MATHS

BOOKS - RD SHARMA MATHS (HINGLISH)

GEOMETRIC PROGRESSIONS

Solved Examples And Exercises

1. Find the sum of $2n$ terms of the series whose every even term is $'a'$ times the term before it and every odd term is $'c'$ times the term before it, the first term being unity.

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2. Let a_n be the nth term of a G.P. of positive numbers. Let $\sum_{n=1}^{100} a_{2n} = \alpha$ and $\sum_{n=1}^{100} a_{2n-1} = \beta$, such that $\alpha \neq \beta$, then the common ratio is α/β b.

β/α c. $\sqrt{\alpha/\beta}$ d. $\sqrt{\beta/\alpha}$

3. The fifth term of a G.P. is 81 whereas its second term is 24. Find the series and sum of its first eight terms.

4. Find the sum: $\displaystyle\sum_{n=1}^{10}\left\{\left(\frac{1}{2}\right)^{n-1}+\left(\frac{1}{5}\right)^{n+1}\right\}$.

5. If $S_1, S_2, , S_n$ are the sum of n term of n G.P., whose first term is 1 in each and common ratios are $1, 2, 3, ..., n$ respectively, then prove that

$$S_1 + S_2 + 2S_3 + 3S_4 + (n-1)S_n = 1^n + 2^n + 3^n + + n^n.$$

6. If S_1, S_2, S_3 be respectively the sums of n, $2n$, $3n$ terms of a G.P., then prove that $(S_1)^2 + (S_2)^2 = S_1(S_2 + S_3)$.

7. Find the least value of n for which the sum $1 + 3 + 3^2 + \rightarrow n$ terms is greater than 7000.

8. Prove that the sum of n terms of the series:
$11 + 103 + 1005 + is\dfrac{10}{9}(10^n - 1) + n^2.$

9. How many terms of the sequence $\sqrt{3}, 3, 3\sqrt{3}$ must be taken to make the sum $39 + 13\sqrt{3}$?
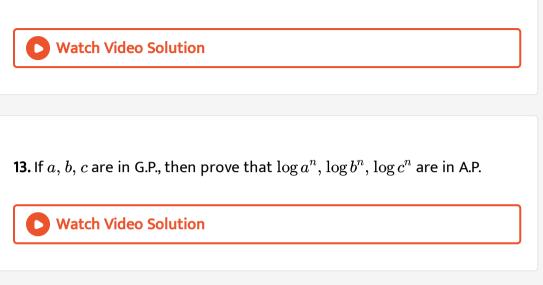
A. 3

B. 6

C. 9

D. not possible

Answer: B

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10. How many terms of the G.P. $3, 3/2, 3/4,$ be taken together to make $\dfrac{3069}{512}$?

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11. If a, b, c, d are in A.P. and x, y, z are in G.P., then show that $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1.$

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12. If a, b, c, d are in G.P., prove that $a + b, b + c, c + d$ are also in G.P.

13. If a, b, c are in G.P., then prove that $\log a^n$, $\log b^n$, $\log c^n$ are in A.P.

14. If p, q, r are in A.P., show that the pth, qth and rth terms of any G.P. are in G.P.

15. If S denotes the sum of an infinite G.P. adn S_1 denotes the sum of the squares of its terms, then prove that the first term and common ratio are respectively $\dfrac{2SS_1}{s^2 + S_1}$ and $\dfrac{S^2 - S_1}{S^2 + S_1}$.

16. The sum of first two terms of an infinite G.P. is 5 and each term is three times the sum of the succeeding terms. Find the G.P.
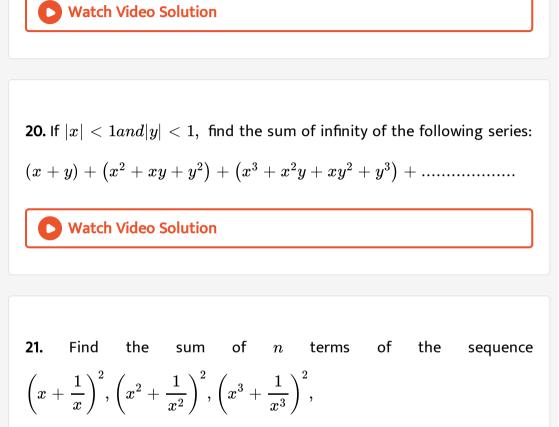
17. One side of an equilateral triangle is 18 cm. The mid-point of its sides are joined to form another triangle whose mid-points, in turn, are joined to form still another triangle. The process is continued indefinitely. Find the sum of the (i) perimeters of all the triangles. (ii) areas of all triangles.

18. If S_p denotes the sum of the series $1 + r^p + r^{2p} + \to \infty$ and S_{2p} the sum of the series $1 - r^p + r^{2p} - \to \infty$, prove that $S_+ S_{2p} = 2S_{2p}$.

19. If each term of an infinite G.P. is twice the sum of the terms following it, then find the common ratio of the G.P.

20. If $|x| < 1 and |y| < 1$, find the sum of infinity of the following series:

$$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots\ldots\ldots$$

21. Find the sum of n terms of the sequence

$$\left(x + \frac{1}{x}\right)^2, \left(x^2 + \frac{1}{x^2}\right)^2, \left(x^3 + \frac{1}{x^3}\right)^2,$$

22. Find the sum to n terms of the sequence given by

$$a_n = \sum 2^n, n \in N.$$

23. Sum the series : $x(x+y) + x^2(x^2+y^2) + x^3(x^3+y^3) + \rightarrow n$ terms.

24. Find the sum of the following series : $0.7 + 0.77 + 0.777 + \rightarrow n$ terms

25. The product of three numbers in G.P. is 216. If 2, 8, 6, be added to them, the results are in A.P. find the numbers.

26. Find the sum of 10 terms of the G.P. 3, 6, 12,...

27. In an increasing G.P. , the sum of the first and the last term is 66, the product of the second and the last but one is 128 and the sum of the terms is 126. How many terms are there in the progression?

28. If S_1, $S_2 and S_3$ be respectively the sum of n, 2n and 3n terms of a G.P., prove that $S_1(S_3 - S_2) = (S_2 - S_1)^2$

29. Determine the number of terms in G.P. if $a_1 = 3, a_n = 96$ and $S_n = 189$.

30. How many terms of the geometric series $1 + 4 + 16 + 64 + $ will make the sum 5461 ?

31. Find the sum of an infinitely decreasing G.P. whose first term is equal to $b + 2$ and the common ratio to $2/c$, where b is the least of the product of the roots of the equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ and c is the greatest value of the sum of its roots.

32. If

`x=sum_(n=0)^oocos^(2n)theta,y=sum_(n=0)^oosin^(2n)varphi,z=sum_(n=0)^oo

h e r e0

33. The sum of an infinite G.P. is 57 and the sum of their cubes is 9457, find the G.P.

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34. Which is the rational number having the decimal expansion $0.3\overline{56}$?

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35. A square is drawn by joining the mid-points of the sides of a square. A third square is drawn inside the second square in the same way and the processes continued indefinitely. If the side of the square is $10 cm$, find the sum of the areas of all the squares so formed.

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36. After striking a floor a certain ball rebounds $\left(\dfrac{4}{5}\right)^{th}$ of the height from which it has fallen. Find the total distance that it travels before coming to rest, if it is gently dropped from a height of $120 metres.$

37. Sum the following geometric series to infinity:

$(a)\left(\sqrt{2}+1\right)+1+\left(\sqrt{2}-1\right)\ldots\ldots\ldots+\infty$

$(b)\dfrac{1}{2}+\dfrac{1}{3^3}+\dfrac{1}{2^3}+\dfrac{1}{3^4}+\dfrac{1}{2^5}+\dfrac{1}{3^6}\ldots\ldots\ldots+\infty$

38. If

$x = a + \dfrac{a}{r} + \dfrac{a}{r^2} + \infty, y = b - \dfrac{b}{r} + \dfrac{b}{r^2} + \infty, and z = c + \dfrac{c}{r^2} + \dfrac{c}{r^4} + \infty$

prove that $\dfrac{xy}{z} = \dfrac{ab}{.}$

39.

$x = 1 + a + a^2 + \infty, where |a| < 1 and y = 1 + b + b^2 + \infty, where |b| < 1$

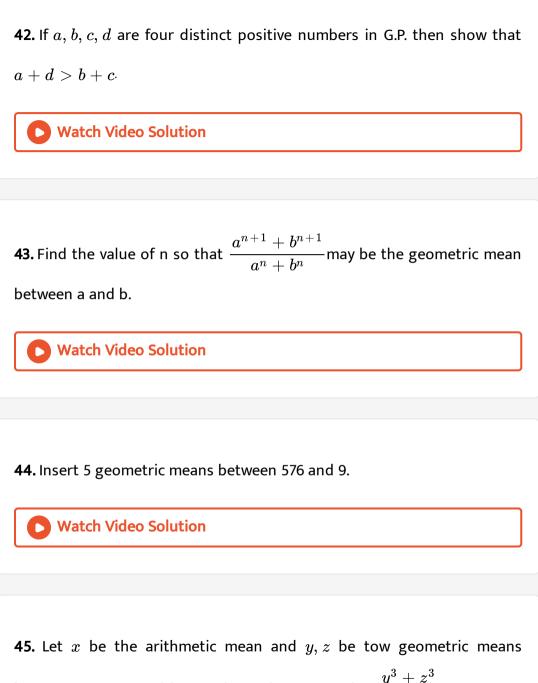prove that: $1 + ab + a^2 b^2 + \infty = \dfrac{xy}{x + y - 1}$

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40. If $A = 1 + r^a + r^{2a} + $ to ∞ and $B = 1 + r^b + r^{2b} + \infty$, prove that

$r = \left(\dfrac{A - 1}{A} \right)^{1/a} = \left(\dfrac{B - 1}{B} \right)^{1/b}$

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41. Prove that the product n geometric means between two quantities is

equal to the nth power of a geometric mean of those two quantities.

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42. If a, b, c, d are four distinct positive numbers in G.P. then show that $a + d > b + c.$

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43. Find the value of n so that $\dfrac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the geometric mean between a and b.

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44. Insert 5 geometric means between 576 and 9.

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45. Let x be the arithmetic mean and y, z be tow geometric means between any two positive numbers. Then, prove that $\dfrac{y^3 + z^3}{xyz} = 2.$

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46. Find two positive numbers whose difference is 12 an whose A.M. exceeds the G.M. by 2.
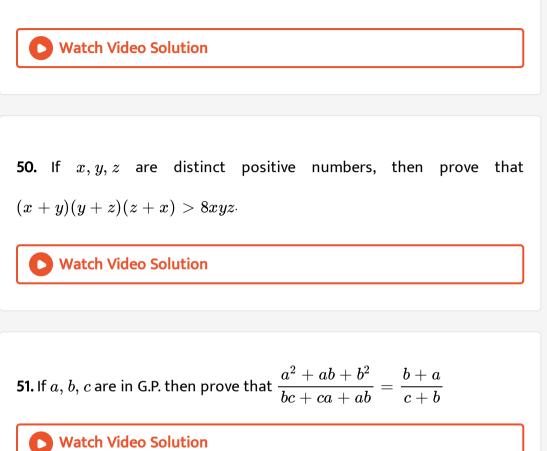
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47. If one geometric mean G and two arithmetic means $A_1 and A_2$ be inserted between two given quantities, prove that $G^2 = (2A_1 - A_2)(2A_2 - A_1) \cdot$

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48. If a is the A.M. of $b and c$ and the two geometric means are $G_1 and G_2$, then prove that $G_1^3 + G_2^3 = 2abc$

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49. If $x \in R$, find the minimum value of the expression $3^x + 3^{1-x}$.

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50. If x, y, z are distinct positive numbers, then prove that $(x + y)(y + z)(z + x) > 8xyz$.

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51. If a, b, c are in G.P. then prove that $\dfrac{a^2 + ab + b^2}{bc + ca + ab} = \dfrac{b + a}{c + b}$

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52. If $\dfrac{1}{a + b}, \dfrac{1}{2b}, \dfrac{1}{b + c}$ are three consecutive terms of an A.P., prove that a, b, c are the three consecutive terms of a G.P.

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53. If a, b, c, d are in G.P. prove that: $\dfrac{ab - cd}{b^2 - c^2} = \dfrac{a + c}{b}$

54. If a, b, c, d are in G.P. prove that: $(a^2 + b^2)$, $(b^2 + c^2)$, $(c^2 + d^2)$ are in G.P. $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in G.P. $\dfrac{1}{a^2 + b^2}$, $\dfrac{1}{b^2 + c^2}$, $\dfrac{1}{c^2 + d^2}$ are in G.P. $(a^2 + b^2 + c^2)$, $(ab + bc + cd)$, $(b^2 + c^2 + d^2)$

55. The sum of three numbers a, b, c in A.P. is 18 . If a and b are each increased by $4 and c$ is increased by $36,$ the new numbers form a G.P. Find a ,b ,c

56. If a, b, c are in G.P., prove that: 1) $a(b^2 + c^2) = c(a^2 + b^2)$

, 2) $A^2 b^2 c^2 \left(\dfrac{1}{a^3} + \dfrac{1}{b^3} + \dfrac{1}{c^3} \right) = a^3 + b^3 + c^3$

, 3) $\dfrac{(a + b + c)^2}{a^2 + b^2 + c^2} = \dfrac{a + b + c}{a - b + c}$

, 4) $\dfrac{1}{a^2 - b^2} + \dfrac{1}{b^2} = \dfrac{1}{b^2 - c^2}$

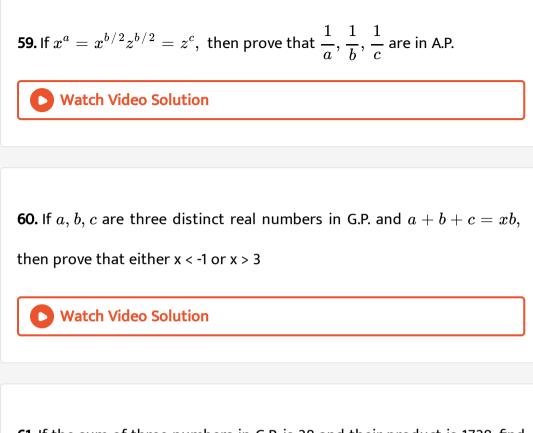,5) $(a + 2b = 2c)(a - 2b + 2c) = a^2 + 4c^2.$

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57. If mth, nth and pth terms of a G.P. form three consecutive terms of a G.P. Prove that $m, n,$ and p form three consecutive terms of an arithmetic system.

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58. If a, b, c are in G.P. and x, y are the arithmetic means of $a, b and b, c$ respectively, then prove that $\dfrac{a}{x} + \dfrac{c}{y} = 2 and \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{2}{b}$

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59. If $x^a = x^{b/2} z^{b/2} = z^c$, then prove that $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in A.P.

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60. If a, b, c are three distinct real numbers in G.P. and $a + b + c = xb$, then prove that either x < -1 or x > 3

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61. If the sum of three numbers in G.P. is 38 and their product is 1728, find them.

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62. If the pth and q^{th} terms of a G.P. are q and p respectively, show that $(p + q)^{th}$ term is $\left(\dfrac{q^p}{p^q} \right)^{\frac{1}{p-q}}$.

63. In a finite G.P. the product of the terms equidistant from the beginning and the end is always same and equal to the product of first and last term.

64. The product of first three terms of a G.P. is 1000. If 6 added to its second term and 7 added to its third term, the terms become in A.P. Find the G.P.

65. Find three numbers in G.P. whose sum is 52 and the sum of whose products pairs is 624.

A. 2, 12, 32

B. 2,12, 72

C. 4, 12, 36

D. None of these

Answer: C

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66. Three numbers are in G.P. whose sum is 70. If the extremes be each multiplied by 4 and the means by 5, they will be in A.P. Find the numbers.

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67. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.
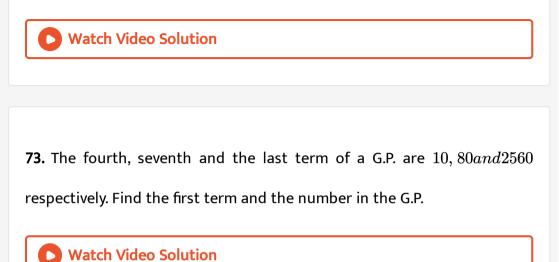
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68. Find four numbers in G.P. whose sum is 85 and product is 4096.

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69. Find three numbers in G.P. whose sum is 13 and the sum of whose squares is 91.

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70. Find $4th$ term from the end of the G.P. 3, 6, 12, 24, , 3072.

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71. Which term of the G.P. 2, $\frac{1}{2}$, $\frac{1}{4}$, is $\frac{1}{128}$?

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72. Which term of the G.P. 5, 10, 20, 40, $is 5120$?

73. The fourth, seventh and the last term of a G.P. are $10, 80 and 2560$ respectively. Find the first term and the number in the G.P.

74. If a, b, c are respectively the $p^{th}, q^{th} and r^{th}$ terms of a G.P. show that $(q - r)\log a + (r - p)\log b + (p - q)\log c = 0.$

75. In a G.P. of positive terms if any terms is equal to the sum of next tow terms, find the common ratio of the G.P.

76. The $(m+n)th$ and $(m-n)th$ terms fa G.P. ae p and q respectively. Show that the mth and nth terms are $\sqrt{pq}$ and $p\left(\dfrac{q}{p}\right)^{m/2n}$ respectively.

77. If the pth ,q^{th} and r^{th} terms of a GP are a, b and c, respectively. Prove that $a^{q-r}b^{r-p}c^{p-q} = 1$.

78. Find all sequences which are simultaneously A.P. and G.P.

79. Let S be the sum, P be the product and R be the sum of the reciprocals of 3 terms of a G.P. then $P^2R^3 : S^3$ is equal to

(a) $1:1$

(b) $(\text{common ratio})^n : 1$

(c) $(\text{First term})^2 (\text{common ratio})^2$

(d) None of these

80. If the A.M. of two positive numbers a and $b (a > b)$ is twice their geometric mean. Prove that : $a : b = (2 + \sqrt{3}) : (2 - \sqrt{3})$.

81. If one A.M., A and tow geometric means G_1 and G_2 inserted between any two positive numbers, show that $\dfrac{G1^2}{G_2} + \dfrac{G2^2}{G_1} = 2A$.

82. If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$, then xyz are in AP (b) GP (c) HP (d) none of these

83. If A be one A.M. and p, q be two G.M. $'s$ between two numbers, then $2A$ is equal to (a) $\dfrac{p^3 + q^3}{pq}$ (b) $\dfrac{p^3 - q^3}{pq}$ (c) $\dfrac{p^2 + q^2}{2}$ (d) $\dfrac{pq}{2}$

84. If p, q be two A.M. $'s$ and G be one G.M. between two numbers, then $G^2 =$ a)$(2p - q)(p - 2q)$ (b) $(2p - q)(2q - p)$ c)$(2p - q)(p + 2q)$ (d) none of these

85. If x is positive, the sum to infinity of the series

$$\frac{1}{1+x} - \frac{1-x}{(1+x)^2} + \frac{(1-x)^2}{(1+x)^3} - \frac{(1-x)^3}{(1+x)^4} + is \ 1/2 \ \text{(b)} \ 3/4 \ \text{(c)} \ 1 \ \text{(d)}$$

none of these

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86. If $\left(4^3\right)\left(4^6\right)\left(4^9\right)\left(4^{12}\right)\left(4^{3x}\right) = (0.\,0625)^{-54}$, the value of x is 7 (b) 8 (c) 9 (d) 10

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87. The sequence 4,12,36,108, is a G.P., because $\dfrac{12}{4} = \dfrac{36}{12} = \dfrac{108}{36} = \ = 3$, which is constant.

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88. The sequence $\dfrac{1}{3}, -\dfrac{1}{2}, \dfrac{3}{4}, -\dfrac{9}{8}$ is a G.P. with first term $\dfrac{1}{3}$ and common ratio equal to $\left(-\dfrac{1}{2}\right) \div \left(\dfrac{1}{3} = -\dfrac{3}{2}\right)$.

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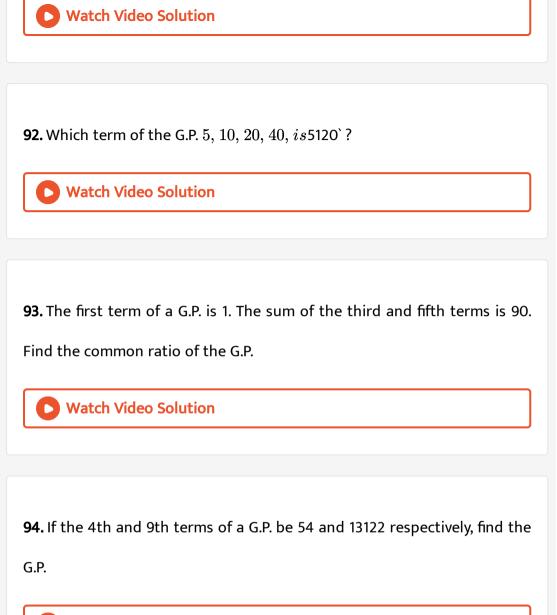**89.** Show that the sequence given by $a_n = 3\,(2^n)$, for all $n \in N$, is a G.P. Also find its common ratio.

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90. Find the 9th term and the general term of the progressions:
$\dfrac{1}{4}, -\dfrac{1}{2}, 1, -2$

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91. Find the 5th term of the progression
$1, \dfrac{\left(\sqrt{2}-1\right)}{2\sqrt{3}}, \left(\dfrac{3-2\sqrt{2}}{12}\right), \left(\dfrac{5\sqrt{2}-7}{24\sqrt{3}}\right), \;..$

92. Which term of the G.P. 5, 10, 20, 40, $is 5120$` ?

93. The first term of a G.P. is 1. The sum of the third and fifth terms is 90. Find the common ratio of the G.P.

94. If the 4th and 9th terms of a G.P. be 54 and 13122 respectively, find the G.P.

95. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

96. (ii)If the third term of G.P.is 4, then find the product of first five terms

97. If the $pth, \ qth \ and \ rth$ terms of a G.P. are $a, \ b, \ c$ respectively, prove that: $a^{q-r} . \ b^{r-p} . \ c^{p-q} = 1$

98. Find four numbers forming a geometric progression in which the third term is greater than the first terms by 9, and second term is greater than the 4th by 18.

99. The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour?

100. What will Rs. 500 amounts to in 10 years after its deposit in a bank which pays annual interest are 10% compounded annually?

101. A manufacturer reckons that the value of a machine, which costs him Rs. 15625, will depreciate each year by 20%. Find the estimated value at the end of 5 years.

102. If the first and the nth terms of a G.P., are $a and b$, respectively, and if P is hte product of the first n terms prove that $P^2 = (ab)^n$.

103. Show that the products of the corresponding terms o the sequences $a, \ ar, \ ar^2, \ ; \ ar^{n-1} \ and \ A, \ AR, \ AR^2, \ , \ AR^{n-1}$ form a G.P. and find the common ratio.

104. If $p^{th}, \ q^{th}, \ r^{th} and \ s^{th}$ terms of an A.P. are in G.P., then show that $(p-q), \ (q-r), \ (r-s)$ are also in G.P.

105. Show that each of one of the following progression is a G.P. Also, find the common ratio in each case: $4, \ -2, \ 1, \ -1/2$

106. Show that each of one of the following progression is a G.P. Also, find the common ratio in each case: $a, \ \dfrac{3a^2}{4}, \ \dfrac{9a^3}{16}, \ ,$

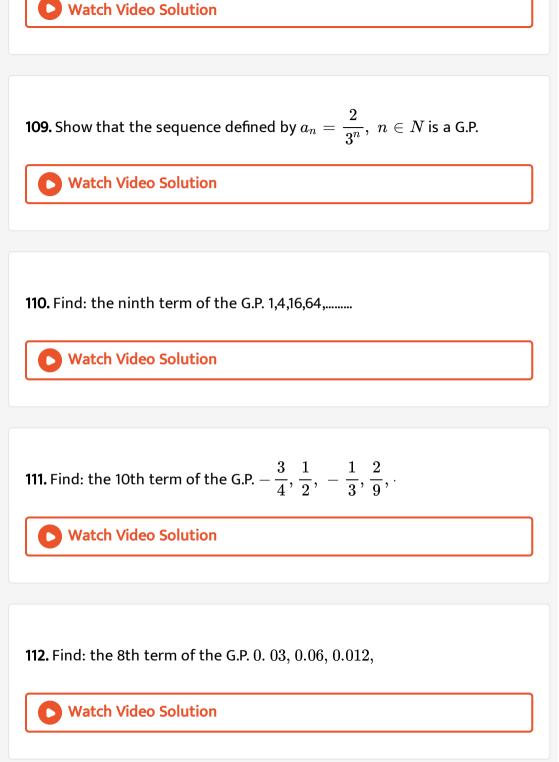**107.** Show that each of one of the following progression is a G.P. Also, find the common ratio in each case: $-2/3 - 6, \ -54.$

108. Show that each of one of the following progression is a G.P. Also, find the common ratio in each case: $1/2, \ 1/3, \ 2/9, \ 4/27.$

109. Show that the sequence defined by $a_n = \dfrac{2}{3^n}$, $n \in N$ is a G.P.

110. Find: the ninth term of the G.P. 1,4,16,64,.........

111. Find: the 10th term of the G.P. $-\dfrac{3}{4}, \dfrac{1}{2}, -\dfrac{1}{3}, \dfrac{2}{9}, \cdot$

112. Find: the 8th term of the G.P. 0. 03, 0.06, 0.012,

113. Find: the 12th term of the G.P. $\dfrac{1}{a^3x^3}, \ ax, \ a^5x^5, \ \cdot$

114. Find: n th term of the G.P. $\sqrt{3}, \ \dfrac{1}{\sqrt{3}}, \ \dfrac{1}{3\sqrt{3}},$

115. Find: the 10th term of the G.P. $\sqrt{2}, \ \dfrac{1}{\sqrt{2}}, \ \dfrac{1}{2\sqrt{2}},$

116. Find the 4th term from the end of the G.P. $\dfrac{2}{27}, \dfrac{2}{9}, \dfrac{2}{3}, , , , , , , \ 162.$

117. Which term of the progression 0.004,0.02,0.1,... is 12.5.?

118. Which term of the G.P.: $\sqrt{2}, \dfrac{1}{\sqrt{2}}, \dfrac{1}{2\sqrt{2}}, \dfrac{1}{4\sqrt{2}}, is \dfrac{1}{512\sqrt{2}}$?

119. Which term of the G.P.: $2, \ 2\sqrt{2}, \ 4, \ is \ 128$?

120. Which term of the G.P.: $\sqrt{3}, 3, 3\sqrt{3}, is \ 729$?

121. Which term of the G.P.: $\dfrac{1}{3}, \dfrac{1}{9}, \dfrac{1}{27} is \dfrac{1}{19683}$?

122. Find the 4th term from the end of the G.P. $\dfrac{1}{2}, \dfrac{1}{6}, \dfrac{1}{18}, \dfrac{1}{54}, , \dfrac{1}{4374}$

123. The fourth term of a G.P. is 27 and the 7th term is 729, find the G.P.

124. The 7th term of a G.P. is 8 times the 4th term and the 5th term is 48. Find the G.P.

125. If the G.Ps 5 , 10, 20, and 1280, 640, 320 have their nth terms equal, find the value of n.

126. The 5th, 8th and 11th terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

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127. The 4th term of a G.P. is square of its second term and the first term is -3. Find its 7th term.

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128. In a GP the 3rd term is 24 and the 6th term is 192. Find the 10th term.

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129. If a, b, c, d and p are different real numbers such that: $(a^2 + b^2 + c^2)p^2 - 2(ab + bc + cd)p + (b^2 + c^2 + d^2) \leq 0$, then show that a, b, c and d are in G.P.

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130. If $\dfrac{a + bx}{a - bx} = \dfrac{b + cx}{b - cx} = \dfrac{c + dx}{c - dx}(x \neq 0)$, then show that $a, \ b, \ c \ and \ d$ are in G.P.

131. the sum of three numbers in G.P. is 56. If we subtract 1,7,21 from these numbers in that order then we obtain an A.P. the three numbers is

132. Find three numbers in G.P. whose sum is 65 and whose product is 3375.

133. If the sum of three numbers in G.P. is 38 and their product is 1728, find them.

134. The sum of first three terms of a G.P. is 13/12 and their product is -1. Find the G.P.

135. The product of the three numbers in G.P. is 125 and sum of their product taken in pairs is $\dfrac{175}{2}$. Find them.

136. The sum of first three terms of a G.P. is $\dfrac{39}{10}$ and their product is 1. Find the common ratio and the terms.

137. The sum of three numbers in G.P. is 14. If the first two terms are each increased by 1 and the third term decreased by 1, the resulting numbers are in A.P. find the numbers.

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138. Find three numbers in G.P. whose product is 729 and the sum of their products in pairs is 819.
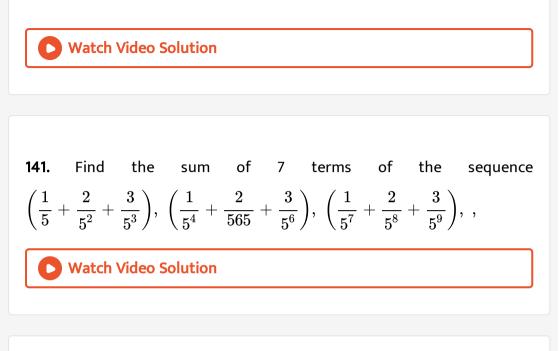
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139. The sum of three numbers in G.P. is 21 and the sum of their squares is 189. Find the numbers.

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140. Find the sum of 7 terms of the G.P. 3,6,12...

141. Find the sum of 7 terms of the sequence $\left(\dfrac{1}{5} + \dfrac{2}{5^2} + \dfrac{3}{5^3} \right)$, $\left(\dfrac{1}{5^4} + \dfrac{2}{565} + \dfrac{3}{5^6} \right)$, $\left(\dfrac{1}{5^7} + \dfrac{2}{5^8} + \dfrac{3}{5^9} \right)$, ,

142. Find the sum of the series $2 + 6 + 18 + + 4374$

143. The sum of first three terms of a G.P is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

144. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

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145. The sum of some terms of a G.P. is 315 whose first term and the common ratio are 5 and 2, respectively. Find the last term and the number of terms.

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146. Find the sum of the products of the corresponding terms of the sequences $2, 4, 8, 16, 32 \ and \ 128, 32, 8, 2, \dfrac{1}{2}$.

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147. if S is the sum , P the product and R the sum of reciprocals of n terms in $G.\,P.$ prove that $P^2 R^n = S^n$
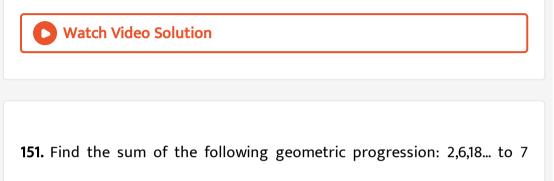
148. A person writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with instruction that they move the chain similarly. Assuming that the chain is not broken and that it costs 50 paise to mail one letter. Find the amount spend on the postage when 8th set of letter is mailed.

149. If f is a function satisfying $f(x + y) = f(x) \times f(y)$ for all $x, y \in N$ such that $f(1) = 3$ and $\sum_{x=1}^{n} f(x) = 120,$ find the value of n .

150. Find the natural number a for which $\sum_{k=1}^{n} f(a + k) = 16(2^n - 1)$, where the function f satisfies the relation $f(x + y) = f(x)f(y)$ for all natural number $x, y and, further, f(1) = 2.$

151. Find the sum of the following geometric progression: 2,6,18... to 7 terms

152. Find the sum of the following geometric progression: 1,3,9,27,... to 8 terms.

153. Find the sum of the following geometric progression:

$1, \; -1/2, \; 1/4, \; -1/8.$

154. Find the sum of the following geometric progression:

$\left(a^2 - b^2\right), \; (a - b), \; \left(\dfrac{a - b}{a + b}\right), \; \rightarrow \; n$ terms.

155. Find the sum of the following geometric progression:

$4, 2, 1, 1/2, \ldots .$ 10 terms.

156. Find the sum of the following geometric series: $0.15 + 0.015 + 0.0015 +$

to..... 8 t e r m s

157. Find the sum of the following geometric series:

$$\sqrt{2} - \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} - \rightarrow 8\ terms;$$

158. Find the sum of the following geometric series:2/9-1/3+1/2-3/4+ to 5 terms.

159. Find the sum of n terms of series

$$(x + y) + (x^2 + xy + y^2) + (x^3 + x^2 y + xy^2 + y^3) + \ldots\ldots\ldots\ldots$$

160. Find the sum of the series upto infinite terms

$$\frac{3}{4} - \frac{5}{4^2} + \frac{3}{4^3} - \frac{5}{4^4} + \ldots$$

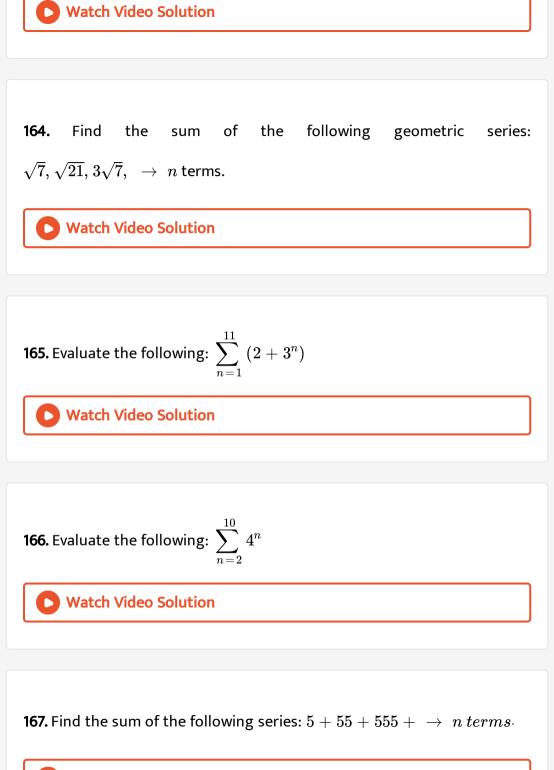**161.** Find the sum of the following geometric series:

$$\frac{a}{1+i} + \frac{a}{(1+i)^2} + \frac{a}{(1+i)^3} + \ldots + \frac{a}{(1+i)^n}.$$

162. Find the sum of the following geometric series:

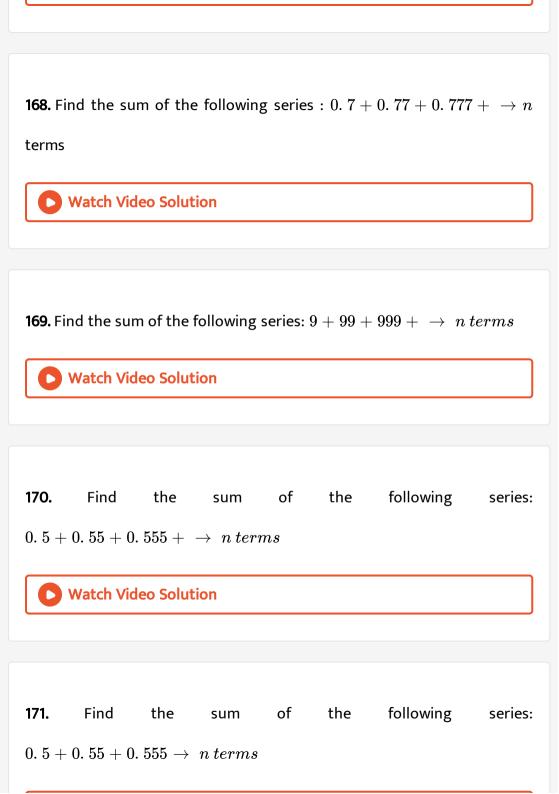$$1, \ -a, \ a^2, \ -a^3, \ \rightarrow \ n \text{ terms } (a \neq 1)$$

163. Find the sum of the following geometric series: $x^3, x^5, x^7, \ \rightarrow \ n$ terms

164. Find the sum of the following geometric series:

$\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \quad \rightarrow \quad n$ terms.

165. Evaluate the following: $\sum\limits_{n=1}^{11} (2 + 3^n)$

166. Evaluate the following: $\sum\limits_{n=2}^{10} 4^n$

167. Find the sum of the following series: $5 + 55 + 555 + \quad \rightarrow \quad n \ terms.$

168. Find the sum of the following series : $0.7 + 0.77 + 0.777 + \rightarrow n$ terms

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169. Find the sum of the following series: $9 + 99 + 999 + \rightarrow n \ terms$

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170. Find the sum of the following series: $0.5 + 0.55 + 0.555 + \rightarrow n \ terms$

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171. Find the sum of the following series: $0.5 + 0.55 + 0.555 \rightarrow n \ terms$

172. How many terms of the series 3,6,12,.. having sum 381?

173. .Find the G.P., if the common ratio of G.P. is 3, nth term is 486 and sum of first n terms is 728.

174. The ratio of the sum of first three terms is to that of first 6 terms of a G.P. is 125:152. Find the common ratio.

175. The 4th and 7th terms of a G.P. are $\dfrac{1}{27}$ and $\dfrac{1}{729}$ respectively. Find the sum of n terms of the G.P.

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176. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\dfrac{1}{r^n}$

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177. If $a \; and \; b$ are the roots of $x^2 - 3x + p = 0 \; and \; c, \; d$ are the roots $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. Prove that $(q+p):(q-p) = 17:15$.

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178. How many terms of the G.P. $3, 3/2, 3/4,$ be taken together to make $\dfrac{3069}{512}$?

179. A person has 2 parents, 4 grandparents, 8 great grand parents, and so on. Find the number of his ancestors during the ten generations preceding his one.
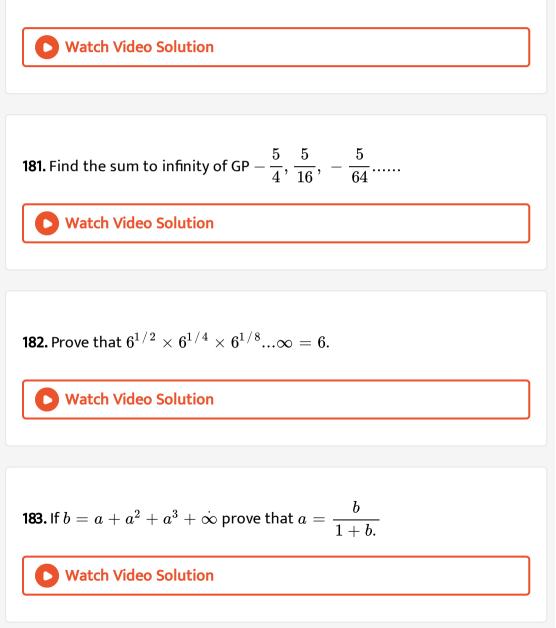
    A. 1023

    B. 1024

    C. 2046

    D. None of the above

Answer: C

180. A. G.P. consists of an even number of terms. If the sum of all the terms is 5 times the sum of the terms occupying the odd places. Find the common ratio of the G.P.

181. Find the sum to infinity of GP $-\dfrac{5}{4}, \dfrac{5}{16}, -\dfrac{5}{64}......$

182. Prove that $6^{1/2} \times 6^{1/4} \times 6^{1/8}...\infty = 6$.

183. If $b = a + a^2 + a^3 + \dot\infty$ prove that $a = \dfrac{b}{1+b}.$

184. The first term of G.P. is 2 and the sum to infinity is 6. Find the common ratio.

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185. The sum of an infinite G.P. is 57 and the sum of their cubes is 9747, then the common ratio of the G.P. is

A. 1//2`

B. $2/3$

C. $1/6$

D. none of these

Answer: null

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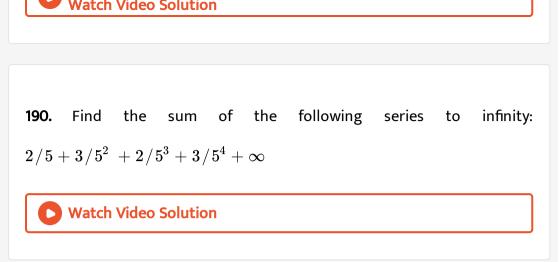**186.** Use geometric series to express $0.555... = 0.\bar{5}$ as a rational number

187. The inventor of the chess board suggested a reward of one grain of wheat for the first square, 2 grains for the second, 4 grains for the third and so on, doubling the number of the grains for subsequent squares. How many grains would have to be given to inventor? (There are 64 square in the chess board).

188. The sum of an infinite geometric series is 15 and the sum of the squares of these terms is 45. Find the series.
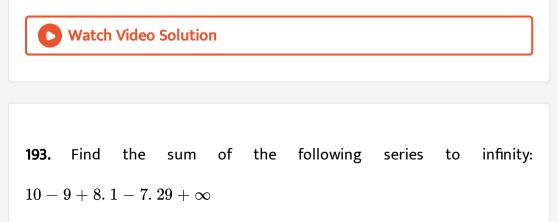
189. Find the sum of the following series to infinity:

$$1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} + \infty$$

190. Find the sum of the following series to infinity:

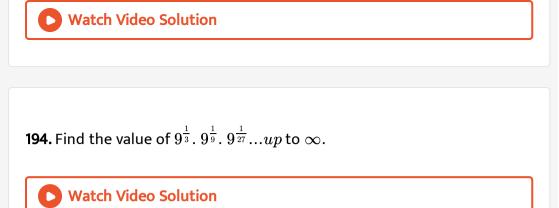$$2/5 + 3/5^2 + 2/5^3 + 3/5^4 + \infty$$

191. Find the sum of the following series to infinity:

$$\frac{1}{3} + \frac{1}{5^2} + \frac{1}{3^3} + \frac{1}{5^4} + \frac{1}{3^5} + \frac{1}{5^6} + \infty$$

192. Find the sum of the following series to infinity: $8 + 4\sqrt{2} + 4 + \infty$

193. Find the sum of the following series to infinity:

$$10 - 9 + 8.\,1 - 7.\,29 + \infty$$

194. Find the value of $9^{\frac{1}{3}}.\ 9^{\frac{1}{9}}.\ 9^{\frac{1}{27}}...up$ to ∞.

195. Prove that: $2^{\frac{1}{4}}.4^{\frac{1}{8}},\ 8^{\frac{1}{16}}.\ 16^{\frac{1}{32}}.........\infty = 2.$

196. The terms of an infinitely decreasing $G.\ P.$ in which all the terms are positive, the first term is 4 and the difference between the third and fifth term is $\dfrac{81}{32}$, then find the common ratio.

197. Express the recurring decimal $0.125125125...$ as a rational number.

198. Find the rational numbers having the following decimal expansion:

$0.\overline{231}$

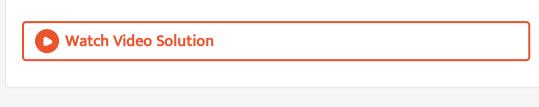**199.** Find the rational numbers shaving the following decimal expansion:

$3.5\overline{2}$

200. Find the rational numbers having the following decimal expansion:

0.231

201. Find an infinite G.P. whose first term is 1 and each term is the sum of all the terms which follow it.

202. Show that in an infinite G.P. with common ratio $r(|r| < 1)$, each term bears a constant ratio to the sum of all terms that follow it.

203. Three numbers whose sum is 15 are in A.P. If they are added by $1,4$ and 19 respectively, they are in GP. The numbers are
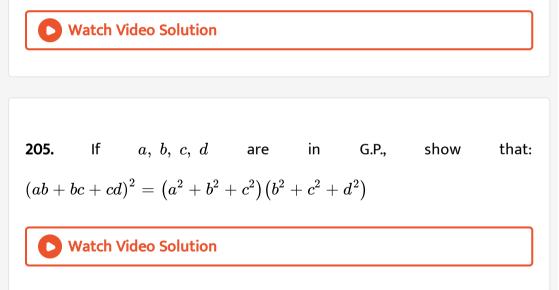
204. If a, b, c, d are in G.P, then $(b - c)^2 + (c - a)^2 + (d - b)^2$ is equal to

A. $(a-d)^2$

B. $(ad)^2$

C. $(a+d)^2$

D. $(a/d)^2$

Answer: A

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205. If a, b, c, d are in G.P., show that:

$(ab + bc + cd)^2 = (a^2 + b^2 + c^2)(b^2 + c^2 + d^2)$

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206. If a, b, c, d are in G.P. prove that $(a^n + b^n), (b^n + c^n), (c^n + d^n)$ are in G.P.

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207. If a, b, c are in $G.P.$ and $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$ prove that x, y, z are in $A.P.$

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208. If $a^2 + b^2$, $ab + bc$ and $b^2 + c^2$ are in G.P., then a, b, c are in

A. A.P.

B. G.P.

C. H.P.

D. None of these

Answer: B

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209. If a, b, c are in G.P. prove that $\log a, \log b, \log c$ are in A.P.

210. If $a, \ b, \ c$ are in G.P. prove that $\dfrac{1}{(\log)_a m}, \ \dfrac{1}{(\log)_b m}, \ \dfrac{1}{(\log)_c m}$ are in A.P.

211. Find k such that $k + 9, \ k - 6 \ \text{and} \ 4$ from three consecutive terms of a G.P.

212. Three numbers are in A.P. and their sum is 15. If $1, 3, 9$ be added to them respectively they form a G.P. Find the numbers.

213. The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 and the third is increased by 1 we obtain three consecutive terms of a G.P. Find the numbers.

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214. The sum of three numbers in G.P. is 56. If we subtract $1, 7, 21$ from these numbers in that order then we obtain an A.P. the three numbers is
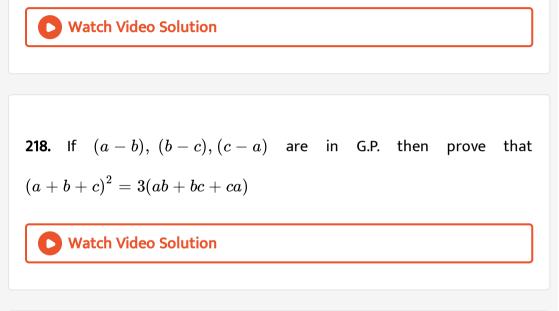
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215. If $a,\ b\ c$ are in G.P., prove that the following are also in G.P.: a^2, b^2, c^2

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216. If a, b, c are in G.P., prove that the following are also in G.P.: a^3, b^3, c^3

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217. If a, b, c are in $G.P.$, prove that $a^2 + b^2$, $ab + bc$, $b^2 + c^2$ are also in $G.P$

218. If $(a - b), (b - c), (c - a)$ are in G.P. then prove that $(a + b + c)^2 = 3(ab + bc + ca)$

219. If the $4th$, $10th$ and $16th$ terms of a G.P. are $x, y,$ and z respectively. Prove that x, y, z are in G.P.

220. If a, b, c are in $A.P$ and a, b, d are in $G.P,$ prove that $a, a - b, d - c$ are in $G. P.$

221. If the $pth, qth, rth,$ and sth terms of an A.P. are in G.P., then $p - q, q - r, r - s$ are in

A. A.P.

B. G.P.

C. H.P.

D. None of these

Answer: null

222. If $a, \ b, \ c$ are in A.P. $b, \ c, \ d$ are in G.P. and $\dfrac{1}{c}, \dfrac{1}{d}, \dfrac{1}{e}$ are in A.P. prove that $a, \ c, \ e$ are in G.P.?

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223. If a, b, c are in $A.P$ and a, b, d are in $G.P,$ prove that $a, a - b, d - c$ are in $G. P.$

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224. If $p^{th}, \ q^{th}, \ and \ r^{th}$ terms of an A.P. and G.P. are both $a, \ b \ and \ c$ respectively show that $a^{b-c}b^{c-a}c^{a-b} = 1.$

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225. Find two numbers whose arithmetic mean is 34 and the geometric mean is $16.$

226. If A.M. and G.M. between two numbers is in the ratio $m:n$ then prove that the numbers are in the ratio $\left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$
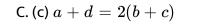
227. a, b, c are positive real numbers forming a G.P. If $ax^2 + 2bx + c = 0$ and $x^2 + 2ex + f = 0$ have a common root, then prove that $\dfrac{d}{a}, \dfrac{e}{b}, \dfrac{f}{c}$ are in A.P.

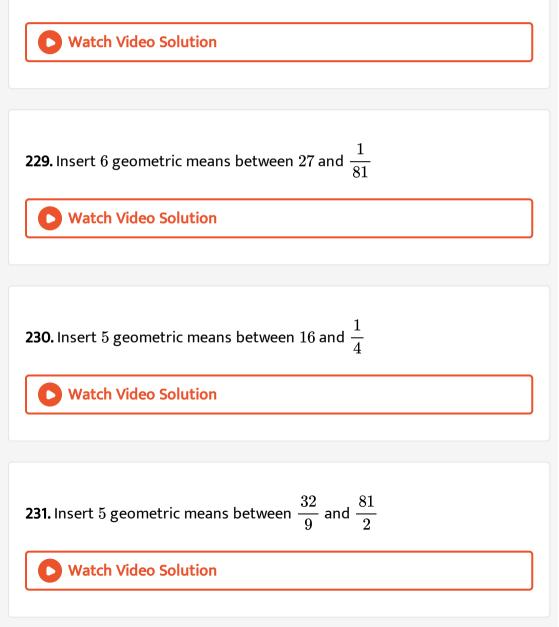**228.** If a, b, c and d are distinct positive no in AP. then

A. (a) $ad < bc$

B. (b) $a + c = b + d$

C. (c) $a + d = 2(b + c)$

D. (D) $a + d = 3(b + c)$

Answer: (b) $a + c = b + d$

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229. Insert 6 geometric means between 27 and $\dfrac{1}{81}$

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230. Insert 5 geometric means between 16 and $\dfrac{1}{4}$

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231. Insert 5 geometric means between $\dfrac{32}{9}$ and $\dfrac{81}{2}$

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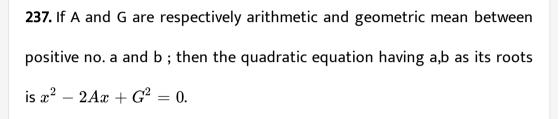
232. Find the geometric means of the following pairs of number: 2 and 8.

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233. Find the geometric means of the following pairs of number: a^3b and ab^3

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234. Find the geometric means of the following pairs of number: -8 and -2.

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235. If a is the G.M. of 2 and $\dfrac{1}{4}$, find a

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236. Find the two numbers whose A.M. is 25 and GM is 20.

237. If A and G are respectively arithmetic and geometric mean between positive no. a and b ; then the quadratic equation having a,b as its roots is $x^2 - 2Ax + G^2 = 0$.

238. The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3 + 2\sqrt{2}\right) : \left(3 - 2\sqrt{2}\right)$.

239. If A.M. and GM. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

240. If AM and GM of two positive numbers a and b are 10 and 8 respectively, find the numbers.

241. If the fifth term of a G.P. is 2, then write the product of its 9 terms.

242. If $(p+q)^{th}$ and $(p-q)^{th}$ terms of a G.P. re m and n respectively, then write it p^{th} term.

243. If $(\log)_x a$, $a^{x/2}$ and $(\log)_b x$ are in G.P., then write the value of x

(a)$\log a(\log_b a)$

(b) $\log a(\log_e a) - \log a(\log_e b)$

(c) $-\log a(\log_a b)$

(d) none of these

244. If the sum of an infinite decreasing G.P. is 3 and the sum of the squares of its term is $\dfrac{9}{2}$, then write its first term and common difference.

245. If p^{th}, q^{th} and r^{th} terms of G.P. are x, y, x respectively then write the value of $x^{q-r}y^{r-p}z^{p-q}$.

246. If A_1 and A_2 are two A.M.s between a and b and G_1 and G_2 are two G.M between the same numbers then what is the value of $\dfrac{A_1 + A_2}{G_1 G_2}$

247. If second, third and sixth terms of an A.P. are consecutive terms of a G.P. write the common ratio of the G.P.

248. If A and G are respectively arithmetic and geometric mean between positive no. a and b ; then the quadratic equation having a,b as its roots is $x^2 - 2Ax + G^2 = 0$.

249. If n geometric means are inserted between two quantities ; then the product of n geometric means is the nth power of the single geometric mean between two quantities.

250. If $a = 1 + b + b^2 + b^3 + \rightarrow \infty$, then write b in terms of a given that $|b| < 1$.

251. If in an infinite G.P., first term is equal to 10 times the sum of all successive terms then its common ratio is

a.$1/10$

b. $1/11$

c. $1/9$

d. $1/20$

252. If the first term of a G.P. $a_1, a_2, a_3, \ldots$ is unity such that $4a_2 + 5a_3$ is least then the common ratio of G.P. is

a. $-2/5$

b. $-3/5$

c. $2/5$

d. none of these

253. If a, b, c are in A.P. and x, y, z are in G.P., then show that $x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1.$

254. The first three of four given numbers are in G.P. and their last three are in A.P. with common difference 6. If first and fourth numbers are equal, then the first number is

a. 2

b. 4

c. 6

d. 8

255. If S be the sum P the product and R be the sum of the reciprocals

of n terms of a GP then P^2 is equal to

(a). S/R

(b). R/S

(c). $(R/S)^n$

(d). $(S/R)^n$