





### MATHS

## NCERT - NCERT MATHEMATICS(ENGLISH)

## THREE DIMENSIONAL GEOMETRY

Exercise 11 2

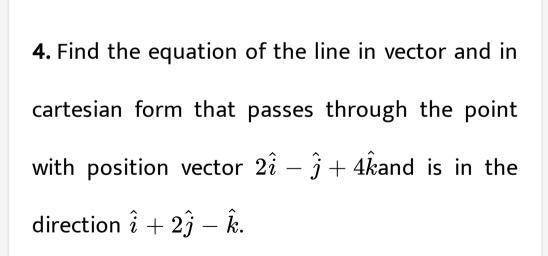
**1.** Show that the three lines with direction cosines

 $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}, \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.



**2.** Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (1, 2, 1), (1, 2, 5).

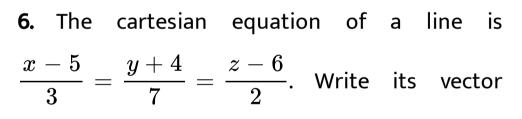
**3.** Show that the line through the points (1, 1, 2), (3, 4, 2) is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).



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5. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector  $3\hat{i} + 2\hat{j} - 2\hat{k}$ .

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form.

7. Find the cartesian equation of the line which

passes through the point (2, 4, 5) and parallel to

the line given by 
$$rac{x+3}{3}=rac{y-4}{5}=rac{z+8}{6}.$$



**8.** Find the vector and the cartesian equations of the line that passes through the points (3, 2, 5), (3, 2, 6).

**9.** Find the vector and the cartesian equations of the lines that passes through the origin and (5, 2, 3).



10. Find the values of p so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$$
 and  $\frac{7y-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$  are at right angles.

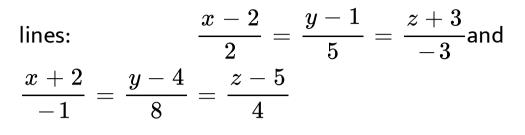
11. Show that the lines  $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  are perpendicular to each other.

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**12.** Find the angle between the following pairs of lines:(i)

$$egin{aligned} &
ightarrow r=2\hat{i}-5\hat{j}+\hat{k}+\lambda\Big(3\hat{i}+2\hat{j}+6\hat{k}\Big) ext{and} \ &
ightarrow r=7\hat{i}-6\hat{k}+\mu\Big(\hat{i}+2\hat{j}+2\hat{k}\Big) ext{(ii)} \ &
ightarrow r=3\hat{i}+\hat{j}-2\hat{k}+\lambda\Big(\hat{i}-\hat{j}-2\hat{k}\Big) ext{and`-} \end{aligned}$$

13. Find the angle between the following pair of

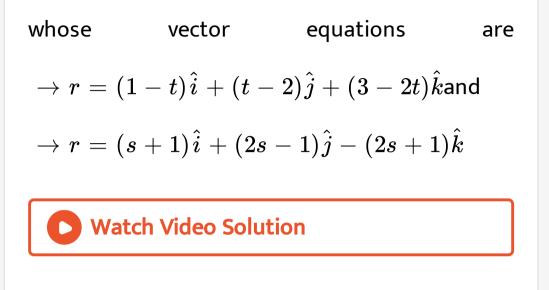




#### 14. Find the shortest distance between the lines

whose vector equations are
$$ightarrow r = ig(\hat{i}+2\hat{j}+3\hat{k}ig) + \lambdaig(\hat{i}-3\hat{j}+2\hat{k}ig)$$
and $ightarrow r = 4\hat{i}+5\hat{j}+6\hat{k}+\muig(2\hat{i}+3\hat{j}+\hat{k}ig).$ 

15. Find the shortest distance between the lines



16. Find the shortest distance between the lines

$$egin{aligned} &
ightarrow r = \left(\hat{i}+2\hat{j}+\hat{k}
ight)+\lambdaig(\hat{i}-\hat{j}+\hat{k}ig)$$
and  $&
ightarrow r = 2\hat{i}-\hat{j}-\hat{k}+\muig(2\hat{i}+\hat{j}+2\hat{k}ig) \end{aligned}$ 

17. Find the shortest distance between the lines

$$rac{x+1}{7} = rac{y+1}{-6} = rac{z+1}{1}$$
and $rac{x-3}{1} = rac{y-5}{-2} = rac{z-7}{1}$ 



#### Miscellaneous Exercise

1. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes  $\dot{j} + \hat{j} + \hat{k} = 5$ and  $\dot{j} + \hat{j} + \hat{k} = 6$ .

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2. Find the distance of the point (1, 5, 10) from the point of intersection of the line  $ightarrow r = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda \left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$  and the plane  $ightarrow r = \left(\hat{i} - \hat{j} + \hat{k}\right) = 5.$ 

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**3.** Find the equation of the plane passing through the intersection of the planes  $\rightarrow r\hat{i} + \dot{\hat{j}} + \hat{k} = 1$  and  $\rightarrow r2\hat{i} + 3\hat{j} - \hat{k} + 4 = 0$ and parallel to x-axis.



4. If the points (1, 1, p)and(3, 0, 1)be equidistant from the plane  $\overrightarrow{r} \cdot \left(3\hat{i} + 4\hat{j} - 12\hat{k}\right) + 13 = 0$ , then find the value of p.

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5. Find the equation the plane which contain the

line of intersection of the planes $ec{r}\,\hat{i}+2\dot{\hat{j}}+3\hat{k}-4=0$ and $ec{r}\,2\hat{i}+\dot{\hat{j}}-\hat{k}+5=0$ 

and which is perpendicular to the plane $ec{r}\left(5\hat{i}+3\hat{j}-6\hat{k}
ight)+8=0\,.$ 



**6.** If O be the origin and the coordinates of P be

(1, 2, -3), then find the equation of the plane

passing through P and perpendicular to OP.



**7.** Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.



**8.** Find the coordinate of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the i. yz-plane ii. zx-plane.

**9.** Find the equation of the plane passing through the point (1, 3, 2) and perpendicular to each of the planes x + 2y + 3z = 5 and 3x + 3y + z = 0.

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10. Find the coordinates of the point where the line through (3, 4, 5) and (2, 3, 1) crosses the plane2x + y + z = 7.

11. The planes 2x - y + 4z = 5 and

$$5x-2.5y+10z=6$$
 are



12. Find the equation of the plane passing through (a, b, c) and parallel to the plane  $ightarrow r\hat{i}+\dot{\hat{j}}+\hat{k}=2.$ 

13. Find the shortest distance between lines 
$$ightarrow r=6\hat{i}+2\hat{j}+\hat{k}+\lambda\Big(\hat{i}-2\hat{j}+2\hat{k}\Big)$$
and  $ightarrow r=-4\hat{i}-\hat{k}+\mu\Big(3\hat{i}-2\hat{j}-2\hat{k}\Big).$ 

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14. If the lines 
$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$$
 and  $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$  are perpendicular,

find the value of k.

A. 
$$-\frac{11}{19}$$
  
B.  $-\frac{8}{7}$ 

$$C_{-} = \frac{10}{7}$$

$$D_{-} = \frac{9}{7}$$
Answer: C
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#### 15. Find the vector equation of the line passing

through (1, 2, 3) and perpendicular to the plane

$$\overrightarrow{r}.\left(\hat{i}+2\hat{j}-5\hat{k}
ight)+9=0.$$

16. Find the equation of a line parallel to x-axis

and passing through the origin.



**17.** If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (4, 3, 6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

**18.** If  $l_1$ ,  $m_1$ ,  $n_1$  and  $l_2$ ,  $m_2$ ,  $n_2$  are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are  $m_1n_2 - m_2n_1$ ,  $n_1l_2 - n_2l_1$ ,  $l_1m_2 - l_2m_1$ .

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**19.** Find the angle between the lines whose direction ratios are a, b, c and b - c, c - a, a - b.

**20.** Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, -1), (4, 3, -1).

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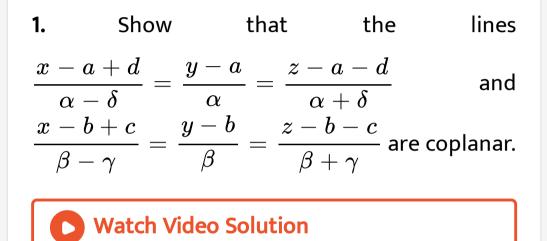
21. Find the vector equation of the line passing through the point (1, 2, -4) and perpendicular to the two lines:  $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$  and  $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ 

22. Prove that if a plane has the intercepts a, b, c and is at a distance of p units from theorigin, then  $\frac{1}{a^2} = \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$ .

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23. Distance between the two planes: 2x + 3y + 4z = 4 and 4x + 6y + 8z = 12 is (A) 2 units (B) 4 units (C) 8 units (D)  $\frac{2}{\sqrt{29}}$  units

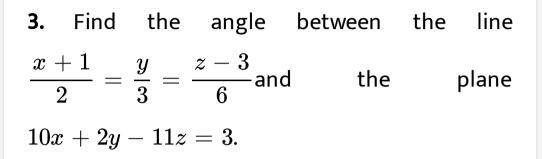
#### **Solved Examples**



**2.** Find the distance between the point P(6, 5, 9) and the plane determined by the points A(3, 1, 2), B(5, 2, 4) and C(1, 1, 6).



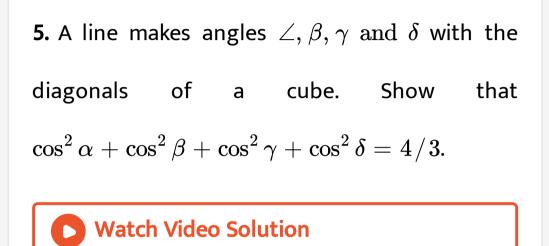




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**4.** Find the equation of the plane that contains the point (1, -1, 2) and is perpendicular to each of the planes 2x + 3y - 2z = 5 and x + 2y - 3z = 8.





6. Find the coordinates of the point where the

line through the points

A(3, 4, 1) and B(5, 1, 6) crosses the XY-plane.

7. Find the vector equation for the line passing

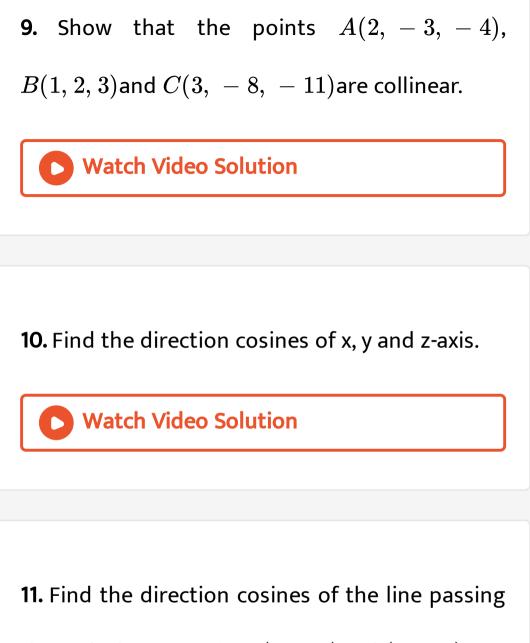
through the points (1, 0, 2) and (3, 4, 6).



8. Find the vector and the Cartesian equations of

the line through the point (5, 2, 4) and which is

parallel to the vector  $3\hat{i} + 2\hat{j} - 8\hat{k}$ .



through the two points (2, 4, 5) and (1, 2, 3).

12. If a line has direction ratios 2, 1, 2.determine

its direction cosines.

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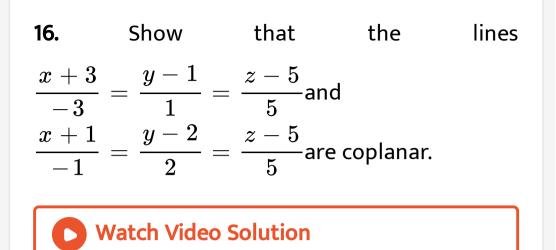
**13.** If a line makes angle  $90^{\circ}$ ,  $60^{\circ}$  and  $30^{\circ}$  with the positive direction of x, y and z-axis respectively, find its direction cosines.

14. Find the angle between the pair of lines  
given by 
$$\overrightarrow{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$$
  
and  $\overrightarrow{r} = 5\hat{i} - 2\hat{j} + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ .

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# 15. The Cartesian equation of a line is $rac{x+3}{2}=rac{y-5}{4}=rac{z+6}{2}.$ Find the vector

equation for the line.



## 17. Find the vector equation of the plane passing through the intersection of the planes $ightarrow r\hat{i} + \dot{\hat{j}} + \hat{k} = 6$ and $ightarrow r2\hat{i} + 3\hat{j} + 4\hat{k} = -5$ and the point (1, 1, 1).

**18.** Find the angle between the two planes 3x - 6y + 2z = 7 and 2x + 2y - 2z = 5. Watch Video Solution

19. Find the angle between the two planes 2x + y - 2z = 5 and 3x - 6y - 2z = 7 using vector method.

**20.** Find the distance of a point (2, 5, 3) from the

plane 
$$\overrightarrow{r}.\left(6\hat{i}-3\hat{j}+2\hat{k}
ight)=4.$$

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21. Find the angle between the pair of lines  $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$  and  $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}.$ 

22. Find the shortest distance between the lines 11 and 12 whose vector equations are  $\rightarrow r = \hat{i} + \hat{j} + \lambda \left(2\hat{i} - \hat{j} + \hat{k}\right)$  (1)and  $\rightarrow r = 2\hat{i} + \hat{j} - k + \mu \left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$  (2)

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**23.** Find the distance between the lines  $l_1$  and  $l_2$ 

given by 
$$\overrightarrow{r}=\hat{i}+2\hat{j}-4k+\lambda\Big(2\hat{i}+3\hat{j}+6\hat{k}\Big)$$
  
and  $\overrightarrow{r}=3\hat{i}+3\hat{j}-5k+\mu\Big(2\hat{i}+3\hat{j}+6\hat{k}\Big).$ 

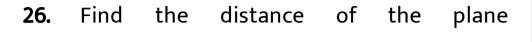
**24.** Find the vector equation of the plane which is at a distance of  $\frac{6}{\sqrt{29}}$  from the origin and its normal vector from the origin is  $2\hat{i} - 3\hat{j} + 4\hat{k}$ . Also, find its Cartesian form.

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25. Find the direction cosines of the unit vector

perpendicular to the plane $\dot{j} \rightarrow r6\hat{i} - 3\hat{j} - 2\hat{k} + 1 = 0$ passing through the

origin.



2x - 3y + 4z - 6 = 0 from the origin.

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27. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane 2x - 3y + 4z - 6 = 0.

**28.** Find the vector and cartesian equations of the plane which passes through the point (5, 2, 4) and perpendicular to the line with direction ratios (2, 3, 1).

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29. Find the vector equations of the plane passing through the points R(2, 5, -3), S(-2, -3, 5) and T(5, 3, -3).

**30.** Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z-axis respectively.

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### Exercise 11 3

**1.** Find the equation of the plane through the line of intersection of the planes x + y + z = 1 and 2x + 3y + 4z = 5, which is perpendicular to the plane x - y + z = 0.



2. Find the vector equation of the plane passing through the intersection of the planes  $\rightarrow r2\hat{i} + 2\hat{j} - 3\hat{k} = 7, \rightarrow r2\hat{i} + 5\hat{j} + 3\hat{k} = 9$ and through the point (2, 1, 3).

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**3.** In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between

them.(a)

7x + 5y + 6z + 30 = 0 and 3x - y - 10z + 4 = 0(b) 2x + y + 3z - 2 = 0 and x - 2y + 5z = 0(c) 2x - 2y + 4z + 5 = 0 and 3x - 3y + 6z - 1 = 0(d) 2x - y + 3z - 1 = 0 and 2x - y + 3z + 3 = 0(e) 4x + 8y + z - 3 = 0 and y + z - 4 = 0

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4. Find the angle between the planes whose vector equations are  $\overrightarrow{r}.\left(2\hat{i}+2\hat{j}-3\hat{k}
ight)=5$ 

and 
$$\overrightarrow{r}.\left(3\hat{i}-3\hat{j}+5\hat{k}
ight)=3.$$

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5. In the following cases, find the distance of each of the given points from the corresponding given plane. Point Plane(a) (0, 0, 0)3x - 4y + 12z = 3 (b) (3, -2, 1)2xy + 2z + 3 = 0 (c) (2, 3, -5)x + 2y - 2z = 9 (d)(-6, 0, 0)2x - 3y + 6z - 2 = 0

6. Find the equation of the plane with intercept

3 on the y-axils and parallel to ZOX plane.



7. Find the equation of the plane through the intersection of the planes 3x - y + 2z - 4 = 0 and x + y + z - 2 = 0 and the point (2, 2, 1).

**8.** Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector  $3\hat{i} + 5\hat{j} - 6\hat{k}$ .

A.  $6\sqrt{70}$ B.  $5\sqrt{70}$ C.  $8\sqrt{70}$ 

D.  $7\sqrt{70}$ 

#### Answer: D



9. Find the Cartesian equation of the following planes: (a)  $\rightarrow r\hat{i} + \dot{\hat{j}} - \hat{k} = 2$ (b)  $\rightarrow r2\hat{i} + 3\hat{j} - 4\hat{k} = 1$ (c)  $\rightarrow r(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k} = 15$ Watch Video Solution

10. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.(a) z = 2 (b) x + y + z = 1 (c) 2x + 3yz = 5(d) 5y + 8 = 0

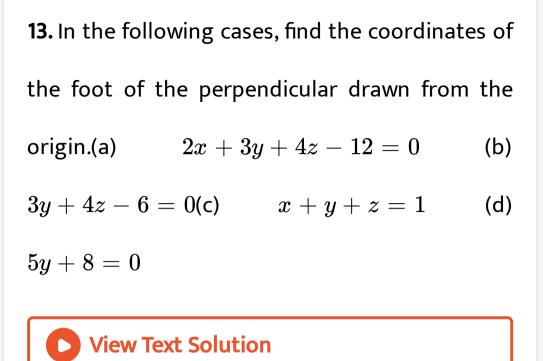
**11.** Find the equations of the planes that passes

through three points.(a)(1,1,-1),(6,4,-5),(-4,-2,3)



# **12.** Find the intercepts cut off by the plane 2x + y - z = 5.





14. Find the vector and cartesian equations of the planes(a) that passes through the point (1,0,-2)and the normal to the plane is  $\hat{i}+\hat{j}-\hat{k}$ (b) that passes through the point (1,4, 6) and the normal vector to the plane is  $\hat{i}-2\hat{j}+\hat{k}$ 



**1.** Find the direction cosines of a line which makes equal angles with the coordinate axes.



**2.** Show that the points(2, 3, 4), (-1, -2, 1), (5, 8, 7) are collinear. $\bigcirc$  Watch Video Solution

**3.** Find the direction cosines of the sides of the triangle whose vertices are (3, 5, 4), (1, 1, 2) and (5, 5, 2).

**4.** If a line makes angles  $90^{\circ}$ ,  $135^{\circ}$ ,  $45^{\circ}$  with the x, y and z-axes respectively, find its direction cosines.



# 5. If a line has the direction ratios 18, 12, 4, then

what are its direction cosines?