MATHS

BOOKS - XII BOARDS PREVIOUS YEAR

BOARD PAPER SOLUTIONS

Others

1. If the mean and variance of a binomial distribution are respectively 9 and 6, find the distribution.

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2. There are two bags I and II. Bag I contains 3 white and 3 red balls and Bag II contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag II.

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3. A card is drawn at random from a well-shuffled pack of 52 cards. Find the probability that it is neither a king nor a heart.

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4. Evaluate : $\int\left(e^x \frac{2+\sin 2x}{2\cos^2 x}\, dx\right.$

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5. Solve the following differential equation : $(1+y^2)(1+\log x)dx + x\backslash\ dy = 0$

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6. Form the differential equation of the family of curves $y = A\,e^{Bx}$ where A and B are constants.

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7. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\, dx$

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8. Two forces act at a point and are such that if the direction of one is reversed, the resultant is turned through a right angle. Show that the two forces must be equal in magnitude.

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9. Find the value of λ , which makes the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ coplanar, where $\vec{a} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{b} = 2\hat{i} + \lambda\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} - 7\hat{j} + 10\hat{k}$

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10. Find the point on the curve $y^2 = 4x$ which is nearest to the point (2, 1).

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11. If $y = a\cos(\log x) + \sin(\log x),$ prove that $\frac{x^2 d^2}{dx^2} + x\frac{dy}{dx} + y = 0$

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12. Find the derivative of $\cos(2x + 1)$ w.r.t. x from first principle.

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13. An open box, with a square base, is to be made out of a given quantity of metal sheet of area C^2. Show that the maximum volume of the box is

$\frac{C^3}{6\sqrt{3}}$

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14. Find the intervals in which the function $f(x) = x^3 - 12x^2 + 36x + 17$ is (a) increasing, (b) decreasing.

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15. Evaluate : $\int \frac{x^2 + 1}{(x + 1)^2} dx$

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16. Find the area of the region bounded by the parabola $x^2 = 4y \backslash$ and the line $x = 4y - 2$

A. $\frac{3}{8}$sq units

B. $\frac{5}{8}$ sq unit

C. $\frac{7}{8}$ sq unit

D. $\frac{9}{8}$ sq units

Answer: null

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17. Find the coordinates of the point where the line $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$ meets the plane $x+y+4z=6$.

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18. Using properties of determinants, prove the following

$$\begin{vmatrix} 3a & -a+b & -a+c \\ a-b & 3b & c-a \\ a-c & b-c & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$$

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19. Express the matrix as $A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ the sum of a symmetric and a skew-symmetric matrix

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20. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of number of jacks.

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21. A and B toss coin alternately till one of them gets a head and wins the game. If A starts first, find the probability the B will win the game.

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22. If $y = \sin(\log x)$, prove that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

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23. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 4$ on [1, 4].

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24. If $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$, show the $A^2 - 6A + 17I = 0$. Hence find A^{-1}

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25. An urn contains 7 red and 4 blue balls. Two balls are drawn at random with replacement. Find the probability of getting (a) 2 red balls (b) 2 blue balls (c) one red and one blue ball.

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26. Solve the following differential equation : $\frac{dy}{dx} + 2y = 6e^x$

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27. Form the differential equation of the family of curves $y = A\cos 2x + B\sin 2x$, where A and B are constants.

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28. Prove that the curves $\mathrm{x} = \mathrm{y}^2$ and $\mathrm{x\,y=k}$ intersect at right angles if $8\mathrm{k}^2 = 1$.

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29. Let T be the set of all triangles in a plane with R as relation in T given by $\mathrm{R} = \left\{\left(\mathrm{T}_1, (\backslash\ \mathrm{T})_2\right) : (\backslash\ \mathrm{T})_1 \cong \mathrm{T}_2\right\}$. Show that R is an equivalence relation.

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30. If $x\sqrt{1+y}+y\sqrt{1+x}=0,$ find $\frac{\mathrm{dy}}{\mathrm{dx}}$. To prove $\frac{dy}{dx}=-\frac{1}{(1+x)^2}$

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31. Form the differential equation representing the parabolas having vertex at the origin and axis along positive direction of x-axis.

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32. A and B throw a pair of die turn by turn. The first to throw 9 is awarded a prize. If A starts the game, show that the probability of A getting the prize is $\frac{9}{17}$

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33. Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$.

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34. Prove that

$$\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) = \frac{2b}{a}$$

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35. Let $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$ Express A as sum of two matrices such that one is symmetric and the other is skew symmetric.

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36. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

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37. Find the equation of tangent to the curve $x = \sin 3t, y = \cos 2t$ at $t = \frac{\pi}{4}$

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38. For what value of k is the following function continuous at $x = 2$?

$f(x) = \{2x + 1; x < 2k; x = 23x - 1; x > 2\}$

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39. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.

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40. Using properties of determinants, prove the following

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$$

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41. Evaluate $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

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42. Find the point on the line $\dfrac{x+2}{3} = \dfrac{y+1}{2} = \dfrac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1,2,3).

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43. Write the adjoint of the following matrix: $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

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44. Write the distance of the following plane from the origin: $2x - y + 2z + 1 = 0$

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45. Write a vector of magnitude 9 units in the direction of vector $-2\hat{i} + \hat{j} + 2\hat{k}$

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46. Three bags contain balls as shown in the table below: Bag Number of White balls Number of Black balls Number of Red balls I 1 2 3 II 2 1 1 III 4 3 2 A bag is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they came from the III bag?

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47. If $x = a\sin t$ and $y = a\left(\cos t + \frac{\log\tan t}{2}\right)$, find $\frac{d^2 y}{dx^2}$

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48. Using properties of determinants, prove that $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ c+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

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49. Show that the height of a closed right circular cylinder of given surface and maximum volume, is equal to the diameter of its base.

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50. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x .

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51. Prove the following:

$$\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$$

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52. Find the value of a if $\begin{bmatrix} a-b & 2a+c \\ 2a-b & c+d \end{bmatrix}=\begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

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53. Find the Cartesian equation of the line which passes through the point (2, 4, 5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

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54. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the value of θ.

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55. For what value of x is the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ 1 & 0 & 3 \\ x & 3 & 0 \end{bmatrix}$ a skew-symmetric matrix?

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56. Write the differential equation representing the family of curves $y = mx$, where m is an arbitrary constant.

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57. Sketch the graph of $y = |x + 3|$ and evaluate the area under the curve $y = |x + 3|$ above x-axis and between $x = 6$ to $x = 0$.

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58. Evaluate: $\int \frac{x^2+1}{(x-1)^2(x+3)}\,dx$

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59. Using differentials, find the approximate value of $f(2.01)$, where $f(x) = 4x^3 + 5x^2 + 2$.

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60. Prove the following: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x\ (0, 1)$

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61. Prove the following: $\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

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62. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

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63. Evaluate : $\int_{-1}^{2} |x^3 - x| \, dx$

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64. Write the vector equation of the following line: $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$

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65. If $y = \cos^{-1}\left(\frac{2^x+1}{1+4^x}\right)$, find $\frac{dy}{dx}$

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66. From the following matrix equation, find the value of $x: \begin{bmatrix} x+y & 4 \\ -5 & 3y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -5 & 6 \end{bmatrix}$

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67. Prove the following: $\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] = \frac{2b}{a}$

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68. If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$

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69. Write the direction cosines of a line equally inclined to be three coordinate axes.

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70. Let* be a binary operation on N given by a*b = HCF$(a,\ b)$ $a,\ b\ N$.

Write the value of 22*4.

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71. Write the value of the following determinant $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

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72. Find the value of x, from the following: $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$

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73. Write the principal value of, $\cos^{-1}\left(\frac{\cos(7\pi)}{6}\right)$

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74. From a lot of 10 bulbs, which includes 3 defectives, a sample of 2 bulbs is drawn at random. Find the probability distribution of the number of defective bulbs.

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75. Solve the following differential equation: $(x^3 + y^3)dy - x^2 y dx = 0$

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76. Find the distance of the plane $3x - 4y + 12z = 3$ from the origin.

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77. Using properties of determinants, show that

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$$

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78. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

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79. Show that the relation R in the set of real numbers, defined as $R = \{a, b) : a \leq b^2\}$ is neither reflexive, nor symmetric, nor transitive.

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80. In a certain college, 4% of boys and 1% of girls are taller than 1.75 metres. Furthermore, 60% of the students in the college are girls. A student is selected at random from the college and is found to be taller than 1.75 metres. Find the probability that the selected student is a girl.

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81. Prove that: $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$

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82. If $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}3 & 1\\ 2 & 5\end{bmatrix}=\begin{bmatrix}7 & 11\\ k & 23\end{bmatrix}$, then write the value of k.

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83. If $A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$, then for what value of α is A an identity matrix?

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84. Using matrices, solve the following system of equations:

$x - y + z = 4;\ 2x + y - 3z = 0;\ x + y + z = 2$

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85. Using vectors, find the area of the triangle with vertices A (1, 1, 2), B (2, 3, 5) and C (1, 5, 5).

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86. Find the intervals in which the function f given by $f(x) = x^3 + \frac{1}{x^3},\ x \neq 0$ is (i) increasing (ii) decreasing.

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87. Evaluate: $\int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$

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88. Solve for x: $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x),\ x \neq \frac{\pi}{2}$

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89. Prove the following:

$$\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{x}{2},\ x\left(0, \frac{\pi}{4}\right)$$

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90. Using properties of determinants, prove the following

$$\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$$

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91. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular?

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92. Write the direction cosines of the vector $-2\hat{i}+\hat{j}-5\hat{k}$

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93. The length x of a rectangle is decreasing at the rate of 5 cm/minute and the width y is increasing at the rate of 4 cm/minute. When $x=8$ cm and $y=6$, find the rate of change of (a) the perimeter, (b) the area of the rectangle.

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94. Find the coordinates of the point where the line through the points A (3, 4, 1) and B (5, 1, 6) crosses the XY-plane.

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95. Two cards are drawn simultaneously (without replacement) from a well-shuffled pack of 52 cards. Find the mean and variance of the number of red cards.

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96. If the lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{k}=\frac{y-2}{1}-\frac{z-3}{5}$ are perpendicular, find the value of k and hence find the equation of plane containing these lines.

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97. If $y = 3\cos(\log x) + 4\sin(\log x)$, then show that $x^2 \frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = 0$

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98. If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

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99. Using principal value, evaluate the following: $\sin^{-1}\left(\sin\left(\frac{3\pi}{5}\right)\right)$

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100. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

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101. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$

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102. Write the position vector of the mid-point of the vector joining the points P (2, 3, 4) and Q (4, 1,-2).

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103. Write the principal value of $\tan^{-1}(-1)$

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104. Evaluate: $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

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105. Write a square matrix of order 2, which is both symmetric and skew symmetric.

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106. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then find the value of k if $|2A| = k\,|A|$

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107. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of the number of successes and hence find its mean.

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108. Prove that:

$$\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x, \quad -\frac{1}{\sqrt{2}} \le x \le 1$$

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109. Solve the following differential equation: $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$

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110. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$, what is the probability that the student knows the answer, given that he answered it correctly?

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111. An urn contains 4 white and 3 red balls. Let X be the number of red balls in a random draw of three balls. Find the mean and variance of X.

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112. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just guessing?

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113. Solve the following differential equation: $\cos^2 x \frac{dy}{dx} + y = \tan x$

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114. Solve the following differential equation: $x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$

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115. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point $P(1, 3, 3,)\cdot$

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116. Using properties of determinants, prove that

$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$$

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117. Write the direction cosines of a line parallel to z-axis.

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118. Evaluate: $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx\cdot$

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119. If $x=a(\theta-\sin\theta),\ y=a(1+\cos\theta),$ find $\frac{d^2y}{dx^2}$

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120. Show that the total surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

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121. Show that the function $f(x)=2x-|x|$ is continuous but not differentiable at $x=0$

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122. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}x,$ when $x\neq 0.$

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123. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

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124. Find the vector equation of the line passing through the point (1,2,3) and parallel to the planes $\overrightarrow{r}\hat{i} - \dot{\hat{j}} + 2\hat{k} = 5$ and $\overrightarrow{r}3\hat{i} + \dot{\hat{j}} + \hat{k} = 6.$

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125. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equation: $x - y + 2z = 1; 2y - 3z = 1; 3x - 2y + 4z = 2$

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126. Solve the following differential equation:

$e^{x}\tan y\,dx+(1-e^{x})\sec^{2}y\,dy=0$

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127. Consider the binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a*b=\min.\{a, b\}$. Write the operation table of the operation $*$.

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128. Using properties of determinants, prove that

$$\begin{vmatrix}-a^2 & ab & ac\\ ba & -b^2 & bc\\ ca & cb & -c^2\end{vmatrix}=4a^2b^2c^2$$

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129. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is atmost 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs. 300 and that on a chain is Rs 190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an L.P.P. and solve it graphically.

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130. A binary operation * on the set {0,1,2,3,4,5} is defined as: $a \cdot b = \{a + ba + b - 6\ \\ \\ \text{if}\ \ a + b < 6\ \\ \text{if}\, a + b \geq 6$ Show that zero is the identity for this operation and each element a of the set is invertible with 6a, being the inverse of a.

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131. If $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$, then find the value of x.

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132. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

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133. An open box with a square base is to be made out of a given quantity of card board of area c2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.

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134. Find the particular solution of the differential equation: $(1+e^{2x})dy+(1+y^2)e^x dx=0,$ given that $y=1,$ when $x=0.$

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135. Prove, using properties of determinants:

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

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136. Find the value of k so that the function f defined by $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x}, 3 \end{cases}$, $\text{if } x \neq \frac{\pi}{2} \text{if } x = \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$

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137. Prove that : $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}.$

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138. What is the principal value of $\cos^{-1}\cos\left(\frac{2\pi}{3}\right) + \sin^{-1}\sin\left(\frac{2\pi}{3}\right)$?

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139. Let $A = \{1, 2, 3\}, B = \{4, 5, 6, 7\}$and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one-one.

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140. If a matrix has 5 elements, write all possible orders it can have.

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141. Using properties of determinants, prove the following:

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1 - x^3\right)^2$$

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142. Differentiate the following function with respect to x:

$(\log x)^x + x^{\log x}$

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143. Probabilities of solving a specific problem independently by A and B are $\frac{1}{2}\ and \frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

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144. If the binary operation * on the set Z of integers is defined by $a \cdot b = a + b - 5$, then write the identity element for the operation * in Z.

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145. If I is the identity matrix and A is a square matrix such that $A^2 = A$, then what is the value of $(I + A)^2 - 3A$?

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146. Prove that the relation R in the set $A = \{5, 6, 7, 8, 9\}$ given by $R = \{(a, b) : |a - b|,$ is divisible by 2}, is an equivalence relation. Find all elements related to the element 6.

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147. Prove that $\tan^{-1}\left(\frac{\cos x}{1 + \sin x}\right) = \frac{\pi}{4} - \frac{x}{2}, \ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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148. If $C = 0.003x^3 + 0.02x^2 + 6x + 250$ gives the amount of carbon pollution in air in an area on the entry of x number of vehicles, then find the marginal carbon pollution in the air, when 3 vehicles have entered in the area and write which value does the question indicate.

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149. Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$

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150. Find the point on the curve $y=x^3-11x+5$ at which the equation of tangent is $y=x-11$

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151. Using differentials, find the approximate value of $\sqrt{49.5}$

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152. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$.

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153. If $y = \sin(\log x)$, then prove that $\frac{x^2 d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0$

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154. Find the value of k, for which $f(x) = \left\{\left\{\frac{\sqrt{1+kx} - \sqrt{1-kx}}{x^{\frac{2x+1}{x-1}}}, \quad \text{if}, \ -1 \leq x < 0 \text{if } 0 \leq x < 1\right.\right.$ is continuous at $x = 0$

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155. Find the area of the greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

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156. Find a unit vector in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

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157. If [[y+2x,5],[-x,3]]=[[7, 5],[-2 ,3]],$f \in dthevalueofy$\`

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158. Find the Cartesian equation of the plane passing through the points $A\ (0, 0, 0)$ and $b(3, -1, 2)$ and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

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159. On a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

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160. Let Z be the set of all integers and R be the relation on Z defined as $R = \{(a, b); a,\ b\ \in Z,\ \text{and } (a - b) \text{ is divisible by } 5\}$. Prove that R is an equivalence relation.

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161. If $\sin y = x \sin(a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

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162. If $A^T = [34 - 1201]$ and $B = [-121123]$, then find $A^T - B^T$

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163. In a game, a man wins rupees five for a six and loses rupee one for any other number, when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.

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164. Solve: $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

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165. Find the value of $\cos^{-1}\left(\frac{x}{2} + \frac{\sqrt{3 - 3x^2}}{2}\right)$

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166. The elements a_{ij} of a 3×3 matrix are given by $a_{ij} = \frac{1}{2}|-3i+j|\cdot$

Write the value of element a_{32}

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167. Write the value of $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

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168. Find $\int \frac{\sin^6 x}{\cos^8 x} dx\cdot$

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169. Find the unit vector perpendicular to plane ABC where the position vector of A,B and C are $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{k}$ respectively.

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170. Solve the differential equation $x\frac{dy}{dx}+y=x\cos x+\sin x,$ given $y\left(\frac{\pi}{2}\right)=1$

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171. Let $f, g: R-R$ be two functions defined as $f(x)=|x|+x$ and $g(x)=|x|-x$, for all xR. Then find fog and gof.

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172. Write the equation of the straight line through the point (α, β, γ) and parallel to z-axis.

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173. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. Find the rate at which the area increases, when the side is 10 cm.

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174. Find the particular solution of the differential equation $e^x\sqrt{1-y^2}dx+\frac{y}{x}dy=0,$ given that $y=1$ when $x=0$

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175. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

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176. Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls ? Given that (i) the youngest is a girl. (ii) atleast one is a girl.

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177. If $2[345x] + [1y01] = [70105]$, find $(x - y)$.

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178. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.

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179. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin} + \sqrt{1-\sin x}}{1\sqrt{1+\sin} - \sqrt{1-\sin x}}\right) = \frac{x}{2}; x \in \left(0, \frac{\pi}{4}\right)$.

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180. Using differentials, find the approximate value of $(3.968)^{\frac{3}{2}}$

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181. Write the derivative of $\sin x$ w.r.t. $\cos x$.

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182. Write the principal value of $\cos^{-1}[\cos(680^\circ)]$

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183. Write a $x2x$ matrix which is both symmetric and skew-symmetric.

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184. Write the value of $\cos^{-1}\left(-\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

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185. Let $R=\{a,a^3):a$ is a prime number less than $5\}$ be a relation. Find the range of R.

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186. Find the intervals in which the function $f(x)=\frac{3}{2}x^4-4x-45x+51$ is (a) strictly increasing. (b) strictly decreasing.

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187. Find the approximate value of $f(3.02)$, upto 2 places of decimal, where $f(x)=3x^2+5x+3$.

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188. Solve the differential equation $\frac{dy}{dx} + y \cot x = 2 \cos x$, given that $y = 0$, when $x = \frac{\pi}{2}$.

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189. Using properties of determinants, prove that

$$\begin{vmatrix} a+x & y & z \\ x & a+y & z \\ x & y & a+z \end{vmatrix} = a^2(a+x+y+z)$$

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190. $x = a \cos \theta + b \sin \theta$ and $y = a \sin \theta - b \cos \theta$, show that $y^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = 0$

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191. If $x^m y^n = (x+y)^{m+n}$, Prove that $\frac{dy}{dx} = \frac{y}{x}$.

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192. Using properties of determinants, solve for $x: \begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

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193. In the interval `2/pi

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194. Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right)$

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195. If $x = ae^t(\sin t + \cos t)$ and $y = ae^t(\sin t - \cos t)$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$.

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196. Two schools A and B decided to award prizes to their students for three values, team spirit, truthfulness and tolerance at the rate of $Rs\dot{x}$, $Rs\dot{y}$ and $Rs\dot{z}$ per student respectively. School A , decided to award a total of Rs. 1,100 for the three values to 3, 1 and 2 students respectively while school B decided to award Rs. 1,400 for the three values to 1, 2 and 3 students respectively. If one prize for all the three values together amount to Rs. 600 then (i) Represent the above situation by a matrix equation after forming linear equations. (ii) Is it possible to solve the system of equations so obtained using matrices ? (iii) Which value you prefer to be rewarded most and why ?

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197. If a line makes angles α, β, γ with the positive direction of coordinate axes, then write the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

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198. Find the area of the region in the first quadrant enclosed by the y-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$, using integration.

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199. Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

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200. If $x = a\sec^3\theta, y = a\tan^3\theta,$ find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$

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201. Solve : $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

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202. Prove that : $\frac{\tan^{-1}(63)}{16}=\frac{\sin^{-1}5}{13}+\frac{\cos^{-1}3}{5}$

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203. A tank with rectangular base and rectangular sides open at the top is to be constructed so that its depth is 3 m and volume is 75 m3. If building of tank costs Rs. 100 per square metre for the base and Rs. 50 per square metres for the sides, find the cost of least expensive tank.

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204. Let $f: N \rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow S$, where S is the range of f is invertible. Also find the inverse of f

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205. Write the sum of the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$.

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206. Find the sum of the intercepts cut off by the plane $2x + yz = 5$, on the coordinate axes.

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207. If

$$\tan^{-1}\left(\frac{1}{1+1.\ 2}\right)+\tan^{-1}\left(\frac{1}{1+2.\ 3}\right)+\ldots+\tan^{-1}\left(\frac{1}{1+n(n+1)}\right)=$$

then find the value of θ.

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208. Three machinesE1, E2 and E3 in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines E1 and E2 are defective and that 5% of those produced by machine E3are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

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209. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.

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210. Write the number of all possible matrices of order 2×2 with each entry $1, 2, \text{ or } 3.$

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211. An urn contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution.

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212. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at Rs. 7 profit and that of B at a profit of Rs. 4. Find the production level per day for maximum profit graphically.

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213. Matrix $A = [02b - 23133a3 - 1]$ is given to be symmetric, find values of a and b.

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214. If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

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215. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$

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216. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4R}{3}$. Also find maximum volume in terms of volume of the sphere.

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217. A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

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218. Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by $(x+y+z)$, and hence find the quotient.

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219. Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1 : 2 : 4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3 respectively. If the change does not take place, find the probability that it is due to the appointment of C.

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220. Prove that : $\frac{\tan^{-1} 1}{5} + \frac{\tan^{-1} 1}{7} + \frac{\tan^{-1} 1}{3} + \frac{\tan^{-1} 1}{8} = \frac{\pi}{4}$

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221. Find : $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta} d th\eta\theta.$

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222. Differentiate

$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) wrt \sin^{-1}\left(\frac{2x}{1+x^2}\right)\dot{o}fx(-1, 1)$

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223. If a, b and c are all non-zero and $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = 0,$ then prove that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + 1 = 0$

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224. A coaching institute of English (subject) conducts classes in two batches I and II and fees for rich and poor children are different. In batch I, it has 20 poor and 5 rich children and total monthly collection is Rs.

9,000, whereas in batch II, it has 5 poor and 25 rich children and total monthly collection is Rs. 26,000. Using matrix method, find monthly fees paid by each child of two types. What values the coaching institute is inculcating in the society ?

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225. If $A = (3579)$ is written as A = P + Q, where P is a symmetric matrix and Q is skew symmetric matrix, then write the matrix P.

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226. Using integration find the area of the region bounded by the curves $y = \sqrt{4 - x^2}, x^2 + y^2 - 4x = 0$ and the x-axis.

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227. If $f, g: R \rightarrow R$ be two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$. Find fog and gof. Hence find $fog(-3)$, $fog(5)$ and $gof(-2)$.

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228. The sum of the surface areas of a cuboid with sides x, $2x$ and $\frac{x}{3}$ and a sphere is given to be constant. Prove that the sum of their volumes is minimum, if x is equal to three times the radius of sphere. Also find the minimum value of the sum of their volumes.

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229. A box has 20 pens of which 2 are defective. Calculate the probability that out of 5 pens drawn one by one with replacement, at most 2 are defective.

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