MATHS

NCERT - NCERT MATHEMATICS(HINGLISH)

APPLICATION OF DERIVATIVES

Solved Examples

1. Use differential to approximate $\sqrt{36.6}$

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2. Find the equation of tangent to the curve given by $x = a\sin^3 t, y = b\cos^3 t$ ... (1)at a point where $t = \frac{\pi}{2}$.

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3. Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

A. 45.46

B. 37.46

C. 27.56

D. 39.40

Answer: A

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4. The line y=mx+1 is a tangent to the curve $y^2 = 4x$ if the value of m is(A) 1 (B) 2(C) 3(D) 1/2.

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5. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

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6. Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%.

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7. Find the maximum and minimum values of f , if any, of the function given by $f(x) = |x|, x \in R.$

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8. Find the maximum and the minimum values, if any, of the function f given by $f(x) = x^2, x \in R.$

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9. Find all points of local maxima and local minima of the function f given by $f(x) = x^3 - 3x + 3.$

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10. Find the maximum and the minimum values, if any, of the function given by $f(x) = x, x \in (0, 1)$

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11. Find local maximum and local minimum values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12.$

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12. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5.$

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13. Find all the points of local maxima and local minima of the function f given by $f(x) = 2x^3 - 6x^2 + 6x + 5$

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14. Find local minimum value of the function f given by $f(x) = 3 + |x|, x \in R.$

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15. Let AP and BQ be two vertical poles at points A and B, respectively. If $AP = 16m$, $BQ = 22m and AB = 20m$, then find the distance of a point R on AB from the point A such that $RP^2 + RQ^2$ is minimum.

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16. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

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17. Find two positive numbers whose sum is 15 and the sum of whose squares is minimum.

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18. Find the shortest distance of the point (0, c) from the parabola $y = x^2$, where $0 \leq c \leq 5$.

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19. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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20. Find the absolute maximum and minimum values of a function f given by

$f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval $[1, 5]$.

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21. Prove that the function given by $f(x) = \cos x$ is

(a) strictly decreasing in $(0, \pi)$

(b) strictly increasing in $(\pi, 2\pi)$, and

(c) neither increasing nor decreasing in $(0, 2\pi)$

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22. Show that the function f given by $f(x) = x^3 - 3x^2 + 4x, x \in R$is strictly

increasing on R.

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23. Show that the function given by $f(x) = 7x - 3$ is strictly increasing on R.

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24. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal

revenue, when $x = 15$, where by marginal revenue we mean the rate of change of total revenue with respect to the number of items sold at an instant

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25. The total cost C(x) in Rupees, associated with the production of x units of an item is given by $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000.$ Find the marginal cost when 3 units are

produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

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26. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When $x = 10\text{cm}$ and $y = 6\text{cm}$, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

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27. A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing?

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28. The volume of a cube is increasing at a rate of 9 cubic centimetres per second. How fast is

the surface area increasing when the length of an edge is 10 centimetres?

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29. Find the rate of change of the area of a circle per second with respect to its radius r when $r = 5$cm.

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30. Find the equation of the tangent to the curve $y = \frac{x-7}{(x-2(x-3)}$ at the point where it cuts the x-axis.

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31. Find the equations of the tangent and normal to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$at (1, 1)

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32. Find the intervals in which the function f given by $f(x) = x^2 - 4x + 6$ is (a) strictly increasing (b) strictly decreasing

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33. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$is (a) strictly increasing (b) strictly decreasing

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34. Find the intervals in which the function $\sin 3x, x \in \left[0, \frac{\pi}{2}\right]$, is (a) increasing (b) decreasing

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35. Find the intervals in which the function $f(x) = \sin x + \cos x, x \in [0, 2\pi]$ is

(i) strictly increasing, (ii) strictly decreasing.

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36. Find the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$.

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37. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius r is 4r/3..

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38. Find the equation of all lines having slope 2 and being tangent to the curve $y + \frac{2}{x-3} = 0.$

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39. Find points on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$at which the tangents are (i) parallel to x-axis (ii) parallel to y-axis.

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40. A circular disc of radius 3 cm is being heated. Due to expansion, its radius increases at the rate of 0.05 cm/s. Find the rate at which its area is increasing when radius is 3.2 cm.

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41. Show that the function f given by $f(x) = \tan^{-1}(\sin x + \cos x), x > 0$is always an strictly increasing function in $\left(0, \frac{\pi}{4}\right)$.

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42. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m.

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43. A car starts from a point P at time t = 0 seconds and stops at point Q. The distance x, in metres, covered by it, in t seconds is given by $x = t^2\left(2 - \frac{t}{3}\right)$ Find the time taken by it to reach Q and also find distance between P and Q.

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44. An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier,

placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance.

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45. Find absolute maximum and minimum values of a function f given by $f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}, x \in [-1, 1]$.

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46. Find intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5} + 11$is (a) strictly increasing (b) strictly decreasing.

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47. Find the equation of tangents to the curve $y = \cos(x + y), \ -2\pi \leq x \leq 2\pi$ that are parallel to the line $x + 2y = 0$.

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48. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).

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49. A man 2 metres high walks at a uniform speed of 5 km/hr away from a lamp-post 6 metres high. Find the rate at which the length of his shadow increases.

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50. An open topped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

A. $\frac{200}{3}m^3$

B. $\frac{250}{3}m^3$

C. $\frac{100}{3}m^3$

D. $\frac{350}{3}m^3$

Answer: A

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51. Manufacturer can sell x items at a price of rupees $\left(5-\frac{x}{100}\right)$ each. The cost price of x items is Rs $\left(\frac{x}{5}+500\right)$. Find the number of items he should sell to earn maximum profit

A. $x=140$

B. $x=240$

C. $x=340$

D. $x=440$

Answer: B

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Exercise 6 3

1. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$.

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2. Find the equation of all lines having slope 2 which are tangents to the curve $y = \frac{1}{x-3}, x \neq 3$

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3. Find the equation of all lines having slope 1 that are tangents to the curve $y = \frac{1}{x-1}, x \neq 1.$

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4. Find points on the curve $\frac{x^2}{9}+\frac{y^2}{16}=1$ at which the tangents are

(i) parallel to x-axis

(ii) parallel to y-axis.

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5. Find the equations of all lines having slope 0 which are tangent to the curve $y=\frac{1}{x^2-2x+3}$.

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6. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$which is(a) parallel to the line $2xy + 9 = 0$(b) perpendicular to the line $5y - 15x = 13$.

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7. Find the equations of the tangent and normal to the given curve at the indicated point: $y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at $(1, 3)$

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8. Find the points on the curve $y = x^3$ at which the slope of the tangent is equal to the y-coordinate of the point.

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9. Show that the tangents to the curve $y = 7x^3 + 11$ at the points where $x = 2$ and $x = -2$ are parallel.

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10. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$at which the tangents are parallel to the x-axis.

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11. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin

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12. Find the slope of the tangent to the curve $y = \frac{x-1}{x-2}, x \neq 2$ at $x = 10$.

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13. Find the slope of the tangent to curve $y = x^3 - x + 1$ at the point whose x-coordinate is 2.

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14. Find the slope of the tangent to the curve

$y = 3x^4 - 4x$ at $x = 4.$

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15. Find the slope of the normal to the curve

$x = 1 - a\sin\theta, y = b\cos^2\theta$ at $\theta = \frac{\pi}{2}.$

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16. Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$is parallel to the x-axis.

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17. Find the slope of the tangent to the curve $y = x^3 - 3x + 2$ at the point whose x-coordinate is 3.

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18. Find the slope of the normal to the curve $x = a\cos^3\theta, y = \sin^3\theta$ at $\theta = \frac{\pi}{4}$.

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19. Find the point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points (2,0) and (4,4).

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20. Find the point on the curve $y = x^3 - 11x + 5$at which the tangent is $y = x - 11$.

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21. Find the equation of the normals to the curve $y = x^3 + 2x + 6$which are parallel to the line $x + 14y + 4 = 0$.

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22. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles* if $8k^2 = 1$.

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23. Find the equations of the tangent and normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. at the point (x_0, y_0)

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24. Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$which is parallel to the line $4x - 2y + 5 = 0.$

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25. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$is(A) 3 (B) $\frac{1}{3}$ (C)-3 (D) $-\frac{1}{3}$

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26. The line $y = x + 1$is a tangent to the curve $y^2 = 4x$at the point(A) $(1, 2)$ (B)$(2, 1)$ (C) $(1, 2)$ (D) $(1, 2)$

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27. Find the equation of the normal at the point $(am^2, am^3)$for the curve $ay^2 = x^3$.

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Miscellaneous Exercise

1. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

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2. Find the absolute maximum and minimum values of the function f given by $f(x) = \cos^2 x + \sin x, x \in [0, \pi]$

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3. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

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4. Let f be a function defined on [a, b] such that $f'(x) > 0$, for all $x \in (a, b)$. Then prove that f is an increasing function on (a, b).

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5. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

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6. The sum of the perimeter of a circle and square is k, where k is some constant. Prove that the sum of their areas is least when the

side of square is double the radius of the circle.

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7. Find the points at which the function f given by $f(x) = (x-2)^4(x+1)^3$ has

(i) local maxima

(ii) local minima

(iii) point of inflexion

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8. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the maximum length of the hypotenuse is $\left(a^{\frac{2}{3}}+b^{\frac{2}{3}}\right)^{\frac{3}{2}}$.

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9. A cylindrical tank of radius 10 m is being filled with wheat at the rate of 314 cubic metre per hour. Then the depth of the wheat is increasing at the rate of(A) 1 m^3/h (B) 0.1 m^3/h (C) 1.1 m^3/h (D) 0.5 m^3/h

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10. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

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11. Find the intervals in which the function f given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is (i) increasing (ii) decreasing.

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12. Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3}, x \neq 0$ is

(i) increasing

(ii) decreasing.

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13. Find the equation of the normal to curve $x^2 = 4y$which passes through the point (1, 2).

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14. Show that the normal at any point $\theta$to the curve$x = a\cos\theta + a\theta\sin\theta$,

$y = a\sin\theta - a\theta\cos\theta$is at a constant distance from the origin.

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15. Show that the function given by $f(x) = \frac{\log x}{x}$ has maximum at $x = e$.

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16. The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

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17. Find the maximum area of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

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18. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2 m and volume is $8m^3$. If building of tank costs Rs 70 per square metre for the base and Rs 45 per

square metre for sides, what is the cost of least expensive tank?

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19. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$at the point $(2, 1)$is(A) $\frac{22}{7}$ (B) $\frac{6}{7}$ (C) $\frac{7}{6}$ (D) $\frac{-6}{7}$

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20. The line $y = mx + 1$is a tangent to the curve $y^2 = 4x$if the value of m is(A) 1 (B) 2 (C) 3 (D) $\frac{1}{2}$

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21. The normal at the point (1,1) on the curve $2y + x^2 = 3$is(A) $x + y = 0$ (B) $x - y = 0$ (C) $x + y + 1 = 0$(D) $x - y = 0$

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22. The normal to the curve $x^2 = 4y$passing (1,2) is(A) $x + y = 3$ (B) $x - y = 3$ (C) $x + y = 1$ (D) $x - y = 1$

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23. The points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes are

(A) $\left(4, \pm \frac{8}{3}\right)$

(B) $\left(4, \frac{8}{3}\right)$

(C) $\left(4, \pm \frac{3}{8}\right)$

(D) $\left(\pm 4, \frac{3}{8}\right)$

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Exercise 6 4

1. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

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2. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

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3. Find the approximate value of $f(5.001)$, where $f(x) = x^3 - 7x^2 + 15$.

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4. Find the approximate value of $f(2.01)$, where $f(x) = 4x^2 + 5x + 2$.

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5. Using differentials, find the approximate value of each of the following up to 3 places of decimal.(i) $\sqrt{25.3}$ (ii) $\sqrt{49.5}$ (iii) $\sqrt{0.6}$ (iv) $(0.009)^{\frac{1}{3}}$ (v) $(0.999)^{\frac{1}{10}}$ (vi) $(15)^{\frac{1}{4}}$

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6. The approximate change in the volume of a cube of side x meters caused by increasing the side by 3% is

(A) 0.06 x^3m^3 (B) 0.6 x^3m^3 (C) 0.09 x^3m^3 (D) 0.9 x^3m^3

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7. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$is

(A) 47.66

(B) 57.66

(C) 67.66

(D) 77.66

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8. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

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9. Find the approximate change in the volume V of a cube of side x metres caused by increasing the side by 1%.

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Exercise 6 5

1. Show that of all the rectangles inscribed in a given fixed circle, the square has the maximum area.

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2. For all real values of x, the minimum value of $\frac{1-x+x^2}{1+x+x^2}$ is

(A) 0

(B) 1

(C) 3

(D) $\frac{1}{3}$

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3. The maximum value of $[x(x-1)+1]^{\frac{1}{3}}, 0 \leq x \leq 1$is(A) $\left(\frac{1}{3}\right)^{\frac{1}{3}}$ (B) $\frac{1}{2}$ (C) 1 (D) 0

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4. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

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5. A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible?

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6. Find two positive numbers whose sum is 16 and the sum of whose cubes is minimum.

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7. Find two positive numbers x and y such that their sum is 35 and the product x^2y^5 is a maximum.

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8. Find two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum.

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9. Find two numbers whose sum is 24 and whose product is as large as possible.

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10. Find the maximum and minimum values of $x + \sin 2x$ on $[0, 2\pi]$.

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11. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0,2]. Find the value of a.

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12. Find the maximum value of $2x^3 - 24x + 107$ in the interval [1, 3]. Find the maximum value of the same function in $[3, 1]$.

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13. Find the maximum and minimum values, if any, of the following functions given by

(i) $f(x) = (2x - 1)^2 + 3$

(ii) $f(x) = 9x^2 + 12x + 2$

(iii) $f(x) = -(x - 1)^2 + 10$

(iv) $g(x) = x^3 + 1$

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14. Find the maximum and minimum values, if any, of the following functions given by

(i) $f(x) = |x + 2| - 1$

(ii) $g(x) = -|x+1|+3$

(iii) $h(x) = \sin(2x)+5$

(iv) $f(x) = |\sin 4x + 3|$

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15. Find the local maxima and local minima, if any, of the following functions. Find also the local maximum and the local minimum values, as the case may be:

(i) $f(x) = x^2$

(ii) $g(x) = x^3 - 3x$

(iii) $h(x) = \sin x + \cos x, 0 < x < \frac{\pi}{2}$

(iv) $f(x) = \sin x - \cos x, 0 < x < 2\pi$

(v) f(x) = $x^3 - 6x^2 + 9x + 15$

(vi) g(x) = $\frac{x}{2} + \frac{2}{x}, x > 0$

(vii) g(x) = $\frac{1}{x^2 + 2}$

(viii) $f(x) = x\sqrt{1 - x}, x > 0$

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16. Prove that the following functions do not have maxima or minima:(i) f (x) = e^x (ii) $g(x) = \log x$(iii) $h(x) = x^3 + x^2 + x + 1$

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17. Find the absolute maximum value and the absolute minimum value of the following functions in the given intervals:(i)

$f(x) = x^3, x \in [-2, 2]$ (ii)

$f(x) = \sin x + \cos x, x \in [0, \pi]$(iii)

$f(x) = 4x - \frac{1}{2}x^2, x \in \left[-2, \frac{9}{2}\right]$

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18. Find the maximum profit that a company can make, if the profit function is given by $p(x) = 41 - 24x - 18x^2$

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19. Find both the maximum value and minimum value of $3x^4 - 8x^3 - 48x - 25$ on the interval [0, 3].

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20. At what points in the interval $[0, 2\pi]$, does the function $\sin 2x$ attain its maximum value?

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21. What is the maximum value of the function $\sin x + \cos x$?

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22. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box without top, by cutting off square from each corner and folding up the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

A. $x = 18$

B. $x = 24$

C. $x = 5$

D. None of these

Answer: C

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23. A wire of length 28 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of the square and the circle is minimum?

A. $L_1 = \frac{112}{\pi + 4}, L_1 = \frac{28\pi}{\pi + 4}$

B. $L_1 = \frac{142}{\pi + 4}, L_1 = \frac{28\pi}{\pi + 4}$

C. $L_1 = \frac{112}{\pi + 4}, L_1 = \frac{\pi}{\pi + 4}$

D. None of these

Answer: A

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24. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

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25. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

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26. Of all the closed cylindrical cans (right circular), of a given volume of 100 cubic

centimetres, find the dimensions of the can which has the minimum surface area?

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27. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

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28. The point on the curve $x^2 = 2y$ which is nearest to the point (0, 5) is(A) $\left(2\sqrt{2}, 4\right)$ (B) $\left(2\sqrt{2}, 0\right)$ (C) (0, 0) (D) (2, 2)

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29. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$time the radius of the base.

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Exercise 6 2

1. Prove that $y = \frac{4\sin\theta}{(2+\cos\theta)} - \theta$ is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$.

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2. Find the values of x for which $y = [x(x-2)]^2$ is an increasing function

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3. Show that the function given by $f(x) = 3x + 17$is strictly increasing on R.

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4. Show that the function given by $f(x) = \sin x$ is

(a) strictly increasing in $\left(0, \frac{\pi}{2}\right)$ (b) strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$(c) neither increasing nor decreasing in $(0, \pi)$

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5. Show that the function given by $f(x) = e^{2x}$ is strictly increasing on R.

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6. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$is (a) strictly increasing (b) strictly decreasing

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7. Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$is

(a) strictly increasing (b) strictly decreasing

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8. Show that $y = \log(1+x) - \frac{2x}{2+x}, x > -1,$ is an increasing function of x throughout its domain.

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9. Find the intervals in which the following functions are strictly increasing or decreasing:

(a) $x^2 + 2x - 5$ (b) $10 - 6x - 2x^2$ (c) $-2x^3 - 9x^2 - 12x + 1$ (d) $6 - 9x - x^2$(e) $(x+1)^3(x-3)^3$

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10. Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$is increasing in R.

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11. The interval in which $y = x^2 e^{-x}$ is increasing is(A) $(-\infty, \infty)$ (B) $(2, 0)$ (C) $(2, \infty)$ (D) (0, 2)

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12. Which of the following functions are strictly decreasing on $\left(0, \frac{\pi}{2}\right)$

(A) $\cos x$

(B)$\cos 2x$

(C) $\cos 3x$

(D) $\tan x$

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13. On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ strictly decreasing ?

(A) (0, 1)

(B) $\left(\frac{\pi}{2}, \pi\right)$

(C) $\left(0, \frac{\pi}{2}\right)$

(D) None of these

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14. Prove that the logarithmic function is strictly increasing on $(0, \infty)$.

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15. Prove that the function f given by $f(x) = x^2 - x + 1$is neither strictly increasing nor strictly decreasing on $(1, 1)$.

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16. Prove that the function f given by $f(x) = \log \sin x f(x) = \log \sin x$is strictly increasing on $\left(0, \frac{\pi}{2}\right)$and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

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17. Prove that the function f given by $f(x) = \log \cos x$ is strictly decreasing on $\left(0, \frac{\pi}{2}\right)$and strictly increasing on $\left(\frac{\pi}{2}, \pi\right)$

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18. Find the least value of a such that the function f given by $f(x) = x^2 + ax + 1$is strictly increasing on $(1, 2)$.

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19. Let I be an interval disjointed from $[-1, 1]$. Prove that the function $f(x) = x + \frac{1}{x}$ is increasing on I .

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Exercise 6 1

1. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5.$ The marginal revenue, when $x = 15$is (A) 116 (B) 96 (C) 90 (D) 126

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2. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x + 1)$.Find the

rate of change of its volume with respect to x.

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3. The radius of an air bubble is increasing at the rate of $\frac{1}{2} cm/s$. At what rate is the volume of the bubble increasing when the radius is 1 cm?

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4. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate

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5. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall

decreasing when the foot of the ladder is 4 m away from the wall ?

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6. The rate of change of the area of a circle with respect to its radius r at $r = 6$cm is(A) 10π (B) 12π (C) 8π (D) 11π

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7. The total revenue in Rupees received from the sale of x units of a product is given by $R(x) = 13x^2 + 26x + 15$. Find the marginal revenue when$x = 7$.

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8. The total cost C (x) in Rupees associated with the production of x units of an item is given by $C(x) = 0.007x^3 - 0.003x^2 + 15x + 4000$.

Find the marginal cost when 17 units are produced

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9. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4cm.

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10. An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing when the edge is 10 cm long?

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11. A stone is dropped into a quiet lake and waves move in circles at the speed of $5cm/s$. At the instant when the radius of the circular

wave is $8cm$, how fast is the enclosed area increasing?

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12. The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

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13. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When x = 8cm and y = 6cm, find the rates of change of (a) the perimeter, and (b) the area of the rectangle

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14. Find the rate of change of the area of a circle with respect to its radius r when(a)

$r = 3cm$ (b) $r = 4cm$

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15. The volume of a cube is increasing at the rate of $8cm^3/s$. How fast is the surface area increasing when the length of an edge is 12 cm?

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16. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm.

17. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimetres of gas per second. Find

the rate at which the radius of the balloon increases when the radius is 15 cm.

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18. A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing when the radius when the later is 10 cm

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