MATHS

BOOKS - RD SHARMA MATHS (HINGLISH)

MATHEMATICAL INDUCTION

Solved Examples And Exercises

1. Prove that for $n \in N, 10^n + 3. 4^{n+2} + 5$ is divisible by 9 .

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2. Prove by induction that the sum of the cubes of three consecutive natural numbers is divisible by 9.

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3. A sequence $x_1, x_2, x_3, \ldots.$ is defined by letting $x_1 = 2$ and $x_k = \frac{x_{k-1}}{k}$ for all natural numbers k, $k \geq 2$ Show that $x_n = \frac{2}{n!}$ for all $n \in N$.

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4. Show by the Principle of Mathematical induction that the sum S_n, of the nterms of the series $1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2 + 7^2 + \ldots.$ is given by $S_n = \begin{cases} \frac{n(n+1)^2}{2} \end{cases}$, if n is even , then $\frac{n^2(n+1)}{2}$, if n is odd

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5. Using the principle of mathematical induction prove that : $1.3 + 2.3^2 + 3.3^3 + \quad + n.3^n = \frac{(2n-1)3^{n+1} + 3}{4}$ for all $n \in N$.

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6. Prove by the principle of mathematical induction that for all $n \in N, n^2 + n$ is even natural number.

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7. Using the principle of mathematical induction prove that $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \quad + \frac{1}{1+2+3+ \quad + n} = \frac{2n}{n+}$

for all $n \in N$

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8. Prove by induction that the sum $S_n = n^3 + 2n^2 + 5n + 3$ is divisible by 3 for all $n \in N$.

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9. Using the principle of mathematical induction prove that $\frac{1}{1.\,2.\,3}+\frac{1}{2.\,3.\,4}+\frac{1}{3.\,4.\,5}++\frac{1}{n(n+1)(n+2)}=\frac{n(n+3)}{4(n+1)(n+2)}$ for all $n \in N$

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10. Using the principle of mathematical induction. Prove that (x^n-y^n) is divisible by (x+y) for all $n \in N$.

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11. Using the principle of mathematical induction prove that 41^n-14^n is a multiple of 27 .

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12. Using the principle of mathematical induction, prove that $\left(2^{3n}-1\right)$ is divisible by 7 for all $n \in N$

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13. Using principle of mathematical induction prove that $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}}$ for all natural numbers $n \geq 2$.

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14. Prove that: $1+2+3+n < \frac{(2n+1)^2}{8}$ for all$n \in N$.

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15. Prove that $1^2+2^2+\vdots+n^2 > \frac{n^2}{3}, n \in N$

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16. Prove by the principle of mathematical induction that $n < 2^n$ for all $n \in N$.

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17. Prove by the principle of mathematical induction that for all $n \in N, 3^{2n}$ when divided by 8 , the remainder is always 1.

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18. Using the principle of mathematical induction, prove that : $1.\ 2.\ 3 + 2.\ 3.\ 4 + \ + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$ for all $n \in N$.

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19. Prove the following by using the principle of mathematical induction for all $n \in N : 1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

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20. Using principle of MI prove that $2.7^n + 3.5^n - 5$ is divisible by 24

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21. Prove by the principle of mathematical induction that $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number for all $n \in N$.

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22. For all positive integer n , prove that $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2}{3}n^3 - \frac{n}{105}$ is an integer

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23. If $P(n)$ is the statement $2^{3n} - 1$. Is an integral multiple $7,$ and if $P(r)$ is true, prove that $P(r+1)$ is true.

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24. Let $P(n)$ be the statement $3^n > n$. If $P(n)$ is true, $P(n+1)$ is also true.

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25. If P(n) is the statement $n^2 > 100$, prove that whenever $P(r)$ is true, $P(r+1)$ is also true.

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26. Prove by the principle of mathematical induction that for all $n \in N$:

$1^2 + 2^2 + 3^2 + \quad + n^2 = \frac{1}{6}n(n+1)(2n+1)$

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27. Prove by the principle of mathematical induction that for all $n \in N$:

$1 + 4 + 7 + \quad + (3n-2) = \frac{1}{2}n(3n-1)$

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28. Prove by the principle of mathematical induction that:

$n(n+1)(2n+1)$ is divisible by 6 for all $n \in N$.

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29. Prove by the principle of mathematical induction that for all $n \in N$, $n^2 + n$ is even natural number.

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30. Prove that : $\cos^2\alpha + \cos^2(\alpha+\beta) - 2\cos\alpha\cos\beta\cos(\alpha+\beta) = \sin^2\beta$

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31. Prove that $\frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} > \frac{13}{24}$,for all natural number $n > 1.$

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32. Prove the following by using the principle of mathematical induction for all $n \in N : (2n+7) < (n+3)^2$.

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33. Prove by induction the inequality $(1+x)^n \geq 1+nx$ whenever x is positive and n is a positive integer.

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34. If $P(n)$ is the statement n^3+n is divisible 3 is the statement $P(3)$ true ? Is the statement $P(4)$ true?

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35. If $P(n)$ is the statement $n(n+1)(n+2)$ is divisible is 12 prove that the statements $P(3)$ and $P(4)$ are true, but that $P(5)$ is not true.

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36. Let $P(n)$ be the statement 7 divides $(2^{3n}-1)$. What is $P(n+1)$?

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37. If $P(n)$ is the statement $n(n+1)$, then what is $P(3)$?

A. even

B. odd

C. prime

D. none of these

Answer: A

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38. If $P(n)$ is the statement $n^3 + n$ is divisible by 3, prove that $P(3)$ is true but $P(4)$ is not true.

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39. If $P(n)$ is the statement $n^2 + n$ is even, and if $P(r)$ is true then $P(r+1)$ is

A. True

B. False

C. Not determined

D. None of these

Answer: A

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40. If $P(n)$ is the statement $2^n \geq 3n,$ and if $P(r)$ is true, prove that $P(r+1)$ is true.

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41. Given an example of a statement $P(n)$ such that it is true of all $n \in N$.

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42. If $P(n)$ is the statement $n^2 - n + 41$ is prime. Prove that $P(1), \ P(2)$ and $P(3)$ are true. Prove also that $P(41)$ is not true.

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43. Given an example of a statement $P(n)$ which is true for all $n \geq 4$ but $P(1), \ P(2) and \ P(3)$ are not true. Justify your answer.

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44. Prove by the principle of mathematical induction that for all nN:

$$\frac{1}{1.\,3} + \frac{1}{35} + \frac{1}{57} + + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$$

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45. Using principle of mathematical induction prove that $x^{2n} - y^{2n}$ is divisible by $x + y$ for all $n \in N$.

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46. Prove by the principle of induction that for all nN, $\left(10^{2n-1} + 1\right)$ is divisible by 11.

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47. Prove by induction that $4 + 8 + 12 + \quad + 4n = 2n(n + 1)$ for all nN.

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48. Using principle of mathematical induction prove that $\cos\alpha\cos 2\alpha\cos 4\alpha\cos\left(2^{n-1}\alpha\right) = \dfrac{\sin 2^n\alpha}{2^n\sin\alpha}$ for all $n \in N$.

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49. Let $U_1 = 1,\ U_2 = 1\ and\ U_{n+2} = U_{n+1} + U_n f\text{ or }\ n \geq 1.$ use mathematical induction to show that: $U_n = \dfrac{1}{\sqrt{5}}\left\{\left(\dfrac{1+\sqrt{5}}{2}\right)^n - \left(\dfrac{1-\sqrt{5}}{2}\right)^n\right\} f\text{ or }\ all\ n \geq 1.$

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50. Prove the following by the principle of mathematical induction: $1 + 2 + 3 + \ + n = \dfrac{n(n+1)}{2}\dot{ie}$, the sum o the first n natural numbers is $\dfrac{n(n+1)}{2}$.

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51. Prove the following by the principle of mathematical induction:

$$1^2 + 2^2 + 3^2 + \quad + n^2 = \frac{n(n+1)(2n+1)}{6}$$

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52. Prove the following by the principle of mathematical induction:

$$1 + 3 + 3^2 + \quad + 3^{n-1} = \frac{3^n - 1}{2}$$

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53. Using the principle of mathematical induction, prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{(n+1)}.$$

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54. Prove the following by the principle of mathematical induction:

$1 + 3 + 5 + \quad + (2n-1) = n^2 i\dot{e};$ the sum of first n odd natural numbers is n^2.

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55. Prove the following by the principle of mathematical induction:

$$\frac{1}{2.5}+\frac{1}{5.8}+\frac{1}{8.11}++\frac{1}{(3n-1)(3n+2)}=\frac{n}{6n+4}$$

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56. Prove the following by the principle of mathematical induction:

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}++\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

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57. Prove the following by the principle of mathematical induction:

$$\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\frac{1}{(2n+1)(2n+3)}=\frac{n}{3(2n+3)}$$

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58. Prove the following by the principle of mathematical induction:

$$\frac{1}{3.\ 7}+\frac{1}{7.\ 11}+\frac{1}{11.\ 15}++\frac{1}{(4n-1)(4n+3)}=\frac{n}{3(4n+3)}$$

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59. Prove the following by the principle of mathematical induction:

$$1.\ 2+2.\ 2^2+3.\ 2^3++n.2^n=(n-1)2^{n+1}+2$$

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60. Prove the following by the principle of mathematical induction:

$$2+5+8+11++(3n-1)=\frac{1}{2}n\,(3n+1)$$

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61. Prove the following by the principle of mathematical induction:

$$1.\ 3+3.\ 5++(2n-1)(2n+1)=\frac{n\left(4n^2+6n-1\right)}{3}$$

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62. Prove the following by the principle of mathematical induction:

$$1.\ 2 + 2.\ 3 + 3.\ 4 + \quad + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

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63. Prove the following by the principle of mathematical induction:

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \quad + \frac{1}{2^n} = 1 - \frac{1}{2^n}$$

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64. Prove the following by the principle of mathematical induction:

$$1^2 + 3^2 + 5^2 + \quad + (2n-1)^2 = \frac{1}{3}n\left(4n^2 - 1\right)$$

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65. Prove the following by the principle of mathematical induction:

$a + (a + d) + (a + 2d) + \quad + (a + (n - 1)d) = \frac{n}{2}[2a + (n - 1)d]$

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66. Prove the following by the principle of mathematical induction:

$5^{2n} - 1$ is divisible by 24 for all $n \in N$.

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67. Prove the following by the principle of mathematical induction:

$3^{2n} + 7$ is divisible by 8 for all $n \in N$.

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68. Prove the following by the principle of mathematical induction:

$5^{2n+2} - 24n - 25$ is divisible 576 for all $n \in N$.

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69. Prove the following by the principle of mathematical induction:

$3^{2n+2} - 8n - 9$ is divisible 8 for all $n \in N$.

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70. Prove the following by the principle of mathematical induction:

$(ab)^n = a^n b^n$ for all $n \in N$.

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71. Prove the following by using the principle of mathematical induction

for all $n \in N$: $n(n+1)(n+5)$ is a multiple of 3.

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72. Prove the following by the principle of mathematical induction:

$7^{2n} + 2^{3n-3}.\, 3^{n-1}$ is divisible 25 for all $n \in N$.

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73. Prove the following by the principle of mathematical induction:

$2.\, 7^n + 3.\, 5^n - 5$ is divisible 24 for all $n \in N$.

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74. Prove the following by the principle of mathematical induction:

$11^{n+2} + 12^{2n+1}$ is divisible 133 for all $n \in N$.

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75. Prove the following by the principle of mathematical induction:

$n^3 - 7n + 3$ is divisible 3 for all $n \in N$.

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76. Prove the following by the principle of mathematical induction: $1 + 2 + 2^2 \ldots\ldots + 2^n = 2^{n+1} - 1$ for all $n \in N$.

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77. Prove the following by the principle of mathematical induction: $7 + 77 + 777 + \; + 777 + \; + \ddot{n} - digits7 = \frac{7}{81}\left(10^{n+1} - 9n - 10\right)$ for all $n \in N$

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78. Prove the following by the principle of mathematical induction: $\frac{n^7}{7} + \frac{n^5}{5} + \frac{n^3}{3} + \frac{n^2}{2} - \frac{37}{210}n$ is a positive integer for all $n \in N$.

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79. Prove the following by the principle of mathematical induction: $\frac{n^{11}}{11} + \frac{n^5}{5} + \frac{n^3}{3} + \frac{62}{165}n$ is a positive integer for $n \in \mathbb{N}$.

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80. Prove the following by the principle of mathematical induction: $\frac{1}{2}\tan\left(\frac{x}{2}\right) + \frac{1}{4}\tan\left(\frac{x}{4}\right) + \ + 1/2^n \tan\left(\frac{x}{2^n}\right) = \frac{1}{2^n}\cot\left(\frac{x}{2^n}\right) - \cot x$

for all $n \in N$ and 0

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81. Prove the following by the principle of mathematical induction: $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$ or all natural numbers $n \geq 2$.

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82. Prove the following by the principle of mathematical induction: $\frac{(2n)!}{2^{2n}(n!)^2} \leq \frac{1}{\sqrt{3n+1}}$ for all $n \in N$.

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83. Prove the following by the principle of mathematical induction: $x^{2n-1} + y^{2n-1}$ is divisible by $x + y$ for all $n \in \mathbb{N}$.

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84. Prove that: $\sin x + \sin 3x + + \sin(2n-1)x = \frac{\sin^2 nx}{\sin x}$ for all $n \in \mathbb{N}$

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85. Given $a_1 = \frac{1}{2}\left(a_0 + \frac{A}{a_0}\right)$, $a_2 = \frac{1}{2}\left(a_1 + \frac{A}{a_1}\right)$ and $a_{n+1} = \frac{1}{2}\left(a_n + \frac{A}{a_n}\right)$ for $n \geq 2$, where $a > 0$, $A > 0$. prove that

$$\frac{a_n - \sqrt{A}}{a_n + \sqrt{A}} = \left(\frac{a_1 - \sqrt{A}}{a_1 + \sqrt{A}}\right)2^{n-1}.$$

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86. Let $P(n)$ be the statement: $2^n \geq 3n$. If $P(r)$ is true, show that $P(r+1)$ is true. Do you conclude that $P(n)$ is true for all $n \in N$

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87. The distributive law from algebra states that for all real numbers c,a1 and a2,we have $c(a_1 + a_2) = ca_1 + ca_2$ Use this law and mathematical induction to prove that,for all natural numbers,$n \geq 2$,if $c, a_1, a_2, \ldots., $ an are any real numbers,then $c(a_1 + a_2 + \ldots.. + a_n) = ca_1 + ca_2 + \ldots. + ca_n$

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88. State First principle of mathematical induction.

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89. Write the set of values of n for which the statement $P(n): 2n < n!$ is true.

A. 4

B. 8

C. 12

D. 16

Answer: A

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90. State Second principal of mathematical induction.

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91. If $P(n): 2 \times 4^{2n+1} + 3^{3n+1}$ is divisible by λ for all $n \in N$ is true, then find the value of λ.

A. 9

B. 11

C. 13

D. 15

Answer: B

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92. If $x^n - 1$ is divisible by $x - \lambda$, then the least prositive integral value of λ is 1 b. 3 c. 4 d. 2

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93. For all $n \in N$, $3 \times 5^{2n+1} + 2^{3n+1}$ is divisible by a.19 b. 17 c. 23 d. 25

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94. If $10^n + 3 \times 4^{n+2} + \lambda$ is divisible by 9 or all natural numbers, then the least positive integral value of λ is

a. 5

b. 3

c. 7

d. 1

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95. Let $P(n) : 2^n < (1 \times 2 \times 3 \times \quad \times n)$. Then the smallest positive integer for which $P\ (n)$ is true is

A. 1

B. 2

C. 3

D. 4

Answer: D

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96. A student was asked to prove a statement by induction. He proved (i) P(5) is true and (ii) truth of P(n) => truth of P(n+1), n $\in$ N . On the basis of this, he could conclude that P(n) is true

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97. If $P(n): 49^n + 16^n + \lambda$ is divisible by 64 for $n \in N$ is true, then the least negative integral value of λ is

A. -3

B. -2

C. -1

D. -4

Answer: C

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Others

1. Prove that the number of subsets of a set containing n distinct elements is 2^n for all $n \in N$.

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