MATHS

NCERT - NCERT MATHEMATICS(HINGLISH)

RELATIONS AND FUNCTIONS

Miscellaneous Exercise

1. Given a non-empty set X, consider the binary operation $*: P(X) \times P(X) \to P(X)$ given by $A * B = A \cap B, \forall A, B \in P(X)$ is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in $P(X)$ with respect to the operation $*$

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2. Given a non-empty set X , consider $P(X)$ which is the set of all subjects of X . Define a relation in $P(X)$ as follows: For subjects $A, \ B$ in $P(X), \ \ A \ R \ B$ if $A \subset B$. Is R an equivalence relation on $P(X)$? Justify your answer.

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3. Given examples of two functions $f: N \rightarrow N$ and $g: N \rightarrow N$such that gof is onto but f is not onto. (Hint: Consider $f(x)=x+1$ and $g(x)=|x|$) .

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4. Give examples of two functions $f: N \rightarrow Z$ and $g: Z \rightarrow Z$such that gof is injective but g is not injective. (Hint: Consider $f(x)=x$ and $g(x)=|x|$)

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5. Show that function $f: R \rightarrow \{x \in R: -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}, x \in R$ is one one and onto function

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6. If $f: R \rightarrow R$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

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7. Let $f: W \rightarrow W$ be defined as $f(n) = n - 1$, if n is odd and $f(n) = n + 1$, if n is even. Show that f is invertible. Find the inverse of f. Here, W is the set of all whole numbers.

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8. Let $f: R \rightarrow R$ be defined as $f(x) = 10x + 7$. Find the function $g: R \rightarrow R$ such that $gof = fog = I_R$

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9. Let $f: R \to R$ be the Signum Function defined as $f(x) = \{1, x > 0; 0, x = 0; -1, x < 1$ and $g: R \to R$ be the Greatest Integer Function given by $g(x) = [x]$, where [x] is greatest integer less than or equal to x. Then does fog and gof coincide in (0,1]

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10. Show that the function $f: R \to R$ given by $f(x) = x^3$ is injective.

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11. Let $A = \{1, 2, 3\}$ Then number of relations containing $(1, 2)$ and $(1, 3)$ which are reflexive and symmetric but not transitive is

A. 1

B. 2

C. 3

D. 4

Answer: A

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12. Find the number of all onto functions from the set $A = \{1, 2, 3, , n\}$ to itself.

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13. Let $S = \{a, b, c\}$ and $T = \{1, 2, 3\}$. Find F^{-1} of the following functions F from S to T, if it exists.

(i) $F = \{(a, 3), (b, 2), (c, 1)\}$

(ii) $F = \{(a, 2), (b, 1), (c, 1)\}$

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14. Consider the binary operations $\cdot : R \times R \rightarrow R$ and $o : R \times R \rightarrow R$ defined as $a \cdot b = |a - b|$ and $aob = a, \forall a, b \in R$.

Show that $*$ is commutative but not associative, o is associative but not commutative.

Further, show that $\forall a, b, \in R, a \cdot (boc) = (a \cdot b)o(a \cdot c)$.

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15. Given a non -empty set X, let $* : P(X) \times P(X) \rightarrow P(X)$ be defined as $A * B = (A - B) \cup (B - A), \forall A, B \in P(X)$. Show that the empty set ϕ is the identity for the operation $*$ and all the elements A of P(A) are invertible with $A^{-1} = A$.

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16. Define a binary operation $*$ on the set $\{0, 1, 2, 3, 4, 5\}$ as

$$a * b = \begin{cases} a + b & \text{if } \; a + b < 6; \\ a + b - 6 & \text{if } \; a + b \geq 6 \end{cases}.$$

Show that zero is the identity for this operation and each element $a \neq 0$ of the set is invertible with $6 - a$ being the inverse of a

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17. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$and $f, g: A \to B$be functions defined by $f(x) = x^2 - x, x \in A$and $g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$. Are f and g equal? Justify your answer.

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18. Number of binary operations on the set {a, b} are

(A) 10

(B) 16

(C) 20

(D) 8

A. 10

B. 16

C. 20

D. 8

Answer: B

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19. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is

(A) 1

(B) 2

(C) 3

(D) 4

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Solved Examples

1. Show that if $f: A \rightarrow B$and $g: B \rightarrow C$are onto, then $gof: A \rightarrow C$is also onto.

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2. Show that if $f: A \rightarrow B$and $g: B \rightarrow C$are one-one, then $gof: A \rightarrow C$is also one-one.

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3. Let $f: \{2, 3, 4, 5\} \rightarrow \{3, 4, 5, 9\}$and $g: \{3, 4, 5, 9\} \rightarrow \{7, 11, 15\}$be functions defined as $f(2) = 3,$ $f(3) = 4,$ $f(4) = f(5) = 5$and $g(3) = g(4) = 7 \text{ and } g(5)$

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4. Show that a one-one function $f: \{1, 2, 3\} \rightarrow \{1, 2, 3\}$must be onto.

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5. Show that if $f: R - \left\{\frac{7}{5}\right\} \rightarrow R - \left\{\frac{3}{5}\right\}$ is defined by $f(x) = \frac{3x+4}{5x-7}$ and $g: R - \left\{\frac{3}{5}\right\} \rightarrow R - \left\{\frac{7}{5}\right\}$ is define by $g(x) = \frac{7x+4}{5x-3}$, then $fog = I_A$ and $gof = I_B$, where $A = R - \left\{\frac{3}{5}\right\}, B = R - \left\{\frac{7}{5}\right\}; I_A(x) = x, \forall x \in A, I_B(x) = x, \forall x \in I$

are called ideal

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6. Find gof and fog, if $f: R \rightarrow R$ and $g: R \rightarrow R$ are given by $f(x) = \cos x$ and $g(x) = 3x^2$. Show that $gof \neq fog$.

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7. Show that the function $f: R \rightarrow R$, defined as $f(x) = x^2$, is neither one-one nor onto.

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8. Show that the function $f: N \rightarrow N$ given by $f(1)=f(2)=1$ and $f(x)=x-1$ for every $x \geq 2$, is onto but not one-one.

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9. Show that an onto function $f:\{1,2,3\} \rightarrow \{1,2,3\}$ is always one-one.

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10. Show that $f: N \rightarrow N$ given by

$$f(x)=\begin{cases} x+1 & \text{if x is odd} \\ x-1 & \text{if x is even} \end{cases}$$

is both one-one and onto.

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11. Show that the function $f: N \to N$, given by $f(x) = 2x$, is one-one but not onto.

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12. Prove that the function $f: R \to R$, given by $f(x) = 2x$, is one-one and onto.

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13. Let R be the relation defined on the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b):$ both a and b are either odd or even$\}$. Show that R is an equivalence relation. Further, show that all the elements of the subset {1, 3, 5, 7} are related to each other and all the elements of the subset {2, 4, 6} are related to each other, but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.

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14. Let A be the set of all 50 students of class XII in a central school. Let $f: A \to N$ be a function defined by $f(x) = \text{Roll number of student x}$ Show that f is one-one but not onto.

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15. Show that the relation R in the set $\{1, 2, 3\}$given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$is reflexive but neither symmetric nor transitive.

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16. Show that the relation R on the set Z of integers, given by $R = \{(a,\ b) : 2 \text{ divides } a - b\}$, is an equivalence relation.

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17. Let T be the set of all triangles in a plane with R as relation in T given by

$\mathrm{R} = \{(\mathrm{T}_1, \mathrm{T}_2) : (\mathrm{T})_1 \text{congruent to} \mathrm{T}_2\}$

. Show that R is an equivalence relation.

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18. Let L be the set of all lines in a plane and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{is perpendicular to } L_2\}$ Show that R is symmetric but neither reflexive nor transitive.

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19. Let A be the set of all students of a boys school. Show that the relation R in A given by R = {(a, b) : a is sister of b} is the empty relation and R'= {(a, b) : the difference between heights of a and b is less than 3 meters} is the universal relation.

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20. Show that – a is the inverse of a for the addition operation '+' on R and $\frac{1}{a}$ is the inverse of $a \neq 0$for the multiplication operation X on R.

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21. Show that zero is the identity for addition on R and 1 is the identity for multiplication on R. But there is no identity element for the operations $- : R \times R \to R$and $\div : R_{.} \times R_{.} \to R_{.}$.

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22. Show that the $\vee : R \to R$ given by $(a, b) \to max\{a, b\}$and the $\wedge : R \to R$ given by $(a, b) \to \min\{a, b)$are binary operations.

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23. Let P be the set of all subsets of a given set X. Show that $\cup : P \times P \rightarrow P$given by $(A, B) \rightarrow A \cup B$and $\cap : P \times P \rightarrow P$given by $(A, B) \rightarrow A \cap B$are binary operations on the set P.

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24. Show that $* : R \times R \rightarrow R$ given by $(a, b) \rightarrow a + 4b^2$is a binary operation.

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25. Show that subtraction and division are not binary operations on N.

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26. Show that $\cdot : R \times R \rightarrow R$given by $a \cdot b = a + 2b$is not associative.

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27. Show that addition and multiplication are associative binary operation on R. But subtraction is not associative on R. Division is not associative on R*.

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28. Show that $\cdot : R \times R \to R$defined by $a \cdot b = a + 2b$is not commutative.

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29. Show that $+ : R \times R \to R$ and $\times : R \times R \to R$ are commutative binary operations, but $: R \times R \to R$ and $\div : R_{.} \times R_{.} \to R_{.}$ are not commutative.

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30. Let $Y = \{n^2 : n \in N\} \in N$. Consider $f : N \rightarrow Y$ as $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

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31. Let $f : N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f : N \rightarrow S$, where, S is the range of f, is invertible. Find the inverse of f.

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32. Consider $f : N \rightarrow N$, $g : N \rightarrow N$ and $h : N \rightarrow R$ defined as $f(x) = 2x$, $g(y) = 3y + 4$ and $h(z) = s \in z$, $\forall$x, y and z in N. Show that ho(gof) = (hog) of.

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33. Consider $f: \{1, 2, 3\} \to \{a, b, c\}$and $g: \{a, b, c\} \to \{apple, ball, cat\}$ defined as $f(1) = a$, $f(2) = b$, $f(3) = c$, $g(a) = apple$, $g(b) = ball$ and $g(c) = cat$. Show that f, g and gof are invertible .Find out f^{-1}, g^{-1} and $(gof)^{-1}$ and show that $(gof)^{-1} = f^{-1}og^{-1}$

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34. Consider functions f and g such that composite gof is defined and is one-one.Are f and g both necessarily one-one.

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35. Are f and g both necessarily onto, if $gof$is onto?

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36. Let $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$be one-one and onto function given by $f(1) = a$, $f(2) = b$and $f(3) = c$. Show that there exists a function $g: \{a, b, c\} \rightarrow \{1, 2, 3\}$such that $gof = I_x$and `fog=

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37. Let $f: N\overrightarrow{Y}$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N: y = 4x + 3$ for some $x \in N\}$. Show that f is invertible and its inverse is

(1) $g(y) = \dfrac{3y + 4}{3}$

(2) $g(y) = 4 + \dfrac{y + 3}{4}$

(3) $g(y) = \dfrac{y + 3}{4}$

(4) $g(y) = \dfrac{y - 3}{4}$

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38. Let $S = \{1, 2, 3\}$. Determine whether the functions $f: S \rightarrow S$ defined as below have inverses. Find f^{-1}, if it exists

.(a) $f = \{(1, 1), (2, 2), (3, 3)\}$

(b) $f = \{(1, 2), (2, 1), (3, 1)\}$

(C) $f = \{(1, 3), (3, 2), (2, 1)\}$

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39. Show that addition, subtraction and multiplication are binary operations on R, but division is not a binary operation on R. Further, show that division is a binary operation on the set R of nonzero real numbers.

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40. Consider the identity function $I_N : N \to N$ defined as, $I_N(x) = x$ for all $x \in N$. Show that although I_N is onto but $I_N + I_N : N \to N$ defined as $(I_N + I_N)(x) = I_N(x) + I_N(x) = x + x = 2x$ is not onto.

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41. Let R be a relation on the set A of ordered pairs of positive integers defined by $(x, y)R(u, v)$if and only if $xv = yu$. Show that R is an equivalence relation.

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42. Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let R be a relation in X given by $R_1 = \{(x, y) : x - y$ is divisible by $3\}$ and R_2 another on X given by $R = \{(x, y) : (x, y) \cup \{1, 4, 7\}\}$ or $\{x, y\} \cup \{2, 5, 8\}$ or $\{x, y\} \cup \{3, 6, 9\}\}$ Show that $R_1 = R_2$.

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43. Show that $-a$is not the inverse of $a \in N$for the addition operation + on N and $\frac{1}{a}$ is not the inverse of $a \in N$for multiplication operation $\times$ on N, for $a \neq 1$.

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44. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

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45. Find the number of all one-one functions from set $A = \{1, 2, 3\}$to itself.

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46. Let $A = \{1,\ 2,\ 3\}$. Then, show that the number of relations containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric is three.

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47. Let $f: X \to Y$be a function. Define a relation R in X given by $R = \{(a, b): f(a) = f(b)\}$. Examine if R is an equivalence relation.

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48. Determine which of the following binary operations on the set N are associative and which are commutative.

$(a) a \cdot b = 1 \forall a, b \in N$

$(b) a \cdot b = \left(\frac{a+b}{2}\right) \forall a, b \in N$

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49. Show that the number of equivalence relation in the set $\{1, 2, 3\}$ containing $(1, 2)$and $(2, 1)$is two.

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50. Show that the number of binary operations on $\{1, 2\}$having 1 as identity and having 2 as the inverse of 2 is exactly one.

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Exercise 1 4

1. Let A = N×N and · be the binary operation on A defined by(a, b) ∗(c, d) = (a + c, b + d) . Show that · is commutative and associative. Find the identity element for · on A, if any.

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2. Show that none of the operations given below has identity.(i) $a * b = a - b$ (ii) $a * b = a^2 + b^2$ (iii) $a * b = a + ab$ (iv) $a * b = (a - b)^2$ (v) $a * b = \frac{ab}{4}$ (vi) $a * b = ab^2$

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3. Consider a binary operation. on N defined $a * b = a^3 + b^3$. Choose the correct answer.

A. (A) Is $*$ both associative and commutative?

B. (B) Is $*$ commutative but not associative?

C. (C) Is $*$ associative but not commutative?

D. (D) Is $*$ neither commutative nor associative?

Answer: (B) Is $*$ commutative but not associative?

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4. State whether the following statements are true or false. Justify.

(i) For an arbitrary binary operation $*$ on a set N, $a * a = a \forall a \in N$.

(ii) If $*$ is a commutative binary operation on N, then $a * (b * c) = (c * b) * a$

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5. Let $*$ be the binary operation on N given by $a * b = LCM$of a and b. Find

(i) $5 * 7, 20 * 16$

(ii) Is $\cdot$ commutative?

(iii) Is $*$ associative?

(iv) Find the identity of $*$ in N

(v) Which elements of N are invert

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6. Is $*$ defined on the set $\{1, 2, 3, 4, 5\}$ by $a * b = LCM$of a and b , a binary operation? Justify your answer.

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7. Consider a binary operation * on the set {1, 2, 3, 4, 5} given by the following multiplication table Compute (2*3) *4 and 2* (3*4) Is * commutative? (iii) Compute (2*3)*(4*5)

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8. Let $*'$be the binary operation on the set $\{1, 2, 3, 4, 5\}$defined by $a*'b = HCF$of a and b. Is the operation $*'$same as the operation $*$ defined Justify your answer.

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9. For each binary operation * defined below, determine whether * is commutative or associative.

(i) On Z, define $a*b = a - b$

(ii) On Q, define $a*b = ab + 1$

(iii) On Q, define $a*b = \frac{ab}{2}$

(iv) On Z^+, define $a*b = 2^{ab}$

(v) On Z^+, define $a*b = a^b$

(vi) On $R - \{-1\}$, define $a*b = \frac{a}{b+1}$

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10. Consider the binary operation $\wedge$ on the set $\{1, 2, 3, 4, 5\}$defined by $a \wedge b = \min \{a, b\}$. Write the operation table of the operation $\wedge$.

11. Determine whether or not each of the definition of given below gives a binary operation. In the event that * is not a binary operation, give justification for this.

(i) On Z^+, define $*$ by $a * b = a - b$

(ii) On Z^+, define $*$ by $a * b = ab$

(iii) On R, define $*$ by $a * b = ab^2$

(iv) On Z^+, define $*$ by $a * b = |a - b|$

(v) On Z^+, define $*$ by $a * b = a$

12. Let $*$ be the binary operation on N defined by $a * b = H\dot{C}\dot{F}$ of a and b. Is $*$ commutative? Is $*$ associative? Does there exist identity for this binary operation on N?

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13. Let $*$ be a binary operation on the set Q of rational numbers as follows: (i) $a * b = a - b$ (ii) $a * b = a^2 + b^2$ (iii) $a * b = a + ab$ (iv) $a * b = (a - b)^2$ (v) $a * b = \frac{ab}{4}$ (vi) $a * b = ab^2$

Find which of the binary operations are commutative and which are associative

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Exercise 1 2

1. Let A and B be two sets. Show that $f: A \times B \to B \times A$ defined by $f(a,\ b) = (b,\ a)$ is a bijection.

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2. Let $f: N \to N$be defined by $f(n) = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$ for all $n \in N$. State whether the function f is bijective. Justify your answer.

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3. Show that the Modulus Function f : R → R, given by f(x) = |x|, is neither oneone nor onto, where | x | is x, if x is positive or 0 and |x| is – x, if x is negative.

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4. Show that the Signum Function $f: R \rightarrow R$, given by $f(x)=\begin{Bmatrix}1 & \text{if} & x>0 \\ 0 & \text{if} & x=0 \\ -1 & \text{if} & x<0\end{Bmatrix}$ is neither one-one nor onto.

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5. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) $f: R \rightarrow R$, defined by $f(x)=3-4x$

(ii) $f: R \rightarrow R$, defined by $f(x)=1+x^2$

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6. Show that the functions $f: R. \rightarrow R.$ defined by $f(x)=\frac{1}{x}$ is one-one and onto. where R* is the set of all non-zero real numbers. Is the result true, if the domain R* is replaced by N with co-domain being same as R*.

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7. Check the injectivity and surjectivity of the following functions:

(i) $f: N \rightarrow N$ given by $f(x) = x^2$

(ii) $f: Z \rightarrow Z$ given by $f(x) = x^2$

(iii) $f: R \rightarrow R$ given by $f(x) = x^2$

(iv) $f: N \rightarrow N$ given by $f(x) = x^3$

(v) $f: Z \rightarrow Z$ given by $f(x) = x^3$

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8. Prove that the Greatest Integer Function $f: R \rightarrow R$, given by $f(x) = [x]$, is neither one-one nor onto, where [x] denotes the greatest integer less than or equal to x.

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9. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one.

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10. Let $f : R \rightarrow R$be defined as $f(x) = 3x$. Choose the correct answer.

(A) f is one-one onto

(B) f is many-one onto

(C) f is one-one but not onto

(D) f is neither one-one nor onto.

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11. Let $f : R \rightarrow R$be defined as $f(x) = x^4$. Choose the correct answer.

A. f is one-one onto

B. f is many-one onto

C. f is one-one but not onto

D. f is neither one-one nor onto

Answer: D

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12. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$.

Is f is one-one and onto? Justify your answer

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Exercise 1 1

1. Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3, \ldots, 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$

(ii) Relation R in the set N of natural numbers defined as $R = \{(x, y) : y = x + 5$ and $x < 4\}$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as $R = \{(x, y) : y$ is divisible by $x\}$

(iv) Relation R in the set Z of all integers defined as $R = \{(x, y) : x - y$ is an integer$\}$

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x, y) : x$ and y work at the same place$\}$

(b) $R = \{(x, y) : x$ and y live in the same locality$\}$

(c) $R = \{(x, y) : x$ is exactly 7 cm taller than $y\}$

(d) $R = \{(x, y) : x$ is wife of $y\}$

(e) $R = \{(x, y) : x$ is father of $y\}$

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2. Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$as $R = \{(a, b) : b = a + 1\}$is reflexive, symmetric or transitive.

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3. Show that the relation R in the set R of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$is neither reflexive nor symmetric nor transitive.

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4. Check whether the relation R in R defined by $R = \{(a, b) : a \leq b^3\}$is reflexive, symmetric or transitive.

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5. Show that the relation R in R defined as $R = \{(a, b) : a \leq b\}$, is reflexive and transitive but not symmetric.

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6. Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y) : x and y have same number of pages} is an equivalence relation.

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7. Show that the relation R in the set $\{1, 2, 3\}$given by $R = \{(1, 2), (2, 1)\}$is symmetric but neither reflexive nor transitive.

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8. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by

(i) $R = \{(a, b) : |ab| \text{ is a multiple of } 4\}$

(ii) $R = \{(a, b) : a = b\}$is an equivalence relation. Find the set of all elements related to 1 in each case.

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9. Show that the relation R in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation.

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10. Show that the relation R defined on the set A of all triangles in a plane as $R = \{(T_1, T_2) : T_1$ is similar to $T_2)$ is an equivalence relation. Consider three right angle triangle T_1 with sides $3, 4, 5$; T_2 with sides $5, 12, 13$ and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

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11. Show that the relation R , defined on the set A of all polygons as $R = \{(P_1, P_2) : P_1$ and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

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12. Give an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive.

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13. Show that the relation R on the set A of points in a plane, given by $R = \{(P, Q):$ Distance of the point P from the origin is same as the distance of the point Q from the origin$\}$, is an equivalence relation. Further show that the set of all points related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

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14. Let R be the relation in the set N given by $R = \{(a, b): a = b - 2, b > 6\}$. Choose the correct answer.

(A) $(2, 4) \in R$

(B) $(3, 8) \in R$

(C) $(6, 8) \in R$

(D) $(8, 7) \in R$

A. $(2, 4) \in R$

B. $(3, 8) \in R$

C. $(6, 8) \in R$

D. $(8, 7) \in R$

Answer: C

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15. Let L be the set of all lines in $XY = plane$ and R be the relation in L defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

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16. Let R be the relation on the set $A = \{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Then,

(a) R is reflexive and symmetric but not transitive

(b) R is reflexive and transitive but not symmetric

(c) R is symmetric and transitive but not reflexive

(d) R is an equivalence relation

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Exercise 1 3

1. Consider $f: R \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \dfrac{\left(\sqrt{y+6}\right) - 1}{3}$.

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2. Consider $f: R_+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse $f^{-1}$of given f by $f^{-1}(y) = \sqrt{y-4}$ where R_+

is the set of all non-negative real numbers.

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3. Find fog and gof , if

(i) $f(x) = |x|$ and $g(x) = |5x - 2|$

(ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

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4. Let f, g and h be functions from R to R. Show that

$(f+g)oh = foh + goh(f.\,g)oh = (foh).\,(goh)$

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5. Let $f: \{1, 3, 4\} \to \{1, 2, 5\}$and $g: \{1, 2, 5\} \to \{1, 3\}$ be given by

$f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(1, 3), (2, 3), (5, 1)\}$.

Write down gof.

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6. Consider $f: R \rightarrow R$given by $f(x) = 4x + 3$. Show that f is invertible. Find the inverse of f.

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7. Show that $f: [-1, 1] \rightarrow R$, given by $f(x) = \frac{x}{x+2}$ is one- one . Find the inverse of the function $f: [-1, 1] \rightarrow \text{Range} f$.

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8. State with reason whether following functions have inverse

(i) $f: \{1, 2, 3, 4\} \rightarrow \{10\} \text{ with } f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$

(ii) $g: \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\} \text{ with } g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

(iii)

$h: \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\} \text{ with } h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

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9. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$, show that $fof(x) = x$ for all $x \neq \frac{2}{3}$. What is the inverse of f?

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10. Let $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$be a function as $f(x) = \frac{4x}{3x+4}$. The inverse of f is map, $g:\ \ \text{Range}\ \ f \rightarrow R - \left\{-\frac{4}{3}\right\}$given by.

(a) $g(y) = \frac{3y}{3-4y}$

(b) $g(y) = \frac{4y}{4-3y}$

(c) $g(y) = \frac{4y}{3-4y}$

(d) $g(y) = \frac{3y}{4-3y}$

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11. Let $f: X \rightarrow Y$be an invertible function. Show that f has unique inverse. (Hint : suppose g_1 and $g_2$are two inverses of f. Then for all

$y \in Y$, $fog_1(y) = I_Y(y) = fog_2(y)$. Use one-one ness of f).

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12. Consider $f: \{1, 2, 3\} \to \{a, b, c\}$given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Find $f^{-1}$and show that $(f^{-1})^{-1} = f$.

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13. Let $f: X \to Y$ be an invertible function. Show that the inverse of f^{-1} is f, i.e., $(f^{-1})^{-1} = f$.

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14. If $f: R \to R$be given by $f(x) = (3 - x^3)^{1/3}$, then $fof(x)$is

(a) $\dfrac{1}{x^3}$

(b) x^3

(c) x

(d) $(3 - x^3)$

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