined as .

A. $F(x) = \int_{-\infty}^{x} f(x)dx$

B. $F(x) = \int_{1}^{x} f(x)dx$

C. $F(x) = \int_{-\infty}^{\infty} f(x)dx$

D. $F(x) = P[X = x]$

**Answer:** $F(x) = \int_{-\infty}^{x} f(x)dx$

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**2.** The expected value of the number of heads obtained when three coins are tossed simultaneously is .....

A. 1

B. 1.5

C. 0

D. $-1$

**Answer: A**

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**3.** Let the p.m.f. of a random variable X be -

$$P(x) = \frac{3-x}{10} \quad \text{for} \quad x = -1, 0, 1, 2$$

= 0 otherwise

Then E(X) is ..... .

A. 1

B. 2

C. 0

D. $-1$

**Answer:**

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**4.** A random variable X has the following probability distribution:

| $X=x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $P(x)$ | 0.1 | 0.1 | 0.2 | 0.2 | 0.3 | 0.1 |

Then E(X) = .....

A. 0.8

B. 0.9

C. 0.7

D. 1.1

**Answer:**

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**5.** For a random variable X, if $E\left(X^2\right)$ = 31 , Var(X) = 6, then E(X)=.....

A. 2

B. 4

C. 5

D. 25

**Answer:**

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**6.** If the function $f(x) = \frac{x^2}{3}, \ -1 < x < 2$

= 0 , otherwise is a p.d.f of X, then $P(X < 0)$ is

A. $\frac{1}{9}$

B. $\frac{2}{9}$

C. $\frac{3}{9}$

D. $\frac{4}{9}$

**Answer: A**

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**7.** If X is the a random variable with probability mass function $P(x) = kx$, for x = 1,2,3

= 0 , otherwise

then k = .....

A. $\frac{1}{5}$

B. $\frac{1}{4}$

C. $\frac{1}{6}$

D. $\frac{2}{3}$

**Answer: C**

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**8.** The p.d.f of a continuous random variable X is $f(x) = \frac{x^2}{3}, -1 < x < 2$ 0 = otherwise Then the c.d.f of X is

A. $\frac{x^3}{9} + \frac{1}{9}$

B. $\frac{x^3}{9} - \frac{1}{9}$

C. $\frac{x^2}{4} + \frac{1}{4}$

D. $\frac{1}{9x^3} + \frac{1}{9}$

**Answer: A::C**

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**9.** Detemine k such that the following funciton is a p.m.f

$$P(X = x) = k\left(\frac{2^x}{x!}\right), x = 0, 1, 2, 3$$

=0 otherwise .

A. $\frac{1}{19}$

B. $\frac{2}{19}$

C. $\frac{3}{19}$

D. $\frac{4}{19}$

**Answer: A::C**

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