MATHS

NCERT - NCERT MATHEMATICS(HINGLISH)

CONTINUITY AND DIFFERENTIABILITY

Miscellaneous Exercise

1. Differentiate w.r.t. x the function $\sin^3 x + \cos^6 x$

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2. Differentiate w.r.t. x the function $\frac{\cos^{-1}\left(\frac{x}{2}\right)}{\sqrt{2x+7}}$, $-2 < x < 2$

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3. Differentiate w.r.t. x the function $\sin^{-1}\left(x\sqrt{x}\right), 0 \leq x \leq 1$

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4. If $f(x) = |x|^3$, show that $f''(x)$exists for all real x and find it.

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5. Using mathematical induction prove that $\frac{d}{dx}(x^n) = nx^{n-1}$ for all positive integers n.

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6. Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

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7. Using the fact that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and the differentiation, obtain the sum formula for cosines.

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8. If $y = e^{a\cos^{-1}x}, \ -1 \leq x \leq 1$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$

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9. If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, prove that $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

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10. Differentiate w.r.t. x the function $\cos(a\cos x + b\sin x)$, for some constant a and b.

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11. Differentiate w.r.t. x the function. $\left(3x^2 - 9x + 5\right)^9$

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12. Differentiate w.r.t. x the function $(\log x)^{\log x}, x > 1$

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13. Differentiate w.r.t. x the function

$$\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), 0 < x < \frac{\pi}{2}$$

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14. Differentiate w.r.t. x the function $(5x)^{3\cos 2x}$.

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15. If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for, $-1<x<1$, prove that $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$.

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16. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b.

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17. If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \left(\frac{\cos^2(a+y)}{\sin a}\right)$.

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18. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, find $\frac{d^2y}{dx^2}$.

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19. Differentiate w.r.t. x the function $x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0$.

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20. Differentiate w.r.t. x the function $x^{x^2-3}+(x-3)^{x^2}$ for $x>3$.

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21. Find $\frac{dy}{dx}$, if $y=12(1-\cos t), x=10(t-\sin t), -\frac{\pi}{2}<t<\frac{\pi}{2}$

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22. Find $\frac{dy}{dx}$, if $y = \sin^{-1} x + \sin^{-1} \sqrt{1 - x^2}, \ -1 \leq x \leq 1.$

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23. Differentiate w.r.t. x the function $(\sin x - \cos x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < \frac{3\pi}{4}$

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Solved Examples

1. Is it true that $x = e^{\log x}$ for all real

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2. Differentiate the following w.r.t. x:(i) e^{-x} (ii) $\sin(\log x), x > 0$ (iii) $\cos^{-1}(e^x)$ (iv) $e^{\cos x}$

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3. Show that the function f defined by $f(x) = |1 - x + x|$, where x is any real number, is a continuous function.

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4. Find the derivative of the function given $byf(x) = \sin(x^2)$.

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5. Find the derivative of tan $(2x + 3)$.

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6. Differentiate $\sin(\cos(x^2))$ with respect to x.

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7. Find $\frac{dy}{dx}$ if $x - y = \pi$

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8. Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$

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9. Find the derivative of f given by $f(x) = \sin^{-1} x$ assuming it exists.

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10. Find the derivative of / given by $f(x) = \tan^{-1} x$ assuming it exists.

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11. Find df/dx if f(x) $=(\sin x)^{\sin x}$ for all $o < x < \pi$.

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12. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$.

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13. Differentiate the following w.r.t x.(i) $\sqrt{3x+2}+\left(\frac{1}{\sqrt{2x^2+4}}\right)$ (ii) $e^{\sec^2(x)}+3\cos^{-1}(x)$ (iii) $\log_7(\log x)$

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14. Find $f'(x)$if $f(x)=(\sin x)^{\sin x}$for all $0<x<\pi$

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15. Verify Rolles theorem for the function $y=x^2+2, a=-2$and $b=2.$

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16. Verify the Mean Value Theorm for f(x)=x^2 in the interval [2,4].

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17. If $y = 3e^{2x} + 2e^{3x}$. Prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$

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18. If $y = \sin^{-1} x$, show that $\left(1 - x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$

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19. Find all the points of discontinuity of the function f defined by

$f(x) = \{x + 2, \quad \text{if} \quad x < 10 \quad \text{if} \quad x = 1x - 2 \quad \text{if} \quad x > 1$

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20. Discuss the continuity of the function f defined by

$$f(x) = \begin{cases} x + 2 & \text{if} \quad x < 1 \\ x - 2 & \text{if} \quad x > 1 \end{cases}$$

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21. Discuss the continuity of the function f given by

$f(x) = \{x, \quad \text{if} \quad x \geq 0 x^2, \quad \text{if} \quad x < 0$

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22. Discuss the continuity of the function defined by

$f(x) = \{x + 2, \quad \text{if} \quad x < 0 - x + 2, \quad \text{if} \quad x > 0$

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23. Find all the points of discontinuity of the greatest integer function defined by $f(x) = [x]$, where [x] denotes the greatest integer less than or equal to x.

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24. Show that every polynomial function is continuous.

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25. Discuss the continuity of sine function.

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26. Prove that every rational function is continuous.

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27. Show that the function defined by $f(x) = \sin\left(x^2\right)$ is a continuous function.

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28. Prove that the function defined by $f(x) = tanx$is a continuous function.

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29. Examine whether the function f given by $f(x) = x^2$ is continuous at $x = 0$.

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30. Discuss the continuity of the function f given by $f(x) = |x|$ at $x = 0$.

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31. Check the continuity of the function f given by $f(x) = 2x + 3$ at $x = 1$.

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32. Prove that the identity function on real numbers given by $f(x) = x$ is continuous at every real number.

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33. Is the function defined by $f(x) = |x|$, a continuous function?

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34. Show that the function f given by $f(x) = \{x^3 + 3 \text{ if } x \neq 0; 1 \text{ if } x = 0$ is not continuous at $x = 0$.

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35. Check the points where the constant function $f(x) = k$is continuous.

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36. Discuss the continuity of the function/given by $f(x) = x^3 + x^2 - 1$.

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37. Discuss the continuity of the function f defined by $f(x) = \frac{1}{x}, x \neq 0.$

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38. If $y = A \sin x + B \cos x,$ then prove that $\frac{d^2y}{dx^2} + y = 0.$

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39. Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$.

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40. Find $\frac{dy}{dx}$, if $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

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41. Find $\frac{dy}{dx}$, if $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$.

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42. Find $\frac{dy}{dx}$, if $x = at^2, y = 2at.$

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43. Find $\frac{dy}{dx}$, if $x = a\cos\theta, y = a\sin\theta.$

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44. Find $\frac{dy}{dx}$, if $y^x + x^y + x^x = a^b.$

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45. Differentiate $x^{\sin x}$, $x > 0$w.r.t. x.

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46. Differentiate a^xw.r.t. x, where a is a positive constant.

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47. Differentiate $\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$ w.r.t x.

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Exercise 5 6

1. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\left(\cos t + \log \tan\left(\frac{t}{2}\right)\right), y = a \sin t$

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2. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a \sec \theta, y = b \tan \theta$

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3. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = 4t, y = \frac{4}{t}$

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4. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \cos\theta - \cos 2\theta, y = \sin\theta - \sin 2\theta$

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5. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

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6. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

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7. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = 2at^2, y = at^4$

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8. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a\cos\theta, y = b\cos\theta$

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9. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = \sin t, y = \cos 2t$

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10. If $x = \sqrt{a^{\sin^{-1} t}}, y = \sqrt{a^{\cos^{-1} t}}$, show that $\frac{dy}{dx} = -\frac{y}{x}$

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11. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$.

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

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Exercise 5 2

1. Differentiate the functions with respect to x

$\sin\left(x^2 + 5\right)$

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2. Differentiate the functions with respect to x ,

$\cos(\sin x)$

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3. Differentiate the functions with respect to x

$$\frac{\sin(ax+b)}{\cos(cx+d)}$$

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4. Differentiate the functions with respect to x

$\cos x^3 . \sin^2(x^5)$

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5. Differentiate the functions with respect to x

$2\sqrt{\cot\left(x^2\right)}$

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6. Prove that the greatest integer function defined by

$f(x) = [x], 0 < x < 3$ is not differentiable at

$x = 1$ and $x = 2.$

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7. Differentiate the functions with respect to x

$\cos\left(\sqrt{x}\right)$

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8. Prove that the function f given by

$f(x) = |x - 1|, x \in R$ is not differentiable at $x = 1$

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9. Differentiate the functions with respect to x

$\sin(ax + b)$

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10. Differentiate the functions with respect to x

$\sec\left(\tan\left(\sqrt{x}\right)\right)$

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Exercise 5 1

1. Discuss the continuity of the function f, where f is defined by

$$f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \leq x \leq 1 \\ 4x, & \text{if } x > 1 \end{cases}$$

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2. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} x^{10} - 1 & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$

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3. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} \frac{x}{|x|} & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$

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4. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} \frac{|x|}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

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5. Is the function f defined by $f(x) = \begin{cases} x & \text{if } x \leq 1 \\ 5 & \text{if } x > 1 \end{cases}$ continuous at $x = 0? At x = 1? At x = 2?$

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6. Prove that the function $f(x) = x^n$ is continuous at $x = n$, where n is a positive integer.

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7. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} |x| + 3 & \text{if } x \leq -3 \\ -2x & \text{if } -3 < x < 3 \\ 6x + 2 & \text{if } x \geq 3 \end{cases}$

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8. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} 2x + 3 & \text{if } x \leq 2 \\ 2x - 3 & \text{if } x > 2 \end{cases}$

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9. Prove that the function $f(x) = 5x - 3$ is continuous at $x = 0$, at $x = -3$ and at $x = 5$.

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10. Examine the following functions for continuity.

(a) $f(x) = x - 5$

(b) $f(x) = \dfrac{1}{x - 5}$

(c) $f(x) = \dfrac{x^2 - 25}{x + 5}$

(d) $f(x) = |x - 5|$

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11. Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

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12. For what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x) & \text{if } x \leq 0 \\ 4x + 1 & \text{if } x > 0 \end{cases}$$

continuous at $x = 0$? What about continuity at $x = 1$?

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13. Show that the function defined by $g(x) = x - [x]$ is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

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14. Discuss the continuity of the function f, where f is defined by $f(x) = \begin{cases} -2 & \text{if } x \leq -1 \\ 2x & \text{if } -1 < x \leq 1 \\ 2 & \text{if } x > 1 \end{cases}$

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15. Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.

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16. Discuss the continuity of the function f, where f is defined by $f(x) = \begin{cases} 3 & \text{if } 0 \leq x \leq 1 \\ 4 & \text{if } 1 < x < 3 \\ 5 & \text{if } 3 \leq x \leq 10 \end{cases}$

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17. Is the function defined by $f(x) = \begin{cases} x+5 & \text{if } x \leq 1 \\ x-5 & \text{if } x > 1 \end{cases}$ a continuous function?

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18. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} x+1 & \text{if } x \geq 1 \\ x^2+1 & \text{if } x < 1 \end{cases}$

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19. Find all points of discontinuity of f, where f is defined by $f(x) = \begin{cases} x^3-3 & \text{if } x \leq 2 \\ x^2+1 & \text{if } x < 2 \end{cases}$

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20. Find the values of k so that the function f is continuous at the indicated point in

$$f(x) = \begin{cases} kx + 1 & \text{if } x \leq 5 \\ 3x - 5 & \text{if } x > 5 \end{cases} \text{ at } x = 5$$

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21. Find the values of k so that the function f is continuous at the indicated point in

$$f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases} \text{ at } x = \pi$$

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22. Find all points of discontinuity of f, where

$$f(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$$

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23. Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

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24. Discuss the continuity of the following functions:

(a) $f(x) = \sin x + \cos x$

(b) $f(x) = \sin x - \cos x$

(c) $f(x) = \sin x . \cos x$

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25. Is the function defined by $f(x) = x^2 - \sin x + 5$ continuous at $x = \pi$?

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26. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \begin{cases} kx^2 & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$ at $x = 2$.

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27. Find the values of k so that the function f is continuous at the indicated point in

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases} \text{ at } x = \frac{\pi}{2}$$

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28. Examine the continuity of f, where f is defined by

$$f(x) = \begin{cases} \sin x - \cos x & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases}$$

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29. Determine if f defined by $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ is a continuous function?

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30. Find all the points of discontinuity of f defined by $f(x) = |x| - |x + 1|$.

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31. Find the values of a and b such that the function defined by $f(x) = \begin{cases} 5 & \text{if } x \leq 2 \\ ax + b & \text{if } 2 < x < 10 \\ 21 & \text{if } x \geq 10 \end{cases}$ is a continuous function.

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32. Show that the function defined by $f(x) = \cos\left(x^2\right)$ is a continuous function.

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33. Show that the function defined by $f(x) = |\cos x|$ is a continuous function.

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34. The function $f(x) = \sin|x|$ is continuous for all x

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Exercise 5 4

1. Differentiate the following w.r.t. x:

$e^x + e^{x^2} + \ldots + e^{x^5}$

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2. Differentiate the following w.r.t. x:

$\sqrt{e^{\sqrt{x}}}, x > 0$

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3. Differentiate the following w.r.t. x:

$\log(\log x), x > 1$

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4. Differentiate the following w.r.t. x:

$$\frac{\cos x}{\log x}, x > 0$$

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5. Differentiate the following w.r.t. x:

$$\log(\cos e^x)$$

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6. Differentiate the following w.r.t. x:

$$e^{\sin^{-1} x}$$

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7. Differentiate the following w.r.t. x:

e^{x^3}

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8. Differentiate the following w.r.t. x:

$$\frac{e^x}{\sin x}$$

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9. Differentiate the following w.r.t. x:

$\sin\left(\tan^{-1} e^{-x}\right)$

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10. Differentiate the following w.r.t. x:

$\cos(\log x + e^x), x > 0$

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Exercise 5 3

1. Find $\frac{dy}{dx}$ in the following:$\sin^2 y + \cos xy = \pi$

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2. Find $\frac{dy}{dx}$ in the following:$ax + by^2 = \cos y$

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3. Find $\frac{dy}{dx}$ in the following:$2x + 3y = \sin y$

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4. Find $\frac{dy}{dx}$ in the following: $2x + 3y = \sin x$

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5. Find $\frac{dy}{dx}$ in the following:

(a)$x^3 + x^2y + xy^2 + y^3 = 81$

(b) $xy + y^2 = \tan x + y$

(c) $x^2 + xy + y^2 = 100$

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6. Find $\frac{dy}{dx}$ in the following:$x^2 + xy + y^2 = 100$

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7. Find $\frac{dy}{dx}$ in the following:$xy + y^2 = \tan x + y$

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8. Find $\frac{dy}{dx}$ in the following: $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

A. $\frac{2}{1+x^2}$

B. $\frac{5}{4+x^2}$

C. $\frac{1}{3+x^2}$

D. $\frac{5}{6+x^2}$

Answer: A

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9. Find $\frac{dy}{dx}$ in the following:$\sin^2 x + \cos^2 y = 1$

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10. Find $\frac{dy}{dx}$ in the following:

$$y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

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11. Find $\frac{dy}{dx}$ in the following:

$$y = \cos^{-1}\left(\frac{1 - x^2}{1 + x^2}\right), 0 < x < 1$$

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12. Find $\frac{dy}{dx}$ in the following:$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$

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13. Find $\frac{dy}{dx}$ in the following: $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), \ -1 < x < 1$

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14. Find $\frac{dy}{dx}$ in the following: $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

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15. Find $\frac{dy}{dx}$ in the following:$y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$

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Exercise 5 8

1. Examine the applicability of Mean Value Theorem for all three functions given in the above exercise 2.

(i) $f(x) = [x]$ for $x \in [5, 9]$ (ii) $f(x) = [x]$ for $x \in [-2, 2]$
(iii) $f(x) = x^2 - 1$ for $x \in [1, 2]$

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2. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$ in the interval [a, b], where $a = 1$ and $b = 4$.

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3. Verify Mean Value Theorem, if $f(x) = x^3 - 5x^2 - 3x$ in the interval [a, b], where $a = 1$ and $b = 3$. Find all $c \in (1, 3)$ for which $f'(c) = 0$.

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4. Examine if Rolle's theorem is applicable to any of the following functions. Can you say something about the converse of Rolle's theorem from these example?

(i) $f(x) = [x]$ for $x \in [5, 9]$

(ii) $f(x) = [x]$ for $x \in [-2, 2]$

(iii) $f(x) = x^2 - 1$ for $x \in [1, 2]$

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5. Verify Rolle's theorem for the function $f(x) = x^2 + 2x - 8, x \in [-4, 2]$.

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6. If $f:[-5,5] \to R$ is a differentiable function and if $f'(x)$ does not vanish anywhere, then prove that $f(-5) \neq f(5)$.

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Exercise 5 5

1. Differentiate the functions given w.r.t. x:

$x^x - 2^{\sin x}$

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2. Differentiate the functions given w.r.t. x:

$\cos x \cos 2x \cos 3x$

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3. Differentiate the functions given w.r.t. x:

$(\log x)^{\cos x}$

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4. Differentiate the functions given w.r.t. x:

$$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

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5. Differentiate the functions given w.r.t. x:

$(x+3)^2.(x+4)^3.(x+5)^4$

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6. Differentiate the following w.r.t. x:

$(\log x)^x + x^{\log x}$

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7. Differentiate the functions given w.r.t. x:

$$\left(x+\frac{1}{x}\right)^{x}+x^{\left(1+\frac{1}{x}\right)}$$

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8. Differentiate the following w.r.t. x:

$$x^{\sin x}+(\sin x)^{\cos x}$$

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9. Differentiate the following w.r.t. x:

$$(\sin x)^{x}+\sin^{-1}\sqrt{x}$$

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10. Find $\frac{dy}{dx}$ of the functions given by $x^y + y^x = 1$

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11. Find $\frac{dy}{dx}$ of the functions given by $y^x = x^y$

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12. Differentiate the following w.r.t. x:

$$x^{x\cos x} + \frac{x^2+1}{x^2-1}$$

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13. Differentiate the following w.r.t. x:

$(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

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14. Find the derivative of the function given by $f(x) = (1+x)\left(1+x^2\right)\left(1+x^4\right)\left(1+x^8\right)$ and hence find $f'(1)$.

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15. Differentiate $\left(x^2 - 5x + 8\right)\left(x^3 + 7x + 9\right)$ in three ways mentioned below:

(i) by using product rule

(ii) by expanding the product to obtain a single polynomial.

(iii) by logarithmic differentiation.

Do they all give the same answer?

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16. Find $\frac{dy}{dx}$ of the functions given by $(\cos x)^y = (\cos y)^x$

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17. Find $\frac{dy}{dx}$ of the functions given by $xy = e^{(x-y)}$

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18. If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w + u.\frac{dv}{dx}.w + u.v\frac{dw}{dx}$ in two ways - first by repeated application of product rule, second by logarithmic differentiation.

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Exercise 5 7

1. Find the second order derivatives of the functions given.

$\log(\log x)$

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2. Find the second order derivatives of the functions given.

$\tan^{-1} x.$

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3. Find the second order derivatives of the functions given.

$x \cdot \cos x$

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4. Find the second order derivatives of the functions given.

x^{20}

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5. Find the second order derivatives of the functions given

$x^2 + 3x + 2$

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6. Find the second order derivatives of the functions given.

$e^{6x} \cos 3x.$

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7. Find the second order derivatives of the functions given.

$e^x \sin 5x.$

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8. Find the second order derivatives of the functions given.

$x^3 \log x$

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9. Find the second order derivatives of the functions given.

$\log x$

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10. If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

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11. If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$

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12. If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$.

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13. If $y=\left(\tan^{-1}x\right)^2$, show that $\left(x^2+1\right)^2y_2+2x\left(x^2+1\right)y_1=2$

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14. Find the second order derivatives of the functions given.

$\sin(\log x)$

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15. If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

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16. If $y = \cos^{-1} x$,Find $\frac{d^2y}{dx^2}$ in terms of y alone.

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17. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + x y_1 + y = 0.$

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Question

1. Find the number of positive integer which have the characteristic 3 , when the base of the logarithm is 5

A. 499

B. 501

C. 500

D. None of these

Answer:

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2. If

$$x = \log_2\left(\sqrt{56+\sqrt{56+\sqrt{56+\sqrt{56+\ldots\infty}}}}\right)$$

then which of the following statement holds good?

A. $x < 0$

B. $0 < x < 2$

C. $2 < x < 4$

D. $3 < x < 4$

Answer: C

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3. If $n \in N$ such that characteristic of n^2 to the base 8 is 2 , then number of possible values of n is

A. 14

B. 15

C. 448

D. infinite

Answer: B