



MATHS

NCERT - NCERT MATHEMATICS(ENGLISH)

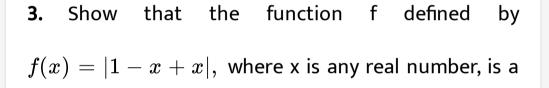
CONTINUITY AND DIFFERENTIABILITY

Solved Examples

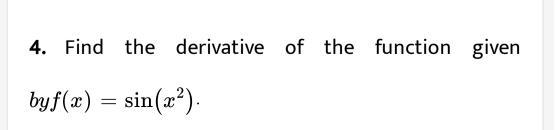
1. Is it true that $x = e^{\log x}$ for all real

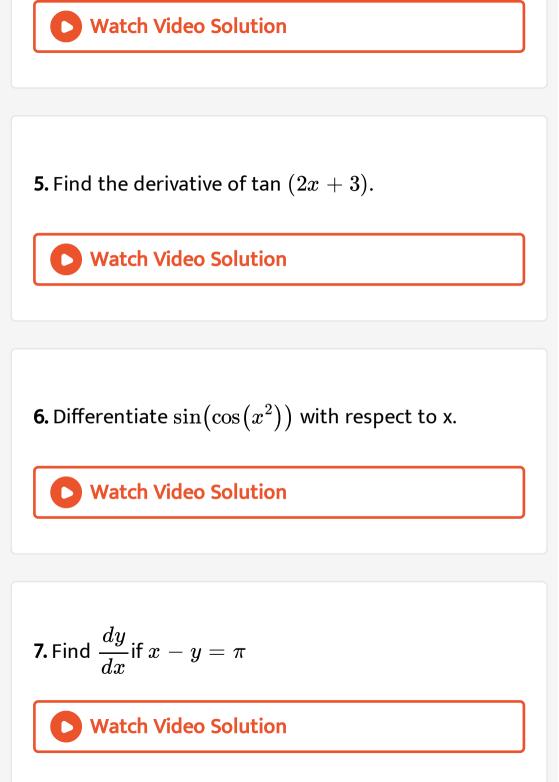
2. Differentiate the following w.r.t. x:(i) e^{-x} (ii) $\sin(\log x), x > 0$ (iii) $\cos^{-1}(e^x)$ (iv) $e^{\cos x}$





continuous function.



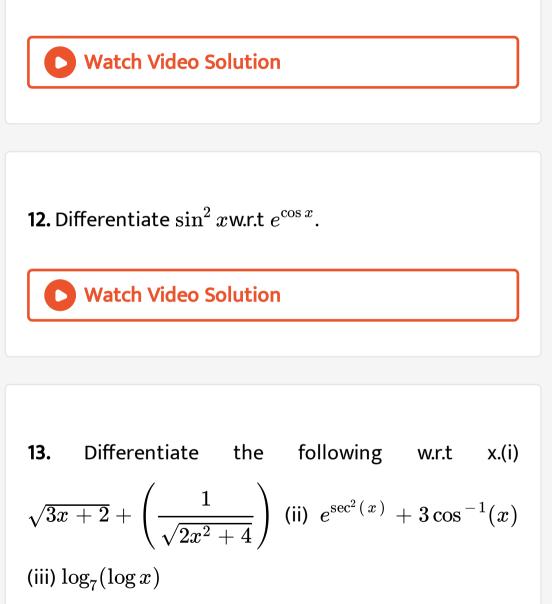


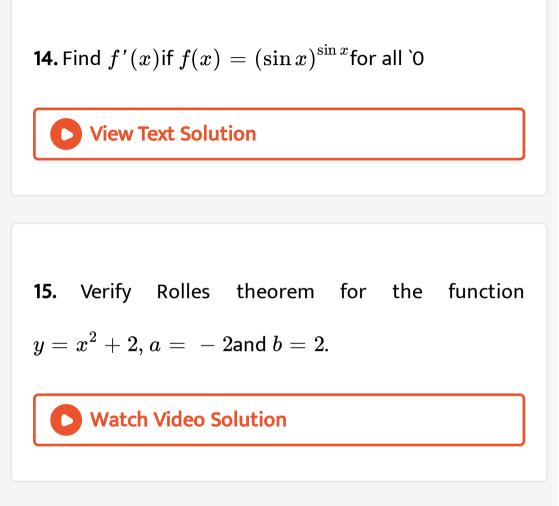
8. Find
$$\frac{dy}{dx}$$
, if $y + \sin y = \cos x$
Watch Video Solution
9. Find the derivative of f given by $f(x) = \sin^{-1} x$
assuming it exists.
Watch Video Solution

10. Find the derivative of / given by $f(x) = an^{-1} x$

assuming it exists.

11. Find df/dx if f(x) = (sin x) ^ sin x for all $o < x < \pi$.





16. Verify the Mean Value Theorm for $f(x)=x^2$ in the interval [2,4].

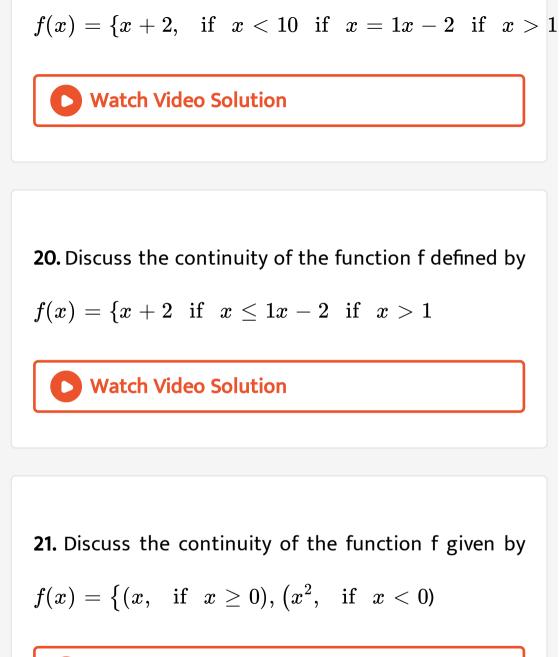


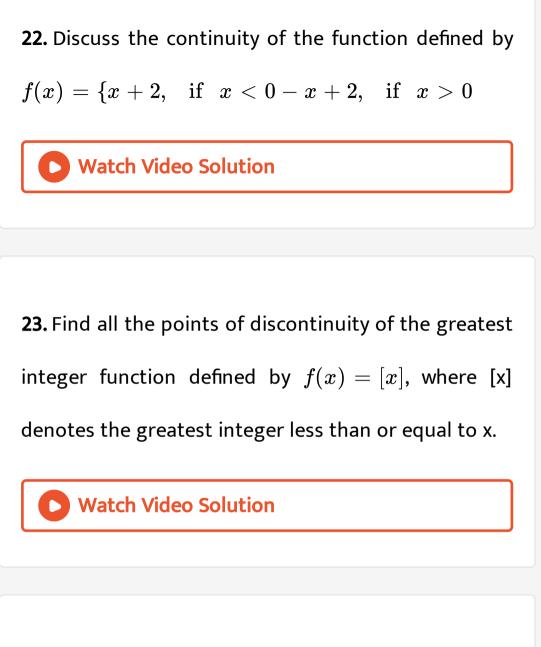
17. If
$$y = 3e^{2x} + 2e^{3x}$$
. Prove that

$$rac{d^2y}{dx^2}-5rac{dy}{dx}+6y=0.$$

18. If
$$y = \sin^{-1} x$$
, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0.$

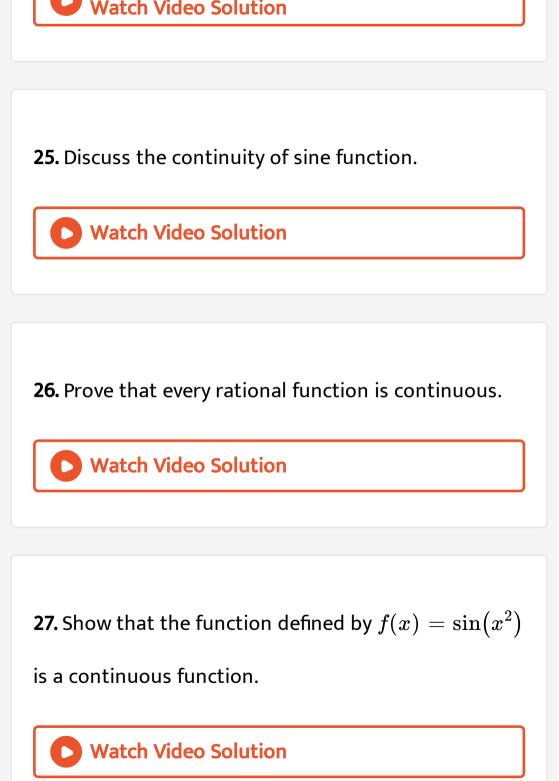
19. Find all the points of discontinuity of the function



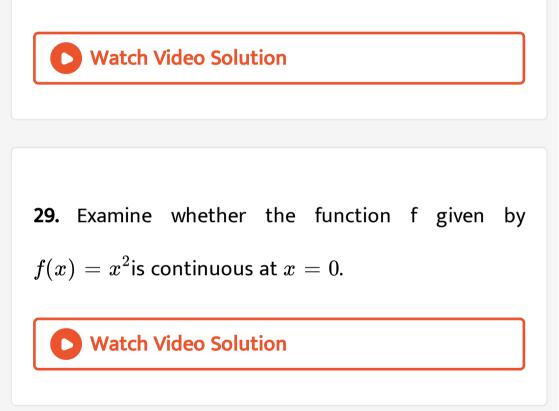


24. Show that every polynomial function is continuous.



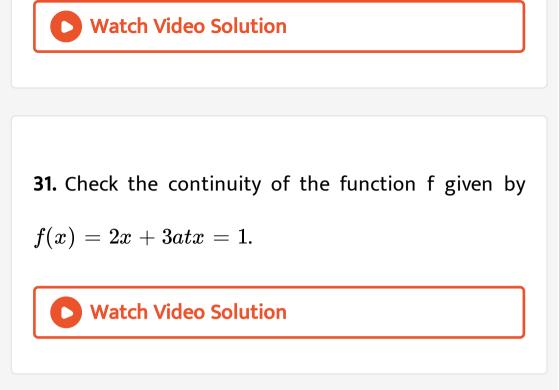


28. Prove that the function defined by f(x) = tanx is a continuous function.



30. Discuss the continuity of the function f given by

$$f(x) = |x|atx = 0.$$

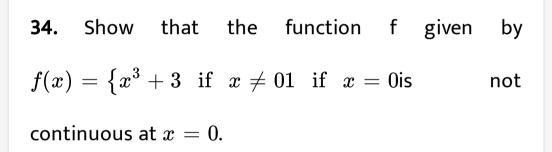


32. Prove that the identity function on real numbers

given by f(x) = x is continuous at every real number.

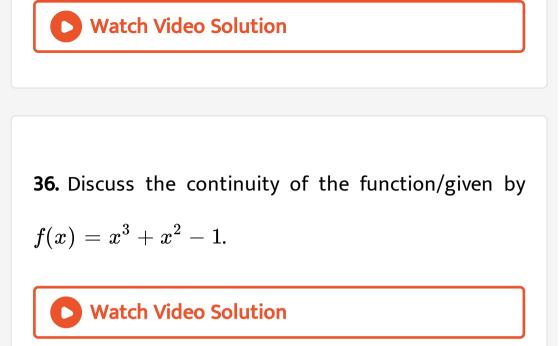


33. Is the function defined by f(x) = |x|, a continuous function? Watch Video Solution



Watch Video Solution

35. Check the points where the constant function f(x) = k is continuous.



37. Discuss the continuity of the function f defined by

$$f(x)=rac{1}{x}, x
eq 0.$$

38. If
$$y = A \sin x + B \cos x$$
, then prove that $\frac{d^2 y}{dx^2} + y = 0.$

39. Find
$$rac{d^2y}{dx^2}$$
, if $y=x^3+ an x.$

Watch Video Solution

40. Find
$$\displaystyle rac{dy}{dx}$$
, if $\displaystyle x^{rac{2}{3}}+y^{rac{2}{3}}=a^{rac{2}{3}}.$

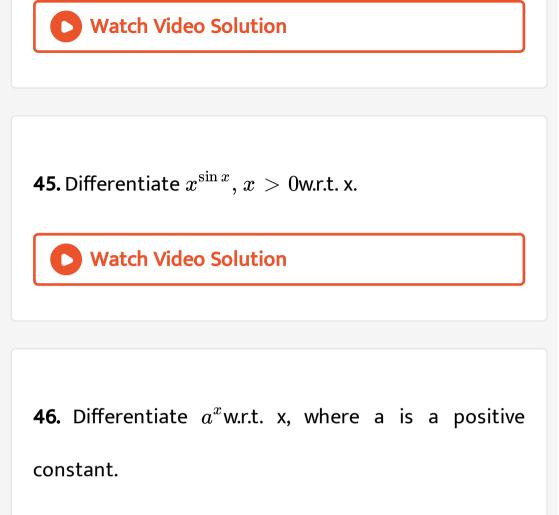
41. Find
$$rac{dy}{dx}$$
, if $x=a(heta+\sin heta), y=1(1-\cos heta).$

42. Find
$$rac{dy}{dx}$$
, if $x=at^2, y=2at.$

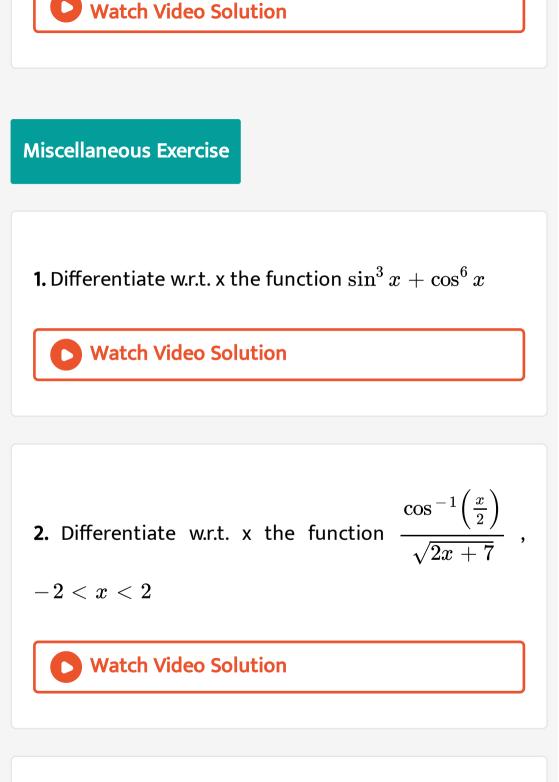
Watch Video Solution

43. Find
$$rac{dy}{dx}$$
, if $x=a\cos heta, y=a\sin heta$.

44. Find
$$rac{dy}{dx}$$
, if $y^x+x^y+x^x=a^b.$



47. Differentiate
$$\sqrt{rac{(x-3)(x^2+4)}{3x^2+4x+5}}$$
 w.r.t x.



3. Differentiate w.r.t. x the function

$$\sin^{-1}ig(x\sqrt{x}ig), 0 \le x \le 1$$

Watch Video Solution

4. If
$$f(x) = \left|x\right|^3$$
 , show that $f(x)$ exists for all real x and find it.

Watch Video Solution

5. Using mathematical induction prove that
$$\frac{d}{dx}(x^n) = nx^{n-1}$$
 for all positive integers n.

6. Does there exist a function which is continuous everywhere but not differentiable at exactly two points? Justify your answer.

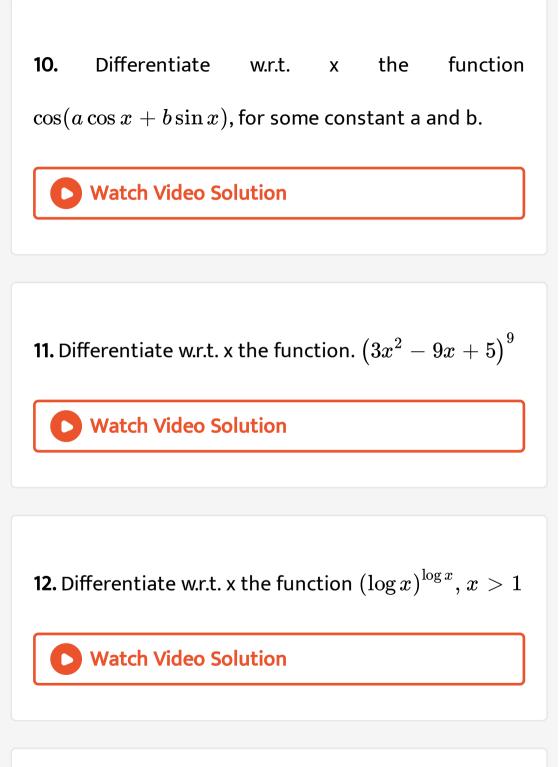
7. Using the fact that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ and the

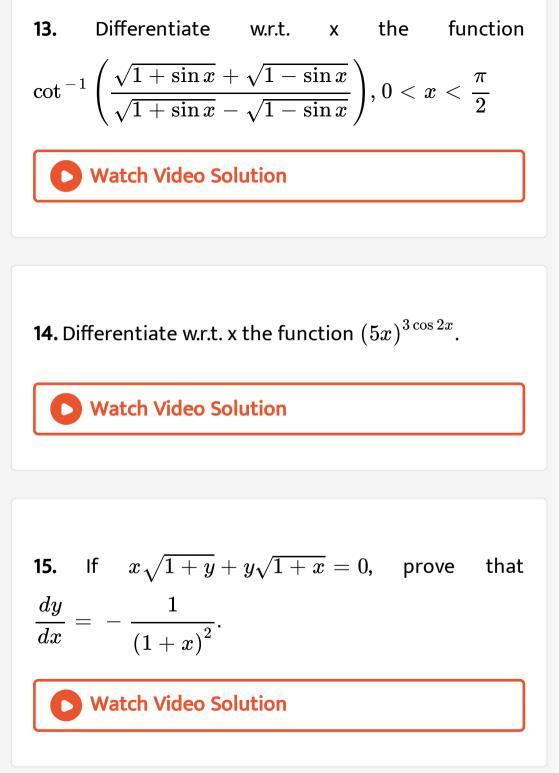
differentiation, obtain the sum formula for cosines.

Watch Video Solution

8. If
$$y = e^{a\cos^{-1}x}$$
, $-1 \le x \le 1$, show that
 $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$

9. If
$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$
, prove that
$$\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$



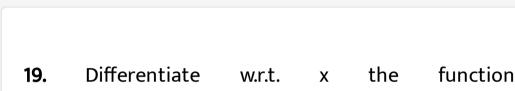


16. If
$$(x-a)^2 + (y-b)^2 = c^2$$
, for some $c > 0$,
prove that $\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant or

independent of a and b.

17. If
$$\cos y = x \cos(a + y)$$
, with $\cos a \neq \pm 1$, prove
that $\frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$.

18. If x=a (cos t + t sin t) and y=a (sin t- t cos t), find $\frac{d^2y}{dx^2}$



 $x^x+x^a+a^x+a^a, ext{ for some fixed } a>0 ext{ and } x>0$

Watch Video Solution

Watch Video Solution

•

20. Differentiate w.r.t. x the function
$$x^{x^2-3} + \left(x-3
ight)^{x^2}$$
 for $x>3.$



21. Find
$$\frac{dy}{dx}$$
 at $t = \left(2\frac{\pi}{3}\right)$ when $x = 10(t - \sin t)$ and $y = 12(1 - \cos t)$.

22. Find
$$\frac{dy}{dx}$$
, if
 $y = \sin^{-1} \left[x \sqrt{1-x} - \sqrt{x} \sqrt{1-x^2} \right]$
Watch Video Solution

23. Differentiate w.r.t. x the function
$$(\sin x - \cos x)^{(\sin x - \cos x)}, \frac{\pi}{4} < x < \frac{3\pi}{4}$$



1. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a\left(\cos t + \log \tan\left(\frac{t}{2}\right)\right), y = a \sin t$

2. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a \sec \theta, y = b \tan \theta$

Watch Video Solution

3. If x and y are connected parametrically by the equations given, without eliminating the parameter,

Find
$$rac{dy}{dx}$$
. $x=4t,y=rac{4}{t}$

4. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = \cos \theta - \cos 2\theta, y = \sin \theta - \sin 2\theta$

Watch Video Solution

5. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a(\theta - \sin \theta), y = a(1 + \cos \theta)$

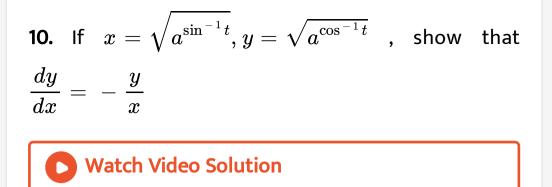
6. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$ Watch Video Solution

7. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = 2at^2, y = at^4$

8. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a \cos \theta, y = b \cos \theta$



9. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = \sin t, y = \cos 2t$



11. If x and y are connected parametrically by the equations given, without eliminating the parameter, Find $\frac{dy}{dx}$. $x = a(\cos \theta + \theta \sin \theta), y = a(\sin \theta - \theta \cos \theta)$



1. Differentiate the functions with respect to x

 $\sin\!\left(x^2+5\right)$



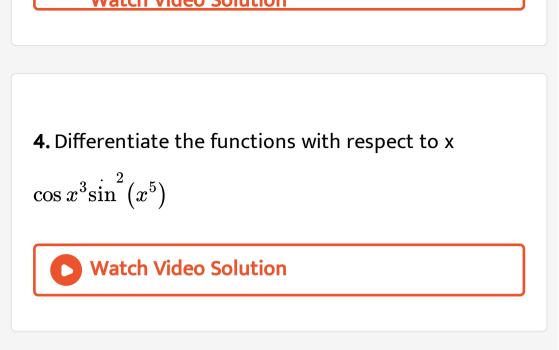
2. Differentiate the functions with respect to x,

 $\cos(\sin x)$

Watch Video Solution

3. Differentiate the functions with respect to x

 $\frac{\sin(ax+b)}{\cos(cx+d)}$



5. Differentiate the functions with respect to x

 $2\sqrt{\cot\left(x^2
ight)}$



6. Prove that the greatest integer function defined by

f(x) = [x], 0 < x < 3 is not differentiable at x = 1 and x = 2.

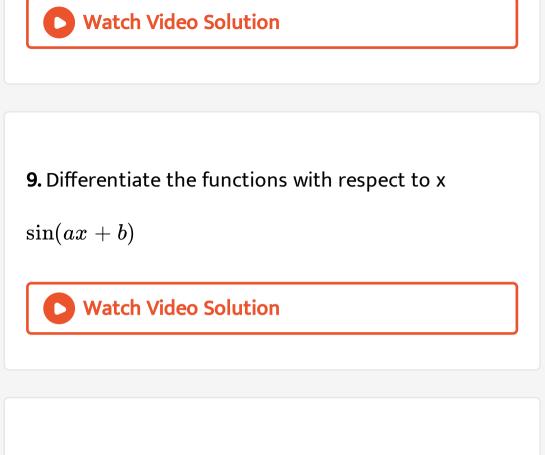
Watch Video Solution

7. Differentiate the functions with respect to x

 $\cos\left(\sqrt{x}\right)$

Watch Video Solution

8. Prove that the function f given by $f(x) = |x-1|, x \in R$ is not differentiable at x = 1



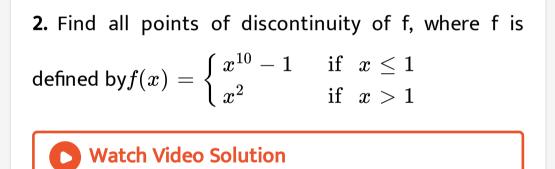
10. Differentiate the functions with respect to x

 $\operatorname{sec}(\operatorname{tan}(\sqrt{x}))$



$${f 1.}\,f(x) egin{cases} 2x, \ {
m if} \ x < 0 \ 0, \ {
m if} \ 0 \le x \le 1 \ {
m Discuss the continuity} \ 4x, \ {
m if} \ x > 1 \end{cases}$$

Watch Video Solution



3. Find all points of discontinuity of f, where f is

defined by
$$f(x) = \left\{egin{array}{ccc} rac{x}{|x|} & ext{if} & x < 0 \ 1 & ext{if} & x \geq 0 \end{array}
ight.$$

Video Colution



4. Find all points of discontinuity of f, where f is

defined by
$$f(x)=\left\{egin{array}{cc} rac{|x|}{x} & ext{ if } x
eq 0 \ 0 & ext{ if } x=0 \end{array}
ight.$$



Watch Video Solution

5. Is the function f defined by
$$f(x) = \begin{cases} x & ext{if } x \leq 1 \\ 5 & ext{if } x > 1 \end{cases}$$
 continuous at

x = 0?Atx = 1?Atx = 2?

6. Prove that the function $f(x) = x^n$ is continuous at

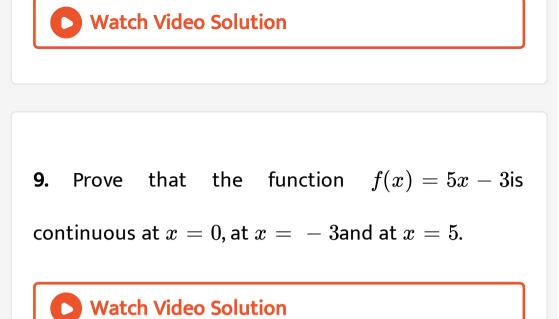
x = n, where n is a positive integer.

7. Find all points of discontinuity of f, where f is

$${\sf defined} \ {\sf by} f(x) = egin{cases} |x|+3 & ext{ if } x \leq -3 \ -2x & ext{ if } -3 < x < 3 \ 6x+2 & ext{ if } x \geq 3 \end{cases}$$

Watch Video Solution

8. Find all points of discontinuity of f, where f is defined by $f(x) = egin{cases} 2x+3 & ext{if} \ x\leq 2 \ 2x-3 & ext{if} \ x>2 \end{cases}$



10. Examine the following functions for continuity.

(a)
$$f(x) = x - 5$$

(b) $f(x) = \frac{1}{x - 5}$
(c) $f(x) = \frac{x^2 - 25}{x + 5}$
(d) $f(x) = |x - 5|$

11. Examine the continuity of the function $f(x) = 2x^2 - 1$ at x = 3.

Watch Video Solution

12. For what value of λ is the function defined by $f(x) = \begin{cases} \lambda (x^2 - 2x) & ext{if } x \leq 0 \\ 4x + 1 & ext{if } x > 0 \end{cases}$ continuous at

x = 0? What about continuity at x = 1?

13. Show that the function defined by g(x) = x[x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

14. Discuss the continuity of the function f, where f is

defined
$$\mathsf{by} f(x) = egin{cases} -2 & ext{if} \quad x \leq -1 \ 2x & ext{if} \quad -1 < x \leq 1 \ 2 & ext{if} \quad x > 1 \end{cases}$$

Watch Video Solution

15. Find the relationship between a and b so that the function f defined by $f(x) = \begin{cases} ax+1 & ext{if } x \leq 3 \\ bx+3 & ext{if } x > 3 \end{cases}$

is continuous at x = 3.

Watch Video Solution

16. Discuss the continuity of the function f, where f is

$${\sf defined} \ {\sf by} f(x) = egin{cases} 3 & ext{if} & 0 \leq x \leq 1 \ 4 & ext{if} & 1 < x < 3 \ 5 & ext{if} & 3 \leq x \leq 10 \end{cases}$$

17.Isthefunctiondefinedby $f(x) = \begin{cases} x+5 & \text{if } x \leq 1 \\ x-5 & \text{if } x > 1 \end{cases}$ a continuous function?Watch Video Solution

18. Find all points of discontinuity of f, where f is

defined by
$$f(x) = egin{cases} x+1 & ext{if} \ x \geq 1 \ x^2+1 & ext{if} \ x < 1 \end{cases}$$

Watch Video Solution

19. Find all points of discontinuity of f, where f is defined by $f(x)=egin{cases} x^3-3 & ext{if} \ x\leq 2 \ x^2+1 & ext{if} \ x<2 \end{cases}$

20. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \begin{cases} kx+1 & \text{if } x \leq 5\\ 3x-5 & \text{if } x > 5 \end{cases}$ at x = 5

Watch Video Solution

21. Find the values of k so that the function f is

continuous at the indicated point in $f(x) = egin{cases} kx+1 & ext{if} \ x \leq \pi \ \cos x & ext{if} \ x > \pi \ \end{array}$ at $x = \pi$

22. Find all points of discontinuity of f, where

$$f(x) = egin{cases} rac{\sin x}{x} & ext{ if } x < 0 \ x+1 & ext{ if } x \geq 0 \end{cases}$$

Watch Video Solution

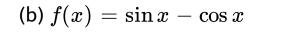
23. Discuss the continuity of the cosine, cosecant,

secant and cotangent functions.



24. Discuss the continuity of the following functions:

(a) $f(x) = \sin x + \cos x$



(c) $f(x) = \sin x \cdot \cos x$

Watch Video Solution

25. Is the function defined by $f(x) = x^2 - \sin x + 5$

continuous at $x = \pi$?

Watch Video Solution

26. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \begin{cases} kx^2 & \text{if } x \leq 2 \\ 3 & \text{if } x > 2 \end{cases}$ at x = 2.

27. Find the values of k so that the function f is continuous at the indicated point in $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases} \text{ at } x = \frac{\pi}{2} \end{cases}$ Watch Video Solution

28. Examine the continuity of f, where f is defined by

$$f(x) = egin{cases} \sin x - \cos x & ext{ if } x
eq 0 \ -1 & ext{ if } x = 0 \end{cases}$$

29. Determine if f defined by
$$f(x) = \begin{cases} x^2 \sin\left(rac{1}{x}
ight) & ext{if } x
eq 0 \\ 0 & ext{if } x = 0 \end{cases}$$
 is a continuous

function?



30. Find all the points of discontinuity of f defined by

$$f(x) = |x| - |x + 1|.$$

31. Find the values of a and b such that the function

defined by $f(x)= egin{cases} 5 & ext{if} & x\leq 2 \ ax+b & ext{if} & 2< x<10 \ ext{is a} \ 21 & ext{if} & x\geq 10 \end{cases}$

continuous function.



32. Show that the function defined by
$$f(x) = \cos\left(x^2
ight)$$
 is a continuous function.

33. Show that the function defined by $f(x) = |\cos x|$

is a continuous function.



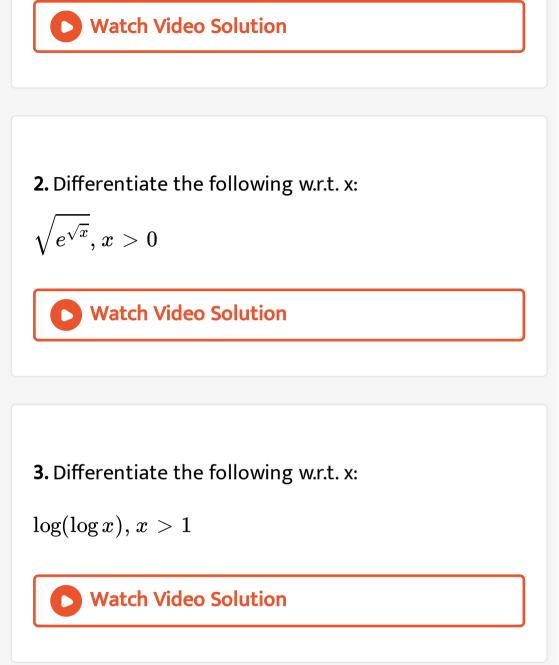
34. The function $f(x) = \sin \lvert x \rvert$ is continuous for all x

Watch Video Solution

Exercise 5 4

1. Differentiate the following w.r.t. x:

 $e^x + e^{x^2} + \ldots + e^{x^5}$



4. Differentiate the following w.r.t. x:

 $\frac{\cos x}{\log x}, x > 0$ **Vatch Video Solution**

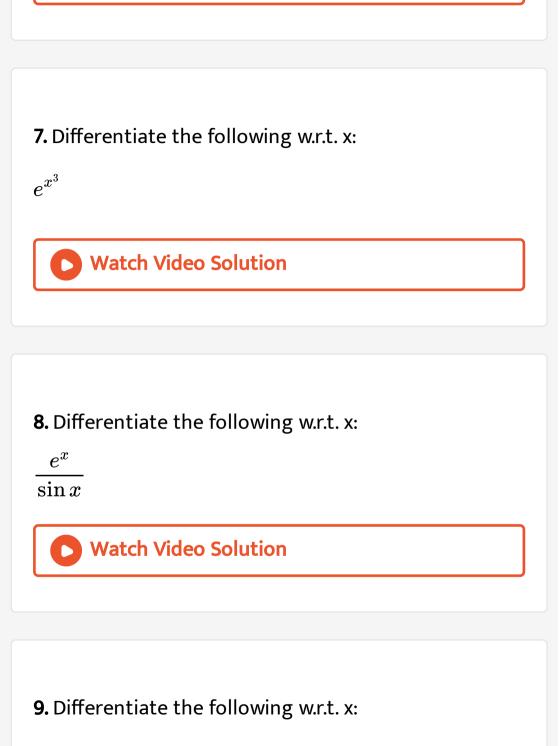
5. Differentiate the following w.r.t. x:

 $\log(\cos e^x)$

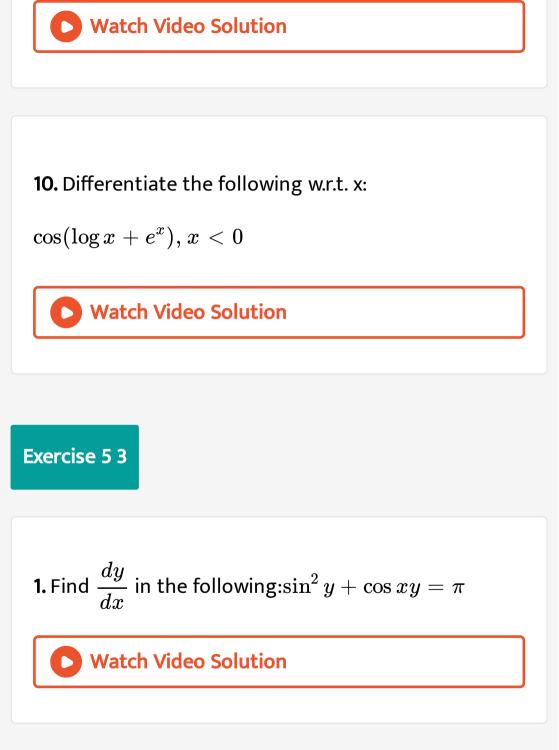
Watch Video Solution

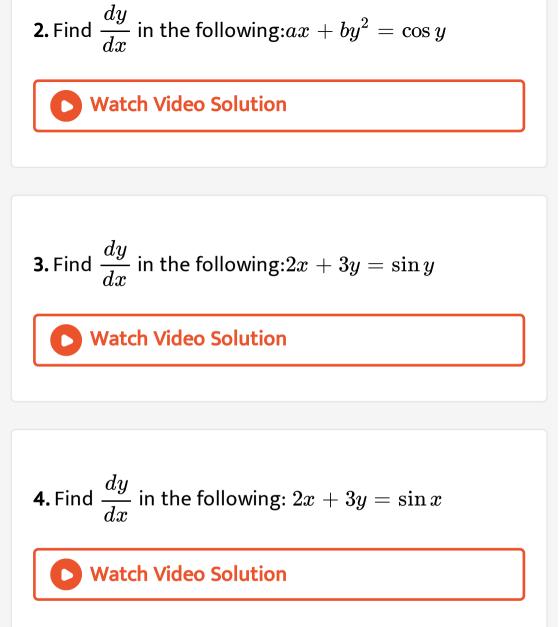
6. Differentiate the following w.r.t. x:

 $e^{\sin^{-1}x}$



$$\sin(\tan^{-1}e^{-x})$$





5. Find
$$\frac{dy}{dx}$$
 in the following:
(a) $x^3 + x^2y + xy^2 + y^3 = 81$
(b) $xy + y^2 = \tan x + y$
(c) $x^2 + xy + y^2 = 100$

Watch Video Solution

6. Find
$$rac{dy}{dx}$$
 in the following: $x^2 + xy + y^2 = 100$

7. Find
$$rac{dy}{dx}$$
 in the following: $xy+y^2= an x+y$

0

8. Find
$$rac{dy}{dx}$$
 in the following: $y=\sin^{-1}\left(rac{2x}{1+x^2}
ight)$

A.
$$\displaystyle rac{2}{1+x^2}$$

B. $\displaystyle rac{5}{4+x^2}$
C. $\displaystyle rac{1}{3+x^2}$
D. $\displaystyle rac{5}{6+x^2}$

Answer: A

9. Find
$$rac{dy}{dx}$$
 in the following: $\sin^2 x + \cos^2 y = 1$



10.
$$y = an^{-1} rac{3x - x^3}{1 - 3(x^2)}, \ -rac{1}{\sqrt{3}} < x < rac{1}{\sqrt{3}}$$



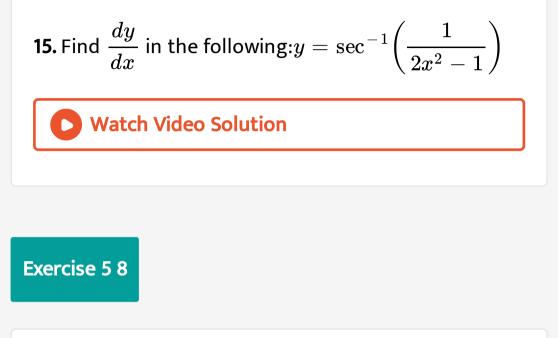
11. If
$$y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$
 , find $\frac{dy}{dx}$.

12. Find
$$\displaystyle rac{dy}{dx}$$
 in the following: $y=\sin^{-1}igg(rac{1-x^2}{1+x^2}igg)$, $0 < x < 1$

13. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$

14. Find
$$\frac{dy}{dx}$$
 in the following:
 $y = \sin^{-1} \left(2x\sqrt{1-x^2} \right), \ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$





1. Examine the applicability of Mean Value Theorem

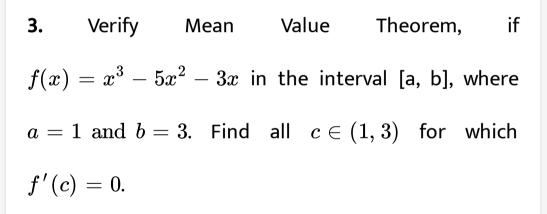
for all three functions given in the above exercise 2.



2. Verify Mean Value Theorem, if $f(x) = x^2 - 4x - 3$

in the interval [a, b], where a = 1 and b = 4.





4. Examine if Rolle's theorem is applicable to any of the following functions. Can you say something about the converse of Rolle's theorem from these example? (i) f(x) = [x] for $x \in [5, 9]$ (ii) f(x) = [x] for $x \in [-2, 2]$ (iii) $f(x) = x^2 - 1$ for $x \in [1, 2]$

View Text Solution

5. Verify Rolle's theorem for the function $f(x)=x^2+2x-8, x\in [-4,2].$

6. If $f\colon [-5,5] o R$ is a differentiable function and if f'(x) does not vanish anywhere, then prove that f(-5)
eq f(5).

Watch Video Solution

Exercise 5 5

1. Differentiate the functions given w.r.t. x:

 $x^x - 2^{\sin x}$

2. Differentiate the functions given w.r.t. x:

 $\cos x \cos 2x \cos 3x$



3. Differentiate the functions given w.r.t. x:

 $(\log x)^{\cos x}$

Watch Video Solution

4. Differentiate the functions given w.r.t. x:

-

$$\sqrt{rac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$



5. Differentiate the functions given w.r.t. x:

$${\left({x + 3} \right)^2}.\,{\left({x + 4} \right)^3}.\,{\left({x + 5} \right)^4}$$



6. Differentiate the following w.r.t. x:

$$\left(\log x
ight)^x+x^{\log x}$$

7. Differentiate the functions given w.r.t. x:

$$\left(x+rac{1}{x}
ight)^x+x^{\left(1+rac{1}{x}
ight)}$$

Watch Video Solution

8. Differentiate the following w.r.t. x:

 $x^{\sin x} + (\sin x)^{\cos x}$

Watch Video Solution

9. Differentiate the following w.r.t. x:

 $(\sin x)^x + \sin^{-1}\sqrt{x}$

Walch Video Solution

10. Find
$$rac{dy}{dx}$$
 of the functions given by $x^y+y^x=1$

Watch Video Solution

11. Find
$$rac{dy}{dx}$$
 of the functions given by $y^x = x^y$

Watch Video Solution

12. Differentiate the following w.r.t. x:

$$x^{x\cos x}+rac{x^2+1}{x^2-1}$$

13. Differentiate the following w.r.t. x:

$$(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$$

Watch Video Solution

14. Find the derivative of the function given by $f(x)=(1+x)ig(1+x^2ig)ig(1+x^4ig)ig(1+x^8ig)$ and hence find f'(1).

15. Differentiate $\left(x^2-5x+8
ight)\left(x^3+7x+9
ight)$ in

three ways mentioned below:

- (i) by using product rule
- (ii) by expanding the product to obtain a single polynomial.
- (iii) by logarithmic differentiation.

Do they all give the same answer?

16. Find
$$\frac{dy}{dx}$$
 of the functions given by $(\cos x)^y = (\cos y)^x$

17. Find
$$rac{dy}{dx}$$
 of the functions given by $xy = e^{(x-y)}$

Watch Video Solution

18. If u, v and w are functions of x, then show that
$$\frac{d}{dx}(u. v. w) = \frac{du}{dx}v. w + u. \frac{dv}{dx}. w + u. v\frac{dw}{dx}$$
 in two ways - first by repeated application of product rule, second by logarithmic differentiation.



given.

 $\log(\log x)$

Watch Video Solution

2. Find the second order derivatives of the functions given.

 $\tan^{-1} x$.



given.

 $x.\cos x$

Watch Video Solution

4. Find the second order derivatives of the functions

given.

 x^{20}



given

 $x^2 + 3x + 2$

Watch Video Solution

6. Find the second order derivatives of the functions

given.

 $e^{6x}\cos 3x.$

given.

 $e^x \sin 5x.$

Watch Video Solution

8. Find the second order derivatives of the functions

given.

 $x^3 \log x$

given.

 $\log x$



10. If
$$y = Ae^{mx} + Be^{nx}$$
, show that
 $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$
Watch Video Solution

11. If
$$y=500e^{7x}+600e^{-7x}$$
 , show that $\displaystyle rac{d^2y}{dx^2}=49y$

Calution



12. If
$$e^y(x+1) = 1$$
, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$. Watch Video Solution

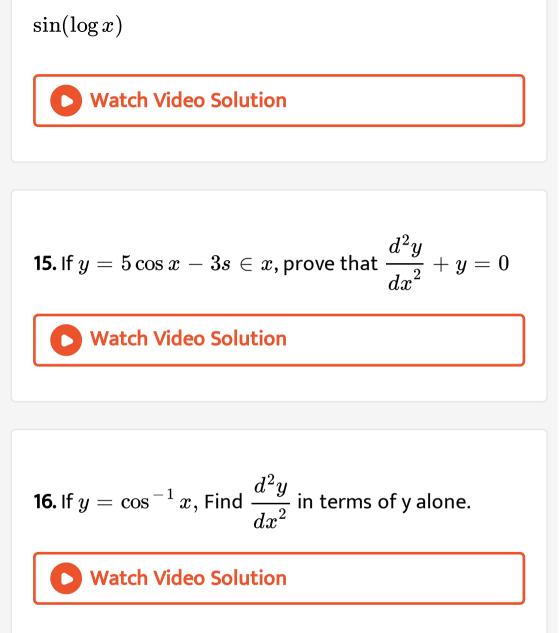
13. If
$$y = \left(an^{-1} x
ight)^2$$
, show that

$$ig(x^2+1ig)^2 y_2 + 2xig(x^2+1ig)y_1 = 2ig)$$

Watch Video Solution

14. Find the second order derivatives of the functions

given.



17. If $y = 3\cos(\log x) + 4\sin(\log x)$, show that

$$x^2y_2 + xy_1 + y = 0.$$