MATHS

NCERT - NCERT MATHEMATICS(ENGLISH)

THREE DIMENSIONAL GEOMETRY

Exercise 11 2

1. Show that the three lines with direction cosines

$\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}, \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

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2. Show that the line through the points $(4, 7, 8), (2, 3, 4)$is parallel to the line through the points $(1, 2, 1),(1, 2, 5)$.

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3. Show that the line through the points $(1, 1, 2), (3, 4, 2)$is perpendicular to the line through the points (0, 3, 2) and (3, 5, 6).

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4. Find the equation of the line in vector and in cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$and is in the direction $\hat{i} + 2\hat{j} - \hat{k}$.

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5. Find the equation of the line which passes through the point (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$.

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6. The cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form.

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7. Find the cartesian equation of the line which passes through the point $(2, 4, 5)$and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

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8. Find the vector and the cartesian equations of the line that passes through the points $(3, 2, 5), (3, 2, 6)$.

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9. Find the vector and the cartesian equations of the lines that passes through the origin and $(5, 2, 3)$.

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10. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

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11. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

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12. Find the angle between the following pairs of lines:(i)

$\rightarrow r = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and

$\rightarrow r = 7\hat{i} - 6\hat{k} + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ (ii)

$\rightarrow r = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda\left(\hat{i} - \hat{j} - 2\hat{k}\right)$ and `-

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13. Find the angle between the following pair of lines: $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ and $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$

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14. Find the shortest distance between the lines whose vector equations are $\rightarrow r=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$ and $\rightarrow r=4\hat{i}+5\hat{j}+6\hat{k}+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right).$

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15. Find the shortest distance between the lines whose vector equations are

$\rightarrow r = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and

$\rightarrow r = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

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16. Find the shortest distance between the lines

$\rightarrow r = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and

$\rightarrow r = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

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17. Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{and}$$
$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

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Miscellaneous Exercise

1. Find the vector equation of the line passing through (1, 2, 3) and parallel to the planes $\rightarrow r\hat{i} - \hat{j} + 2\hat{k} = 5 \text{and} \rightarrow r3\hat{i} + \hat{j} + \hat{k} = 6.$

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2. Find the distance of the point $(1, 5, 10)$from the point of intersection of the line $\rightarrow r = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$and the plane $\rightarrow r = \left(\hat{i} - \hat{j} + \hat{k}\right) = 5.$

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3. Find the equation of the plane passing through the intersection of the planes $\rightarrow r\hat{i} + \dot{\hat{j}} + \hat{k} = 1\, and\ \rightarrow r2\hat{i} + 3\dot{\hat{j}} - \hat{k} + 4 = 0$ and parallel to x-axis.

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4. If the points $(1, 1, p) and (3, 0, 1)$ be equidistant from the plane $\vec{r} \cdot \left(3\hat{i} + 4\hat{j} - 12\hat{k}\right) + 13 = 0$, then find the value of p.

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5. Find the equation the plane which contain the line of intersection of the planes $\vec{r}\,\hat{i} + 2\dot{\hat{j}} + 3\hat{k} - 4 = 0 and \vec{r}\,2\hat{i} + \dot{\hat{j}} - \hat{k} + 5 = 0$

and which is perpendicular to the plane $\vec{r}\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.

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6. If O be the origin and the coordinates of P be $(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

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7. Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.

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8. Find the coordinate of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the i. yz-plane ii. zx-plane.

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9. Find the equation of the plane passing through the point $(1, 3, 2)$and perpendicular to each of the planes $x + 2y + 3z = 5$and $3x + 3y + z = 0$.

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10. Find the coordinates of the point where the line through $(3, 4, 5)$and $(2, 3, 1)$crosses the plane$2x + y + z = 7$.

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11. The planes $2x - y + 4z = 5$ and

$5x - 2.5y + 10z = 6$ are

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12. Find the equation of the plane passing through (a, b, c) and parallel to the plane

$\rightarrow r\hat{i} + \dot{\hat{j}} + \hat{k} = 2.$

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13. Find the shortest distance between lines $\rightarrow r = 6\hat{i} + 2\hat{j} + \hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ and $\rightarrow r = -4\hat{i} - \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right).$

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14. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.

A. $-\frac{11}{19}$

B. $-\frac{8}{7}$

C. $-\frac{10}{7}$

D. $-\frac{9}{7}$

Answer: C

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15. Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}.\left(\hat{i}+2\hat{j}-5\hat{k}\right)+9=0.$

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16. Find the equation of a line parallel to x-axis and passing through the origin.

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17. If the coordinates of the points A, B, C, D be $(1, 2, 3)$, $(4, 5, 7)$, $(4, 3, 6)$and $(2, 9, 2)$ respectively, then find the angle between the lines AB and CD.

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18. If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1, n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$.

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19. Find the angle between the lines whose direction ratios are a, b, c and $b - c, c - a, a - b.$

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20. Show that the line joining the origin to the point $(2, 1, 1)$is perpendicular to the line determined by the points$(3, 5, -1)$,$(4, 3, -1)$.

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21. Find the vector equation of the line passing through the point $(1, 2, -4)$and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

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22. Prove that if a plane has the intercepts $a,\ b,\ c\ and$ is at a distance of p units from theorigin, then $\frac{1}{a^2}=\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

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23. Distance between the two planes: $2x+3y+4z=4$and $4x+6y+8z=12$is(A) 2 units (B) 4 units (C) 8 units (D) $\frac{2}{\sqrt{29}}$units

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Solved Examples

1. Show that the lines $\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$ are coplanar.

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2. Find the distance between the point $P(6,5,9)$ and the plane determined by the points $A(3,1,2), B(5,2,4)$ and $C(1,1,6)$.

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3. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3.$

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4. Find the equation of the plane that contains the point $(1, -1, 2)$ and is perpendicular to each of the planes $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8.$

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5. A line makes angles $\angle, \beta, \gamma$ and δ with the diagonals of a cube. Show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3.$

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6. Find the coordinates of the point where the line through the points $A(3,\ 4,\ 1) and\ B(5,\ 1,\ 6)$ crosses the XY-plane.

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7. Find the vector equation for the line passing through the points $(1, 0, 2)$and $(3, 4, 6)$.

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8. Find the vector and the Cartesian equations of the line through the point $(5, 2, 4)$and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

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9. Show that the points $A(2, -3, -4)$, $B(1, 2, 3)$ and $C(3, -8, -11)$ are collinear.

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10. Find the direction cosines of x, y and z-axis.

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11. Find the direction cosines of the line passing through the two points $(2, 4, 5)$ and $(1, 2, 3)$.

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12. If a line has direction ratios $2, 1, 2$.determine its direction cosines.

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13. If a line makes angle 90°, 60° and 30° with the positive direction of x, y and z-axis respectively, find its direction cosines.

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14. Find the angle between the pair of lines given by $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$.

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15. The Cartesian equation of a line is $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$. Find the vector equation for the line.

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16. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar.

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17. Find the vector equation of the plane passing through the intersection of the planes $\rightarrow r\hat{i} + \hat{j} + \hat{k} = 6$ and $\rightarrow r2\hat{i} + 3\hat{j} + 4\hat{k} = -5$ and the point (1, 1, 1).

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18. Find the angle between the two planes $3x - 6y + 2z = 7$and $2x + 2y - 2z = 5.$

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19. Find the angle between the two planes $2x + y - 2z = 5$and $3x - 6y - 2z = 7$using vector method.

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20. Find the distance of a point $(2, 5, 3)$ from the plane $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$.

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21. Find the angle between the pair of lines $\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$.

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22. Find the shortest distance between the lines l1 and l2 whose vector equations are

$\rightarrow r = \hat{i} + \hat{j} + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right)$ (1)and

$\rightarrow r = 2\hat{i} + \hat{j} - k + \mu\left(3\hat{i} - 5\hat{j} + 2\hat{k}\right)$ (2)

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23. Find the distance between the lines l_1 and l_2 given by $\overrightarrow{r} = \hat{i} + 2\hat{j} - 4k + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ and $\overrightarrow{r} = 3\hat{i} + 3\hat{j} - 5k + \mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$.

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24. Find the vector equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from the origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also, find its Cartesian form.

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25. Find the direction cosines of the unit vector perpendicular to the plane $\rightarrow r6\hat{i} - 3\dot{\hat{j}} - 2\hat{k} + 1 = 0$passing through the origin.

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26. Find the distance of the plane $2x - 3y + 4z - 6 = 0$from the origin.

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27. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$.

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28. Find the vector and cartesian equations of the plane which passes through the point $(5, 2, 4)$and perpendicular to the line with direction ratios $(2, 3, 1)$.

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29. Find the vector equations of the plane passing through the points $R(2, 5, -3)$, $S(-2, -3, 5)$and $T(5, 3, -3)$.

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30. Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z-axis respectively.

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Exercise 11 3

1. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$, which is perpendicular to the plane $x - y + z = 0$.

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2. Find the vector equation of the plane passing through the intersection of the planes $\rightarrow r2\hat{i} + 2\dot{\hat{j}} - 3\hat{k} = 7, \rightarrow r2\hat{i} + 5\dot{\hat{j}} + 3\hat{k} = 9$ and through the point (2, 1, 3).

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3. In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find the angles between

them.(a)

$7x + 5y + 6z + 30 = 0$ and $3x - y - 10z + 4 = 0$

(b) $2x + y + 3z - 2 = 0$ and $x - 2y + 5z = 0$

(c)

$2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$

(d)

$2x - y + 3z - 1 = 0$ and $2x - y + 3z + 3 = 0$

(e)$4x + 8y + z - 3 = 0$ and $y + z - 4 = 0$

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4. Find the angle between the planes whose vector equations are $\vec{r}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) = 5$

and $\vec{r} \cdot \left(3\hat{i} - 3\hat{j} + 5\hat{k}\right) = 3.$

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5. In the following cases, find the distance of each of the given points from the corresponding given plane. Point Plane(a) $(0, 0, 0)$ $3x - 4y + 12z = 3$ (b) $(3, -2, 1)$ $2xy + 2z + 3 = 0$ (c) $(2, 3, -5)$ $x + 2y - 2z = 9$ (d)$(-6, 0, 0)$ $2x - 3y + 6z - 2 = 0$

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6. Find the equation of the plane with intercept 3 on the y-axils and parallel to ZOX plane.

7. Find the equation of the plane through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$and the point (2, 2, 1).

8. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$.

A. $6\sqrt{70}$

B. $5\sqrt{70}$

C. $8\sqrt{70}$

D. $7\sqrt{70}$

Answer: D

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9. Find the Cartesian equation of the following planes: (a) $\rightarrow r\hat{i} + \hat{j} - \hat{k} = 2$(b) $\rightarrow r2\hat{i} + 3\hat{j} - 4\hat{k} = 1$(c) $\rightarrow r(s - 2t)\hat{i} + (3 - t)\hat{j} + (2s + t)\hat{k} = 15$

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10. In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.(a) $z = 2$ (b) $x + y + z = 1$ (c) $2x + 3yz = 5$(d) $5y + 8 = 0$

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11. Find the equations of the planes that passes through three points.(a)(1,1,-1),(6,4,-5),(-4,-2,3)

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12. Find the intercepts cut off by the plane $2x + y - z = 5.$

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13. In the following cases, find the coordinates of the foot of the perpendicular drawn from the origin.(a) $2x + 3y + 4z - 12 = 0$ (b) $3y + 4z - 6 = 0$(c) $x + y + z = 1$ (d) $5y + 8 = 0$

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14. Find the vector and cartesian equations of the planes(a) that passes through the point $(1, 0, -2)$and the normal to the plane is $\hat{i} + \hat{j} - \hat{k}$(b) that passes through the point (1,4,

6) and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$

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Exercise 11 1

1. Find the direction cosines of a line which makes equal angles with the coordinate axes.

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2. Show that the points $(2, 3, 4), (-1, -2, 1), (5, 8, 7)$ are collinear.

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3. Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, 4), (1, 1, 2)$ and $(5, 5, 2)$.

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4. If a line makes angles 90^o, 135^o, 45^o with the x, y and z-axes respectively, find its direction cosines.

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5. If a line has the direction ratios $18, 12, 4$, then what are its direction cosines?

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