



# MATHS

# NCERT - NCERT MATHEMATICS(ENGLISH)

# DIFFERENTIAL EQUATIONS

**Miscellaneous Exercise** 



2. Find a particular solution of the differential equation (x - y)(dx + dy) = dx - dy, given that y = -1, when x = 0. (Hint: put x - y = t).





4. Find a particular solution of the differential equation

$$(x+1)rac{dy}{dx}=2e^{-y}-1$$
given that  $y=0$ when $x=0.$ 

**5.** The population of a village increases continuously at the rate proportional to the number of its inhabitants present at any time. If the population of the village was 20,000 in 1999 and 25000 in the year 2004, what will be the population of the village in 2009?



6. The general solution of the differential equation  ${ydx-xdy\over y}=0$ is(A) xy=C (B)  $x=Cy^2$  (C) y=Cx (D)  $y=Cx^2$ 

7. The general solution of a differential equation of the type

$$\begin{split} \frac{dx}{dy} + P_1 x &= Q_1 \mathsf{is}(\mathsf{A}) \quad y e^{\int P_1 dy} = \int (Q_1 e^{\int P_1 dy}) dy + C \quad (\mathsf{B}) \\ y \dot{e}^{\int P_1 dx} &= \int (Q_1 e^{\int P_1 dx}) dx + C(\mathsf{C}) \\ x e^{\int P_1 dy} &= \int (Q_1 e^{\int P_1 dy}) dy + C \quad (\mathsf{D}) \\ x e^{\int p_1 dx} &= \int Q_1 e^{\int p_1 dx} dx + C \end{split}$$

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8. The general solution of the differential equation  $e^x dy + (ye^x + 2x) dx = 0$ is(A)  $xe^y + x^2 = C$  (B)  $xe^y + y^2 = C$  (C)  $ye^x + x^2 = C$  (D)  $ye^y + x^2 = C$ 

9. Find a particular solution of the differential equation  $\frac{dy}{dx} + y \cot x = 4x \cos ecx (x 
eq 0)$ , given that y = 0 when  $x = \frac{\pi}{2}$ 

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10. Prove that  $x^2 - y^2 = c(x^2 + y^2)^2$  is the general solution of differential equation  $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$ , where c is a parameter.

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11. Find the equation of the curve passing through the point

 $\left(0, \frac{\pi}{4}\right)$  whose differential equation is  $\sin x \cos y dx + \cos x \sin y dy = 0.$ 



12. Show that the general solution of the differentia equation

$$rac{dy}{dx}+rac{y^2+y+1}{x^2+x+1}=0$$
 is given by

x+y+1=A(1-x-y-2xy) where A is a parameter.

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13. Find the general solution of the differential equation

$$rac{dy}{dx}+\sqrt{rac{1-y^2}{1-x^2}}=0.$$

14. Form the differential equation of the family of circles in

the first quadrant which touch the coordinate axes.

• Watch Video Solution 15. Form the differential equation representing the family of curves given by  $(x - a)^2 + 2y^2 = a^2$ , where a is an arbitrary constant.

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**16.** For each of the exercises given below, verify that the given function (implicit or explicit) is a solution of the corresponding differential equation.(i)  $y = ae^x + be^{-x} + x^2$ 

$$+xrac{d^2y}{dx^2}+2yrac{dy}{dx}-xy+x^2-2=0,$$

**17.** For each of the differential equations given below, indicate its order and degree (if defined).(i)

$$\begin{aligned} \frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6xy &= \log x \end{aligned} \tag{ii} \\ \left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y &= \sin x \text{(iii)} \\ \frac{d^4y}{dx^4} - \sin\left(\frac{d^3y}{dx^3}\right) &= 0 \end{aligned}$$

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18. Find the particular solution of the differential equation

$$ig(1+e^{2x}ig)dy+ig(1+y^2ig)e^xdx=0$$
, given that

y = 1 when x = 0.

$$rac{dy}{dx} + \sec xy = an x \Big( 0 \leq x \leq rac{\pi}{2} \Big)$$

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2. Find the general solution of the differential equations:`cos^2x(dx)/(dy)+y=tanx(0<=x</p>

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**3.** Find the general solution of the differential equation : dy

$$xrac{dy}{dx}+2y=x^2\log x$$



4. Find the equation of a curve passing through the point (0,2) given that the sum of the coordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

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**5.** Find the equation of a curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the coordinates of the point.

6. The differential equations, find a particular solution satisfying the given condition:  $\frac{dy}{dx} - 3y \cot x = \sin 2x; y = 2$  when  $x = \frac{\pi}{2}$ 

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7. The differential equations, find a particular solution satisfying the given condition:  $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}; y = 0$ when x = 1• Watch Video Solution

8. For the differential equation, find a particular solution satisfying the given condition:  $\frac{dy}{dx} + 2y \tan x = \sin x; y = 0$  when  $x = \frac{\pi}{3}$ 

$$ig(x+3y^2ig)rac{dx}{dy}=y(y>0)$$

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10. 
$$ydx+ig(x-y^2ig)dy=0$$

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11. Find the general solution of the differential equations:

$$(x+y)rac{dx}{dy}=1$$

$$x\log xrac{dy}{dx}+y=rac{2}{x}\log x$$

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**13.** The Integrating Factor of the differential equation

$$ig(1-y^2ig)rac{dx}{dy}+yx=ay$$

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14. The Integrating Factor of the differential equation  $xrac{dy}{dx}-y=2x^2$ is(A)  $e^{-x}$  (B)  $e^{-y}$  (C)  $rac{1}{x}$  (D) x

$$ig(1+x^2ig) dy + 2xy dx = \cot x dx (x
eq 0)$$

**16.** Find the general solution of the differential equations:

$$xrac{dy}{dx}+y-x+xy \operatorname{cot} x=0 (x
eq 0)$$

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**17.** Find the general solution of the differential equations:

$$rac{dy}{dx} + 2y = \sin x$$



$$rac{dy}{dx} + 3y = e^{-2x}$$

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**19.** Find the general solution of the differential equations:

$$rac{dx}{dy}+rac{y}{x}=x^2$$

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## Exercise 9 2

**1.** The number of arbitrary constants in the particular solution of a differential equation of third order are:



**3.** Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  $y = \sqrt{a^2 - x^2}x \in (-x, a): x + y \frac{dy}{dx} = 0 (y \neq 0)$ 

4. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  

$$y = \sqrt{1 + x^2}$$
:  $y' = \frac{xy}{1 + x^2}$   
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5. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  $y = \sqrt{1 + x^2} : y' = \frac{xy}{1 + x^2}$ Watch Video Solution

**6.** Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:

$$y = x \sin x$$
 :  $xy' = y + x \sqrt{x^2 - y^2} (x 
eq 0$ and  $x > y ext{ or } x < y)$ 

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7. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  $xy = \log y + C : y' = rac{y^2}{1-xy} (xy 
eq 1)$ 

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8. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  $y = e^x + 1: y'' - y' = 0$  9. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:(2)  $y = x^2 + 2x + C$  : y' - 2x - 2 = 0(3)  $y = \cos x + c$ :  $y' + \sin x = 0$ 



10. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  $y = \cos x + C : y' + \sin x = 0$ 



11. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  $y - \cos y = x : (y \sin y + \cos y + x)y' = y$ 



12. Verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation:  $x + y = \tan^{-1} y : y^2 y' + y^2 + 1 = 0$ 



1. Find the general solution of the differential equations 
$$x^5 \frac{dy}{dx} = -y^5$$

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2. Find the general solution of the differential equations  $\frac{dy}{dx} = \sin^{-1} x$ 

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**3.** Find the general solution of the differential equations  $rac{dy}{dx} = \left(1+x^2
ight)\left(1+y^2
ight)$ 

 $\log y \, dx - x \, dy = 0$ 



5. Find the general solution of the differential equations  $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ 

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6. The solution of the differential equation 
$$(e^x + e^{-x})dy - (e^x - e^{-x})dx$$
=0 is

7. Find the general solution of the differential equations  $'(dy)/(dx)=sqrt(4-y^2)(-2$ 

8. Find the general solution of the differential equations  $rac{dy}{dx} + y = 1(y 
eq 1)$ 

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9. Find the general solution of the differential equations

$$rac{dy}{dx} = rac{1-\cos x}{1+\cos x}$$



10. In a bank principal increases at the rate of r% per year. Find the value of r if Rs. 100 double itself in 10 years  $\left((\log)_e 2 = 0.6931.\right)$ 

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11. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years  $\left(e^{0.5}=1.\ 648
ight)$ 

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12. The general solution of the differential equation  $\frac{dy}{dx} = e^{x+y}$  is (A)  $e^x + e^{-y} = C$  (B)  $e^x + e^y = C$  (C)  $e^{-x} + e^y = C$  (D)  $e^{-x} + e^{-y} = C$  **13.** In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

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**14.** The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.



**15.** At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (4, 3). Find the equation of the curve given that it passes through (2, 1).

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**16.** Find the equation of a curve passing through the point (0,

0) and whose differential equation is  $y' = e^x \sin x$ 

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17. The differential equations, find a particular solution satisfying the given condition:  $\frac{dy}{dx} = y \tan x; y = 1$  when x =



**18.** Find the equation of the curve passing through the point (0, -2) given that at any point (x, y) on the curve the product of the slope of its tangent and y coordinate of the point is equal to the x-coordinate of the point.

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19. For the differential equation  $xy \frac{dy}{dx} = (x+2)(y+2)$ ,

find the solution curve passing through the point (1, -1).

**20.** Find a particular solution satisfying the given condition :

$$ig(x^3+x^2+x+1ig)rac{dy}{dx}=2x^2+x,y=1$$
 when  $x=0$ 

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**21.** Find the general solution of the differential equations  $e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$ 

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22. The differential equations, find a particular solution

satisfying the given condition:
$$\cosigg(rac{dy}{dx}igg)=a(a\in R);y=1$$

23. For the differential equation, find a particular solution satisfying the given condition:  $x(x^2-1)\frac{dy}{dx}=1;y=0$  when x=2



is homogeneous and solve it.



**2.** In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs 1000 double itself?

**3.** Show that the differential equation  $2ye^{\frac{x}{y}} dx + (y - 2xe^{\frac{x}{y}}) dy = 0$  is homogeneous. Find the particular solution of this differential equation, given that x = 0 when y = 1.

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4. Show that the differential equation  $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$  is homogeneous and solve it.



6. Find the equation of the curve passing through the point

(1, 1) whose differential equation is

$$xdy=ig(2x^2+1ig)dx(x
eq 0).$$

7. Find the general solution of the differential equation  $\frac{dy}{dx} - y = \cos x$ 

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8. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is  $\frac{x^2 + y^2}{2xy}$ , is given by  $x^2 - y^2 = cx$ .

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9. Form the differential equation representing the family of

ellipses having foci on x-axis and centre at the origin.



10. Form the differential equation of the family of circles

touching the x-axis at origin.



11. Form the differential equation representing the family of

curves y = mx, where, m is arbitrary constant.



12. Find the differential equation representing the family of

curves y=asin (x+b), where a,b are arbitrary constants.



13. Verify that the function  $y = e^{-3x}$  is a solution of the differential equation  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ Vatch Video Solution

14. Verify that the function  $y = a \cos x + b \sin x$ , where, a,

 $b\in R$ is a solution of the differential equation  $\displaystyle rac{d^2y}{dx^2}+y=0.$ 

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**15.** Find the order and degree, if defined, of each of the following differential equations:(i)  $\frac{dy}{dx} - \cos x = 0$  (ii)  $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$  (iii)  $y''' + y^2 + e^{y'} = 0$ 

**16.** Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis.

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17. Find the general solution of the differential equation

$$rac{dy}{dx}=rac{x+1}{2-y}, (y
eq 2)$$

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18. Solve the differential equation 
$$(\tan^{-1}y - x)dy = (1 + y^2)dx.$$

19. Verify that the function  $y = c_1 e^{ax} \cos bx + c_2 e^{ax} \sin bx$ , where  $c_1, c_2$  are arbitrary constants is a solution of the differential equation.  $\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2)y = 0$ 



20. Form the differential equation of the family of circles in

the second quadrant and touching the coordinate axes.



21. Find the particular solution of the differential equation

$$\log\!\left(rac{dy}{dx}
ight)=3x+4y$$
 given that  $y=0$  when  $x=0.$ 

22. Solve the differential equation
$$(xdy - ydx)y\sin\left(\frac{y}{x}\right) = (ydx + xdy)x\cos\left(\frac{y}{x}\right).$$
  
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23. Find the general solution of the differential equation  $x \frac{dy}{dx} + 2y = x^2 (x 
eq 0).$ 

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24. Find the general solution of the differential equation

$$ydx-ig(x+2y^2ig)dy=0.$$

25. Find the particular solution of the differential equation  $rac{dy}{dx}+y\cot x=2x+x^2\cot x(x
eq 0)$  given that y=0 when  $x=rac{\pi}{2}.$ 



**26.** Find the equation of a curve passing through the point (0, 1). If the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x coordinate (abscissa) and the product of the x coordinate and y coordinate (ordinate) of that point.



27. Find the particular solution of the differential equation  $\frac{dy}{dx} = -4xy^2$  given that y = 1, when x = 0.



**28.** Find the general solution of the differential equation

 $rac{dy}{dx} = rac{1+y^2}{1+x^2}.$ 

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**1.** Form a differential equation representing the given family of curves by eliminating arbitrary constants a and b.  $y = ae^{3x} + be^{-2x}$ 



3. Form a differential equation representing the given family

of curves by eliminating arbitrary constants a and b.

$$\frac{x}{a} + \frac{y}{b} = 1$$

4. Form the differential equation of the family of parabolas

having vertex at origin and axis along positive y-axis.

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**5.** Form the differential equation of the family of circles touching the y-axis at origin.



6. Form a differential equation representing the given family

of curves by eliminating arbitrary constants a and b.y = ex (a

 $\cos x + b \sin x$ )



7. Form a differential equation representing the given family of curves by eliminating arbitrary constants a and b.  $y = e^{2x}(a + bx)$ 

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8. Form the differential equation of the family of hyperbolas

having foci on x-axis and centre at origin.



9. Form the differential equation of the family of ellipses

having foci on y-axis and centre at origin.



**10.** Form the differential equation of the family of circles having centre on y-axis and radius 3 units.



12. Which of the following differential equations has y = x as

one of its particular solution?(A) 
$$rac{d^2y}{dx^2} - x^2rac{dy}{dx} + xy = x$$
 (B)

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x \quad (C) \quad \frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 \quad (D)$$
  
$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$$
  
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Exercise 9 5  
1. Show that the given differential equation is homogeneous  
and solve each of them  $x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$ 

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2. Show that the given differential equation is homogeneous

dx

and solve each of them. $ig(x^2-y^2ig)dx+2xydy=0$ 

3. Show that the given differential equation is homogeneous

and solve each of them. $\Big\{x\cos\Big(\frac{y}{x}\Big)+y\sin\Big(\frac{y}{x}\Big)\Big\}ydx=\Big\{y\sin\Big(\frac{y}{x}\Big)-x\cos\Big(\frac{y}{x}\Big)\Big\}xdy$ 

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**4.** Show that the given differential equation is homogeneous and solve each of them. $xdy - ydx = \sqrt{x^2 + y^2}dx$ 

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5. Show that the given differential equation is homogeneous

and solve each of them. $ig(x^2+xyig)dy=ig(x^2+y^2ig)dx$ 

6. Show that the given differential equation is homogeneous

and solve each of them. (x - y)dy - (x + y)dx = 0

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7. Show that the given differential equation is homogeneous

and solve each of them. $y'=rac{x+y}{x}$ 

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8. Show that the given differential equation is homogeneous

and solve each of them.  $ydx + x\log\Bigl(rac{y}{x}\Bigr)dy - 2xdy = 0$ 

9. Show that the given differential equation is homogeneous

and solve it.
$$xrac{dy}{dx}-y+x\sin\Bigl(rac{y}{x}\Bigr)=0$$

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10. A homogeneous differential equation of the from  $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$  can be solved by making the substitution.(A) y = vx (B) v = yx (C) x = vy (D) x = v

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11. Which of the following is a homogeneous differential equation? (A) (4x+6y+5)dy-(3y+2x+4)dx=0 (B)

$$(xy)dx - ig(x^3+y^3ig)dy = 0$$
 (C)  $ig(x^3+2y^2ig)dx + 2xydy = 0$   
(D)  $y^2dx + ig(x^2+xy-y^2ig)dy = 0$ 

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12. For the differential equation , find the particular solution

satisfying the given condition:  $\frac{dy}{dx} - \frac{y}{x} + \cos ec \left( \frac{y}{x} \right) = 0; y = 0$ when x = 1

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13. For the given differential equation , find the particular solution satisfying the given condition:  $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y = 2$ when x = 1

14. For the given differential equation , find the particular solution satisfying the given condition:  $x^2 dy + (xy + y^2) dx = 0; y = 1$  when x = 1

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15. The differential equations , find the particular solution satisfying the given condition:  $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0; y = \frac{\pi}{4}$  when x = 1



16. Show that the given differential equation is homogeneous

and solve each of them.
$$\Big(1+e^{rac{x}{y}}\Big)dx+e^{rac{x}{y}}\Big(1-rac{x}{y}\Big)dy=0$$



1. Determine order and degree (if defined) of differential

equations given $y^{\,\prime\,\prime}+\left(y^{\,\prime}
ight)^{2}+2y=0$ 

2. Determine order and degree (if defined) of differential

equations given  $y' + y = e^x$ 



3. Determine order and degree (if defined) of differential equations given  $rac{d^4y}{dx^4} + \sin(y'\,') = 0$ 

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4. Determine order and degree (if defined) of differential

equations given
$$\left(rac{ds}{dt}
ight)^4+3srac{d^2s}{dt^2}=0$$

5. Determine order and degree (if defined) of differential equations given y'+5y=0

6. Determine order and degree (if defined) of differential

equations given
$$rac{d^2y}{dx^2}=\cos 3x+\sin 3x$$

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7. Determine order and degree (if defined) of differential

equations given
$$\left(rac{d^2y}{dx^2}
ight)^2+\cos\!\left(rac{dy}{dx}
ight)=0$$

8. Determine order and degree (if defined) of differential equations  ${
m given}y^m+2y$ '' +y'=0

9. Determine order and degree (if defined) of differential equations given $(y'\,'\,')^2+(y'\,')^3+(y')^4+y^5=0$ 

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10. The order of the differential equation 
$$2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0$$
 is(A) 2 (B) 1 (C) 0 (D) not defined

11. Determine order and degree (if defined) of differential equations given  $y'\,' + 2y' + \sin y = 0$ 



12. The degree of the differential equation
$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$
(A) 3 (B) 2 (C) 1 (D)

not defined