



MATHS

NCERT - NCERT MATHEMATICS(HINGLISH)

LIMITS AND DERIVATIVES

Exercise 13.1

1. Suppose $f(x) = [a + bx, x < 1; 4, x = 1; b - ax, x > 1$



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2. Let a_1, a_2, \dots, a_n be fixed real numbers and define a function

$$f(x) = (x - a_1)(x - a_2)\dots (x - a_n).$$

What is $\lim_{x \rightarrow x_1} f(x)$? For some $a \neq a_1, a_2, \dots, a_n$, compute

$$\lim_{x \rightarrow a} f(x)$$

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3. $(\lim)_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

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4. $(\lim)_{x \rightarrow 0} \frac{\sin ax + bx}{ax + \sin bx} a, b, a + b \neq 0$

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5. Find $(\lim)_{x \rightarrow 0} f(x)$, where $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

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6. Find $(\lim)_{x \rightarrow 5} f(x)$, where $f(x) = |x| - 5$



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7. Find $(\lim)_{x \rightarrow 1} f(x)$, where $f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ -x^2 - 1 & x > 1 \end{cases}$



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8. Evaluate $(\lim)_{x \rightarrow 0} f(x)$, where

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$



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9. If the function $f(x)$ satisfies $(\lim)_{x \rightarrow 1} \frac{f(x) - 2}{x^2 - 1} = \pi$, evaluate $(\lim)_{x \rightarrow 1} f(x)$

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10. If $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$

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11. If $f(x) = \begin{cases} mx^2 + n & x < 0 \\ nx + m & 0 \leq x \leq 1 \\ nx^3 + m & x > 1 \end{cases}$.

For what integers m and n does both $\lim_{x \rightarrow 0} f(x)$ and

$\lim_{x \rightarrow 1} f(x)$ exist?

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12.

Find

$$\lim_{x \rightarrow 0} f(x) \text{ and } (\lim)_{x \rightarrow 1} f(x), \text{ where } f(x) = \begin{cases} 2x + 3 & x \leq 0 \\ 3(x + 1) & x > 0 \end{cases}$$

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$$13. (\lim)_{x \rightarrow 0} (\cos ecx - \cot x)$$

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$$14. \lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$$

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$$15. \lim_{x \rightarrow 0} \frac{ax + b}{cx + 1}$$

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$$16. \lim_{x \rightarrow 4} \frac{4x + 3}{x - 2}$$



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$$17. \lim_{x \rightarrow -1} \frac{x^{10} + x^5 + 1}{x - 1}$$



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$$18. \lim_{x \rightarrow 0} \frac{(x + 1)^5 - 1}{x}$$



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$$19. \lim_{x \rightarrow 2} \frac{3x^2 - x - 10}{x^2 - 4}$$



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20. $\lim_{x \rightarrow 3} x + 3$

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21. $\lim_{x \rightarrow \pi} \left(x - \frac{22}{7} \right)$

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22. $\lim_{r \rightarrow 1} \pi r^2$

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23. FIND $\lim_{x \rightarrow 0} x \sec x$



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24. Find $\lim_{x \rightarrow 0} \frac{ax + x \cos x}{b \sin x}$



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25. Find $\lim_{x \rightarrow 0} \frac{\cos 2x - 1}{\cos x - 1}$



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26. $\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x}$



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27. $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$



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$$28. \left(\lim \right)_{x \rightarrow 0} \frac{\sin ax}{\sin bx} \quad a, b, \neq 0$$



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$$29. \lim_{x \rightarrow 0} \frac{\sin ax}{bx}$$



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$$30. \lim_{x \rightarrow -2} \frac{\frac{1}{x} + \frac{1}{2}}{x + 2}$$



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31. $\lim_{x \rightarrow 1} \frac{ax^2 + bx + c}{cx^2 + bx + a}, a + b + c \neq 0$



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32. $\lim_{z \rightarrow 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$



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Solved Examples

1. Find the derivative of $f(x)$ from the first principles, where $f(x)$ is (i) $\sin x + \cos x$



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2. Find the derivative of (i) $\frac{x^5 - \cos x}{\sin x}$ (ii) $\frac{x + \cos x}{\tan x}$



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3. Compute derivative of

(i) $f(x) = \sin 2x$ (ii) $g(x) = \cot x$



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4. Find the derivative of $f(x) = x^2$



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5. Find the derivative of the constant function $f(x) = a$ for a fixed real number a .





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6. Find the derivative of $f(x) = \frac{1}{x}$



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7. Compute the derivative of $6x^{100} - x^{55} + x$.



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8. Find the derivative of

$f(x) = 1 + x + x^2 + x^3 + \dots + x^{50}$ at $x = 1$.



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9. Find the derivative of $f(x) = \frac{x + 1}{x}$.



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10. Compute the derivative of $\sin x$ by first principle.



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11. Compute the derivative of $\tan x$.



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12. Compute the derivative of $f(x) = \sin^2 x$.



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13. Find the derivative of f from the first principles, where f is

given by (i) $f(x) = \frac{2x + 3}{x - 2}$ (ii) $f(x) = \frac{1}{x}$

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14. Evaluate (i) $(\lim)_{x \rightarrow 1} \frac{x^{15} - 1}{x^{10} - 1}$ (ii) $(\lim)_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$

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15. Find the limits (i) $\lim_{x \rightarrow 1} \left[\frac{x^2 + 1}{x + 100} \right]$ (ii)

$\lim_{x \rightarrow 2} \left[\frac{x^3 - 4x^2 + 4x}{x^2 - 4} \right]$

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16. Find the limits: (i) $\lim_{x \rightarrow 1} [x^3 - x^2 + 1]$ (iii) $\lim_{x \rightarrow 3} [x(x + 1)]$

(ii) $\lim_{x \rightarrow 1} [1 + x + x^2 + \dots + x^{10}]$



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17. Find the derivative of $\sin x$ at $x = 0$.



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18. Find the derivative of the function $f(x) = 2x^2 + 3x - 5$ at $x = -1$. Also prove that $f'(0) + 3f'(-1) = 0$.



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19. Find the derivative at $x = 2$ of the function $f(x) = 3x$.



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20. Evaluate (i) $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 2x}$ (ii) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$



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21. Find the derivative of $f(x) = 10x$.



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22. Find the derivative of $f(x) = 3x^2$ at $x = 0$ and at $x = 3$.



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23. Evaluate: $\lim_{x \rightarrow 1} \frac{\log_e x}{x - 1}$



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24. Compute $\lim_{x \rightarrow 0} \frac{e^{3x} - \sin x - 1}{x}$



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25. Compute $\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$



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Miscellaneous Exercise

1. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{x}{\sin^n x}$



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2. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x + a)$



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3. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(px + q)\left(\frac{r}{x} + s\right)$



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4. Find the derivative of the following functions from first principle:

(i) x

(ii) $(-x)^{-1}$

(iii) $\sin(x + 1)$

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5. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero

constants and m and n are integers): $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$

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6. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{1}{ax^2 + bx + c}$

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7. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax + b)(cx + d)^2$

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8. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{ax + b}{cx + d}$



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9. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{ax + b}{px^2 + qx + r}$

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10. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{px^2 + qx + r}{ax + b}$

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11. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\sin^n x$

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12. Find derivative of the following functions: $\frac{\sec x - 1}{\sec x + 1}$

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13. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\cos ecx \cot x$

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14. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\sin(x + a)$



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15. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\sin x + \cos x}{\sin x - \cos x}$



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16. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\cos x}{1 + \sin x}$



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17. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $4\sqrt{x} - 2$

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18. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$

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19. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero

constants and m and n are integers): $(ax + b)^n(cx + d)^m$



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20. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(ax + b)^n$



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21. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):
 $(ax^2 + \sin x)(p + q \cos x)$



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22. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x + \cos x)(x - \tan x)$



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23. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{4x + 5 \sin x}{3x + 7 \cos x}$



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24. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$



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25. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{a + b \sin x}{c + d \cos x}$



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26. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $\frac{\sin(x + a)}{\cos x}$



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27. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $x^4(5 \sin x - 3 \cos x)$



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28. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x^2 + 1) \cos x$



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29. Find derivative of the following functions: $\frac{x}{1 + \tan x}$



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30. Find derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers): $(x + \sec x)(x - \tan x)$



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Exercise 13 2

1. Find the derivative of $\cos x$ from first principle.



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2. Find the derivative of the following functions:

(i) $\sin x \cos x$

(ii) $\sec x$

(iii) $5 \sec x + 4 \cos x$

(iv) $\cos ecx$

(v) $3 \cot x + 5 \cos ecx$

(vi) $5 \sin x - 6 \cos x + 7$



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3. Find the derivative of

(i) $2x - \frac{3}{4}$

(ii) $(5x^3 + 3x - 1)(x - 1)$

(iii) $x^{-3}(5 + 3x)$



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4. Find the derivative of $\frac{x^n - a^n}{x - a}$ for some constant a .



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5. Find the derivative of $x^2 - 2$ at $x = 10$.



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6. Find the derivative of x at $x = 1$.



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7. Find the derivative of $99x$ at $x = 100$.



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8. For the function $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \dots + \frac{x^2}{2} + x + 1$.



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9. Find the derivative of the following functions from first principle

(i) $x^3 - 27$

(ii) $(x - 1)(x^2)$

(iii) $\frac{1}{x^2}$

(iv) $\frac{x + 1}{x - 1}$



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10. For some constants a and b , find the derivative of (i)

$(x - a) \cdot (x - b)$ (ii) $(ax^2 + b)^2$ (iii) $\frac{x - a}{x - b}$



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11. Find the derivative of

$x^n + ax^{n-1} + a^2x^{n-2} + \dots + a^{n-1}x + a^n$ for some fixed real number a .

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Exercise 13.3

1. Evaluate the limits, if exist $\lim_{x \rightarrow 0} \frac{x(e^x - 1)}{1 - \cos x}$.

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2. Evaluate the limits, if exist $\lim_{x \rightarrow 0} \frac{\log_e(1 + 2x)}{x}$.

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3. Evaluate the limits, if exist $\lim_{x \rightarrow 0} \frac{e^{\sin x} - 1}{x}$.

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4. Evaluate the limits, if exist $\lim_{x \rightarrow 0} \frac{e^{2+x} - e^2}{x}$.

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5. Evaluate the limits, if exist $\lim_{x \rightarrow 5} \frac{e^x - e^5}{x - 5}$.

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6. Evaluate the limits, if exist $\lim_{x \rightarrow 0} \frac{e^{4x} - 1}{x}$.

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7. Evaluate the limits, if exist $\lim_{x \rightarrow 3} \frac{e^x - e^3}{x - 3}$



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8. Evaluate the limits, if exist $\lim_{x \rightarrow 0} \frac{\log(1 + x^3)}{\sin^3 x}$



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