MATHS

BOOKS - NAVBODH MATHS (HINGLISH)

VECTORS

Solved Examples

1. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A, B, C respectively and $2\bar{a} + 3\bar{b} - 5\bar{c} = \bar{0}$, then find the ratio in which the point C divides line segment AB.

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2. $\bar{a}$ and $\bar{b}$ are non-collinear vectors. If $\bar{c} = (x - 2)\bar{a} + \bar{b}$ and $\bar{d} = (2x + 1)\bar{a} - \bar{b}$ are collinear, then find the value of x.

3. If $\bar{a} = \hat{i} + 2\hat{j}, b = -2\hat{i} + \hat{j}, \bar{c} = 4\hat{i} + 3\hat{j}$ find x and y such $\bar{c} = x\bar{a} + y\bar{b}.$

4. If the origin is the centroid of the triangle whose vertices are A (2,p,-3), B(q,-2,5) and C(-5,1,r), then find the values of p,q and r.

5. Find the corrdinates of the point which divides the line segment joining the points A(2,-6,8) and B (-1,3,-4) externally in the ratio 1:3.

6. If the points A(3,0,p),B(-1,q,3) and C(-3,3,0) are collinear, then find

(1) the ratio in which the points C divides the line segment AB

(2) the value of p and q.

7. Express $-\hat{i} - 3\hat{j} + 4\hat{k}$ as the linear combination of the vectors $2\hat{i} + \hat{j} - 4\hat{k}, 2\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + \hat{j} - 2\hat{k}$.

8. If $\bar{a}, \bar{b}, \bar{c}$ are non - coplanar vectors, then show the vectors $-\bar{a} + 3\bar{b} - 5\bar{c}, \; -\bar{a} + \bar{b} + \bar{c}$ and $2\bar{a} - 3\bar{b} + \bar{c}$ are coplanar.

9. If A, B, C and D are four non-collinear points in the plane such that $\overline{AD} + \overline{BD} + \overline{CD} = \bar{0}$, then prove that the points D is the centroid of the triangle ABC.

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10. If G and G' are the centroids of the triangle ABC and $A'B'C'$, then the value of $\overline{AA'} + \overline{BB'} + \overline{CC'}$ equals

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11. Let $\square PQRS$ be a quadrilateral. If M and N are the mid-points of the sides PQ and RS respectively, then PS+QR=

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12. If $\bar{a} = 3\hat{i} - 2\hat{j} + 7\hat{k}, \bar{b} = 5\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{c} = \hat{i} + \hat{j} - \hat{k}$, then find $\bar{a} \cdot (\bar{b} \times \bar{c})$.

13. Find the volume of the parallelopiped whose coterminus edges are given by vectors $2\hat{i} + 3\hat{j} - 4\hat{k}, 5\hat{i} + 7\hat{j} + 5\hat{k}$ and $4\hat{i} + 5\hat{j} - 2\hat{k}$.

14. If the vectors $-3\hat{i} + 4\hat{j} - 2\hat{k}, \hat{i} + 2\hat{k}, \hat{i} - p\hat{j}$ are coplanar, then the value of p is

15. If $\bar{c} = 3\bar{a} - 2\bar{b}$, then prove that $[\bar{a}\bar{b}\bar{c}] = 0$.

16. If $\bar{a}, \bar{b}$ and $\bar{c}$ are any three vectors, prove that (1) $\left[\bar{a} + \bar{b}\, \bar{b} + \bar{c}\, \bar{c} + \bar{a}\right] = 2\left[\bar{a}\bar{b}\bar{c}\right]$ (2) $\left[\bar{a}\bar{b} + \bar{c}\bar{a} + \bar{b} + \bar{c}\right] = 0$

17. $\bar{a}, \bar{b}, \bar{c}$ are three non-coplanar vectors.

If $\bar{p} = \dfrac{\bar{b} \times \bar{c}}{\bar{a} \cdot \left(\bar{b} \times \bar{c}\right)}, \bar{q} = \dfrac{\bar{c} \times \bar{a}}{\bar{a} \cdot \left(\bar{b} \times \bar{c}\right)}, \bar{r} = \dfrac{\bar{a} \times \bar{b}}{\bar{a} \cdot \left(\bar{b} \times \bar{c}\right)},$

show that $\bar{a} \cdot \bar{p} + \bar{b} \cdot \bar{q} + \bar{c} \cdot \bar{r} = 3.$

18. If A,B,C,D are (1,1,1), (2,1,3), (3,2,2),(3,3,4) respectively, then find the volume of the parallelopiped with AB,AC and AD as the concurrent edges.

19. Show that the points A(2,1,-1),B(0,-1,0),C(4,0,4) and (2,0,1) are coplanar.

20. Find the value of p if the points A(2,-1,1),B(4,0,p),C(1,1,1) and D(2,4,3) are coplanar.

21. Find the value of a tetrahedrn whose vertices are A(-1,2,3),B(3,-2,1),C(2,1,3) and D(-1,-2,4).

22. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar unit vectors each including the angle of measure $30°$ with the other, then find the volume of tetrahedron whose co-terminal edges are $\bar{a}, \bar{b}, \bar{c}$.

23. Prove using vectors: Medians of a triangle are concurrent.

24. Prove that the altitudes of a triangle are concurrent.

25. Show that the perpendicular bisectors of the sides of a triangle are concurrent.

26. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

27. Using vector method, find the incentre of the triangle whose vertices are P(0,4,0),Q(0,0,3) and R(0,4,3).

28. By method, show that the quadrilateral with vertices A(1,2,-1), B(8,-3,-4), C(5,-1,1),D(-2,1,4) is a parallelogram.

29. Prove that the line segment joining the mid points of two side of a triangle is parallel to the third side and equal to half of it.

1. If $\bar{a}$ and $\bar{b}$ any two non-collinear vectors lying in the same plane, then prove that any vector $\bar{r}$ coplanar with them can be uniquely expressed as $\bar{r} = t_1\bar{a} + t_2\bar{b}$, where t_1 and t_2 are scalars.

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2. Theorem 2: If a; b and c are non coplanar vectors; then any vector r can be expressed as linear combination: xa+yb+zc

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3. Three non - zero vectors $\bar{a}, \bar{b}, \bar{c}$ are complanar if and only if there exist scalars x,y,z, not all zero simultaneously such that $x\bar{a} + y\bar{b} + z\bar{c} = \bar{0}$.

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4. Derive the expression for the volume of the prallelopiped whose coterminus edges are vectors $\bar{a}, \bar{b}, \bar{c}$.

Examples For Practics

1. If $\bar{a}, \bar{b}$ and $\bar{c}$ are the position vectors of the points A,B and C respectively, such that

(1) $3\bar{a} + 5\bar{b} = 8\bar{c}$, find the ratio in which C divides line segment AB.

(2) $3\bar{a} + 5\bar{b} - 8\bar{c} = \bar{0}$, find the ratio in. which A divides BC.

2. Find the coordinates of the points R which divides the line segment joining the point $P(2, -2, -4)$ and $Q(3, -2, 5)$ externally in the

ratio 3:2.

3. If the vectors $3\hat{i} - 5\hat{j} + \hat{k}$ and $9\hat{i} - 15\hat{j} + p\hat{k}$ are collinear, then find the value p.

4. If $\bar{p} = \hat{i} - 2\hat{j} + \hat{k}$ and $\bar{q} = \hat{i} + 4\hat{j} - 2\hat{k}$ are position vectors points P and Q. find the position vector of the points R which divides segment PQ internally in the ratio 2:1.

5. Find the position vector of R which divides the line segment joining the points $A(1, -2, 1)$ and $B(1, 4, -2)$ internally in the ratio 2:1

6. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points $A(1, 3, 0), B(2, 5, 0), C(4, 2, 0)$ respectively and $\bar{c} = t_1\bar{a} + t_2\bar{b}$, then find values of t_1 and t_2.

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7. If G(a,2,-1) is the centroid of the triangle with vertices P(1,3,2),Q(3,b,-4) and R(5,1,c), then find the values of a,b and c.

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8. If the points A(2,p,1),B(1,2,q) and C(3,2,1) are collinear, then find

(1) the ratio in which the points C divides the line segment AB

(2) the values of p and q.

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9. Express $\bar{p}$ as a linear combination of $\bar{a}, \bar{b}$ and $\bar{c}$, where $\bar{p} = \hat{i} + 4\hat{j} - 4\hat{k}, \bar{a} = 2\hat{i} - \hat{j} + 3\hat{k}, \bar{b} = \hat{i} - 2\hat{j} + 4\hat{k}, \bar{c} = -\hat{i} + 3\hat{j} - 5\hat{k}$.

10. Express the vector $\bar{a} = 9\hat{i} + \hat{j} + 2\hat{k}$ as a linear combination of the vectors $\bar{q} = -\hat{i} - \hat{j} + 2\hat{k}$ and $\bar{r} = 3\hat{i} + \hat{j} - \hat{k}$.

11. If $\bar{a}, \bar{b}, \bar{c}$ are non-zero, non -coplanar vectors, then show that the vectors $2\bar{a} - 5\bar{b} + 2\bar{c}, \bar{a} + 5\bar{b} - 6\bar{c}$ and $3\bar{a} - 4\bar{c}$ are coplanar.

12. If $\bar{a} + \lambda\bar{b} + 3\bar{c},\ -2\bar{a} + 3\bar{b} - 4\bar{c}, \bar{a} - 3\bar{b} + 5\bar{c}$ are coplanar, then find value of λ

13. D, E, F are the midpoints of the sides BC, CA and AB respectively of $\triangle ABC$ and O is any point in the plane of $\triangle ABC$. Show that (1) $\overline{AD} + \overline{BE} + \overline{CF} = \bar{0}$

14. Show that if $\overline{AB} = \overline{DC}$, then the figure ABCD is a parallelogram.

15. If G is the centroid of $\triangle ABC$ and O is any point in the plane of $\triangle ABC$, show that

(1) $\overline{GA} + \overline{GB} + \overline{GC} = \bar{0}$

(2) $\overline{OA} + \overline{OB} + \overline{OC} = 3\overline{OG}.$

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16. Find $\left[\bar{a}\bar{b}\bar{c}\right]$ where :

(1) $\bar{a} = 2\hat{i} + \hat{j} - \hat{k}, \bar{b} = 3\hat{i} - \hat{j} - \hat{k}, \hat{c} = \hat{j} + 3\hat{k}$

(2) $\bar{a} = 7\hat{i} - \hat{j} + 2\hat{k}, \bar{b} = \hat{i} + 3\hat{j} - \hat{k}, \bar{c} = 4\hat{i} + 5\hat{k}.$

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17. If $\bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{b} = 2\hat{i} + q\hat{j} + \hat{k}, \hat{c} = \hat{i} - \hat{j} + 4\hat{k}$ and $\bar{a} \cdot \left(\bar{b} \times \bar{c}\right) = 1$, then find the value of q.

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18. Show that the vectors $\hat{i} - \hat{j} - 6\hat{k}$, $\hat{i} - 3\hat{j} + 4\hat{k}$ and $2\hat{i} - 5\hat{j} + 3\hat{k}$ are coplanar.

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19. Find the volume of the parallelopiped whose coterminus edges are :

(1) $3\hat{i} + 5\hat{k}$, $4\hat{i} + 2\hat{j} - 3\hat{k}$, $3\hat{i} + \hat{j} + 4\hat{k}$

(2) $\bar{a} = \hat{i} + \hat{j}$, $\bar{b} = \hat{j} + \hat{k}$, $\bar{c} = \hat{k} + \hat{i}$

(3) $2\hat{i} + 5\hat{j} - 4\hat{k}$, $5\hat{i} + 7\hat{j} + 5\hat{k}$, $4\hat{i} + 5\hat{j} - 2\hat{k}$.

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20. Find λ, if the vectors $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} - \hat{j} + \hat{k}$ and $\bar{c} = 2\hat{i} + 3\hat{j} + \lambda\hat{k}$ are coplanar.

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21. If $\bar{a}, \bar{b}, \bar{c}$ are any three vectors, prove that (1) $\begin{bmatrix} \bar{a} + \bar{b} & \bar{a} + \bar{c} & \bar{b} \end{bmatrix} = \begin{bmatrix} \bar{a} & \bar{c} & \bar{b} \end{bmatrix}$ (2) $\begin{bmatrix} \bar{a} - \bar{b} & \bar{b} - \bar{c} & \bar{c} - \bar{a} \end{bmatrix} = 0$.

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22. Find the volume of the parallelopiped with segments AB, AC and AD as concurrent edges, where : (1) $A \equiv (3, 7, 4), B \equiv (5, -2, 3), C \equiv (-4, 5, 6)$ and $D \equiv (1, 2, 3)$ (2) the position vectors of A,B,C,D are $\hat{i} + \hat{j} + \hat{k}, 2\hat{i} - \hat{j}, 3\hat{i} - 2\hat{j} - 2\hat{k}$ and $3\hat{i} + 3\hat{j} + 4\hat{k}$.

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23. Find the volume of the tetrahedron whose vertices are A(3,7,4),B(5,-2,3)C(-4,5,6) and D(1,2,3).

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24. Find the volume tetrahedron whose coterminus edges are $7\hat{i} + \hat{k}, 2\hat{i} + 5\hat{j} - 3\hat{k}$ and $4\hat{i} + 3\hat{j} + \hat{k}$.

25. Show that the following sets of points are coplanar :

(1) (3,9,4),(0,-1,-1),(-4,4,4) and (4,5,1)

(2) (1,-1,-1),(3,1,-1),(0,2,1) and (-2,0,1).

26. Find the value of x, if the points A(3,2,1),B(4,x,5),C(4,2,2)andD(6,5,-1) are coplanar.

27. If the origin and the points P(2,3,4), Q(1,2,3) and R(x,y,z) are coplanar, then

28. If $\bar{u} = \bar{i} - 2\bar{j} + \bar{k}$, $\bar{v} = 3\bar{i} + \bar{k}$ and $\overline{w} = \bar{j} - \bar{k}$, are given vectors, then find (1) $[\bar{u} \times \bar{v} \quad \bar{u} \times \overline{w} \quad \bar{v} \times \overline{w}]$ (2) $(\bar{u} + \overline{w}) \cdot [(\bar{u} \times \bar{v}) \times (\bar{v} \times \overline{w})]$.

Multiple Choice Question

1. If $\bar{a} = \hat{i} + 2\hat{j} = -2\hat{i} + \hat{j}, \bar{c} = 4\hat{i} + 3\hat{j}$ find x and y such $\bar{c} = x\bar{a} + y\bar{b}$.

A. 1, 1

B. 2, − 1

C. − 1, 2

D. 1, 0

Answer: B

2. If the vectors $2\hat{i} - q\hat{j} + 3\hat{k}$ and $4\hat{i} - 5\hat{j} + 6\hat{k}$ are collinear, then of q

is

A. 5

B. 10

C. $\dfrac{5}{2}$

D. $\dfrac{5}{4}$

Answer: C

3. If the points $A(2, 1, 1,)$, $B(0, -1, 4)$ and $C(k, 3, -2)$ are collinear, then k= . . . .

   A. 0

   B. 1

   C. 4

   D. -4

Answer: C

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4. If $\bar{a} = 3\hat{i} - \hat{j} + 4\hat{k}$, $\bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\bar{c} = -5\hat{i} + 2\hat{j} + 3\hat{k}$, then $\bar{a} \cdot \left(\bar{b} \times \bar{c} \right)$ = ...

   A. 100

B. 101

C. 110

D. 109

Answer: C

5. If $\bar{a}, \bar{b}, \bar{c}$ are the position vectors of the points A,B,C respectively such that $3\bar{a} + 5\bar{b} = 8\bar{c}$, the ratio in which A divides BC is

A. 8:5 internally

B. 8:5 externally

C. 5:8 internally

D. 5:8 externally.

Answer: B

6. If the vectors $-3\hat{i} + 4\hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{k}$, $\hat{i} - p\hat{j}$ are coplanar, then the value of p is

A. -2

B. 1

C. -1

D. 2

Answer: D

7. If the vectors $\hat{i} - 2\hat{j} + \hat{k}$, $a\hat{i} + 5\hat{j} - 3\hat{k}$ and $5\hat{i} - 9\hat{j} + 4\hat{k}$ are coplanar, then the value of a is

A. 3

B. -3

C. 2

D. -2

Answer: D

8. If vectors $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}, \hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $a + b + c - abc = \ldots\ldots\ldots$

A. -2

B. -1

C. 2

D. 1

Answer: C

9. The value of x when the points $A(2, -1, 1), B(4, 0, 3), C(x, 1, 1)$ and $D(2, 4, 3)$ are coplanar is

A. 1

B. 0

C. 2

D. $\dfrac{1}{2}$

Answer: A

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10. If $[\bar{a}\bar{b}\bar{c}] \neq 0$ and $\bar{p} = \dfrac{\bar{b} \times \bar{c}}{[\bar{a}\bar{b}\bar{c}]}, \bar{q} = \dfrac{\bar{c} \times \bar{a}}{[\bar{a}\bar{b}\bar{c}]}, \bar{r} = \dfrac{\bar{a} \times \bar{b}}{[\bar{a}\bar{b}\bar{c}]},$ then $\bar{a} \cdot \bar{p} + \bar{b} \cdot \bar{q} + \bar{c} \cdot \bar{r}$ is equal to

A. 0

B. 1

C. 2

D. 3

Answer: D

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