MATHS

NCERT - NCERT MATHEMATICS(HINGLISH)

LINEAR PROGRAMMING

Exercise 12 1

1. Show that the minimum of Z occurs at more than two points : Maximise $Z = x + y$, subject to $x - y \leq -1, -x + y \leq 0, x, y \geq 0.$

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2. Solve the Following Linear Programming Problem graphically : Maximise $Z = 3x + 2y$ subject to $x + 2y \leq 10, 3x + y \leq 15, x, y \geq 0.$

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3. Solve the Following Linear Programming Problem graphically : Minimise $Z = 3x + 5y$

such that $x + 3y \geq 3, x + y \geq 2, x, y \geq 0.$

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4. Show that the minimum of Z occurs at more than two points : Minimise and Maximise $Z = 5x + 10y$subject to $x + 2y \leq 120, x + y \geq 60, x - 2y \geq 0, x, y \geq 0$

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5. Solve the following Linear Programming Problem graphically :

Minimise $Z = x + 2y$ subject to

$3x + y \geq 3, x + 2y \geq 6. x, y \geq 0.$

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6. Solve the Following Linear Programming Problem graphically : Maximise $Z = 3x + 4y$ subject to the constraints :

$x + y \leq 4, x \geq 0, y \geq 0.$

A. 14

B. 15

C. 16

D. 17

Answer: C

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7. Solve the Following Linear Programming Problem graphically : Maximise $Z = 5x + 3y$

subject to

$3x + 5y \leq 15, 5x + 2y \leq 10, x \geq 0, y \geq 0.$

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8. Solve the Following Linear Programming Problem graphically : Minimise $Z = -3x + 4y$

Subject to

$x + 2y \leq 8, 3x + 2y \leq 12, x \geq 0, y \geq 0.$

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9. Maximise $Z = x + 2y$, subject to the constraints:

$x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0.$

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10. Show that the minimum of Z occurs at more than two points : Minimise and Maximise $Z = x + 2y$subject to

$x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200; x, y \geq 0$

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Miscellaneous Exercise

1. A dietician wishes to mix together two kinds of food X and Y m such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food is given below:One kg of food X costs Rs 16 and one kg of food Y costs Rs 20. Find the least cost of the mixture which will produce the required diet?

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2. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B. how many of each should be produced weekly in order to maximise the profit?

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3. A farmer mixes two brands P and Q of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag?

4. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table:

Distance in (km.)		
From / To	A	B
D	7	3
E	6	4
F	3	2

Assuming that the transportation cost of 10 litres of oil is Re 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

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5. Two godowns A and B have gram capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D. E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table:

Transportation cost per quintal (in Rs)		
From/To	A	B
D	6	4
E	3	2
F	2.50	3

How should the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

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6. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

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7. A manufacturer makes two types of toys A and B. Three machines are needed for this purpose and the tune (in minutes) required for each toy on the machines is given below:Each machine is available for a maximum of 6 hours per day. If the profit on each toy of type A is Rs 7.50 and that on each toy of type B is Rs 5. show that 15 toys of type A and 30 of the B should be manufactured m a day to get maximum profit.

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8. Refer to Example 9. How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the maximum amount of vitamin A in the diet?

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9. Refer to Question 8. If the grower wants to maximise the amount of nitrogen added to the garden, how many bags of each brand should be added? What is the maximum amount of nitrogen added?

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10. A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine m a bag of each brand are given m the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimise the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

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Exercise 12 2

1. There are two types of fertilisers $F_1$and F_2. F_1, consists of 10% nitrogen and 6% phosphoric acid and F_2consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a fanner finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If F_1 cost Rs.6/kg and F_2 costs Rs.5/kg, determine how much of each type of fertilizer should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?

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2. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs Rs 60 / kg and Food Q costs Rs 80 / kg. Food P contains 3 units/kg of Vitamin A and 5 units / kg of Vitamin B while food Q contains 4 units kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

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3. One kind of cake requires 200g of flour and 25g of fat and another kind of cake requires 100 g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

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4. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftmans time m its making while a cricket bat takes 3 hour of machine time and 1 hour of craftmans time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsmans time.(i) What number of rackets and bats must be made if the factory is to work at full capacity?(ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.

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5. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at die most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3.

Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?

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6. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours

20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture m order to maximise the profit?

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7. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He

earns a profit of Rsl7.50 per package on nuts and Rs 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?

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8. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs. 7 and screws B at a profit of Rs 1. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.

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9. A merchant plans to sell two types of personal computers - a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.

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10. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods $F_1$and F_2, are available. Food F_1costs Rs 4 per unit food and F, costs Rs 6 per unit. One unit of food F_1, contains 3 units of vitamin A and 4 units of minerals.One unit of foodF_2, contains 6 units of vitamin A and 3 units of minerals.Formulate a LPP. find the minimum cost of diet which matches the minimal requirement

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11. The corner points of the feasible region determined by the following system of linear inequalities:

$2x + y \geq 10, x + 3y \leq 15, x, y \geq 0$ are $(0, 0), (5, 0), (3, 4)$ and $(0, 5)$. Let $Z = px + qy$, where $p, q \geq 0$. Condition on p and q so that the maximum of Z occurs at both $(3, 4)$ and $(0, 5)$ is:

(a) $p = q$ (b) $p = 2q$ (c) $p = eq$ (d) $q = 3p$

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Solved Examples

1. (Transportation problem) There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below :

From/To	Cost (in Rs)		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. What will be the minimum transportation cost?

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2. (Manufacturing problem) A manufacturer has three machines I, II and III installed in his factory.

Machines I and II are capable of being operated for at most 12 hours whereas machine HI must be operated for atleast 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of M and N on the three machines are given m the following table:She makes a profit of Rs 600 and Rs 400 on items M and N respectively. How many of each item should she produce so as to maximise her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

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3. (Manufacturing problem) A manufacturing company makes two models A and B of a product. Each piece of Model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of Model B requires 12 labour hours for fabricating and 3 labour hours for finishing. For fabricating and finishing, the maximum labour hours available are 180 and 30 respectively. The company makes a profit of Rs 8000 on each piece of model A and Rs 12000 on each piece of Model B. How many-pieces of Model A and Model B should be manufactured per week to realise a maximum profit? What is the maximum profit per week?

A. 150000

B. 168000

C. 180000

D. 190000

Answer: B

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4. (Diet problem) A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires atleast 240 units of calcium, atleast 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to minimise the amount of vitamin Am the diet? What is the minimum amount of vitamin A?

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5. Solve the following linear programming problem graphically :Minimise $Z = 200x + 500y$. . . (1)subject to the constraints:$x + 2y \geq 10 \ldots$(2)$3x + 4y \leq 24 \ldots$(3)$x \geq 0, y \geq 0 \ldots$(4)

A. $(Z) = (4, 3) = 2300$

B. $(Z) = (5, 4) = 2400$

C. $(Z) = (6, 2) = 2500$

D. $(Z) = (5, 3) = 2550$

Answer: A

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6. Solve the following problem graphically:Minimise and Maximise $Z = 3x + 9y \ldots$ (1)subject to the constraints: $x + 3y \leq 60 \ldots$ (2) $x + y \geq 10 \ldots$ (3) $x \leq y \ldots$ (4) $x \geq 0, y \geq 0 \ldots$ (5)

7. Solve the following linear programming problem graphically: Maximise $Z = 4x + y \ldots$ (1) , subject to the constraints:$x + y \leq 50 \ldots$(2), $3x + y \leq 90 \ldots$(3) , $x \geq 0$, $y \geq 0 \ldots$(4)

8. (Diet problem): A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contain atleast 8 units of vitamin A and 10 units of vitamin C. Food I contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food II contains 1 unit/kg of vitamin A and 2 unite/kg of vitamin C. It costs Rs 50 per kg to purchase Food T and Rs 70 per kg to purchase Food II. Formulate this problem as a linear programming problem to minimize the cost of such a mixture.

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9. (Allocation problem) A cooperative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs 10,500 and Rs 9,000 respectively. To control weeds, a liquid herbicide has to be used for crops X and Y at rates of 20 litres and 10 litres per hectare. Further, no more than 800 litres of herbicide should be used in order to protect fish and wild life using a pond which collects drainage from this land. How much land should be allocated to each crop so as to maximise the total profit of the society?

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10. Determine graphically the minimum value of the objective function $Z = -50x + 20y \ldots$ (1)subject to the constraints: $2x - y \geq -5 \ldots$ (2) $3x + y \geq 3 \ldots$ (3) $2x - 3y \leq 12 \ldots$ (4) $x \geq 0, y \geq 0 \ldots$ (5)

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11. Minimise $Z = 3x + 2y$ subject to the constraints:

$x + y \geq 8 \ldots$ (1)

$3x + 5y \leq 15 \ldots (2)$

$x \geq 0, y \geq 0 \ldots (3)$

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