



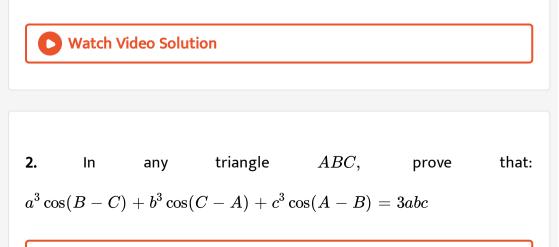
# MATHS

# **BOOKS - RD SHARMA MATHS (HINGLISH)**

# SINE AND COSINE FORMULAE AND THEIR APPLICATIONS

Solved Examples And Exercises

**1.** Two boats leave a place at the same time. One travels 56 km in the direction N  $40^{\circ}$  E, while the other travels 48 km in the direction S  $80^{\circ}$  E. What is the distance between the boats?



**3.** With usual notations, if in a triangle  $ABC\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$ , then prove that:  $\frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$ 

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**4.** If  $a^2, b^2, c^2$  are in A.P., prove that  $\cot A, \cot B, \cot C$  are in AP

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5. If in a triangle  $ABC, \cos A + 2\cos B + \cos C = 2$  prove that the sides

of the triangle are in  ${\cal AP}$ 



6. In a triangle 
$$ABC, \angle C = 60^{\circ}$$
, then prove that:  
 $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ .

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**7.** 
$$b(\cos A - a \cos C) = c^2 - a^2$$

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**8.**  $2(bc \cos A + ca \cos B + ab \cos C) = a^2 + b^2 + c^2$ 

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9. In a ABC , if  $igstarrow B=60^\circ$  , prove that (a+b+c)(a-b+c)=3ca

10. In any 
$$\Delta ABC$$
 , prove that:  $rac{\sin B}{\sin C} = rac{c-a\cos B}{b-a\cos C}$ 

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11. In any 
$$ABC$$
 , prove that:  $2iggl\{arac{\sin^2 C}{2}+crac{\sin^2 A}{2}iggr\}=a+c-b$ 

12. In any 
$$ABC$$
, prove that:  
 $2\left\{b\cos^2\left(\frac{C}{2}\right) + c\cos^2\left(\frac{B}{2}\right)\right\} = a + c + b$   
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13. In any  $ABC$ , prove that:  
 $(b + c)\cos A + (c + a)\cos B + (a + b)\cos C = a + b + c$   
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14. For any triangle ABC, prove that  $a(b\cos C - c\cos B) = b^2 - c^2$ 



15. In a  $\Delta ABC$ , if  $a=2, \angle B=60^\circ and \angle C=75^\circ$  , then b= (a) $\sqrt{3}$  (b)  $\sqrt{6}$  (c)  $\sqrt{9}$  (d)  $1+\sqrt{2}$ 

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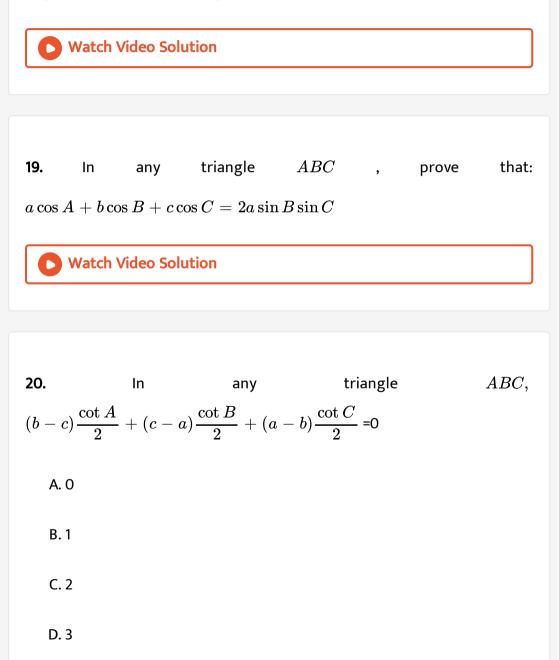
16. If  $a^2, \, b^2, \, c^2$  are in A.P., prove that  $\cot A, \, \cot B, \, \cot C$  are in  $A\dot{P}$ 

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17. In any triangle ABC , prove that:  $rac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B}=rac{a^2+b^2}{a^2+c^2}$ 

18. If in a 
$$ABC, rac{a^2-b^2}{a^2+b^2}=rac{\sin(A-B)}{\sin(A+B)}$$
 , prove that it is either a right

angled or an isosceles triangle.



#### Answer: A



21. If in a 
$$ABC, \frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$$
 , prove that  $a^2, b^2, c^2$  are in  $AP$ .

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**22.** In a triangle *ABC*, if  $a \cos A = b \cos B$ , show that the triangle is

either isosceles or right angled.

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**23.** If in triangle the angles are in the ratio as 1:2:3, prove that the corresponding sides are  $1:\sqrt{3}:2$ .

**24.** In the sides of a triangle are in the ratio  $1:\sqrt{3}:2$ , then the measure of its greatest angle is  $\frac{\pi}{6}$  (b) $\frac{\pi}{3}$  (c)  $\frac{\pi}{2}$  (d)  $\frac{2\pi}{3}$ 

A. 
$$\frac{\pi}{6}$$
  
B.  $\frac{\pi}{2}$   
C.  $\frac{\pi}{3}$   
D.  $\frac{2\pi}{3}$ 

#### Answer: B

**25.** In any triangle *ABC*, prove that:  

$$\frac{a^2 \sin(B-C)}{\sin B + \sin C} + \frac{b^2 \sin(C-A)}{\sin C + \sin A} + \frac{c^2 \sin(A-B)}{\sin A + \sin B} = 0$$
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**26.** The angles of a triangle ABC are in AP and it is being given that

$$b\!:\!c=\sqrt{3}\!:\!\sqrt{2}$$
 , find  $igtriangle A$  .

27. In 
$$ABC, ext{ prove that } (a-b)^2 rac{\cos^2 C}{2} + (a+b)^2 rac{\sin^2 C}{2} = c^2 + c$$

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**28.** In a ABC , if a = 3, b = 5 and c = 7, find  $\cos A, \cos B$  and  $\cos C$ 

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**29.** If the sides 
$$a, b, c$$
 of a  $\triangle ABC$  are in  $HP$ , prove that  $\sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right), \sin^2\left(\frac{C}{2}\right)$  are in  $HP$ 

**30.** A person observes the angle of elevation of the peak of a hill from a station to be  $\alpha$ . He walks c metres along a slope inclined at the angle  $\beta$  and finds the angle of elevation of the peak of the hill to be  $\gamma$ . Show that the height of the peak above the ground is  $\frac{c \sin \alpha \sin(\gamma - \beta)}{\sin(\gamma - \alpha)}$ 

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**31.** If in a triangle 
$$ABC$$
,  $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$ ,

then prove that the triangle is right angled.

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32. In a triangle ABC , if  $\cos A = {\sin B \over 2\sin C}$  , show that the triangle is

isosceles.

**33.** In any 
$$\Delta ABC$$
 , prove that:  

$$\frac{\cos A}{b\cos C + c\cos B} + \frac{\cos B}{c\cos A + a\cos C} + \frac{\cos C}{a\cos B + b\cos A} = \frac{a^2 + b^2 + c^2}{2abc}$$
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**34.** In any  $\Delta ABC$ ,  $\sum a(\sin B - \sin C) = a^2 + b^2 + c^2$  (b)  $a^2$  (c)  $b^2$  (d)  
0
  
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35. In a  $\Delta ABC$  , if  $\sin Aand \sin B$  are the roots of the equation  $c^2x^2-c(a+b)x+ab=0,$  then find  $\angle C\cdot$ 

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**36.** In any ABC, prove that:  $\Delta = \frac{a^2 - b^2}{2} \frac{\sin A \sin B}{\sin (A - B)}$ 

37. In any triangle  $ABC, ext{ prove that:} \Delta = rac{b^2+c^2-a^2}{4\cot a}$  .

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38. In a triangle ABC,  $a=4, b=3, \angle A=60^\circ$  then c is root of the equation  $c^2-3c-7=0$  (b)  $c^2+3c+7=0$  (c)  $c^2-3c+7=0$  (d)  $c^2+3c-7=0$ 

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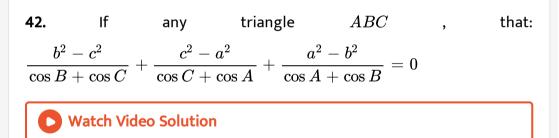
**39.** In any  $\Delta ABC$ ,  $2(bc\cos A + ca\cos B + ab\cos C) =$ 

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**40.** In any  $\Delta ABC$  , the value of  $2ac\siniggl(rac{A-B+C}{2}iggr)$  is (a) $a^2+b^2-c^2$  (b)  $c^2+a^2-b^2$  (c)  $b^2-c^2-a^2$  (d)  $c^2-a^2-b^2$ 

**41.** In a  $\Delta ABC$  , if (c+a+b)(a+b-c)=ab , then the measure of angel C is  $\frac{\pi}{3}$  (b)  $\frac{\pi}{6}$  (c)  $\frac{2\pi}{3}$  (d)  $\frac{\pi}{2}$ 

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**43.** In a right angled triangle ABC, write the value of  $\sin^2 A + \sin^2 B + \sin^2 C$ 

44. If any triangle 
$$ABC$$
, that:  

$$\frac{a\sin(B-C)}{b^2-c^2} = \frac{b\sin(C-A)}{c^2-a^2} = \frac{c\sin(A-B)}{a^2-b^2}$$
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45. In a  $\Delta ABC$ , if  $\cos C = \frac{\sin A}{2\sin B}$ , prove that the triangle is isosceles.  
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#### 46. Problem on sine rule Type:-2

(i)The angle of elevation of the top of the tower from a point A due South of the tower is  $\alpha$  and from B due east of the tower is  $\beta$ . If AB=d Show that the height of the tower is  $\frac{d}{\cot^2 \alpha + \cot^2 \beta}$ (ii) A tree stands vertically on a hill side which makes an angle of  $15^{\circ}$  with the horizontal. From a point on the ground 35m down the hill from the base of the tree ; the angle of elevation of the top of the tree is  $60^{\circ}$ .find the height of the tree ?



47. 
$$rac{\sqrt{\sin A} - \sqrt{\sin B}}{\sqrt{\sin A} + \sqrt{\sin B}} = rac{a+b-2\sqrt{ab}}{a-b}$$

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**48.** The angle of elevation of the top of a tower from a point *A* due south of the tower is  $\alpha$  and from *B* due east of tower is  $\beta$ . If AB = d, show that the height of the tower is  $\frac{d}{\sqrt{\cot^2 \alpha + \cot^2 \beta}}$ .

**49.** 
$$\frac{b \sec B + c \sec C}{\tan B + tanC} = \frac{c \sec C + a \sec A}{\tan C + tanA} = \frac{a \sec A + b \sec B}{\tan A + tanB}$$
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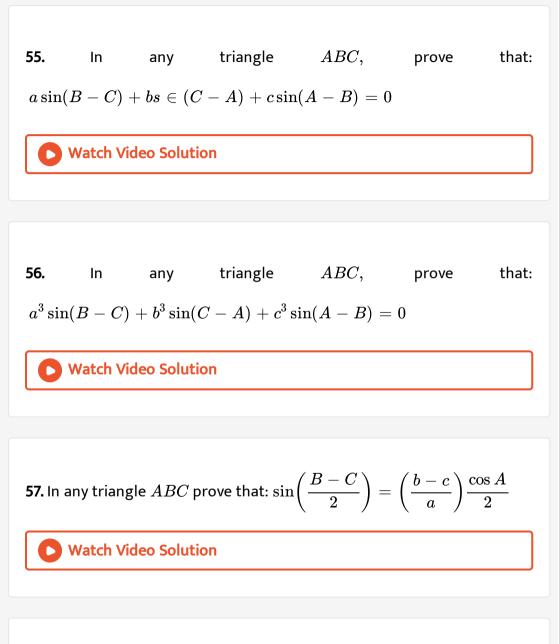
**50.** 
$$a(\cos C - \cos B) = 2(b - c)\frac{\cos^2 A}{2}$$

51.

$$a^2ig(\cos^2 B - \cos^2 Cig) + b^2ig(\cos^2 C - \cos^2 Aig) + c^2ig(\cos^2 A - \cos^2 Big) = 0\,.$$

**S2.** 
$$\frac{\cos(2A)}{a^2} - \frac{\cos(2B)}{b^2} = \frac{1}{a^2} - \frac{1}{b^2}$$
  
**S3.** In a  $\triangle ABC$ , if  $a = 2$ ,  $b = 3$  and  $\sin A = \frac{2}{3}$  then find  $\angle B$   
**S4.** Watch Video Solution

54. In any triangle 
$$ABC, ext{ prove that: } rac{\sin(B-C)}{\sin(B+C)} = rac{b^2-c^2}{a^2}$$



**58.** In any triangle 
$$ABC$$
, prove that:  $\frac{b-c}{b+c} = \frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)}$ 

( \_ ~ )

59. In any triangle 
$$ABC, ext{ prove that: } a\cos\left(rac{B+C}{2}
ight) = (b+c)rac{\sin A}{2}$$



**60.** A tree stands vertically on a hill side which makes an angle of  $15^0$  with the horizontal. From a point on the ground 35 m down the hill from the base of three, the angle of elevation of the top of the tree is  $60^0$ . Find the height of the tree.



**61.** A person, standing on the bank of a river, observes that the angle subtended by a tree on the opposite bank is  $60^{0}$ . When he retreates 20m from the bank, he finds the angle to be  $30^{0}$ . Find the height of the tree and the breadth of the river.

**62.** The angle of elevation of to be top point P of the vertical tower PQ of height h from poin A is  $45^0$  and from a point B, the angle of elevation is  $60^0$ , where B is point at a distance d from the point A measured along the line AB which makes an angle  $30^0$  with AQ. Prove that  $d = (\sqrt{3} - 1)h$ .

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**63.** The elevation of a tower at a station A due North of it is  $\alpha$  and at a station B due West of A is  $\beta$ . Prove that height of the tower is  $\frac{AB\sin\alpha\sin\beta}{\sin^2\alpha - \sin^2\beta}.$ 

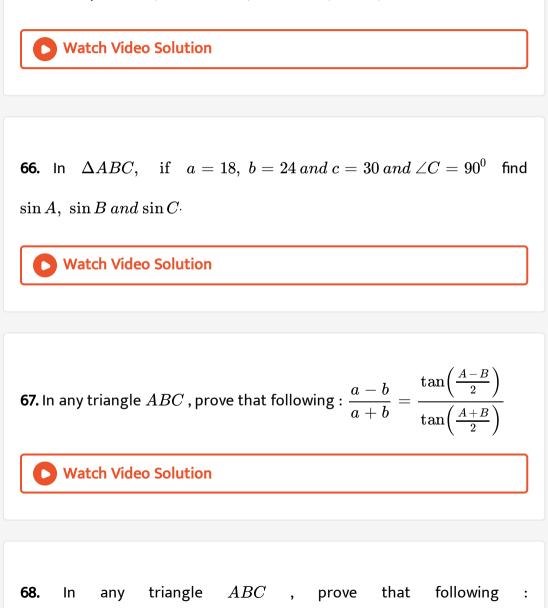


**64.** If in a  $\triangle ABC$ ,  $\angle A = 45^{\circ}$ ,  $\angle B = 60^{\circ}$ ,  $and \angle C = 75^{\circ}$ , find the ratio of its sides.





**65.** If in any  $\Delta ABC$ ,  $\angle C = 105^{0}$ ,  $\angle B = 45^{0}$ , a = 2, then find b.



$$(a-b) \cos \left(rac{C}{2}
ight) = c \sin \left(rac{A-B}{2}
ight)$$

**69.** In any triangle 
$$ABC$$
 , prove that following :  

$$\frac{c}{a-b} = \frac{\tan\left(\frac{A}{2}\right) + \tan\left(\frac{B}{2}\right)}{\tan\left(\frac{A}{2}\right) - \tan\left(\frac{B}{2}\right)}$$
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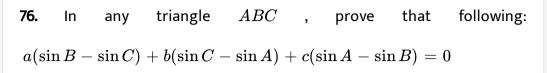
70. In any triangle 
$$ABC$$
 , prove that following :  

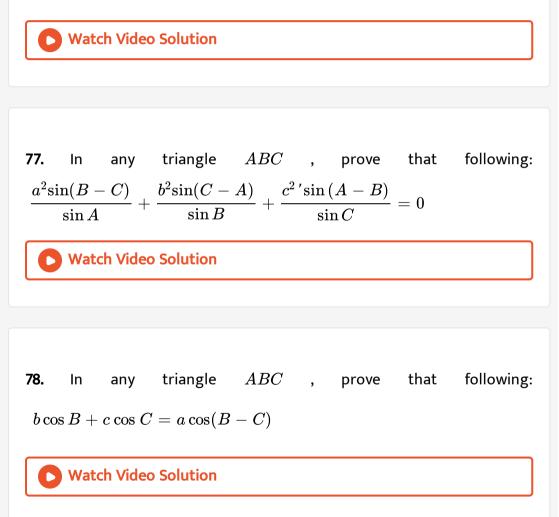
$$\frac{c}{a+b} = \frac{1 - \tan\left(\frac{A}{2}\right) \tan\left(\frac{B}{2}\right)}{1 + \tan\left(\frac{A}{2}\right) \tan\left(\frac{B}{2}\right)}$$

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71. In any triangle ABC , prove that following :  $rac{a+b}{c}=rac{\cos\left(rac{A-B}{2}
ight)}{\sin\left(rac{C}{2}
ight)}$ 

72. In any triangle 
$$ABC$$
, prove that following:  
 $\sin\left(\frac{B-C}{2}\right) = \frac{b-c}{a} \frac{\cos A}{2}$   
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73. In any triangle  $ABC$ , prove that following:  $\frac{a^2-c^2}{b^2} = \frac{\sin(A-C)}{\sin(A+C)}$   
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74. In any triangle  $ABC$ , prove that following:  
 $b\sin B - c\sin C = a\sin(B-C)$   
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75. In any triangle  $ABC$ , prove that following:  
 $a^2\sin(B-C) = (b^2 - c^2)\sin A$ 





79. In any triangle 
$$ABC$$
, prove that following:  

$$\frac{\cos^{2}B - \cos^{2}C}{b + c} + \frac{\cos^{2}C - \cos^{2}A}{c + a} + \frac{\cos^{2}A - \cos^{2}B}{a + b} = 0$$
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80. In any triangle  $ABC$ , prove that following:  
 $a \sin\left(\frac{A}{2}\right)\sin\left(\frac{B - C}{2}\right) + b\sin\left(\frac{B}{2}\right)\sin\left(\frac{C - A}{2}\right) + c\sin\left(\frac{C}{2}\right)\sin\left(\frac{A}{2}\right)$ 
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81. In any triangle  $ABC$ , prove that following:  
 $a \cos A + b \cos B + c \cos C = 2b \sin A \sin C = 2c \sin A \sin B$ 
  
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82. In any triangle  $ABC$ , prove that following:  
 $a (\cos B \cos C + \cos A) = b(\cos C \cos A + \cos B) = c(\cos A \cos B + \cos C)$ 

**83.** In a  $\Delta ABC$ , if  $\sin^2 A + \sin^2 B = \sin^2 C$ , show that the triangle is right angled.

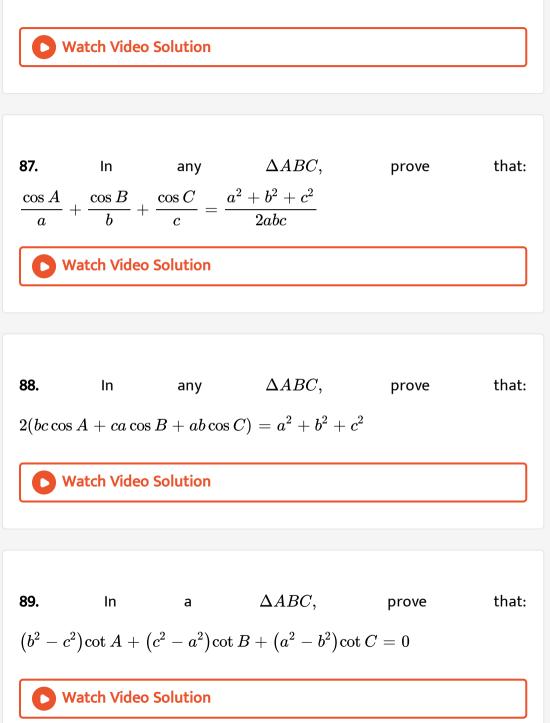
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**84.** The upper part of a tree broken over by the wind makes an angle of  $30^0$  with the ground and the distance from the root to the point where the top of the tree touches the ground is 15m. Using sine rule, fine the height of the tree.

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85. If the sides of a  $\triangle ABC$  are  $a=4,\ b=6\ and\ c=8,$  show that  $4\cos B+3\cos C=2.$ 

**86.** In any  $\Delta ABC$ , prove that:  $a(b\cos C - c\cos B) = b^2 - c^2$ 



**90.** In a 
$$\Delta ABC$$
, prove that:  
 $\left(\frac{b^2-c^2}{a^2}\right)\sin 2A + \left(\frac{c^2-a^2}{b^2}\right)\sin 2B + \left(\frac{a^2-b^2}{c^2}\right)\sin 2C = 0$   
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91. In any  $\Delta ABC$  prove that  $a\cos A + b\cos B + c\cos C = rac{8\Delta^2}{abc}$ 

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**92.** Two ships leave port at the same time. One goes 24 km per hour in the direction  $N \, 45^0 E$  and other travels 32 km per hour in the direction  $S75^0 E$ . Find the distance between the ships at the end of 3 hours.



**93.** A lamp post is situated at the middle point M of the side AC of a triangular plot of ABC with BC = 7m, CA = 8 and AB = 9m. Lamp post subtends an angle of  $15^0$  at the point B. determine the height of the lamp post.

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**94.** Two trees, A and B are on the same side of a river. From a point C in the river the distance of trees A and B are 25 m and 300 m respectively. If the angle C is  $45^0$ , find the distance between the trees  $\left(use~\sqrt{2}=1.~44\right)$ 

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**95.** In a 
$$ABC$$
, prove the following : In a  $\Delta ABC$ , if  $a = 5, m b = 6 \text{ and } C = 60^{0}$ , show that its area is  $\frac{15\sqrt{3}}{2}$  sq.units.

**96.** In a  $\triangle ABC$ , prove the following : If  $a = \sqrt{2}$ ,  $b = \sqrt{3}$  and  $c = \sqrt{5}$ , shos that its area is  $\frac{1}{2}\sqrt{6}$  sq. units.

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**97.** In a  $\Delta ABC$ , prove the following : The sides of a triangle area

 $a=5,\;b=6\;and\;c=8$  , show that  $8\cos A+16\cos B+4c\cos C=17.$ 

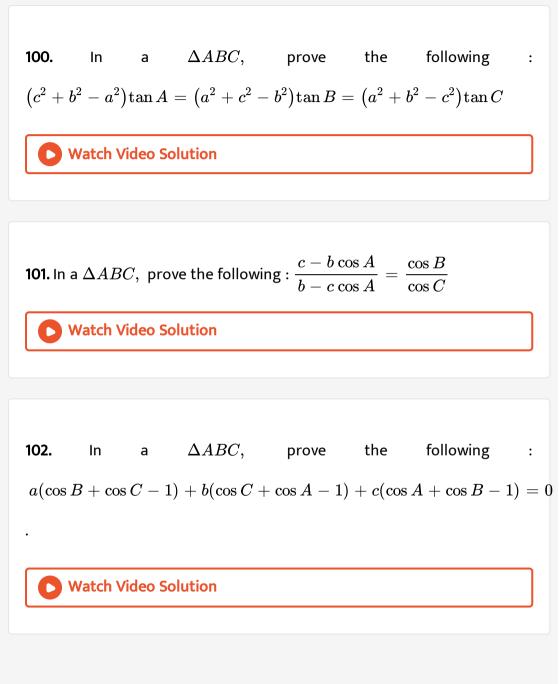
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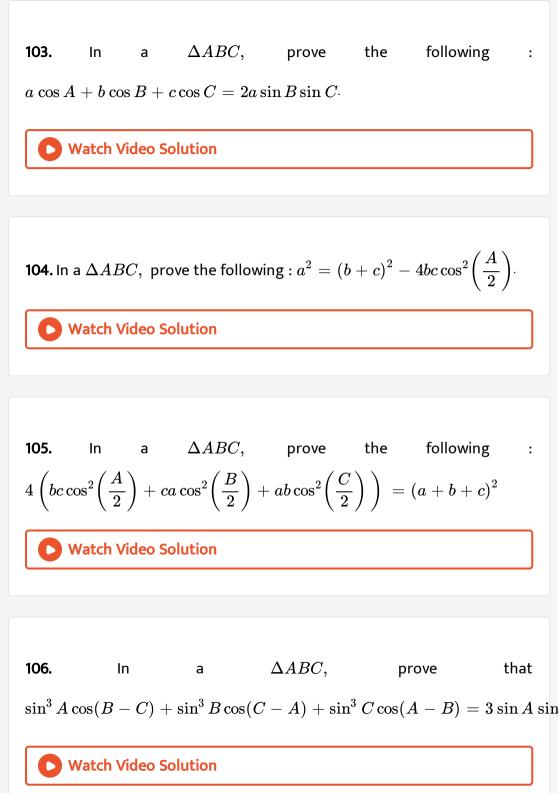
98. In a  $\Delta ABC,\,\,$  prove the following :If  $a=18,\,b=24,\,c=30$  , find

 $\cos A, \ \cos B \ and \ \cos C \cdot$ 



**99.** In a  $\Delta ABC$ , prove the following :  $c \left( a \cos B - b \cos A 
ight) = a^2 - b^2$ .





**107.** In any 
$$\Delta ABC$$
,  $\frac{b+c}{12} = \frac{c+a}{13} = \frac{a+b}{15}$ , then prove that  $\frac{\cos A}{2} = \frac{\cos B}{7} = \frac{\cos C}{11}$ .

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108. If in a  $\Delta ABC, \cos^2 A + \cos^2 B + \cos^2 C = 1$ , prove that the triangle is right angled.

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**109.** Two ships leave a port at the same time. One goes 24 km/hr in the direction  $N38^0E$  and other travels 32 km/hr in the direction  $S52^0$  E. Find the distance between the ships at the end of 3 hrs.



**110.** Find the area of the triangle 
$$\triangle ABC$$
 in which  $a = 1, b = 2$  and  $\angle c = 60^{\circ}$ .

111. In a  $\Delta ABC$ , if  $b=\sqrt{3},\ c=1$  and  $\angle A=30^{0}$ , then find a

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112. In a 
$$\Delta ABC$$
, if  $\cos A = \frac{\sin B}{2\sin C}$ , then show that  $c = a$ .

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**113.** In a 
$$\triangle ABC$$
, if  $b = 20, \ c = 21 \ and \sin A = \frac{3}{5}$  find  $a$ .

114. In  $\triangle ABC$ , if a = 8, b = 10, c = 12 and  $C = \lambda A$ , find the

value of  $\lambda_{\cdot}$ 



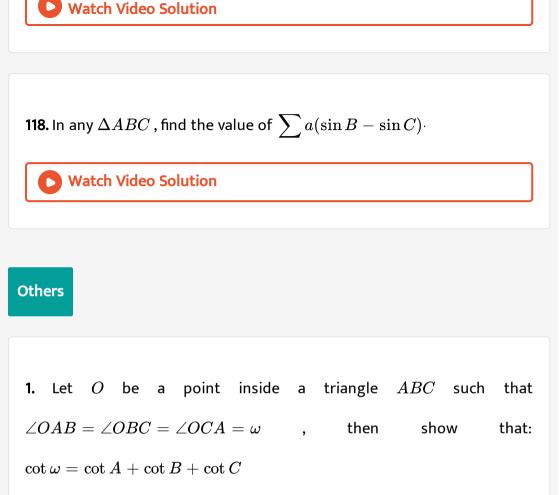
**115.** If the sides of a triangle are proportional to 2,  $\sqrt{6}$  and  $\sqrt{3} - 1$ , find the measure of its greatest angle.

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**116.** If in a Delta
$$ABC$$
,  $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ , then find the measures of angles  $A, B, C$ .

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**117.** If any triangle ABC, find the value of  $a\sin(B-C) + b\sin(C-A) + c\sin(A-B)$ .



$$\cos ec^2\omega = \cos ec^2A + \cos ec^2B + \cos ec^2C$$

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